Effective Lagrangian in de Sitter Spacetime

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Introduction

Quantum gravity in de Sitter spacetime is relevant to deep puzzles we face today

1. Existence of macroscopic spacetime:
   cosmological constant problem

2. Dark Energy at present univese

3. Inflation in the early universe

   Scale invariant metric perturbations are generated at super horizon scale after quantum fluctuations are frozen

   They seed large structures of the universe as CMB temperature fluctuations support such an idea

Scale invariant fluctuation

$$\phi_{\mathbf{p}}(x) = \frac{H\tau}{\sqrt{2p}}\left(1 - i\frac{1}{p\tau}\right)e^{-ip\tau+i\mathbf{p}\cdot\mathbf{x}}.$$

In de Sitter space, scale independent metric fluctuations are universally generated.

$$< \delta g \delta g >= \frac{H^2}{M_P^2}\int_{1/a}^{H} dP/P \sim \frac{H^2}{M_P^2}\log a(t) \sim \frac{H^2}{M_P^2}Ht$$

We assume the initial scale of the universe to be O(H)

de Sitter metric and fluctuation

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2, \quad a(t) = e^{Ht},$$

$$(g_{\mu\nu})_{\rm dS} = a^2(\tau)\eta_{\mu\nu}, \quad a(\tau) = -\frac{1}{H\tau}.$$

$$g_{\mu\nu} = \Omega^2(x)\tilde{g}_{\mu\nu}, \qquad \Omega(x) = a(\tau)e^{\kappa w(x)},$$

$$\det \tilde{g}_{\mu\nu} = -1, \qquad \tilde{g}_{\mu\nu} = (e^{\kappa h(x)})_{\mu\nu},$$

We use the following metric propagator in a particular gauge

$$\langle X(x)X(x')\rangle = -\langle\varphi(x)\varphi(x')\rangle,$$

$$\langle\tilde{h}^i{}_j(x)\tilde{h}^k{}_l(x')\rangle = (\delta^{ik}\delta_{jl} + \delta^i{}_l\delta_j{}^k - \frac{2}{3}\delta^i{}_j\delta^k{}_l)\langle\varphi(x)\varphi(x')\rangle,$$

$$h^{00} \simeq 2w \simeq \frac{\sqrt{3}}{2}X.$$

$$< \varphi^2 >= \frac{H^2}{4\pi^2}\log a(\tau)$$

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We have investigated IR logarithmic effects
in Schwinger-Keldysh formalism where the both
metric and matter are quantized.
The matter fields do not give rise to such effects
unless it is a minimally coupled massless scalar field.
It thus appears to be enough to integrate metric
fluctuations only.

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We construct the effective Lagrangian as

$$\mathcal{L}_{eff} =< \mathcal{L} >_{metric}$$

where the average is taken over the metric only.

We consider a generic renormalizable Lagrangian

$$\mathcal{L}(\lambda, g_Y, e) = \sqrt{-g}\Big[-\frac{1}{2}g^{\mu\nu}D_\mu\phi D_\nu\phi - \frac{1}{2}m^2\phi^2 + i\bar{\psi}e^\mu{}_a\gamma^a D_\mu\psi + m_f\bar{\psi}\psi - \frac{1}{4!}\lambda_4\phi^4 - \lambda_Y\phi\bar{\psi}\psi - \frac{1}{4e^2}g^{\mu\rho}g^{\nu\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}\Big].$$

We rescale

$$\phi \to a^{-1}e^{(\alpha-1)\omega}\phi$$
$$\psi \to a^{-\frac{3}{2}}e^{(\beta-\frac{3}{2})\omega}\psi$$

Gauge invariance keeps the gauge field intact

$$-\frac{1}{2}e^{2\alpha\omega}\tilde{g}^{\mu\nu}D_\mu\phi D_\nu\phi - \frac{1}{2}m^2a^2e^{2(1+\alpha)\omega}\phi^2 + ie^{2\beta\omega}\bar{\psi}\tilde{e}^\mu{}_a\gamma^a D_\mu\psi + m_f a e^{(1+2\beta)\omega}\bar{\psi}\psi - \frac{1}{4!}e^{4\alpha\omega}\lambda_4\phi^4 - \lambda_Y e^{(\alpha+2\beta)\omega}\phi\bar{\psi}\psi - \frac{1}{4e^2}\tilde{g}^{\mu\rho}\tilde{g}^{\nu\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}.$$

We adjust α and β to preserve Lorentz invariance
at sub-horizon scale.

This requirement fixes α and β uniquely
We reproduces the identical results with Schwinger-Keldysh formalism obtained before.
It clearly shows that IR logarithmic effects are local and makes couplings time dependent.

In general relativity, Lorentz symmetry must hold at least locally when the spacetime curvature can be ignored.
In fact the requirement of Lorentz symmetry at this limit follows from the fundamental principle of general relativity.

$$-\frac{1}{2}\tilde{g}^{00}e^{2\alpha\omega}\partial_0\phi\partial_0\phi - \frac{1}{2}\tilde{g}^{ij}e^{2\alpha\omega}\partial_i\phi\partial_j\phi$$

The α=0 contribution is

$$-\frac{3}{16} < \varphi^2 > \partial_0\phi\partial_0\phi - \frac{13}{16} < \varphi^2 > \partial_i\phi\partial^i\phi$$

The linear term in α is

$$-\frac{3\alpha}{8} < \varphi^2 > \partial_0\phi\partial_0\phi - \frac{\alpha}{8} < \varphi^2 > \partial_i\phi\partial^i\phi$$

We find that the requirement of Lorentz invariance fixes α=-2.

The total result including α^2 effect is

$$-\frac{3}{16} < \varphi^2 > \partial_0\phi\partial_0\phi + \frac{3}{16} < \varphi^2 > \partial_i\phi\partial^i\phi$$

We thus find that the IR logarithmic effect can be cancelled by the time dependent wave function renormalization of

$$\phi \to Z\phi \text{ where } Z^2 = (1 + \tfrac{3}{8} < \varphi^2 >)$$

Let us consider the scalar quartic coupling in our parametrization with canonically normalized kinetic term.

$$\lambda\phi^4 \rightarrow \lambda Z^4 e^{-8\omega}\phi^4$$

$$\lambda Z^4 < e^{-8\omega} >= \lambda(1 - \frac{21}{4} < \varphi^2 >)$$

We find that the coupling decreases with time

We next consider the mass term

$$m^2 e^{2\omega}\phi^2 \rightarrow m^2 e^{-2\omega} Z^2 \phi^2$$

Since $< e^{-2\omega} > Z^2 \sim 1$, the mass term is not renormalized after the wave function renormalization.

In this way, we obtain

$$(\lambda_4)_{\text{eff}} \simeq \lambda_4\left\{1 - \frac{21\kappa^2 H^2}{16\pi^2}\log a(\tau)\right\},$$

$$(\lambda_Y)_{\text{eff}} \simeq \lambda_Y\left\{1 - \frac{39\kappa^2 H^2}{128\pi^2}\log a(\tau)\right\}.$$

$$e_{\text{eff}} \simeq e\left\{1 - \frac{3\kappa^2 H^2}{16\pi^2}\log a(\tau)\right\}.$$

They agree with the results obtained in the Schwinger Keldysh formalism where $\alpha=\beta=0$.

Gauge dependence

$$\mathcal{L}_{\rm GF} = -\frac{1}{2}a^2 F^\beta_\mu F^{\beta\mu},$$

$$F^\beta_\mu = \beta(\partial_\rho h_\mu{}^\rho - 2\partial_\mu\omega) + \frac{1}{\beta}(2h_\mu{}^\rho\partial_\rho \log a + 4\omega\partial_\mu \log a).$$

$$\langle h^{00}h^{00}\rangle_\delta = (1-\delta-\frac{5}{2}\delta^2)\langle h^{00}h^{00}\rangle_0,$$

$$\langle \tilde{h}^{ij}\tilde{h}^{kl}\rangle_\delta = (1-\delta+\frac{5}{4}\delta^2)\langle \tilde{h}^{ij}\tilde{h}^{kl}\rangle_0.$$

There are two independent correlators of the metric:
Scalar and spin-2 modes

Under the reparametrization of the matter field

$$\tau \to e^{\gamma}\tau.$$

The way the metric coupes to the matter field changes as

$$h^{00} \to h^{00} + \frac{3}{2}\gamma, \quad \omega \to \omega + \frac{3}{4}\gamma$$

The gauge dependence of the relative magnitude can be canceled by a reparametrization of the matter field

$$\gamma = \tfrac{5}{4}\delta^2 h^{00}$$

This procedure is required by the Lorentz invariance of the matter Dynamics at sub-horizon scale

Effective Lagrangian approach in de Sitter spacetime leads to unique physical predictions by imposing Lorentz invariance at sub-horizon scale.

Matter fields are accompanied by soft metric fluctuations and we need to take such effects properly.
Such a requirement is intimately connected with
Local Lorentz invariance and unitarity.

We show gauge independence of the physical observables such as relative scaling exponents of the couplings

Although this is a small effect at present, it could lead to observable effects as couplings may be incredibly fine tuned.