Symmetries in large scale structure

Lam Hui 許林
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Two applications of Nambu-Goldstone bosons in cosmology

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Outline:

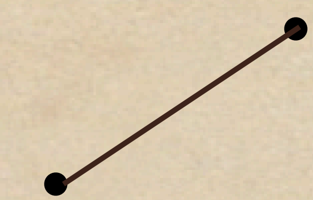
1. Spontaneously broken symmetry in the theory of LSS
   - with Kurt Hinterbichler & Justin Khoury;
     Walter Goldberger & Alberto Nicolis;
     Creminelli, Gleyzes, Simonovic &Vernizzi;
     Bart Horn & Xiao Xiao.

2. Light boson dark matter (mass $\sim 10^{-22}\,\mathrm{eV}$)
   - with Jerry Ostriker, Scott Tremaine, Edward Witten.

Idea 1: non-perturbative consistency relations in LSS

- 1. Consider a familiar example of symmetry: spatial translation.

$$x \to x + \Delta x \quad , \quad \text{where } \Delta x = \text{ const.}$$



Its consequence for correlation function is well known:

$$\langle \phi(x_1)\phi(x_2)\phi(x_3) \rangle = \langle \phi(x_1 + \Delta x)\phi(x_2 + \Delta x)\phi(x_3 + \Delta x) \rangle$$

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Thus, alternatively, we say:

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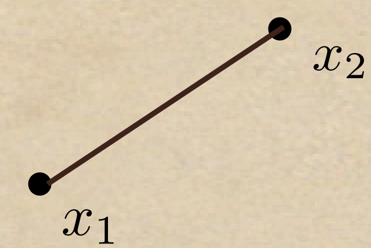

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- 2. Consider a different symmetry: shift in gravitational potential.

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For small c , we have:

$$\langle (\phi_1 + c)(\phi_2 + c)(\phi_3 + c)\rangle \sim \langle \phi_1\phi_2\phi_3\rangle + c\langle \phi_1\phi_2\rangle + c\langle \phi_2\phi_3\rangle + c\langle \phi_1\phi_3\rangle$$

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$$c(\langle \phi_1\phi_2\rangle + \langle \phi_2\phi_3\rangle + \langle \phi_1\phi_3\rangle) = 0 \longleftarrow$$ clearly false!

Conclude : $\langle \phi_1\phi_2\phi_3\rangle$ is not invariant under $\phi \to \phi + c$

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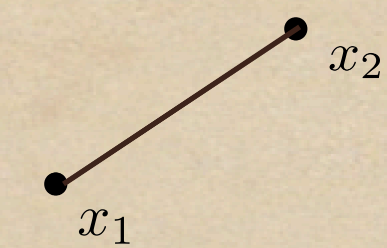

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  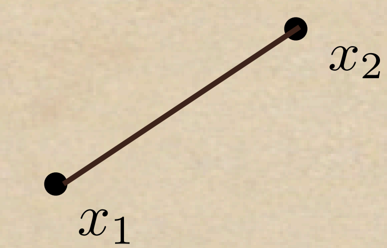

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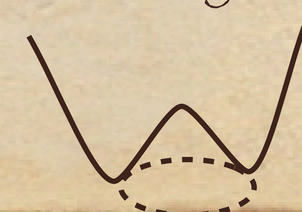
- What makes the second case so different? We generally choose some expectation value for ϕ e.g. $\langle \phi \rangle = 0$. The choice breaks the shift symmetry i.e. spontaneous symm. breaking.

  1. Unbroken symmetries $\longrightarrow$ invariant correlation functions.
  2. Spontaneously broken symmetries $\longrightarrow$ consistency relations.



References:

Maldacena; Creminelli & Zaldarriaga; Creminelli, Norena, Simonovic; Assassi, Baumann & Green; Flauger, Green & Porto; Pajer, Schmidt, Zaldarriaga; Kehagias & Riotto; Peloso & Pietronni; Berezhiani & Khoury; Pimentel; Creminelli, Norena, Simonovic, Vernizzi; Goldberger, LH, Nicolis; Hinterbichler, LH, Khoury; Horn, LH, Xiao.

Relativistic symmetries and consistency relations

comoving gauge $\delta\rho = 0$ $\quad ds^2_{\rm spatial} = a^2 e^{2\zeta} [e^\gamma]_{ij} dx^i dx^j$

dilation symm. $\quad x \to e^{-2\lambda} x \quad , \quad \zeta \to \zeta + \lambda$

$$\lim_{q\to 0} \frac{1}{P_\zeta(q)} \langle \zeta(q) \zeta_{k_1} ... \zeta_{k_m} \rangle' \sim k \cdot \partial_k \langle \zeta_{k_1} ... \zeta_{k_m} \rangle'$$

Maldacena

generalization $\quad x \to x + M \cdot x^{N+1} \quad , \quad \zeta \to \zeta + M \cdot x^N \quad , \quad \gamma \to \gamma + M \cdot x^N$

$$\lim_{q\to 0} \partial_q^N \left(\frac{1}{P_\zeta(q)} \langle \zeta(q) \zeta_{k_1} ... \zeta_{k_m} \rangle' + \frac{1}{P_\gamma(q)} \langle \gamma(q) \zeta_{k_1} ... \zeta_{k_m} \rangle' \right) \sim k \cdot \partial_k^{N+1} \langle \zeta_{k_1} ... \zeta_{k_m} \rangle'$$

$N=0$ dilation , $N=1$ special conformal , etc.

Note:

1. The symmetries originate as diff. But consistency relations are not empty statements i.e. they can be violated (e.g. curvaton); they are a test of initial conditions (e.g. single clock, etc).
2. They are non-perturbative, derived from Ward identities.
3. Testing these requires seeing general relativistic effects, but there exists 2 Newtonian consistency relations (Peloso & Pietroni; Kehagias & Riotto).

Newtonian limit

$$\delta' + \nabla \cdot (1+\delta) v = S$$ mass/number conservation (or lack thereof)

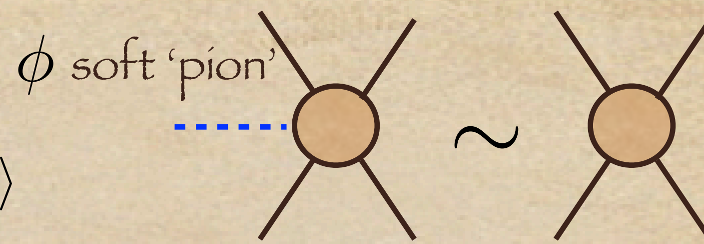
$$v' + v \cdot \nabla v + \mathcal{H} v = -\nabla \phi + F$$ equation of motion

$$\nabla^2 \phi = 4\pi G a^2 \delta \rho_m$$ Poisson equation

Symmetries:

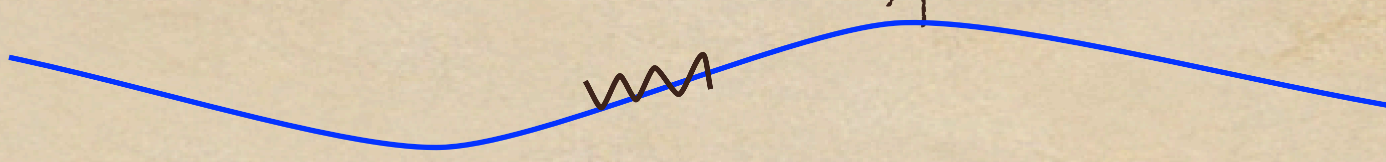
1. $\phi \to \phi + c$
2. $x \to x + n \, , \, v \to v + n' \, , \, \phi \to \phi - (n'' + \mathcal{H} n') \cdot x$

Consistency relations from SSB



- Schematic form: $\lim_{q\to 0} \frac{1}{P_\phi(q)} \langle \phi(q) \mathcal{O}(k_1) ... \mathcal{O}(k_N) \rangle \sim \langle \mathcal{O}(k_1) ... \mathcal{O}(k_N) \rangle$

They are (momentum space) statements about how correlations of observables $\mathcal{O}$ behave in the presence of a long wave-mode Nambu-Goldstone boson/pion.



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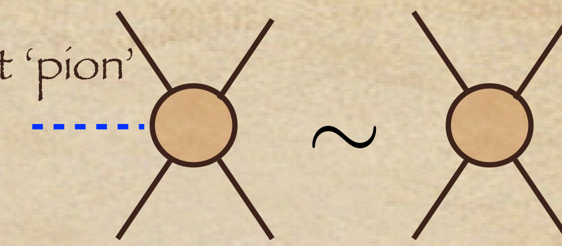

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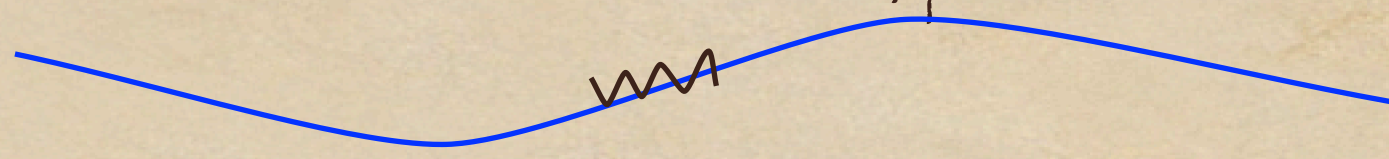
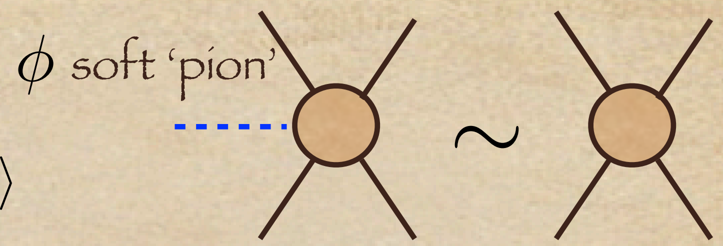


- Why are they interesting?

Consistency relations from SSB



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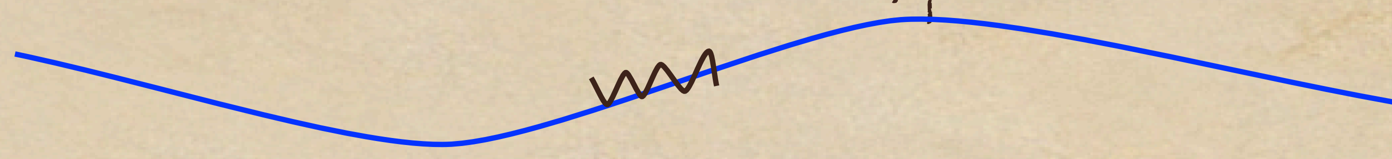


- Why are they interesting?

  1. These are symmetry statements, and are therefore exact, non-perturbative i.e. they hold even if the observables $\mathcal{O}$ are highly nonlinear, and even if they involve astrophysically complex objects, such as galaxies. The main input necessary is how they transform under the symmetry of interest (robust against galaxy mergers, birth, etc.)

  2. In the fully relativistic context, there is an infinite number of consistency relations. Two of them have interesting Newtonian limits (shift and time-dependent translation).

  3. Two assumptions go into these consistency relations, which can be experimentally tested (using highly nonlinear observables!): Gaussian initial condition (or more precisely, single-clock initial condition such as provided by inflation), and the equivalence principle (that all objects fall at the same rate under gravity).

  4. Non-trivial constraints on analytic models.

An open issue:

Connection with asymptotic symmetries (e.g. BMS in the case of scattering amplitudes).

Idea 2: light boson dark matter   mass $\mathrm{m} \sim 10^{-22}\,\mathrm{eV}$

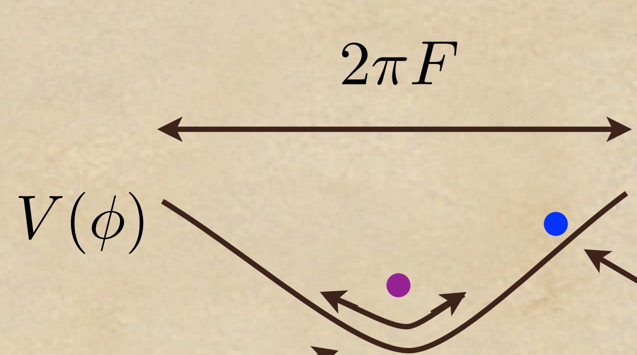
- Invoke shift symmetry to make small mass technically natural
  i.e. a Nambu-Goldstone boson in the $m \to 0$ limit.

- Concrete realization: an angular field of periodicity $2\pi F$ i.e. an axion-like field with a potential from non-perturbative effects (not QCD axion).

$$\mathcal{L} \sim -\frac{1}{2}(\partial\phi)^2 - \Lambda^4(1 - \cos[\phi/F])$$

$$m \sim \Lambda^2/F$$

- Relic abundance:



$\phi \sim F$ at early times until $H \sim m$

$\rho_\phi \sim m^2 F^2 \quad , \quad \rho_{\mathrm{rad.}} \sim H^2 M_{\mathrm{pl}}^2$

subsq. oscill. : $\rho_\phi \propto a^{-3}$
self – interaction can be ignored

$$\Omega_{\mathrm{matter}} \sim \left(\frac{F}{10^{17}\,\mathrm{GeV}}\right)^2 \left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)^{1/2}$$

(standard story)
(low scale inflation)

Dynamics of a free massive scalar

- Ignoring self-interaction:

$$m^{-1} \sim 0.06\,\mathrm{pc}$$

$$(mv)^{-1} \sim 2 \text{ kpc } (10\,\mathrm{km\,s^{-1}}/v)$$

$$-\Box\phi + m^2\phi = 0$$

- Non-relativistic limit:

$$\phi = \frac{1}{\sqrt{2m}}\left[\psi e^{-imt} + \psi^* e^{imt}\right]$$

$$|\ddot{\psi}| \ll m|\dot{\psi}| \quad \longrightarrow \quad i\dot{\psi} = \left[-\frac{\nabla^2}{2m} + m\Phi_{\rm grav.}\right]\psi$$

- High occupancy implies ψ should be thought of as a classical scalar.
  See simulations by Hsi-Yu Schive, Tzihong Chiueh & Tom Broadhurst.

- An alternative viewpoint: ψ as a (classical) fluid.

$$\rho = m\,|\psi|^2 \quad \text{i.e.} \quad \psi = \sqrt{\rho/m}\,e^{i\theta}$$

Recall conservation of probability: current $\propto i(\psi\nabla\psi^* - \psi^*\nabla\psi)$

Reinterpreted as conservation of mass:

$$\dot{\rho} + \nabla\cdot\rho v = 0 \quad \text{where} \quad v = \frac{1}{m}\nabla\theta \qquad \text{i.e. a superfluid.}$$

Fluid formulation (Madelung)

- Euler equation:

$$\dot{v} + v \cdot \nabla v = -\nabla\Phi_{\rm grav.} + \frac{1}{2m^2}\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$$

"quantum pressure"

- More precisely, an unusual form of stress:

$$T_{ij} = \rho v_i v_j + \frac{1}{2m^2}\left[\partial_i\sqrt{\rho}\,\partial_j\sqrt{\rho} - \sqrt{\rho}\,\partial_i\partial_j\sqrt{\rho}\right]$$

- Can be implemented in standard hydrodynamics codes (Mocz & Succi).

- For linear perturbations (on cosmological bgd.):

Jeans scale $\sim 0.03\,{\rm Mpc}$ at z ~ 10

Perturbations suppressed on small scales - could help alleviate small scale problems of standard CDM (Hu, Barkana, Gruzinov: Fuzzy DM).

Typical focus: density profile (cusp versus core), number of satellite galaxies.

Issue: baryonic effects make it hard to draw definitive conclusions.

The unexpected diversity of dwarf galaxy rotation curves

Kyle A. Oman[1,⋆], Julio F. Navarro[1,2], Azadeh Fattahi[1], Carlos S. Frenk[3], Till Sawala[3], Simon D. M. White[4], Richard Bower[3], Robert A. Crain[5], Michelle Furlong[3], Matthieu Schaller[3], Joop Schaye[6], Tom Theuns[3]

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[2] *Senior CIfAR Fellow*
[3] *Institute for Computational Cosmology, Department of Physics, University of Durham, South Road, Durham DH1 3LE, United Kingdom*
[4] *Max-Planck Institute for Astrophysics, Garching, Germany*
[5] *Astrophysics Research Institute, Liverpool John Moores University, IC2, Liverpool Science Park, 146 Brownlow Hill, Liverpool, L3 5RF, United Kingdom*
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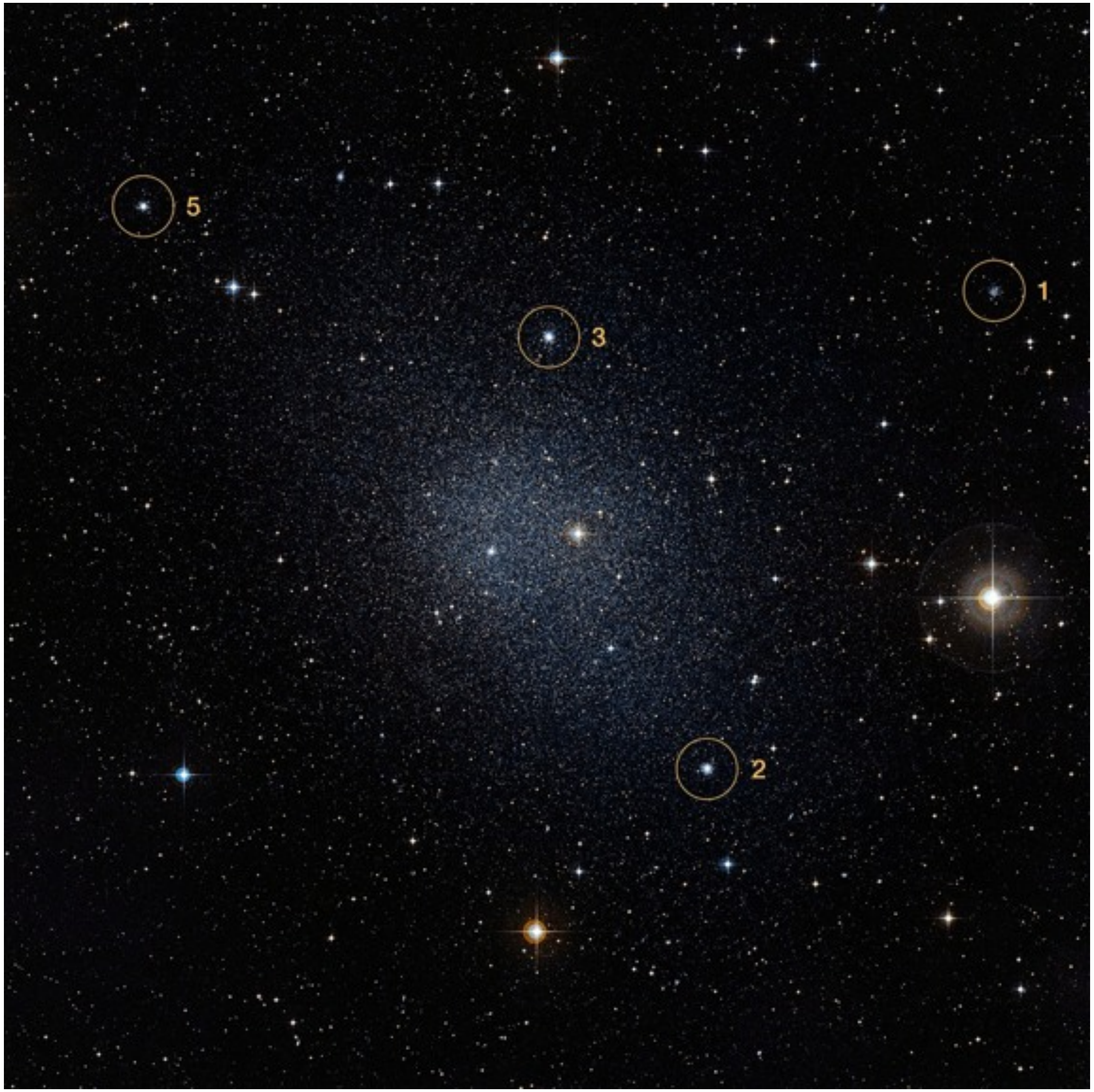
10 July 2015

ABSTRACT
We examine the circular velocity profiles of galaxies in ΛCDM cosmological hydrodynamical simulations from the EAGLE and LOCAL GROUPS projects and compare them with a compilation of observed rotation curves of galaxies spanning a wide range in mass. The shape of the circular velocity profiles of simulated galaxies varies systematically as a function of galaxy mass, but shows remarkably little variation at fixed maximum circular velocity. This is especially true for low-mass dark matter-dominated systems, reflecting the expected similarity of the underlying cold dark matter haloes. This is at odds with observed dwarf galaxies, which show a large diversity of rotation curve shapes, even at fixed maximum rotation speed. Some dwarfs have rotation curves that agree well with simulations, others do not. The latter are systems where the inferred mass enclosed in the inner regions is much lower than expected for cold dark matter haloes and include many galaxies where previous work claims the presence of a constant density "core". The "cusp vs core" issue is thus better characterized as an "inner mass deficit" problem than as a density slope mismatch. For several galaxies the magnitude of this inner mass deficit is well in excess of that reported in recent simulations where cores result from baryon-induced fluctuations in the gravitational potential. We conclude that one or more of the following statements must be true: (i) the dark matter is more complex than envisaged by any current model; (ii) current simulations fail to reproduce the diversity in the effects of baryons on the inner regions of dwarf galaxies; and/or (iii) the mass profiles of "inner mass deficit" galaxies inferred from kinematic data are incorrect.

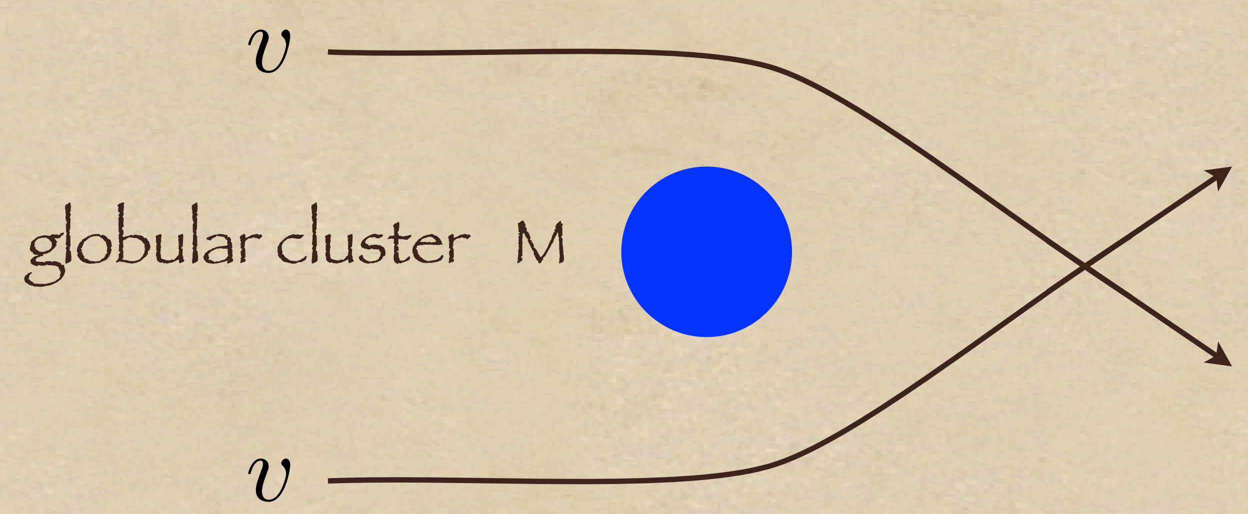
Key words: dark matter, galaxies: structure, galaxies: haloes



Fornax galaxy and its globular clusters

Dynamical friction

- Chandrasekhar's classic calculation:



- Quantum stress smooths out density wake, lowering friction.

- Use known solution for the Coulomb scattering problem:

  $\psi \propto F\,[\,i\beta, 1, ikr(1-\cos\theta)\,]$ where F is the confluent hypergeometric func.

  $\beta \equiv (GM/v^2)/k^{-1}$ with $k^{-1} = (mv)^{-1}$ = de Broglie wavelength

  Small β means quantum stress is important.

- Key - integrate momentum flux to compute friction: $\oint dS_j\, T_{ij}$

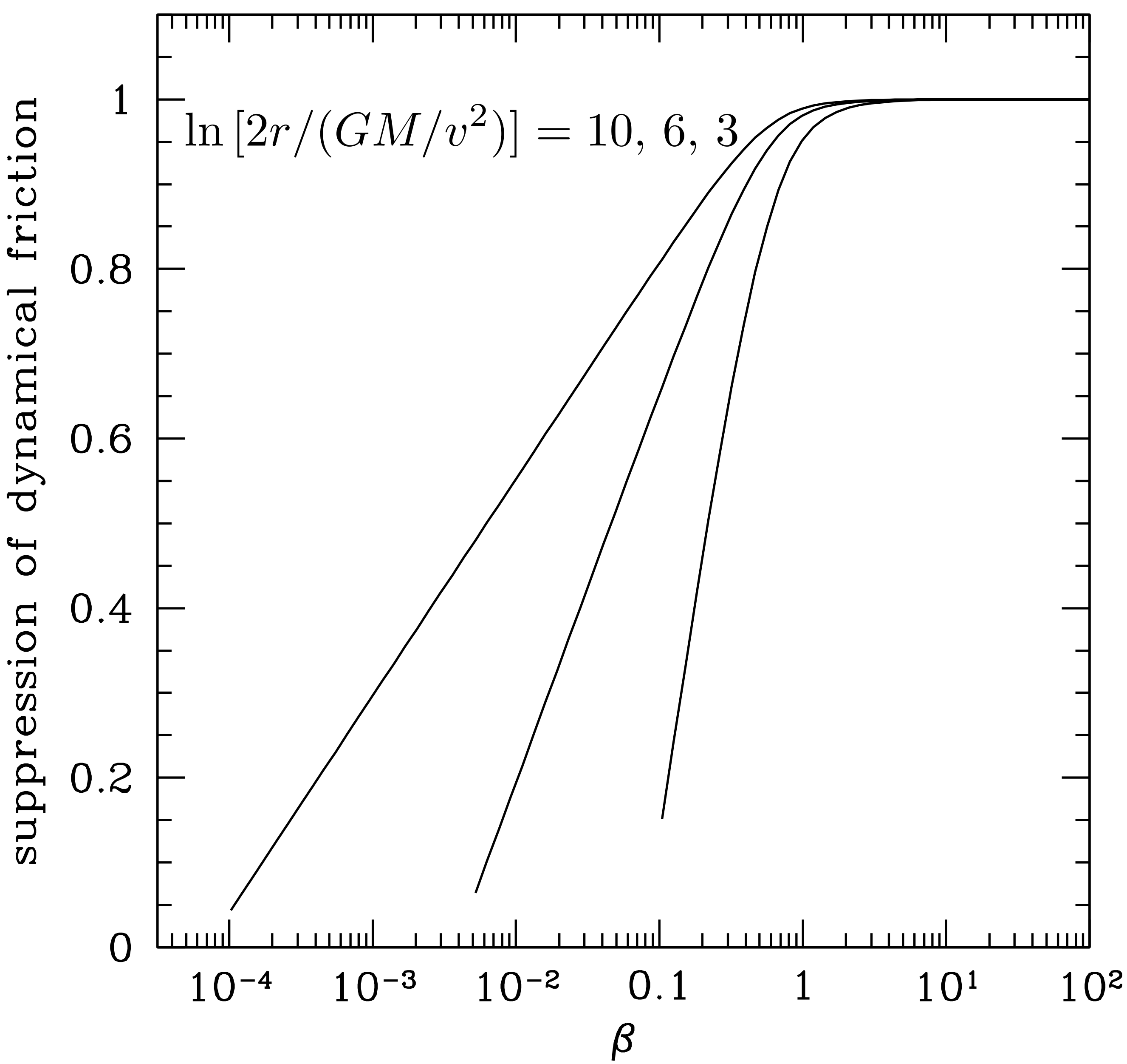
Question: shouldn't the quantum and classical answers be identical?

Recall that for Coulomb differential cross section,

quantum = classical.

- But recall also the integrated cross section has a logarithmic divergence.
- Thus, we expect dynamical friction $\propto \ln{[r/r_c]}$ where $r \sim$ size of galaxy,

  $r_c \sim GM/v^2$ or k^{-1}
- This is borne out by analytic calculation, made possible by obscure identities (some dating to 18th century) involving hypergeometric functions.



$$\beta \equiv (GM/v^2)/k^{-1}$$

Conclusion:

Given the density profile of a galaxy (which can be experimentally determined), standard CDM has a definite prediction for the dynamical friction, which can be checked against observations.

Fuzzy DM of $m \sim 10^{-22}$ eV can lower dynamical friction by an order of magnitude.

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A side remark:

The Newtonian consistency relation simplifies greatly in Lagrangian space:

$$\lim_{\vec{q}\to 0} \frac{1}{P_v(q,\eta)} \langle v^i(\vec{q},\eta)\mathcal{O}(\vec{k}_1,\eta_1)...\mathcal{O}(\vec{k}_m,\eta_m)\rangle = 0$$

Bart Horn, LH, Xiao Xiao

Related: Tanaka; Pajer, Schmidt, Zaldarriaga

Chan & Scoccimarro 2009

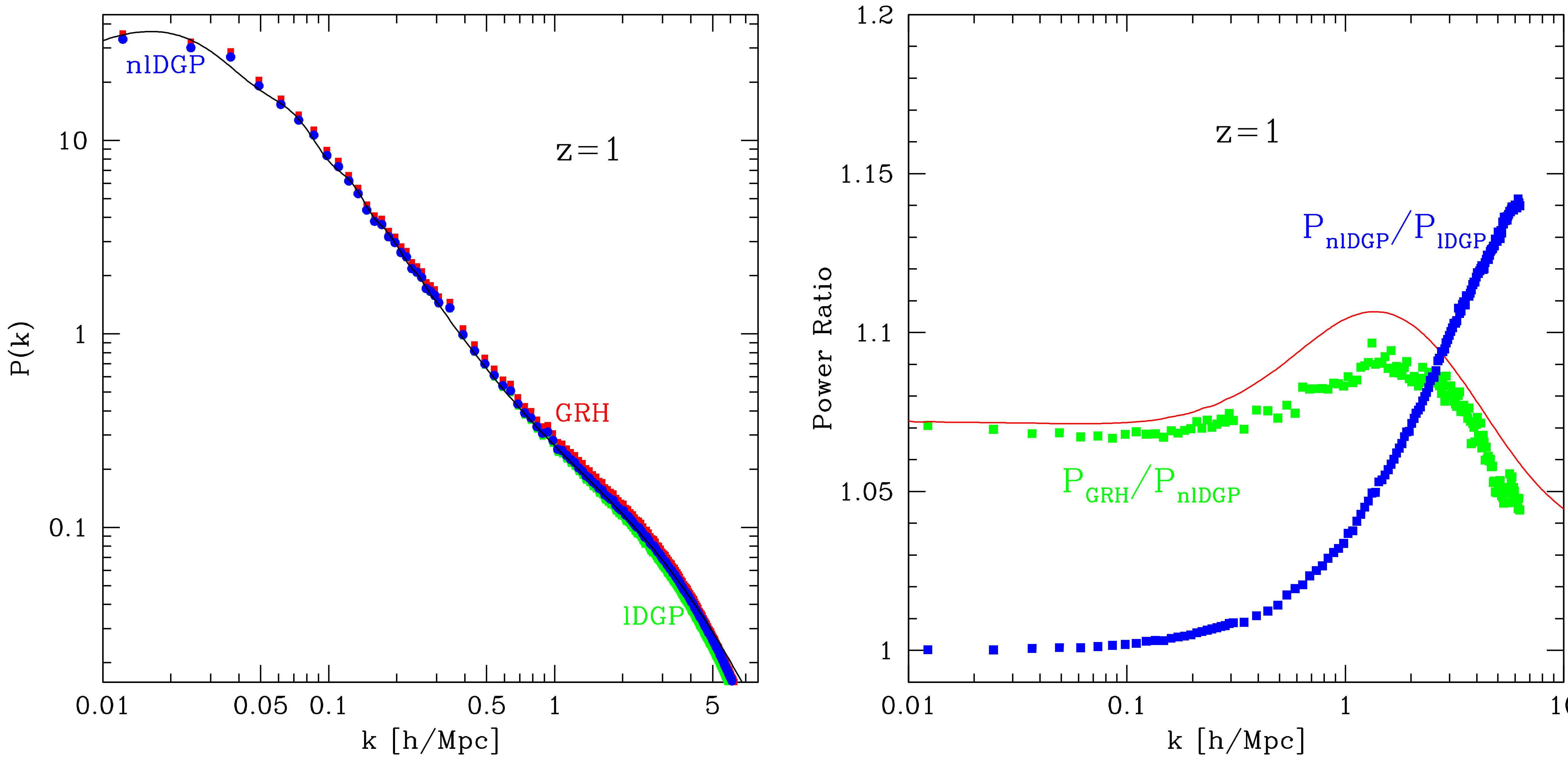


FIG. 5: Dark matter power spectra from the nonlinear DGP model (nlDGP) , linear DGP (lDGP), and GR perturbations with the same expansion history (GRH) at $z = 1$. The left panels show the power spectra, and the right panels shows ratios to better see the differences. Two sets of computational boxes are shown for each case, covering a different range in k (see text). The solid line denotes the predictions from paper I for $P_{\rm nlDGP}$ (left panel) and $P_{\rm GRH}/P_{\rm nlDGP}$ (right panel).

Vainshtein screening e.g. DGP

$$S_{\rm scalar} \sim \int d^4x \left[-\frac{1}{2}(\partial\varphi)^2 - \frac{1}{m^2}(\partial\varphi)^2\Box\varphi + \alpha\varphi T_m{}^\mu{}_\mu \right]$$ (Einstein frame)

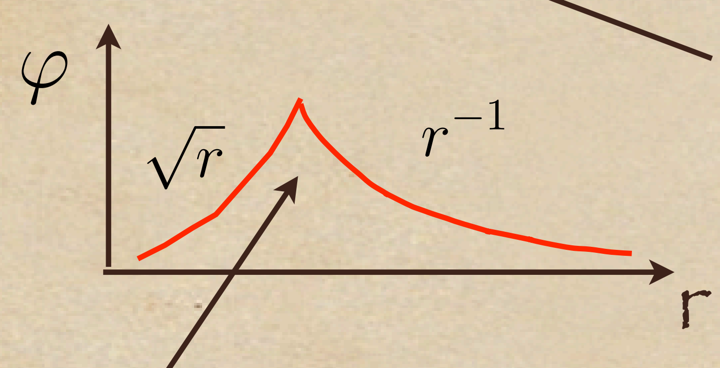
e.o.m.: $$\Box\varphi + \frac{1}{m^2}\left[(\Box\varphi)^2 - \partial^\mu\partial^\nu\varphi\partial_\mu\partial_\nu\varphi\right] \sim \alpha\rho_m$$

$\varphi \propto \frac{1}{r}$ large r

$\varphi \propto \sqrt{r}$ small r

point mass solution

m^2 ← graviton mass



$r_{\rm V} \sim (r_{\rm Schw} m^{-2})^{1/3}$

$\alpha =$ scalar-matter coupling ≈ O(1) generically

Galileon symmetry (Nicolis, Rattazzi, Trincherini): $\varphi \to \varphi + c + b_\mu x^\mu$