Topological defects in open string field theory

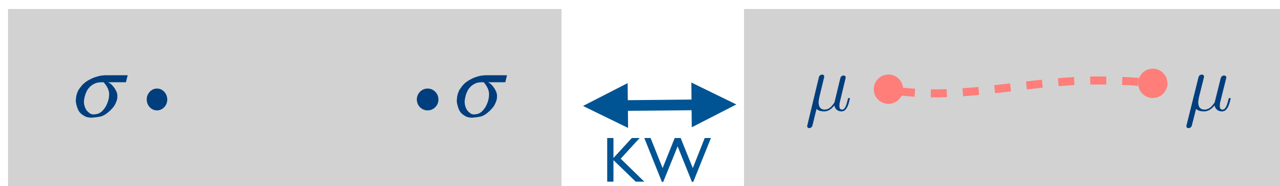
Toru Masuda (Nara Women's University)

Based on the work 1512.****[hep-th] with:

Toshiko Kojita (Masukawa Inst. at Kyoto Sangyo Univ.),
Carlo Maccaferri (Trino University),
Martin Schnabl (AS CR)

Topological defect in 2d CFT: freely deformed

typical example:



disorder lines in 2d critical Ising model

studied in many contexts.. (from CMP, CFT, string theory)

why interesting ?

- generalized concept of conformal b.c.

  (may play important roles in string theory?)

- symmetry / dually of 2d CFT Frohlich-Fuchs-Runkel-Scweigert'04, '07

  e.g. it is used to study non-geometric b.g.

  Satoh-Sugawara'15

and more..

Won-Affleck'94, Oshikawa-Affleck'97, Petkova-Zuber'01, Bachas-Boer-Dijkgraaf-Ooguri'01, Graham-Watts'03, Bachas-Brunner'07…

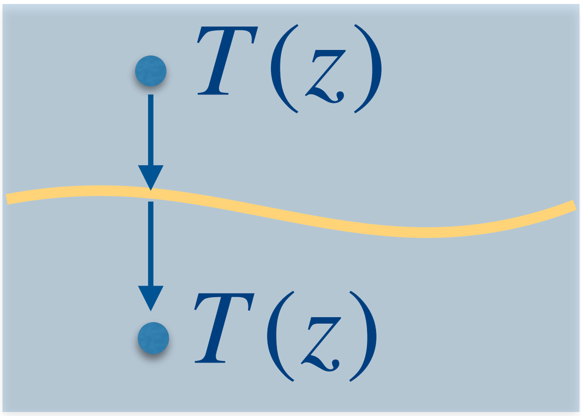
Today's talk: use TD to study Open String Field Theory

Plan

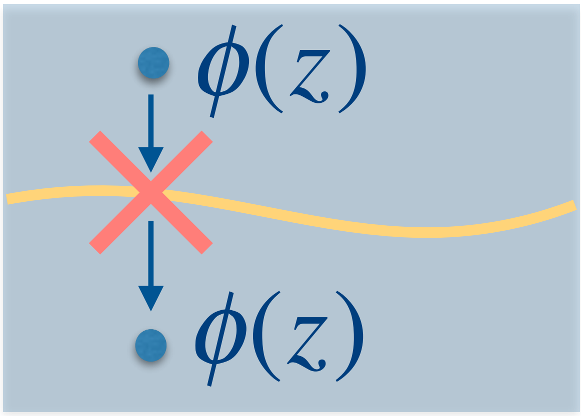
1. review of TD & bulk TD operator
2. construction of boundary TD operator
3. application to open SFT

Topological defects in 2D CFT

The stress tensor is continuous across the defect line:

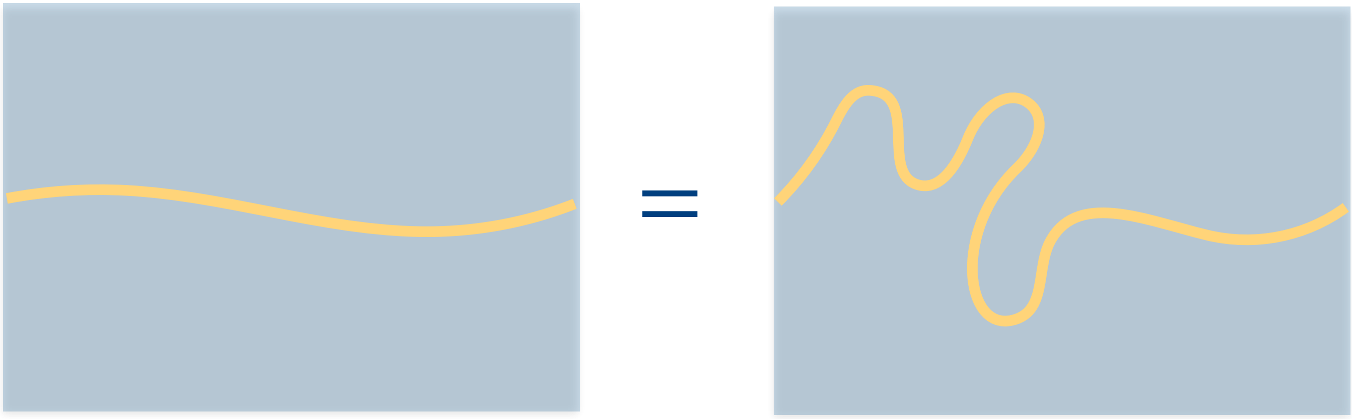


★single-valued wrt z.

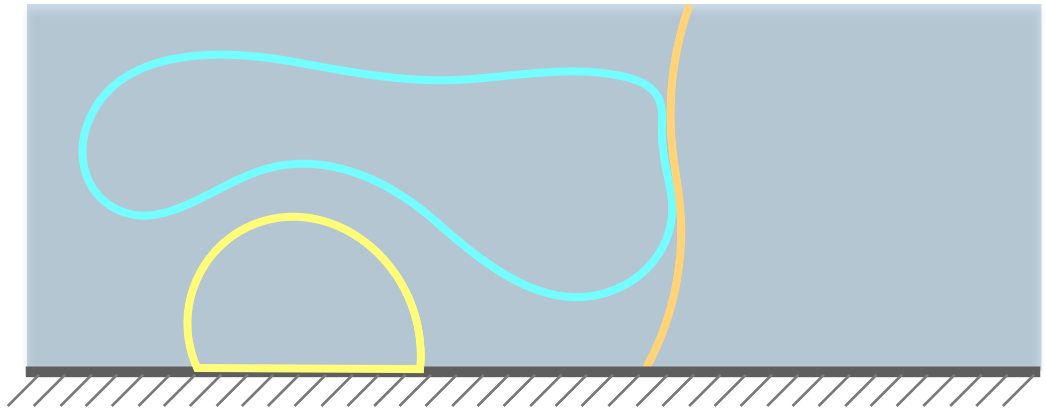


★branch cut wrt z.

Energy does not depend on the shape of TD: tensionless



We can imagine various configurations of TD



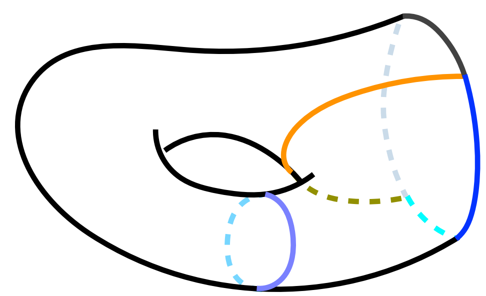
classify TDs

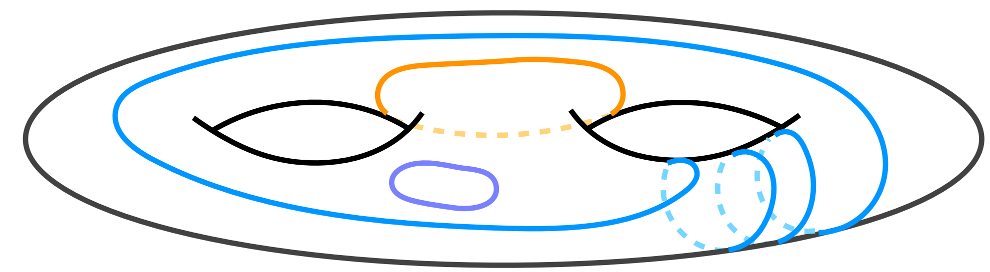
how TDs fuse?

junctions

change of b.c. ?

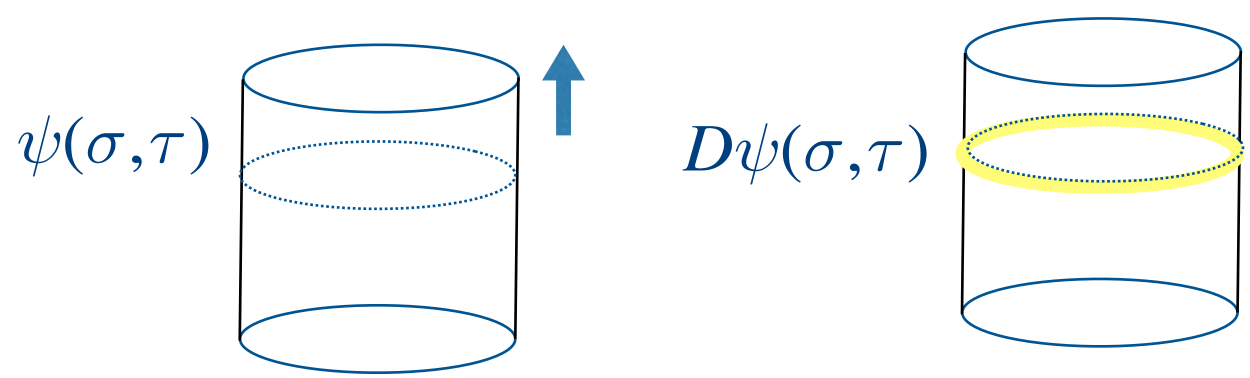
…on any Riemann surfaces in general.

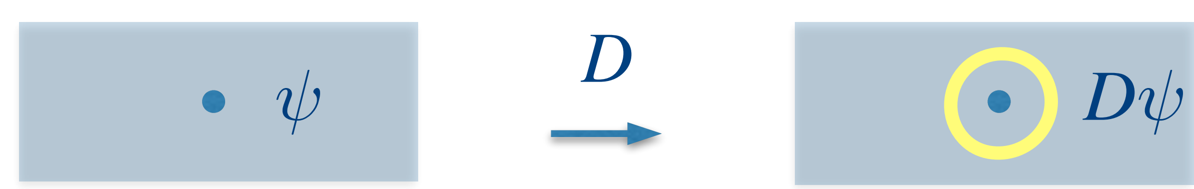




Closed loop of defects: operator acting on closed string states

$$\boxed{[D, L_n] = 0}$$





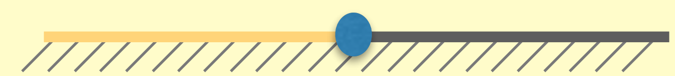
Note: the words "fields / operators" in this talk:

bulk fields

$$\phi_i(z,\bar{z}), \quad T(z)$$

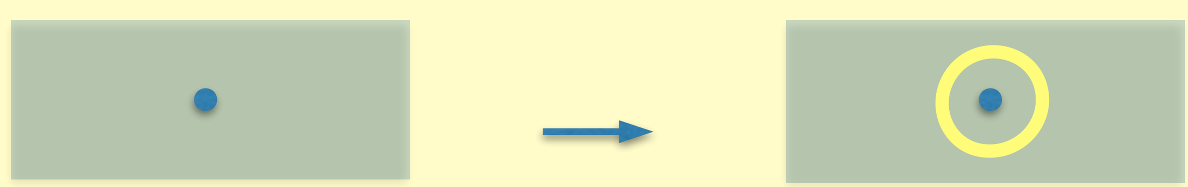
boundary fields

$$\phi_i^{(ab)}(x)$$



defect operators

$$D : \phi_i \mapsto D\phi_i$$



Bulk TD operator

$$D : \mathcal{H}_{\text{closed}} \to \mathcal{H}_{\text{closed}}$$

$[D, L_n] = 0$ → D is constant on each Verma module

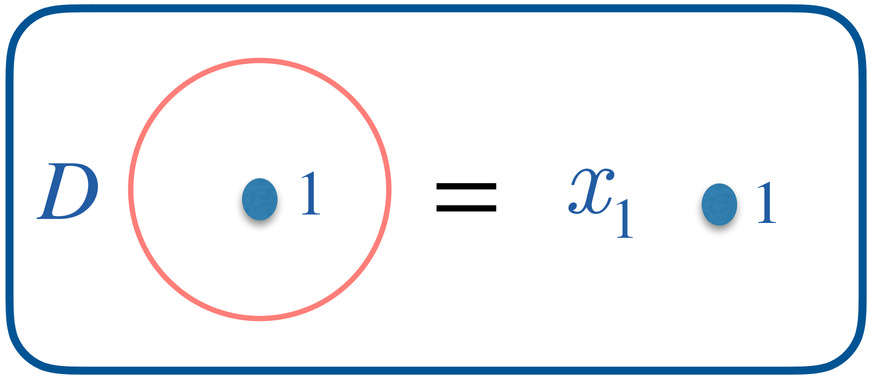
e.g. critical Ising model (c=1/2)

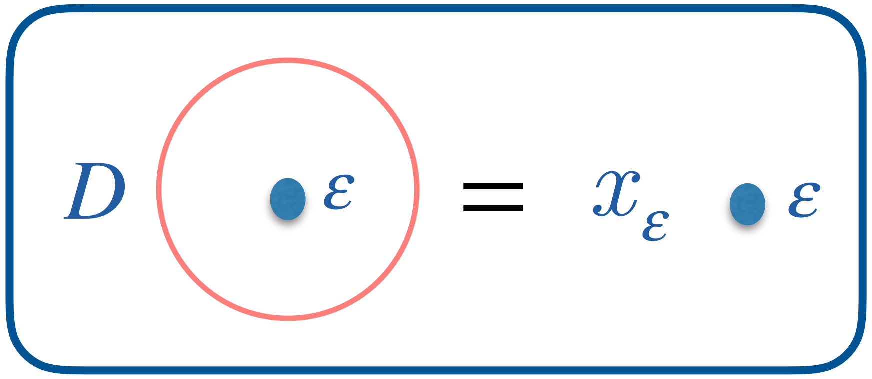
$1,\quad \varepsilon,\quad \sigma$

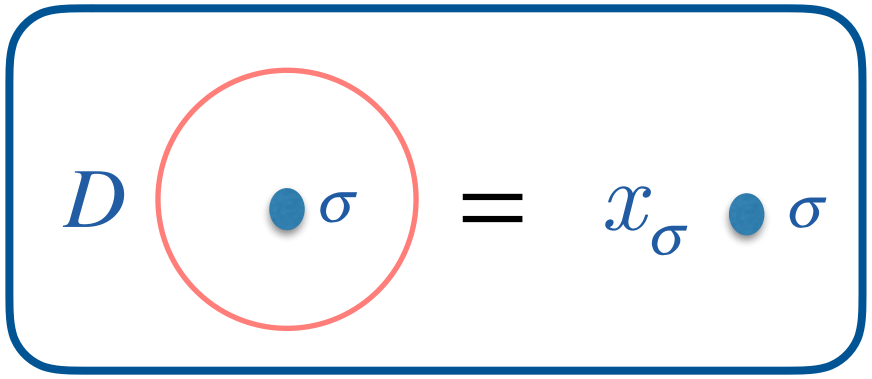
(energy density) (spin)

$$\mathcal{H}_{\text{closed}} = \mathrm{V}_0 \otimes \overline{\mathrm{V}}_0 \oplus \mathrm{V}_{1/2} \otimes \overline{\mathrm{V}}_{1/2} \oplus \mathrm{V}_{1/16} \otimes \overline{\mathrm{V}}_{1/16}$$

$$D = x_1 \cdot id_{\mathrm{V}_{1/2}\otimes\overline{\mathrm{V}}_{1/2}} + x_\varepsilon \cdot id_{\mathrm{V}_{1/2}\otimes\overline{\mathrm{V}}_{1/2}} + x_\sigma \cdot id_{\mathrm{V}_{1/16}\otimes\overline{\mathrm{V}}_{1/16}}$$



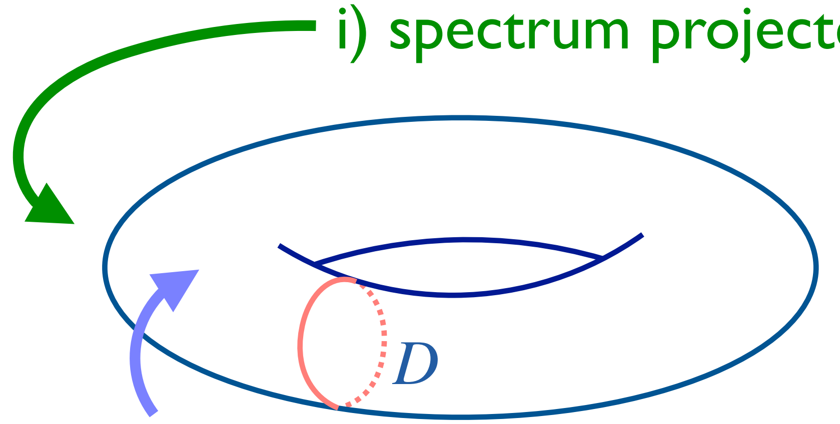




From modular invariance we can fix x :

Petkova-Zuber'00

i) spectrum projected by D



ii) twisted b.c.

$$x_i^{(d)} = \frac{S_{di}}{S_{0i}}$$

for diagonal minimal models

labels for TDs = labels for primary fields

fusion of TD operators

$$D_i D_j = \sum_k N_{ij}{}^k D_k$$

Related to the symmetry / duality of the theory

Frohlich, Fuchs, Runkel, Scweigert'04,'06

action on conformal boundary

$$D_i \| B_j \rangle\rangle = \sum_k N_{ij}{}^k \| B_k \rangle\rangle$$

D_d maps a D-brane system to another

Graham, Watts'03

~fusion rule

$$[\phi_i][\phi_j] = \sum_k N_{ij}{}^k [\phi_k]$$

$\boxed{\text{e.g. critical Ising model}}$

3 primary fields → 3 TD operators

$$D_1 = id$$

trivial defect; nothing

$$D_\varepsilon = id_{R_0 \otimes \bar{R}_0} - id_{R_{1/2} \otimes \bar{R}_{1/2}} + id_{R_{1/16} \otimes \bar{R}_{1/16}}$$

disorder lines

$$D_\sigma = \sqrt{2}\, id_{R_0 \otimes \bar{R}_0} - \sqrt{2}\, id_{R_{1/2} \otimes \bar{R}_{1/2}}$$

(implements KW dual)

Frohlich, Fuchs, Runkel, Scweigert'04,'06

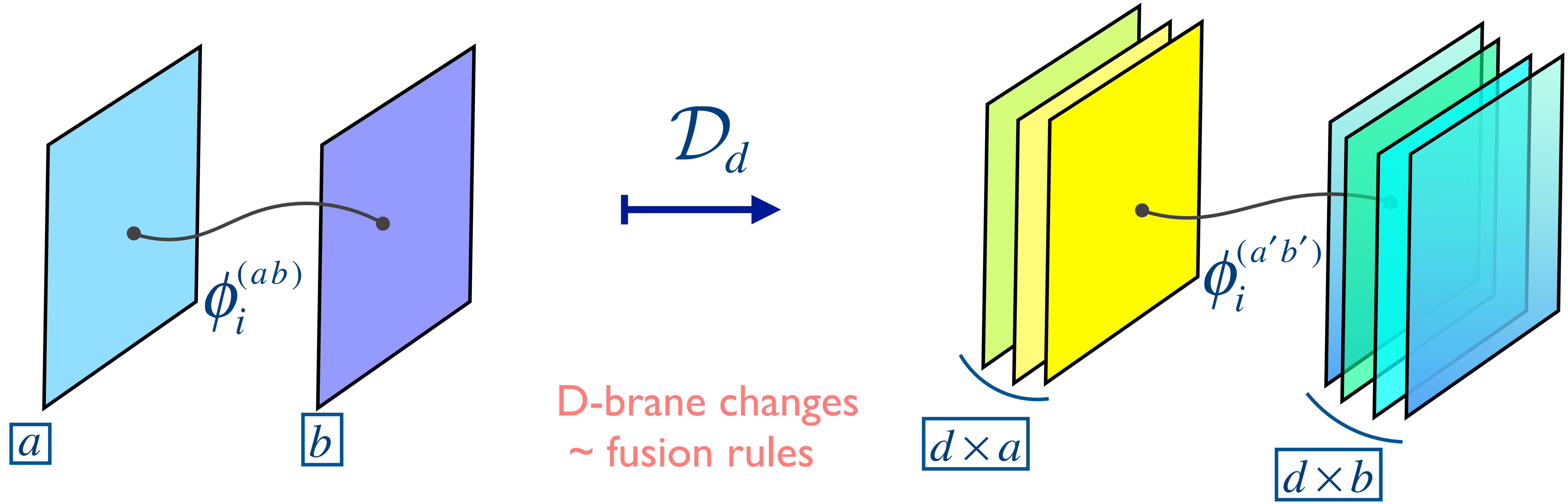
fusion rules of defects:

$$D_\varepsilon \times D_\varepsilon = D_1 \quad (\cdots \mathbb{Z}_2)$$

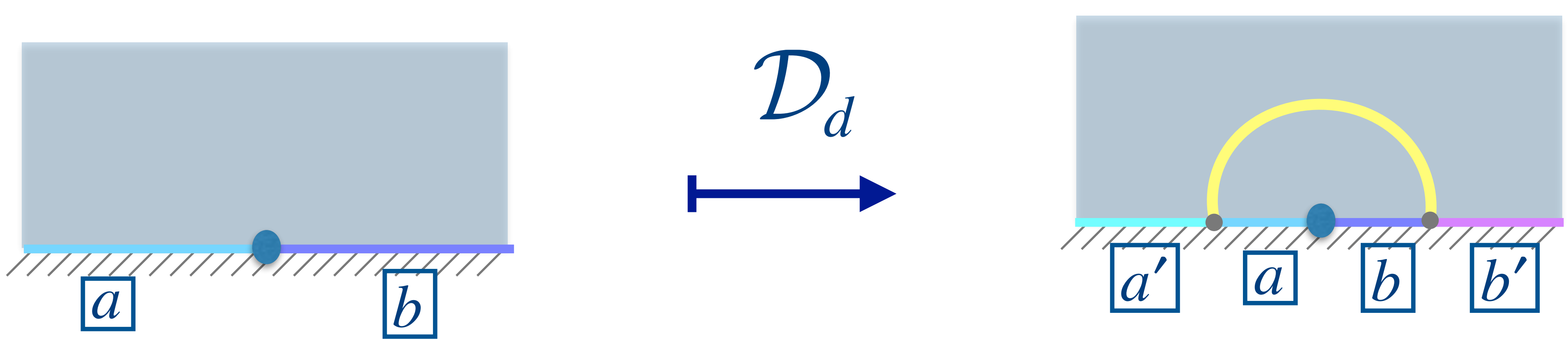
$$D_\sigma \times D_\varepsilon = D_\sigma \qquad D_\sigma \times D_\sigma = D_1 + D_\varepsilon$$

Boundary TD operators

$$\mathcal{D}_d : \mathcal{H}^{(ab)}_{\text{open}} \to \bigoplus_{a',b'} \mathcal{H}^{(a'b')}_{\text{open}}$$



D-brane changes
~ fusion rules



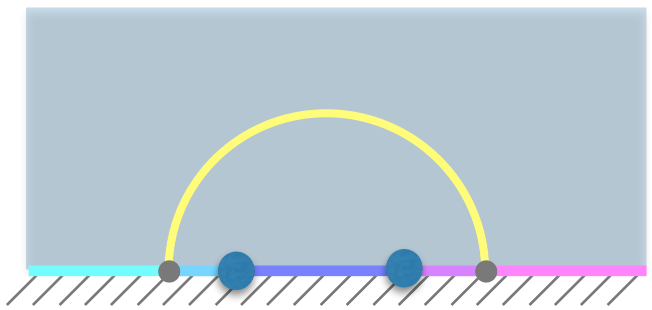
$a' \in d\times a$
$b' \in d\times b$

$$\boxed{\mathcal{D}_d \phi_i^{(ab)} = \sum_{(a'b')\in d\times(ab)} X^{d(ab)}_{i(a'b')} \phi_i^{(a'b')}}$$

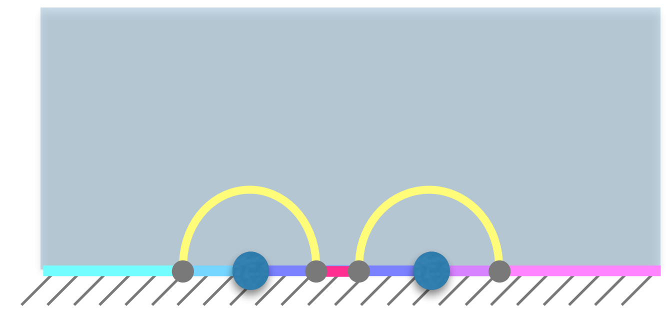
※matrix

How to fix X coefficients?

→require compatibility with the OPE



$$=$$



$$\mathcal{D}_d\,(\phi_i^{(a\,c)}\phi_i^{(c\,b)}) \qquad = \qquad (\mathcal{D}_d\,\phi_i^{(a\,c)})(\mathcal{D}_d\,\phi_i^{(c\,b)})$$

and from some physical requirement we find

$$\boxed{X_{i(a'b')}^{d(ab)} = \sqrt{F_{ba'}\begin{bmatrix} i & b \\ a & d \end{bmatrix}}\sqrt{F_{ab'}\begin{bmatrix} i & a \\ b & d \end{bmatrix}}}$$

Then, let's consider the fusion rules.

Naive fusion rule breaks down due to boundary effects

$$\mathcal{D}_d \mathcal{D}_c \neq \sum_{e \in d \times c} \mathcal{D}_e$$

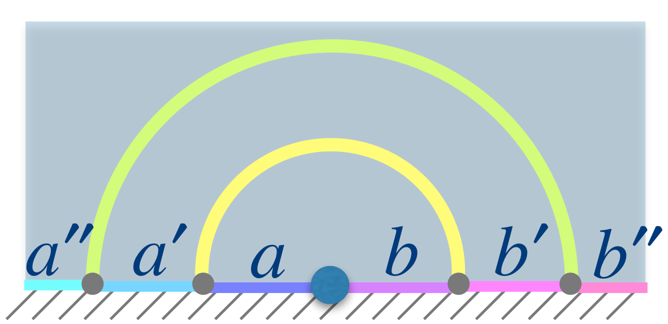
but with the U matrix

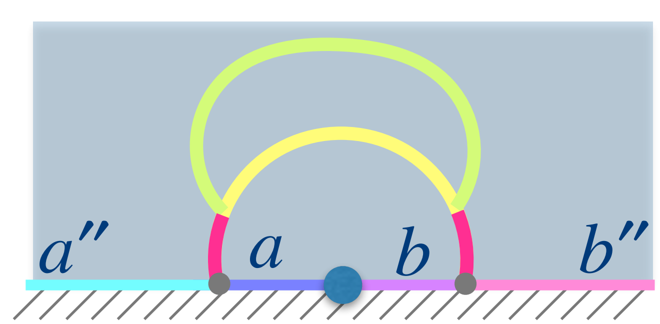
$$\boxed{\left(\mathcal{D}_d \left(\mathcal{D}_c \phi_i^{(ab)} \right)^{(a'b')} \right)^{(a''b'')} = \sum_{e \in d \times c} U_{a'e}\begin{bmatrix} d & c \\ a'' & a \end{bmatrix} \left(\mathcal{D}_e \phi_i^{(ab)} \right)^{(a''b'')} U_{b'e}\begin{bmatrix} c & d \\ b & b'' \end{bmatrix}}$$

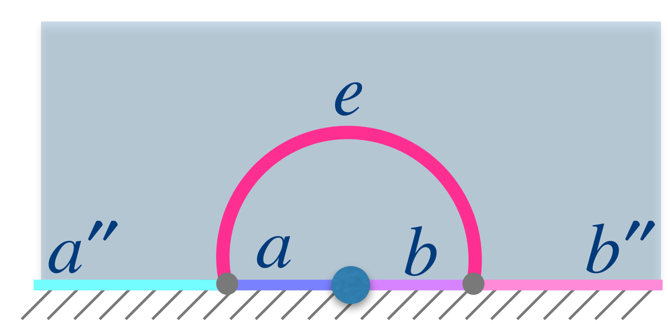
(matrix)

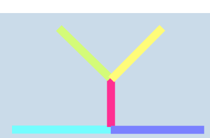
$$U_{a'e}\begin{bmatrix} d & c \\ a'' & a \end{bmatrix} = \sqrt{F_{a'e}\begin{bmatrix} d & c \\ a'' & a \end{bmatrix}} \sqrt{F_{e\,a'}\begin{bmatrix} d & a'' \\ c & a \end{bmatrix}}$$

graphically,







 $= \sum_e F_{a'e}\begin{bmatrix} d & c \\ a'' & a \end{bmatrix}$ 

factors from junction points "•"

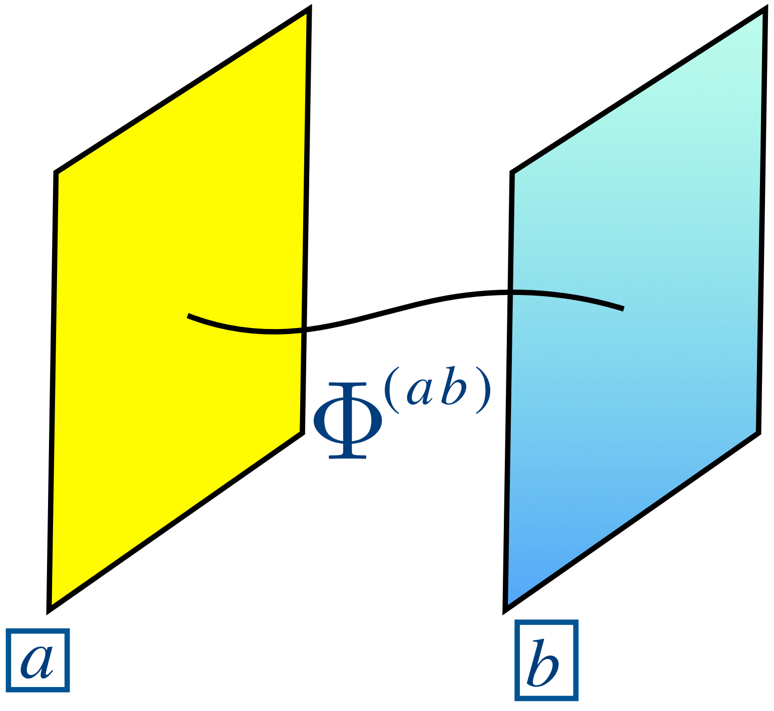
Now, we want to use $\mathcal{D}_d$ to analyze the classical solutions of open string field theory:

open string field: fluctuation around a D-brane system

$$\Phi^{(ab)} = \sum_{i,I} \mathrm{M}_{i,I} L_I \phi_i^{(ab)} |0\rangle$$

$L_I = L_{-i_1} L_{-i_2} \cdots L_{-i_n}$

I : multi-index



eom: $Q\Phi + \Phi * \Phi = 0$

cl. solution: D-brane configuration

Defect operator maps solution to solution:

$\because$

$\mathcal{D}$ commutes with BRS charge:

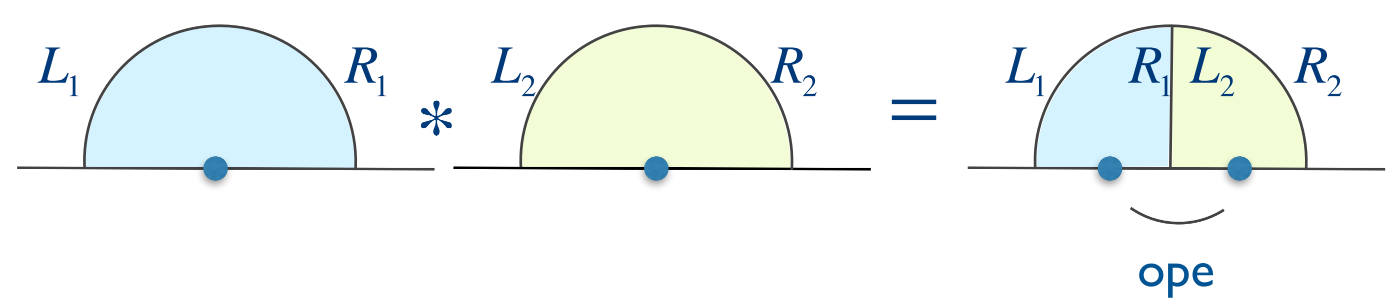
$$[Q, \mathcal{D}] = 0$$

$\because$ $\left(\begin{array}{l} [\mathcal{D}, L_n] = 0 \\ \mathcal{D}: \text{ from matter CFT} \\ [\mathcal{D}, b_n] = [\mathcal{D}, c_n] = 0 \end{array} \right.$

$\mathcal{D}$ is \`distributable' wrt star product:

$$\mathcal{D}(\Phi_1 * \Phi_2) = (\mathcal{D}\Phi_1) * (\mathcal{D}\Phi_2)$$

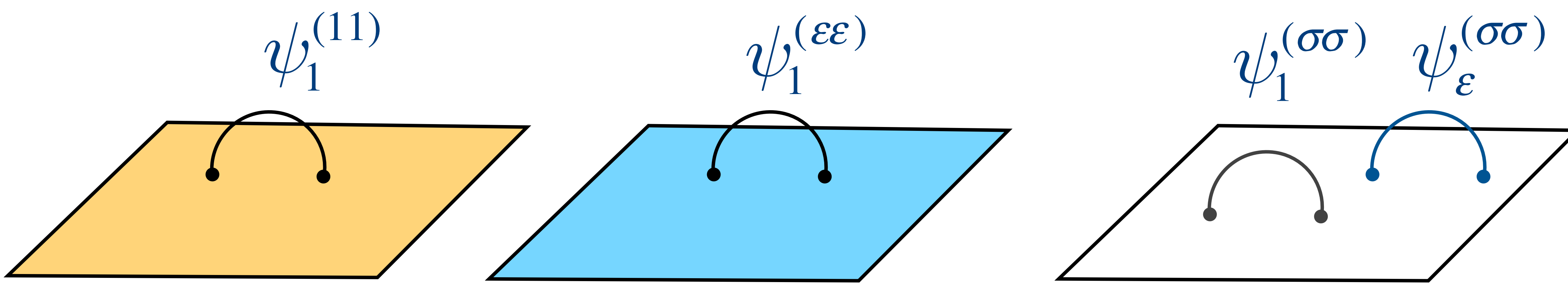
$\because$ star prod. $\leftarrow$ conformal transf. & ope



e.g. critical Ising model $\varepsilon \times \varepsilon = 1, \quad \sigma \times \varepsilon = \sigma, \quad \sigma \times \sigma = 1 + \varepsilon$

fusion rule: $\varepsilon \times \varepsilon = 1, \quad \sigma \times \varepsilon = \sigma, \quad \sigma \times \sigma = 1 + \varepsilon$

three conformal boundary conditions:



1 : fixed(+) $\quad$ ε : fixed(-) $\quad$ σ : free

$\mathbb{Z}_2$ symmetry

defect action on classical solutions:

$$\Psi_{\sigma\to 1} = \psi_1^{(\sigma\sigma)} + \psi_\varepsilon^{(\sigma\sigma)}$$

free → (+)

$$X^{\varepsilon(\sigma\sigma)}_{1(\sigma\sigma)} = 1, \quad X^{\varepsilon(\sigma\sigma)}_{\varepsilon(\sigma\sigma)} = -1$$

$\mathcal{D}_\varepsilon$ $\mathcal{D}_\varepsilon$

$$\Psi_{\sigma\to \varepsilon} = \psi_1^{(\sigma\sigma)} - \psi_\varepsilon^{(\sigma\sigma)}$$

free → (–)

*observed in [Kdurna-Rapcak-Schnabl'13]

$\mathcal{D}_\sigma$ $\big\downarrow$

$$\Psi_{1+\varepsilon\to \sigma} = \begin{pmatrix} \psi_1^{(\sigma\sigma)} & -\psi_\varepsilon^{(1\varepsilon)} \\ -\psi_\varepsilon^{(\varepsilon 1)} & \psi_1^{(\varepsilon\varepsilon)} \end{pmatrix}$$

$\vdots$ $\ddots$

Summary

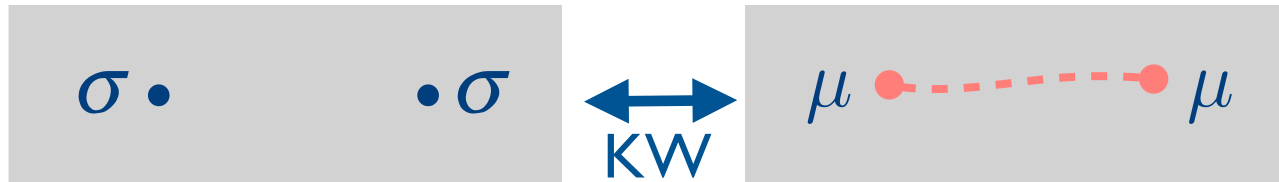
- Construction of boundary topological defect operators
- Application to Open String Field Theory: mapping of classical solutions

and…

- mapping of physical quantities (energies, gauge-invariant obs.)
- U and vacuum structure of OSFT
- Generalization (other RCFTs, etc..)
- graphical calculation of defect networks
- world sheet symmetry and spacetime symmetry..

Topological defect in 2d CFT: freely deformed

typical example:



disorder lines in 2d critical Ising model

studied in many contexts.. (from CMP, CFT, string theory)

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time is limited and cannot explain all, sorry..

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2. construction of boundary TD operator

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