

Inflation and Beyond

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Introduction

What is Inflation?

Brout, Englert & Gunzig '77, Starobinsky '79, Guth '81, Sato '81, ...

- Inflation is a **quasi-exponential expansion** of the Universe at its very early stage; perhaps at $t \sim 10^{-36}$ sec.
- It was meant to solve **the initial condition (singularity, horizon & flatness, etc.) problems** in Big-Bang Cosmology:
 - if any of them can be said to be solved depends on precise definitions of the problems.
- **Quantum vacuum fluctuations** during inflation turn out to play the most important role. They give the initial condition for **all the structures in the Universe**.
- **Cosmic gravitational wave background** is also generated.

more on \checkmark inflation

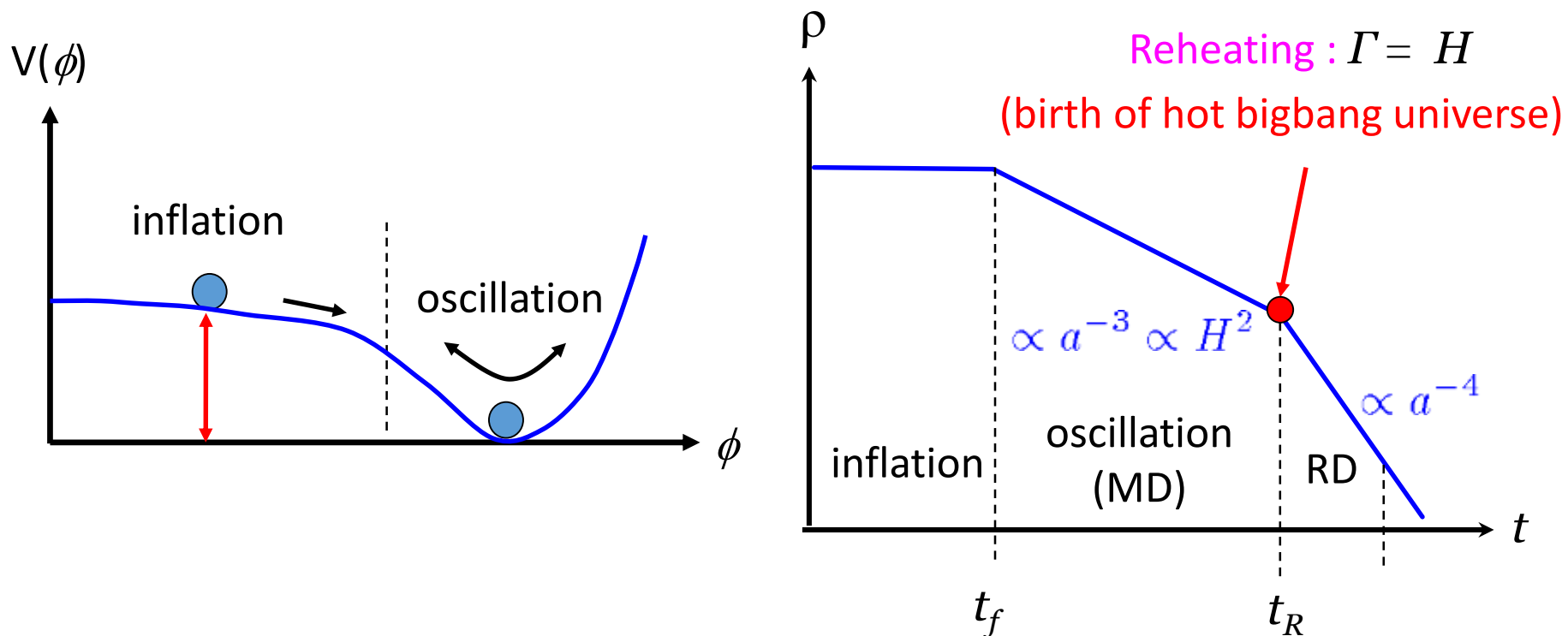
the meaning of

1. Homogeneity and isotropy are the (most important) assumption, **NOT a consequence** of inflation.
2. Quasi-exponential expansion in the “Einstein frame”: **conformal invariant definition**.
3. At least 50-60 e-folds before the end of inflation: solving **“horizon problem”**
4. Don’t care what happened before inflation: predictions are **almost independent** of initial conditions.

1 & 2: basic assumptions/definition of inflation

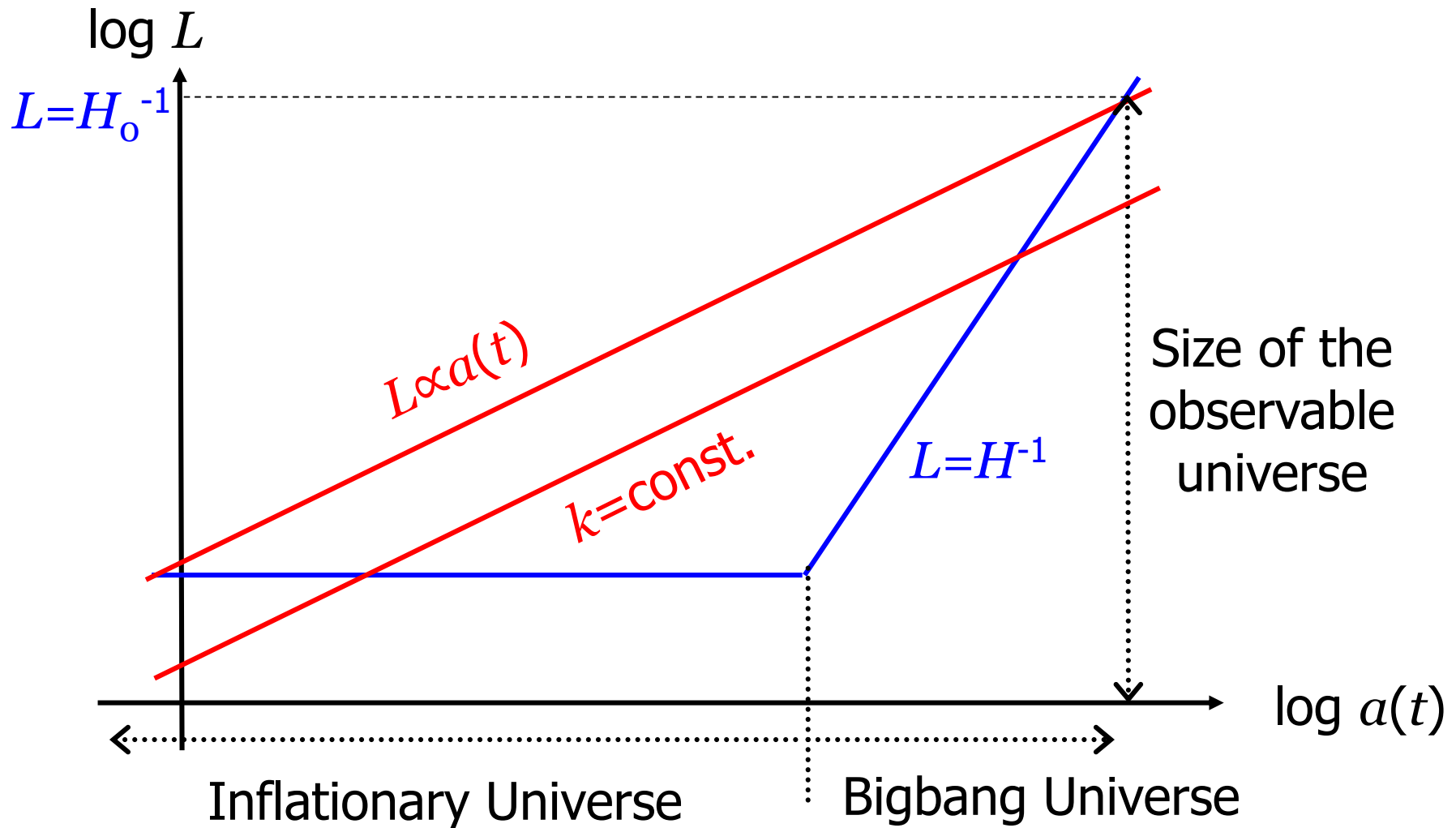
From inflation to bigbang

After inflation, vacuum energy is converted to **thermal energy** (called “**re**”heating) and **hot Bigbang Universe** is realized.

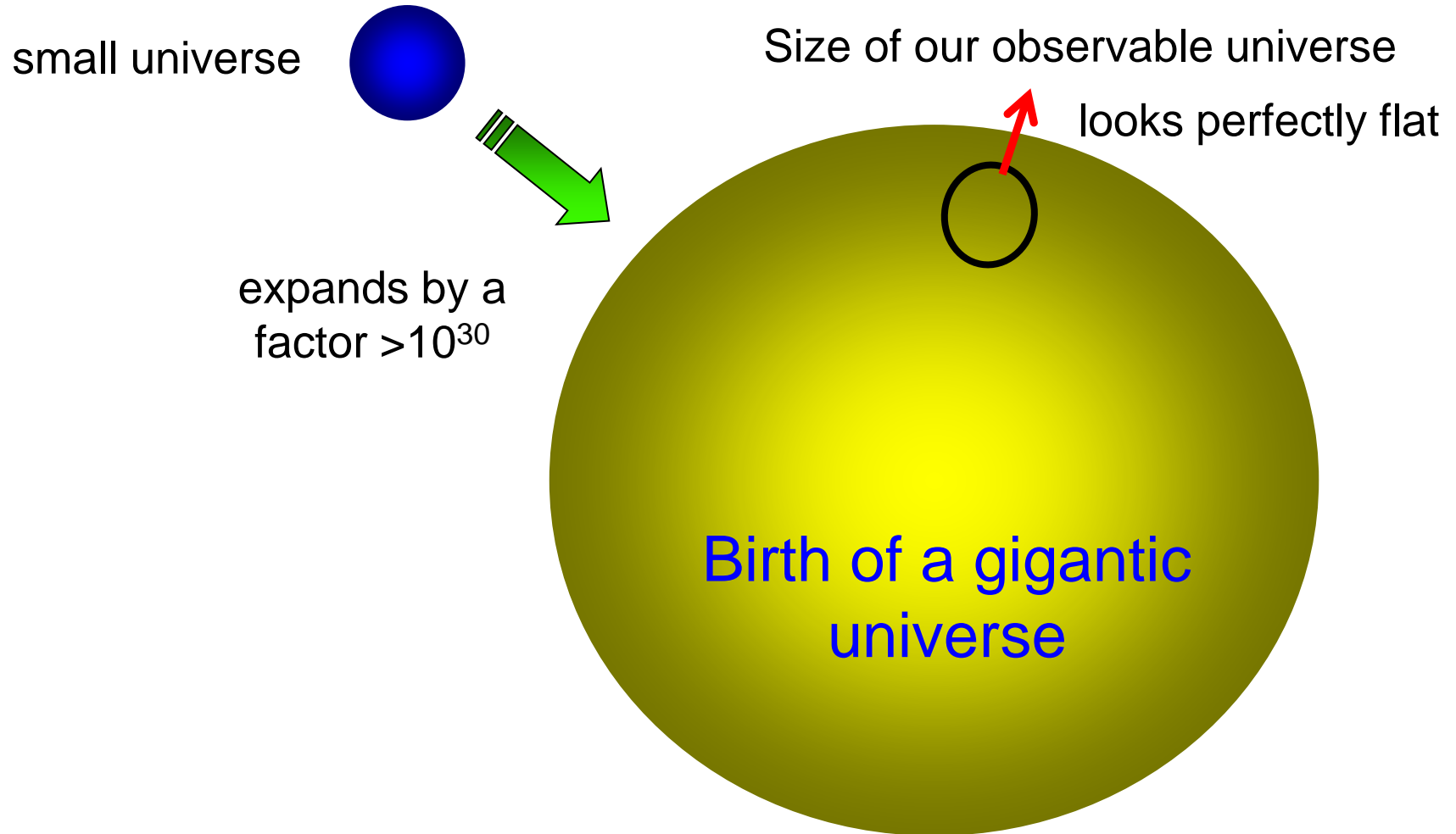


Kinematics

length scales of the inflationary universe



Flatness



Flatness can be explained only by Inflation

NB: Inflation may not always imply flatness

Dynamics

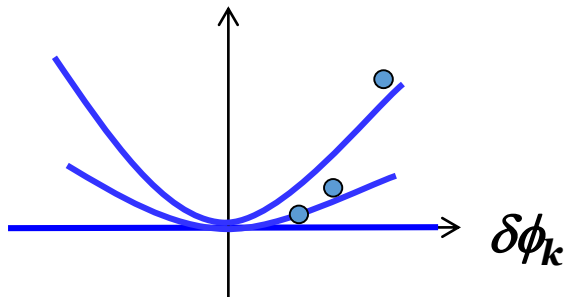
Seed of cosmological perturbations

Mukhanov '81,

Zero-point (vacuum) fluctuations of ϕ : $\delta\phi = \sum_k \delta\phi_k(t) e^{ik \cdot x}$

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \omega^2(t)\delta\phi_k = 0 ; \quad \omega^2(t) = \frac{k^2}{a^2(t)}$$

harmonic oscillator with friction term and time-dependent ω



$$\delta\phi_k \rightarrow \text{const.}$$

... frozen when $\omega < H$
(on superhorizon scales)

tensor (gravitational wave) modes also satisfy the same eq.

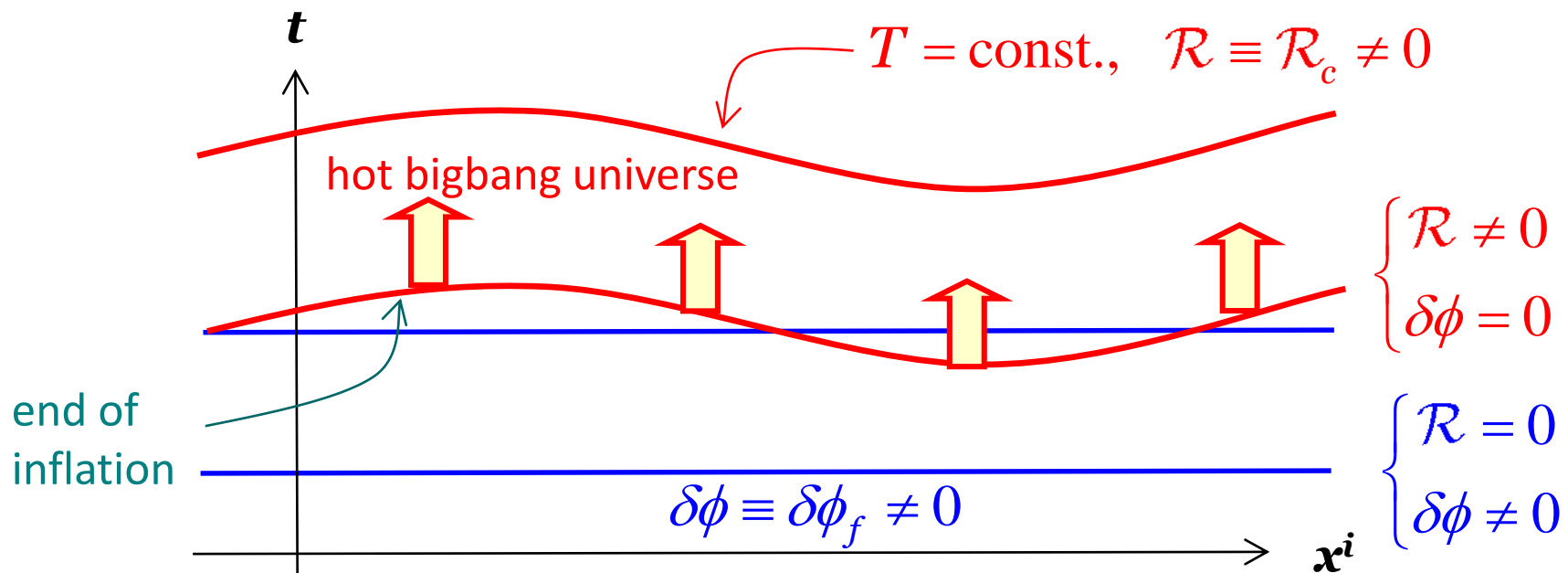
Starobinsky '79

Generation of Curvature Perturbation

curvature (potential) perturbation \mathcal{R} : $\delta R^{(3)} = -\frac{4}{a^2} \nabla^2 \mathcal{R}$

comoving curvature perturbation $\mathcal{R}_c \sim$ - Newton potential

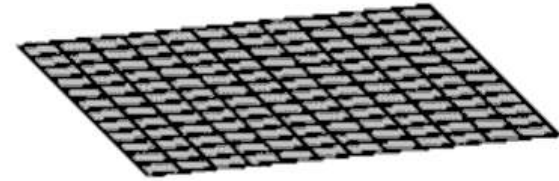
- $\delta\phi$ is frozen on “flat” ($\mathcal{R}=0$) 3-surface ($t=\text{const.}$ hypersurface) $\mathcal{R}_c = -\frac{H}{\dot{\phi}} \delta\phi_f$
- Inflation ends/damped osc starts “comoving” ($\phi=\text{const.}$) on 3-surface.



generic predictions of inflation

slow-roll

- Spatially flat universe
- Almost scale invariant, adiabatic, Gaussian primordial scalar (curvature) perturbations
- Almost scale invariant, Gaussian primordial tensor (gravitational wave) perturbations



Generates CMB anisotropy
Origin of galaxies, stars, ...

Quantitative Predictions

- Amplitude of curvature perturbation:

$$\mathcal{R}_c = \left. \frac{H^2}{2\pi\dot{\phi}} \right|_{k/a=H} \quad \text{Mukhanov (1985), MS (1986)}$$

- Power spectrum index:

$$M_{pl} \equiv \frac{1}{\sqrt{8\pi G}} \sim 2.4 \times 10^{18} \text{ GeV: Planck mass}$$

$$\frac{4\pi k^3}{(2\pi)^3} P_S(k) = \left[\frac{H^2}{2\pi\dot{\phi}} \right]_{k/a=H}^2 = A k^{n_s-1} ; \quad n_s - 1 = M_P^2 \left(2 \frac{V''}{V} - 3 \frac{V'^2}{V^2} \right)$$

Stewart-Lyth (1993)

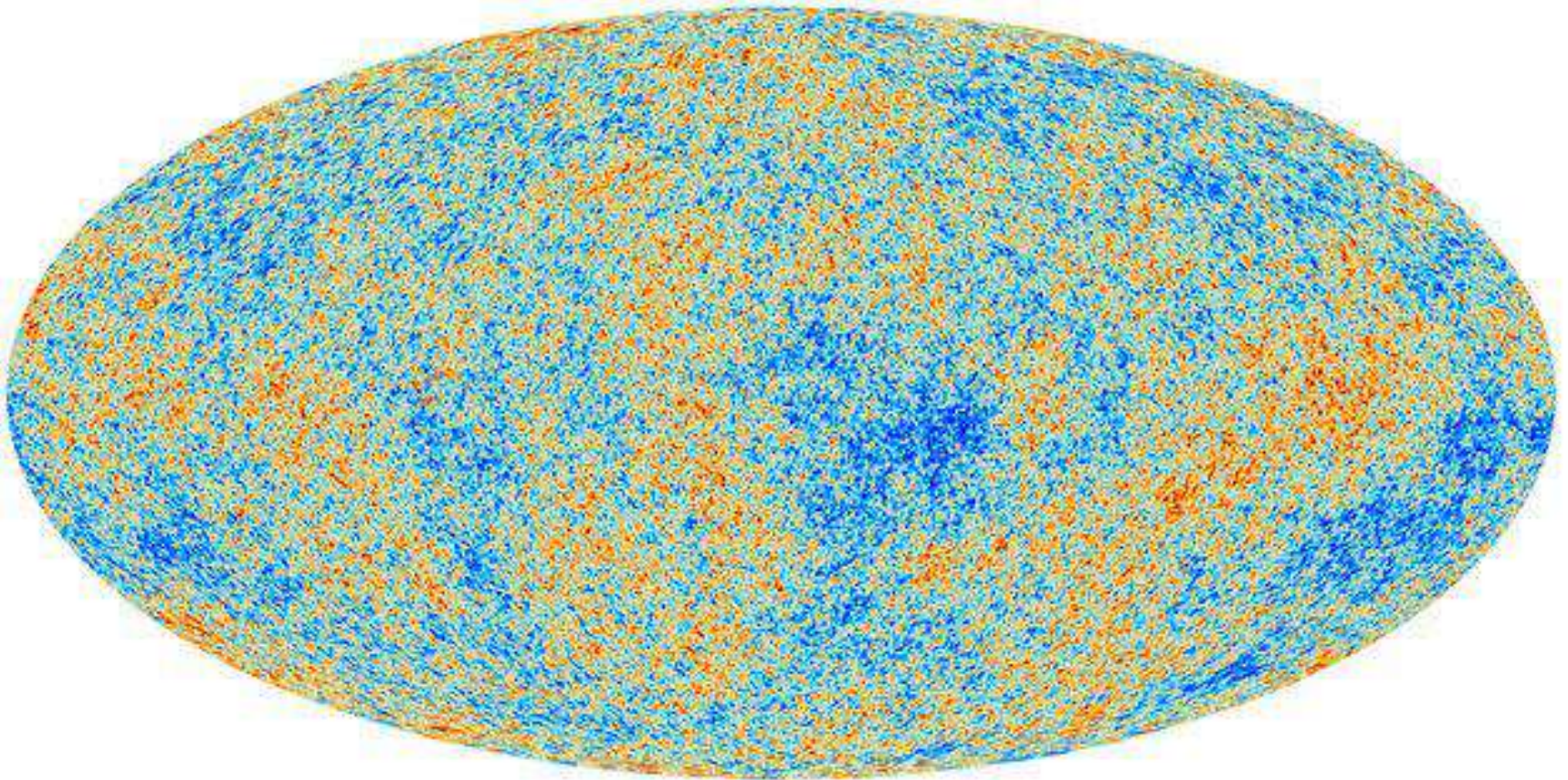
- Tensor (gravitational wave) spectrum:

$$\frac{4\pi k^3}{(2\pi)^3} P_T(k) = A k^{n_T} ; \quad n_T = -\frac{1}{8} \frac{P_S(k)}{P_T(k)} \equiv -\frac{r}{8} \quad \text{Liddle-Lyth (1992)}$$

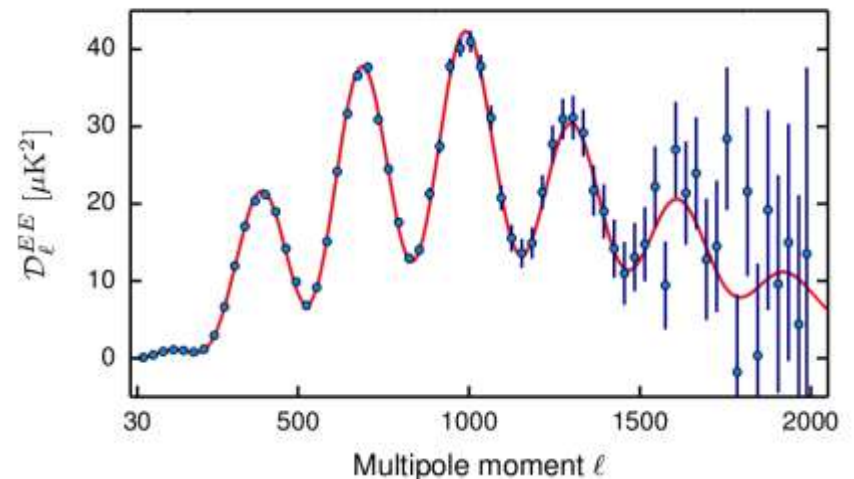
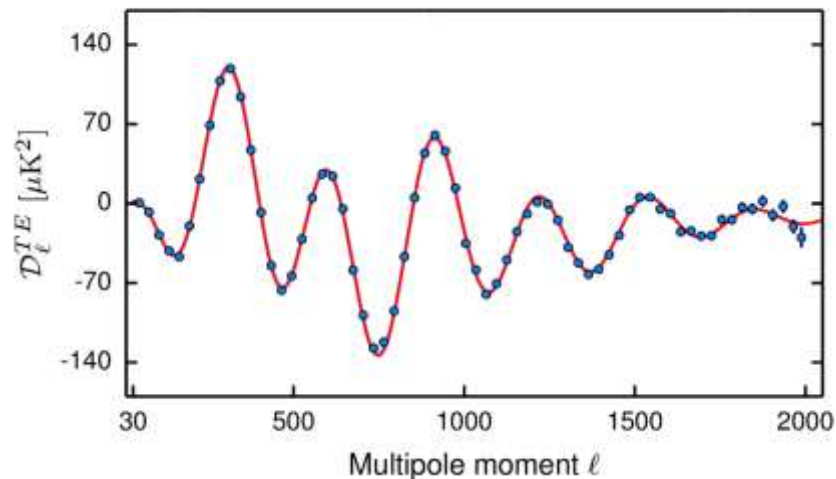
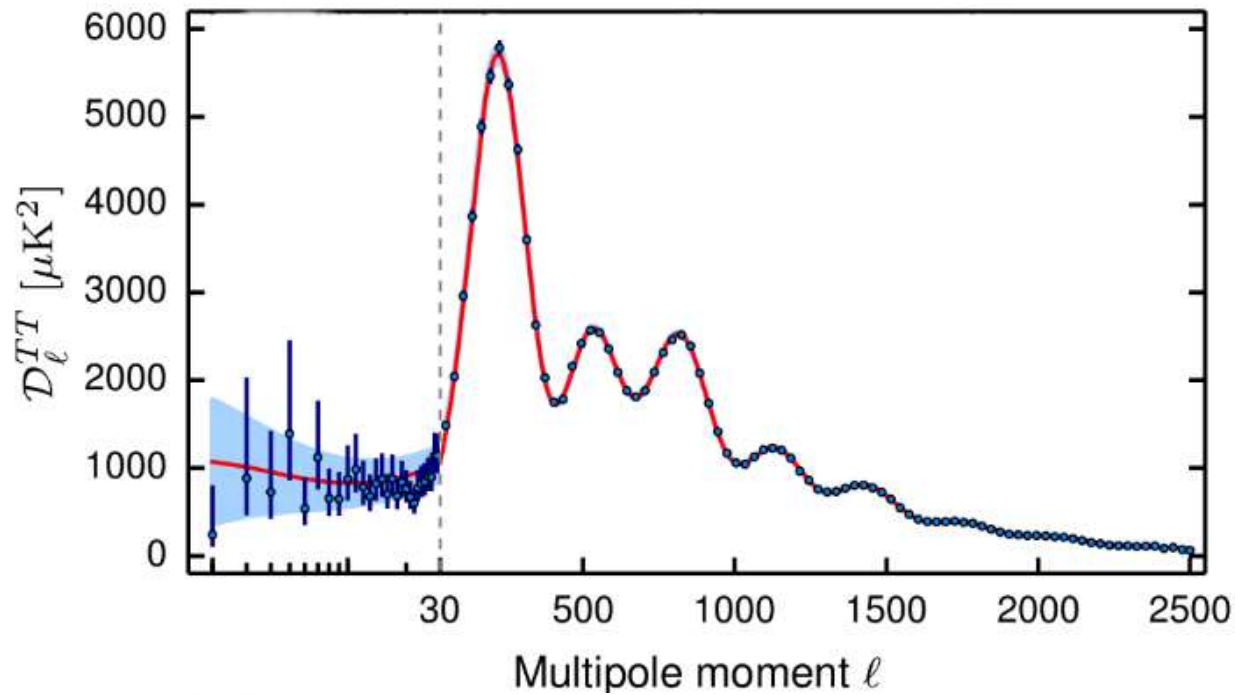
“consistency relation”

Observational results

CMB Full Sky Map by PLANCK



Planck TT, TE & EE spectrum



- Amplitude of curvature perturbation:

$$\mathcal{R}_c = \frac{H^2}{2\pi\dot{\phi}} \Big|_{k/a=H} \quad \text{Mukhanov (1985), MS (1986)}$$

$$\mathcal{R}_{c,\text{obs}} \sim 10^{-5} \Rightarrow V^{1/4}(\phi) \lesssim 10^{16} \text{ GeV} ?$$

- Power spectrum index:

$$M_P \equiv \frac{1}{\sqrt{8\pi G}} \sim 2.4 \times 10^{18} \text{ GeV: Planck mass}$$

$$\frac{4\pi k^3}{(2\pi)^3} P_S(k) = \left[\frac{H^2}{2\pi\dot{\phi}} \right]_{k/a=H}^2 = A k^{n_s-1} ; \quad n_s - 1 = M_P^2 \left(2 \frac{V''}{V} - 3 \frac{V'^2}{V^2} \right)$$

Stewart-Lyth (1993)

$$n_{S,\text{Planck}} - 1 = -0.0355 \pm 0.0049 \Leftrightarrow n_s - 1 \sim -0.04 \text{ for a typical model}$$

Mukhanov & Chibisov (1981)

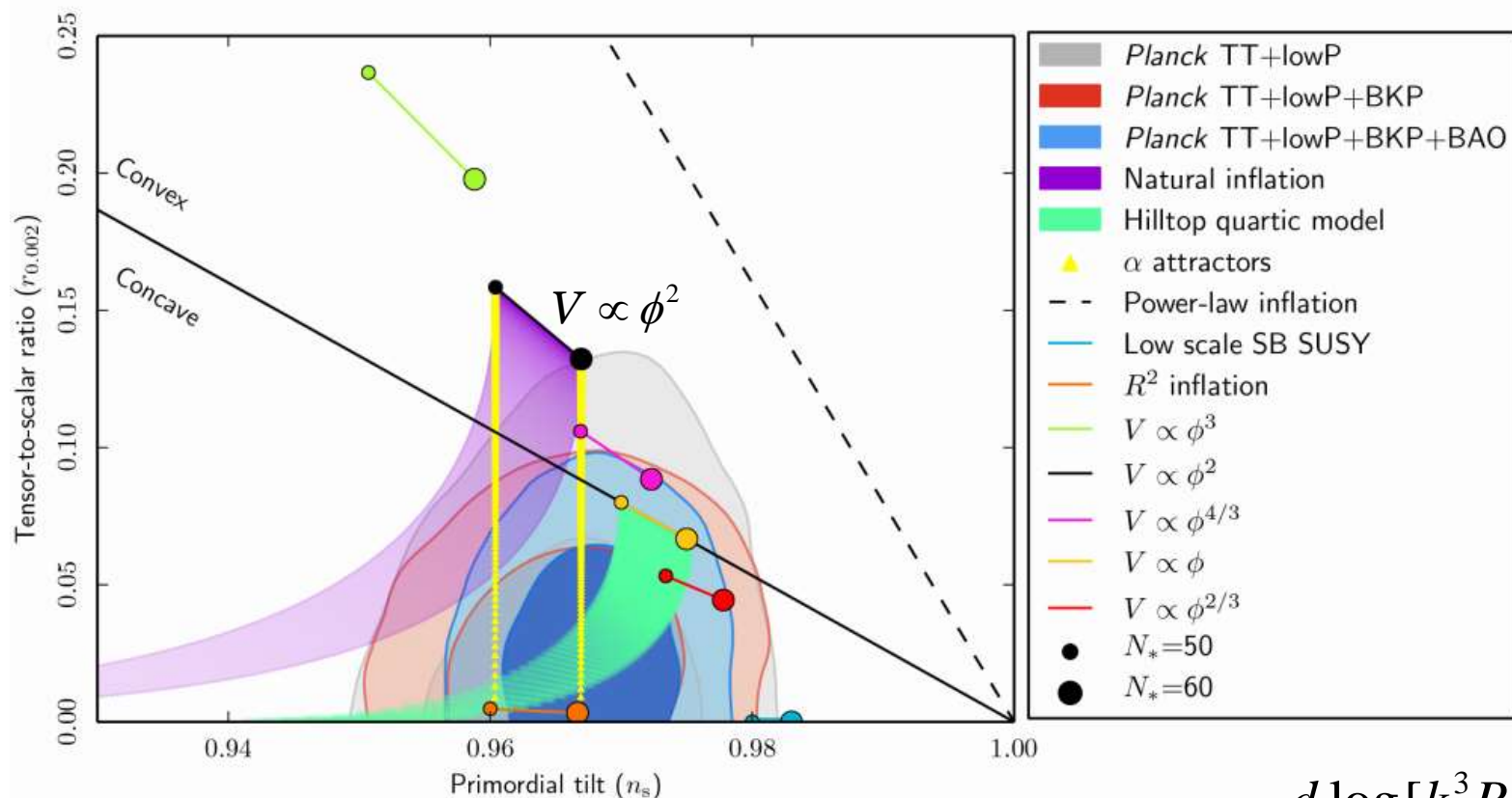
- Tensor (gravitational wave) spectrum:

$$\frac{4\pi k^3}{(2\pi)^3} P_T(k) = A k^{n_T} ; \quad n_T = -\frac{1}{8} \frac{P_T(k)}{P_S(k)} \equiv -\frac{r}{8} \quad \text{Liddle-Lyth (1992)}$$

to be observed ...

Planck constraints on inflation

Planck 2015 XX



- scalar spectral index: $n_s \sim 0.96$
- tensor-to-scalar ratio: $r < 0.1$
- simplest $V \propto \phi^2$ model is almost excluded

$$n_s - 1 \equiv \frac{d \log[k^3 P_s(k)]}{d \log k}$$

$$r \equiv \frac{P_T(k)}{P_s(k)}$$

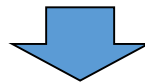
Implications

The most important message is:

Inflation as the Origin of
All Structures
in the Universe

Current status

- scalar spectral index: $n_s < 1$ at $\sim 5 \sigma$
- tensor/scalar ratio: $r < 0.1$ implies $E_{\text{inflation}} < 10^{16} \text{ GeV}$
- simple, **canonical models** are **on verge of extinction**
($m^2\phi^2$ model excluded at $> 2 \sigma$)
- R^2 (Starobinsky) model seems to fit best. **But why?**
(large R^2 correction but negligible higher order terms)
- $f_{\text{NL}}^{\text{local}} < O(1)$ suggests (effectively) **single-field slow-roll**
(but non-slow-roll models with $f_{\text{NL}}^{\text{local}} = O(1)$ not excluded)



element of **non-canonicity** is needed

Beyond
(standard model of)
Inflation

non-canonical models

- Non-canonical kinetic term? ($c_s < 1$?)

$$P_s \propto \frac{1}{c_s} \quad (c_s: \text{sound speed}), \quad f_{\text{NL}}^{\text{equil}} \propto \frac{1}{c_s^2}$$

Planck: $c_s > 0.024$ at 95% CL

- non-minimal coupling to gravity?

$$V(\phi) + \xi \phi^2 R \quad \longrightarrow \quad r = \frac{P_T(k)}{P_s(k)} \propto \frac{1}{\xi} \quad \text{Planck: } \xi > \mathcal{O}(10)?$$

- scalar-tensor with derivative couplings (Hordeski) ?

$$c_s < 1, \quad c_{s,T} < 1, \quad c_s \neq c_{s,T}$$



tensor propagation speed

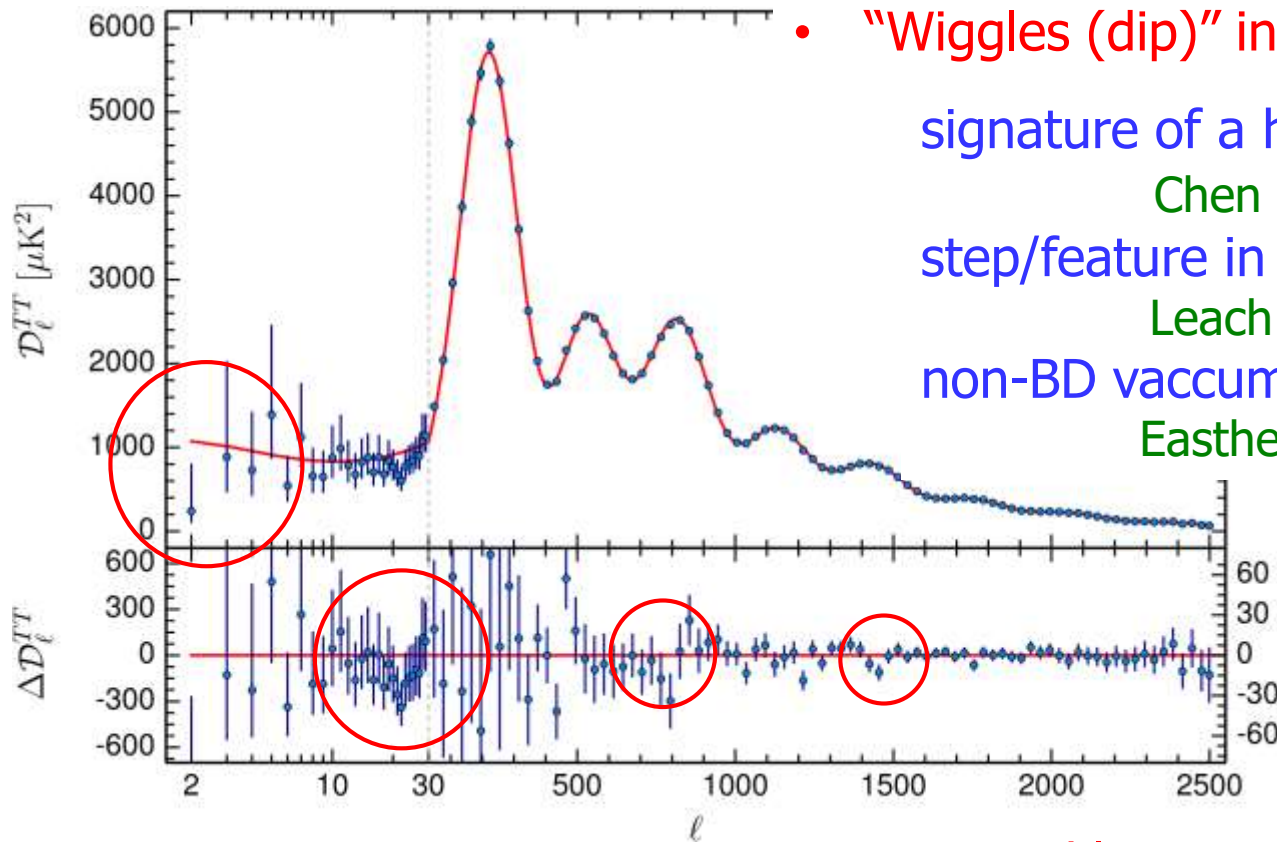
non-existence of
Einstein frame?



definition of
inflation?

- multi-field models, non-attractor inflation, ...

Anomalies & Features



- “Wiggles (dip)” in the power spectrum

signature of a heavy field?

Chen '11, ...

step/feature in potential?

Leach et al. '01,

non-BD vacuum?

Easter et al. '02,

- Suppressed large scale fluctuations

featured potential? open inflation?

Linde, MS & Tanaka '99,

Cosmic Landscape?

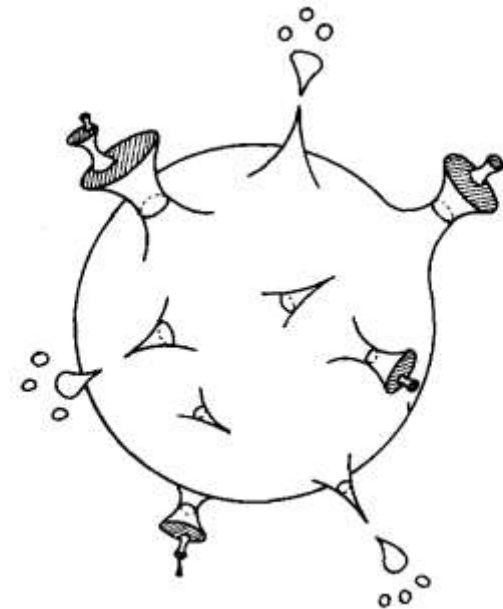
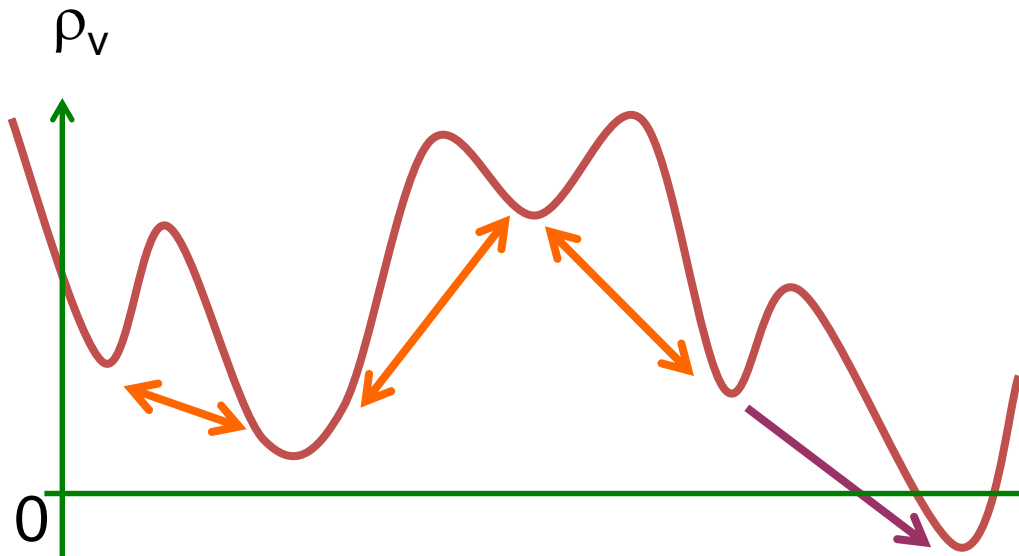
string theory suggests an intriguing picture of the early universe



Maybe we live in one of these vacua...

- Universe jumps around in the landscape by quantum tunneling
 - it can go up to a vacuum with larger ρ_v
 de Sitter (dS) space \sim thermal state with $T = H/2\pi$

expansion rate
↓
 - if it tunnels to a vacuum with negative ρ_v ,
 it collapses within $t \sim M_P/|\rho_v|^{1/2}$.
 - so we may focus on vacua with positive ρ_v : dS vacua



Sato, Kodama, MS & Maeda ('81)

Open Inflation in Cosmic Landscape

Universe inside nucleated bubble = spatially open universe

Friedmann eq.

$$H^2 \equiv \left(\frac{\dot{a}}{a} \right)^2 = \frac{\rho}{3M_P^2} + \frac{1}{a^2}$$

negative
spatial
curvature

$$1 = \frac{\rho}{3M_P^2 H^2} + \frac{1}{a^2 H^2} \equiv \Omega + \Omega_K$$

density parameter

Observational data indicate $1 - \Omega_0 = \Omega_{K,0} \sim < 10^{-2}$: almost flat

("0" stands for current value)

What if this is the case?

➤ two possibilities

1. inflation after tunneling was short enough ($N \sim 60$)

$$\Omega_{K,o} = 1 - \Omega_o = 10^{-2} \sim 10^{-3} \quad \text{“open universe”}$$

➡ signatures in **large angle CMB anisotropies?**

Kanno, MS & Tanaka ('13), White, Zhang & MS ('14), ...

2. inflation after tunneling was long enough ($N \gg 60$)

$$\Omega_{K,o} = 1 - \Omega_o \ll 1 \quad \text{“flat universe”}$$

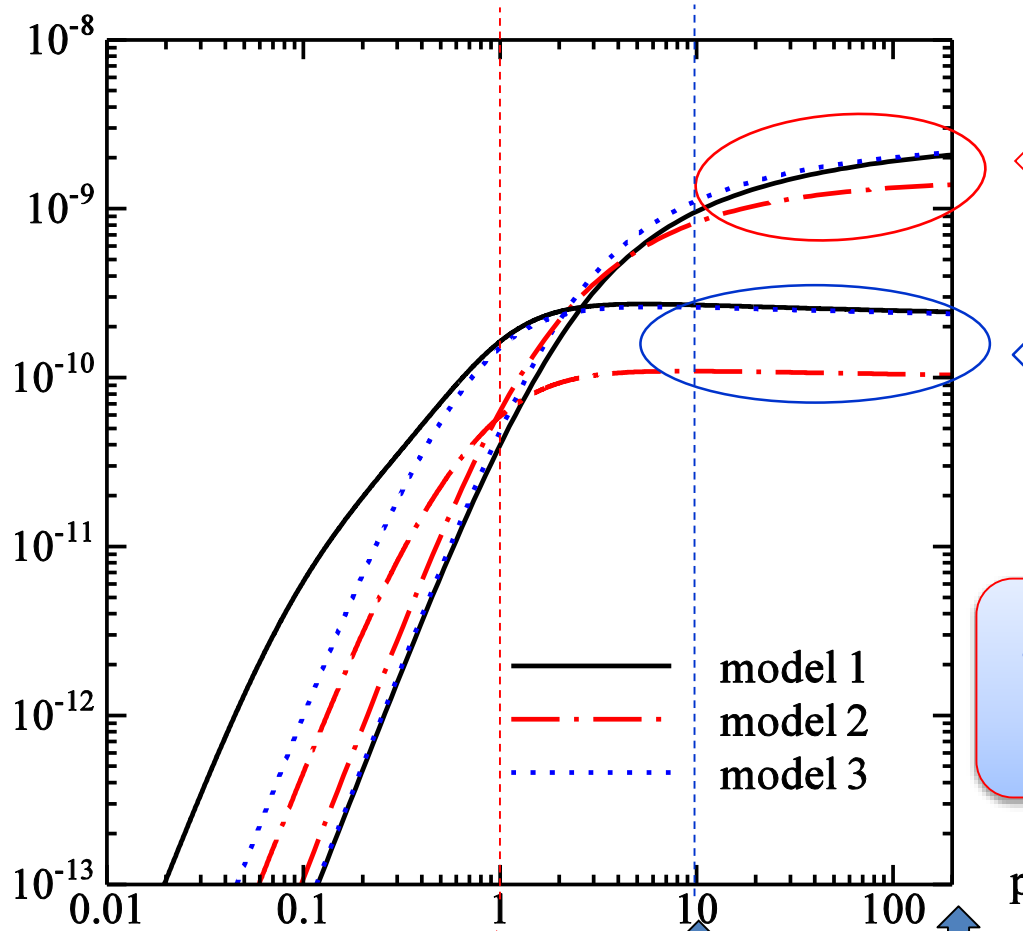
➡ signatures from **bubble collisions**

Sugimura, Yamauchi & MS ('12), ...

scalar suppression on large scales

Linde, MS & Tanaka (1999)
White, Zhang & MS (2014)

$$(|R_p|^2, |U_p|^2) p^3 / (2\pi^2)$$



← scalar

← tensor
(no suppression)

scalar suppression begins at
smaller scale

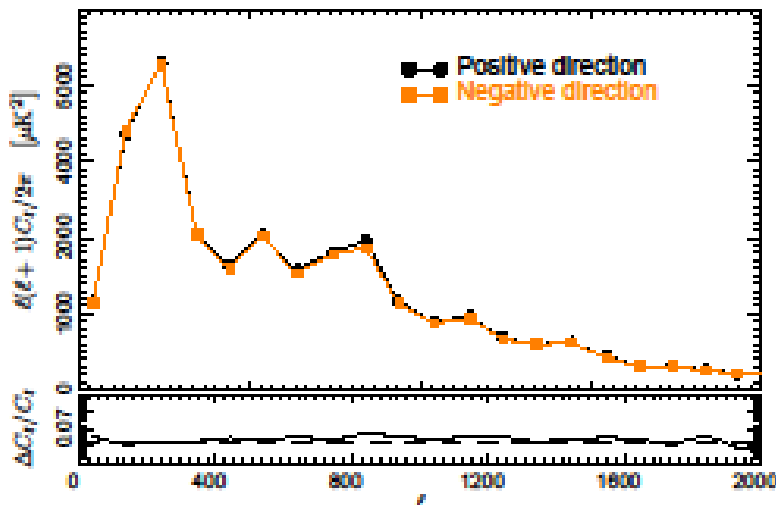
↕
break scale

curvature
radius

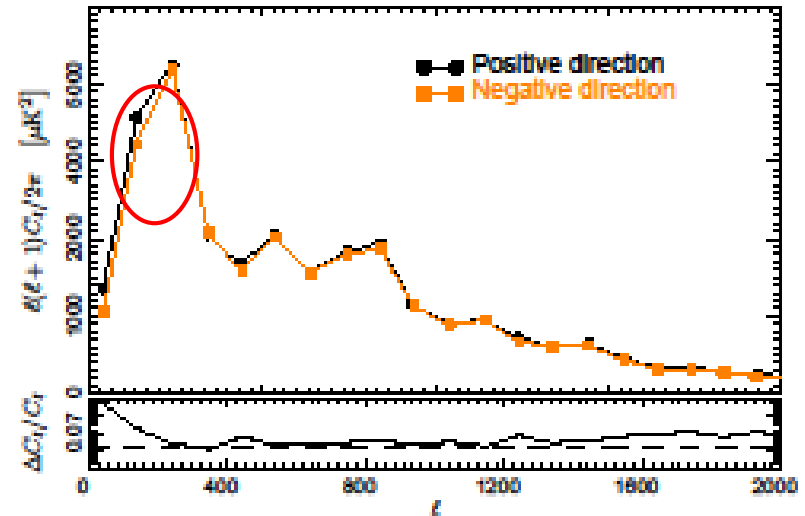
H_0^{-1} if $\Omega_K \approx 0.01$

dipole power asymmetry

$$P(k; \mathbf{n}) = \left(1 + 2 \underbrace{A \hat{\mathbf{d}}}_{\text{dipole}} \cdot \underbrace{\mathbf{n}}_{\text{line of sight}} \right) P_{iso}(k)$$



no asymmetry in the direction of $\ell=1$



dipole asymmetry in the direction
maximizing the asymmetry

Planck 2013 XXIII

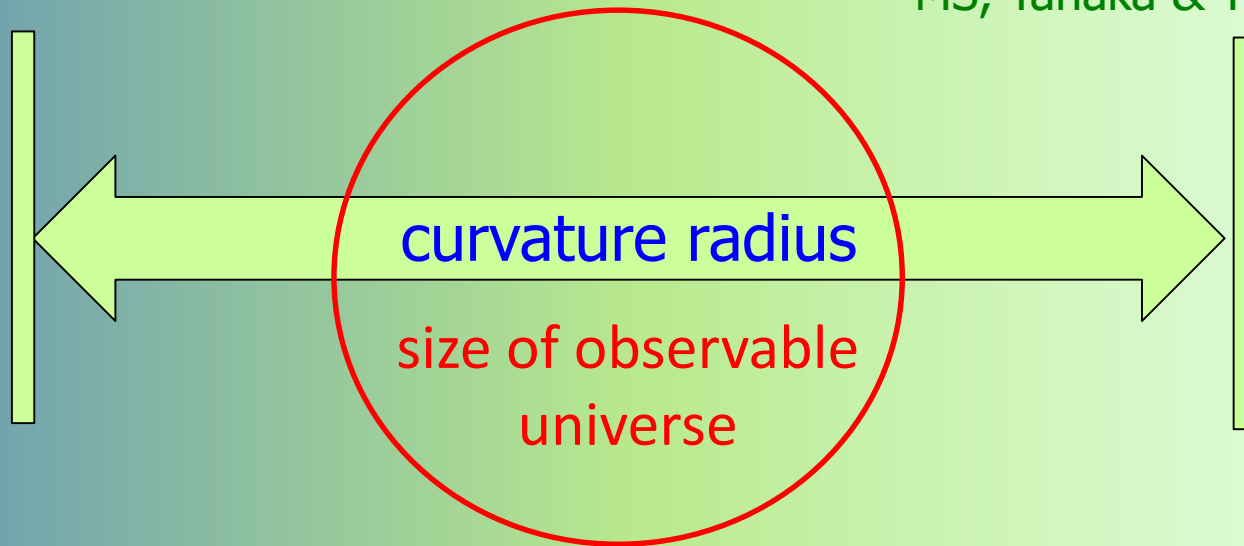
$$\frac{\delta T}{T} = (1 + A \cos \theta) \left(\frac{\delta T}{T} \right)_{iso}$$

$$A \approx 0.07$$

Data set	FWHM [°]	A	(l,b) [°]	$\Delta \ln \mathcal{L}$	Significance
Commander	5	$0.078^{+0.020}_{-0.021}$	$(227, -15) \pm 19$	8.8	3.5σ
NILC	5	$0.069^{+0.020}_{-0.021}$	$(226, -16) \pm 22$	7.1	3.0σ
SEVEM	5	$0.066^{+0.021}_{-0.021}$	$(227, -16) \pm 24$	6.7	2.9σ
SMICA	5	$0.065^{+0.021}_{-0.021}$	$(226, -17) \pm 24$	6.6	2.9σ
WMAP5 ILC	4.5	0.072 ± 0.022	$(224, -22) \pm 24$	7.3	3.3σ
Commander	6	$0.076^{+0.024}_{-0.025}$	$(223, -16) \pm 25$	6.4	2.8σ
NILC	6	$0.062^{+0.025}_{-0.026}$	$(223, -19) \pm 38$	4.7	2.3σ
SEVEM	6	$0.060^{+0.025}_{-0.026}$	$(225, -19) \pm 40$	4.6	2.2σ
SMICA	6	$0.058^{+0.025}_{-0.027}$	$(223, -21) \pm 43$	4.2	2.1σ
Commander	7	$0.062^{+0.028}_{-0.030}$	$(223, -8) \pm 45$	4.0	2.0σ
NILC	7	$0.055^{+0.029}_{-0.030}$	$(225, -10) \pm 53$	3.4	1.7σ
SEVEM	7	$0.055^{+0.029}_{-0.030}$	$(226, -10) \pm 54$	3.3	1.7σ
SMICA	7	$0.048^{+0.029}_{-0.029}$	$(226, -11) \pm 58$	2.8	1.5σ
Commander	8	$0.043^{+0.032}_{-0.029}$	$(218, -15) \pm 62$	2.1	1.2σ
NILC	8	$0.049^{+0.032}_{-0.031}$	$(223, -16) \pm 59$	2.5	1.4σ
SEVEM	8	$0.050^{+0.032}_{-0.031}$	$(223, -15) \pm 60$	2.5	1.4σ
SMICA	8	$0.041^{+0.032}_{-0.029}$	$(225, -16) \pm 63$	2.0	1.1σ
Commander	9	$0.068^{+0.035}_{-0.037}$	$(210, -24) \pm 52$	3.3	1.7σ
NILC	9	$0.076^{+0.035}_{-0.037}$	$(216, -25) \pm 45$	3.9	1.9σ
SEVEM	9	$0.078^{+0.035}_{-0.037}$	$(215, -24) \pm 43$	4.0	2.0σ
SMICA	9	$0.070^{+0.035}_{-0.037}$	$(216, -25) \pm 50$	3.4	1.8σ
WMAP3 ILC	9	0.114	$(225, -27)$	6.1	2.8σ
Commander	10	$0.092^{+0.037}_{-0.040}$	$(215, -29) \pm 38$	4.5	2.2σ
NILC	10	$0.098^{+0.037}_{-0.039}$	$(217, -29) \pm 33$	5.0	2.3σ
SEVEM	10	$0.103^{+0.037}_{-0.039}$	$(217, -28) \pm 30$	5.4	2.5σ
SMICA	10	$0.094^{+0.037}_{-0.040}$	$(218, -29) \pm 37$	4.6	2.2σ

Gradient of a field over the horizon scale
 = **Super-curvature mode** in open inflation

MS, Tanaka & Yamamoto '95



may modulate the amplitude of
 perturbation depending on the direction.

leading order effect is dipolar

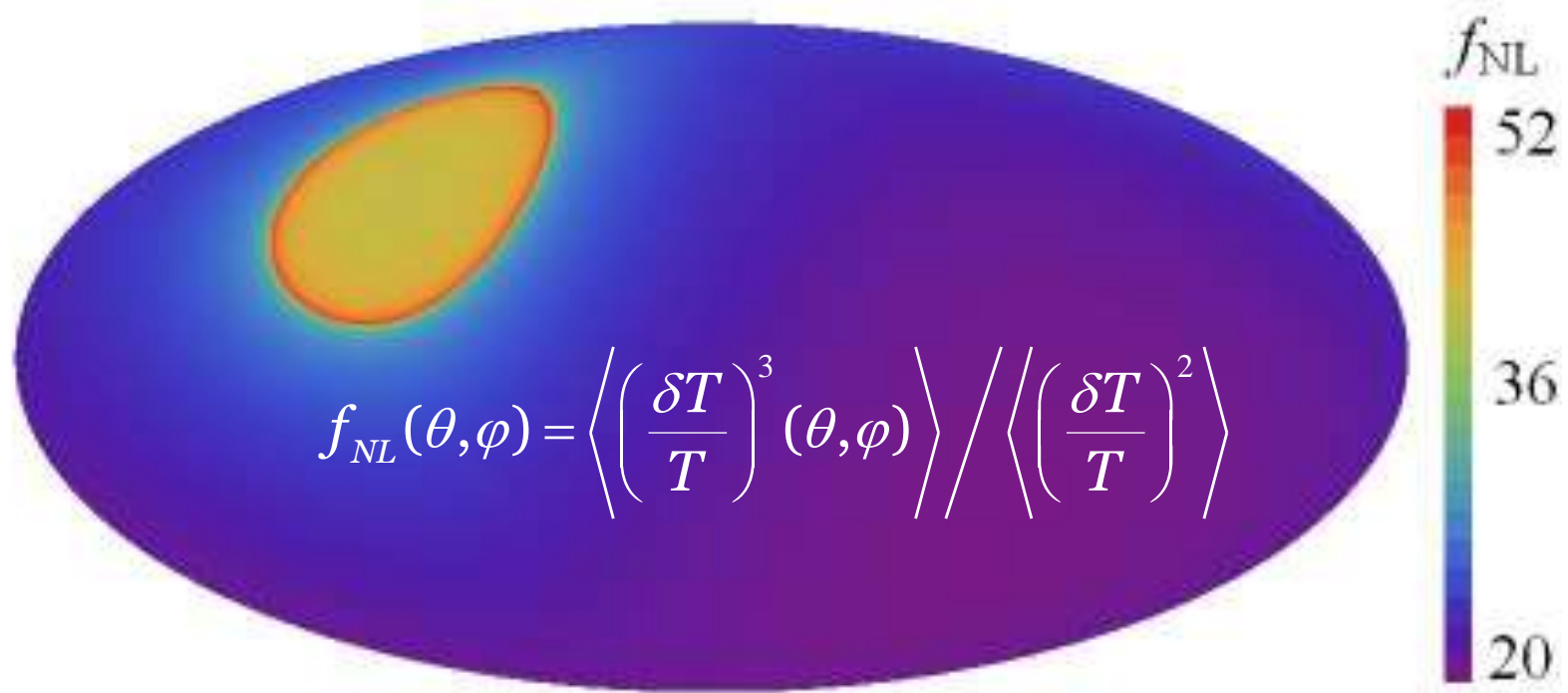
if this is the case, then $\Omega_K \gtrsim 10^{-3}$

Kanno, MS & Tanaka '13

Brynes, Domenech, MS & Takahashi '16

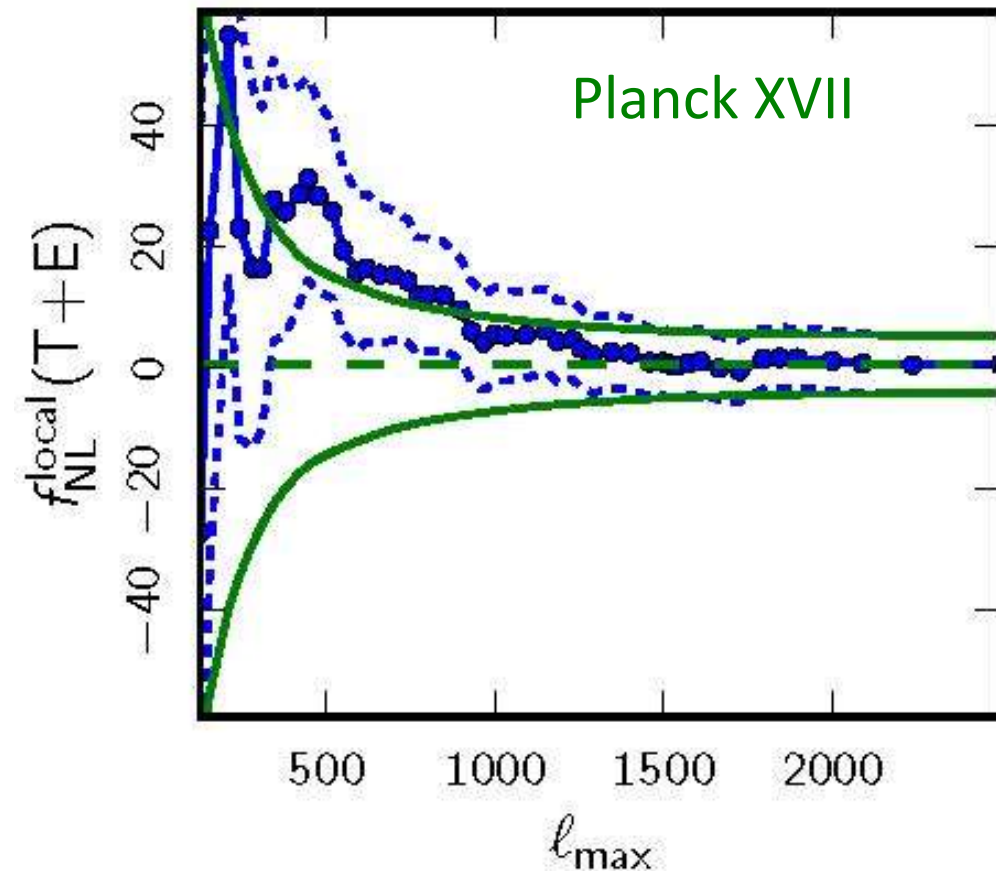
non-Gaussian bubbles in the sky?

Sugimura, Yamauchi & MS '12

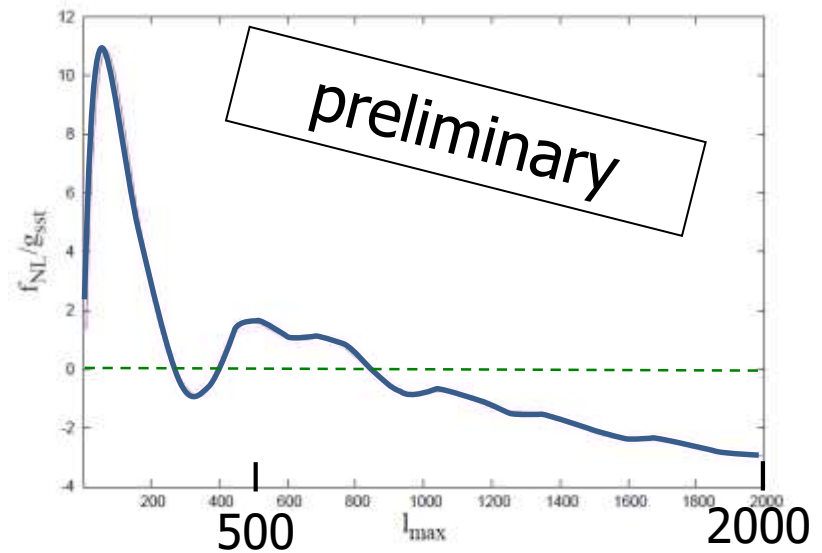


detection of a **spherically symmetric “localized” non-Gaussianity**
will be the first observational **signature of string theory!**

Scale-dependent non-Gaussianity?



- Slightly above 1- σ deviation from zero
- What if this is primordial?

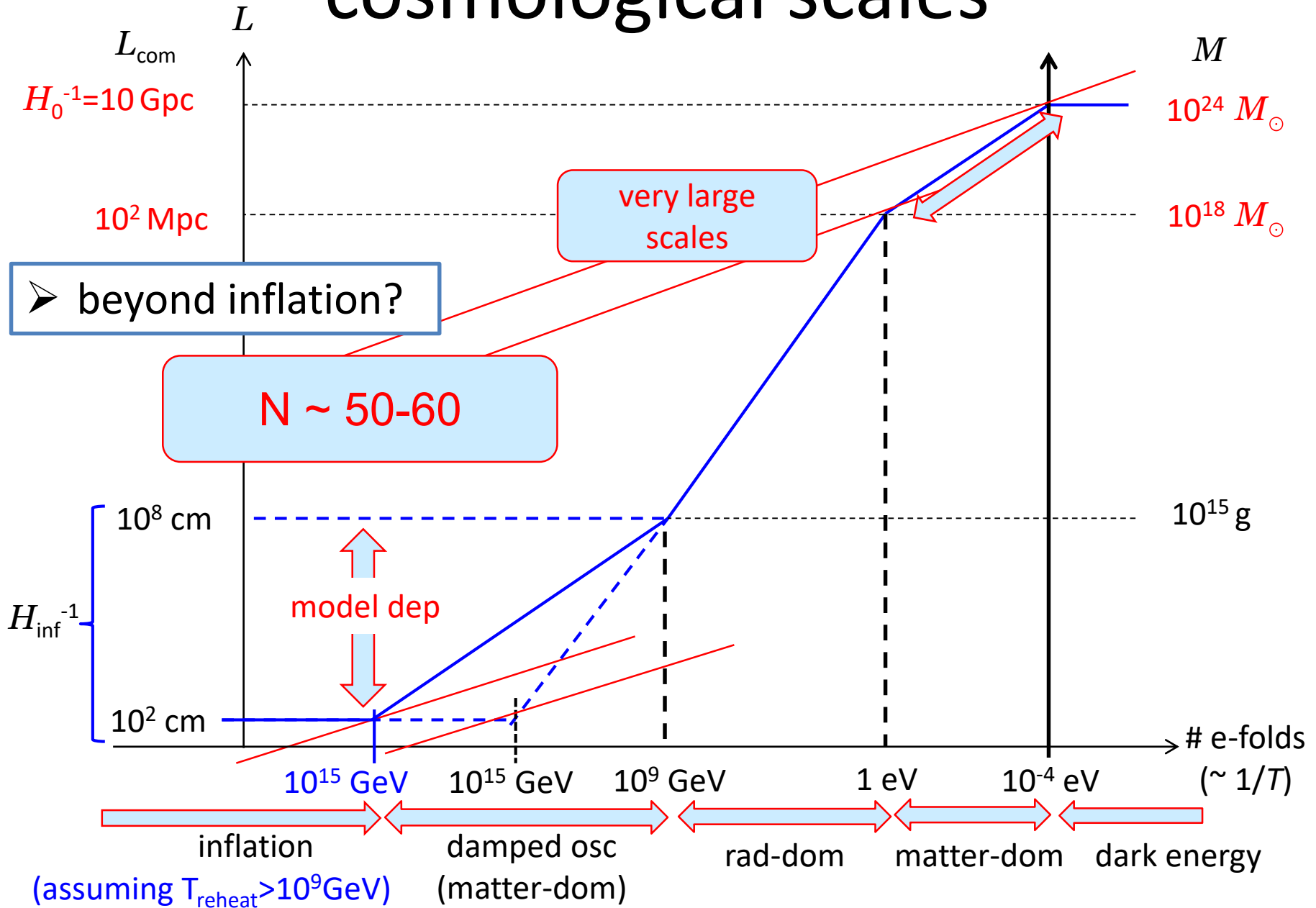


may be due to TSS coupling in a **massive tensor** theory

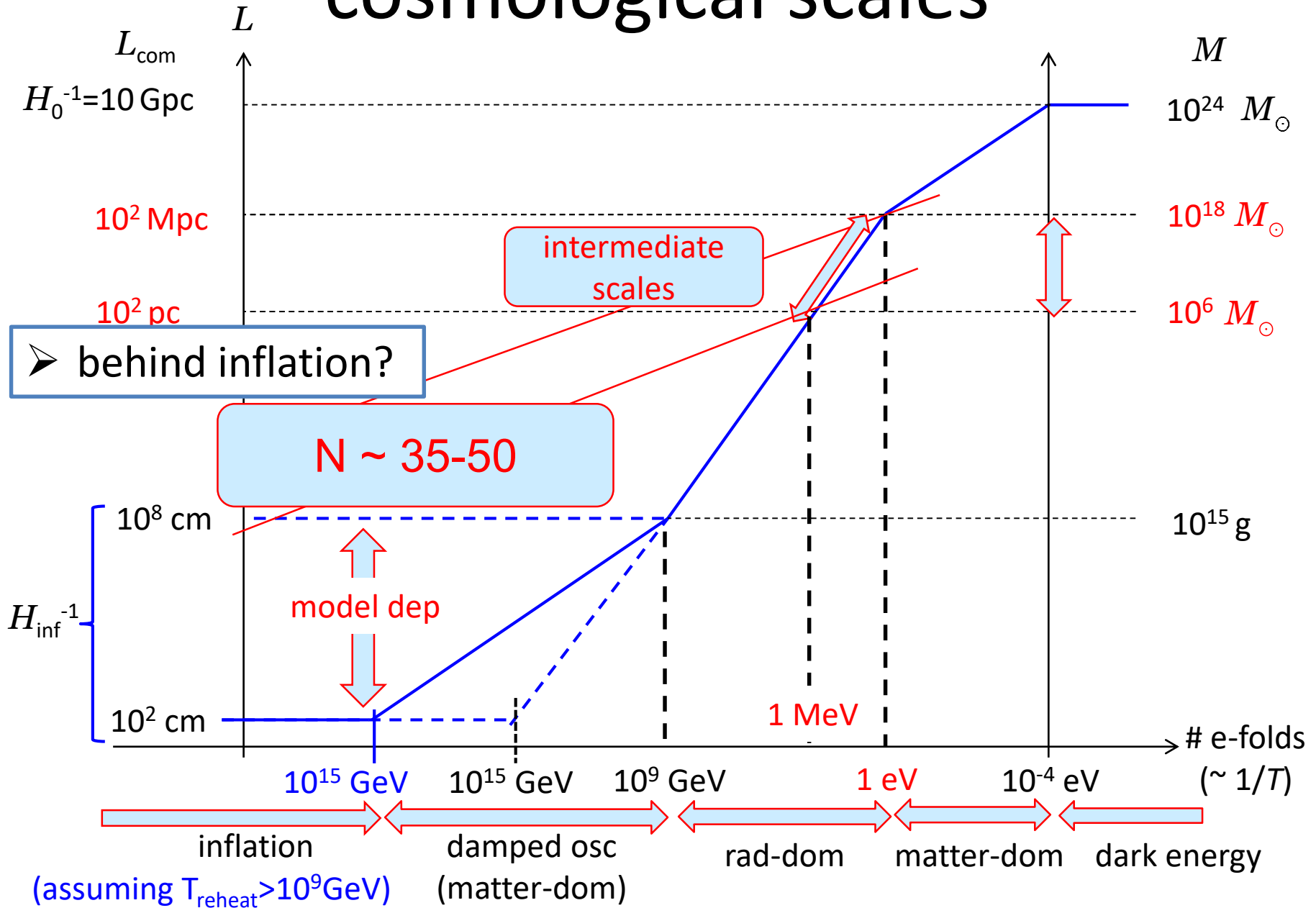
Domenech, Hiramatsu, Lin, MS, Shiraishi, Wang '16

Future issues

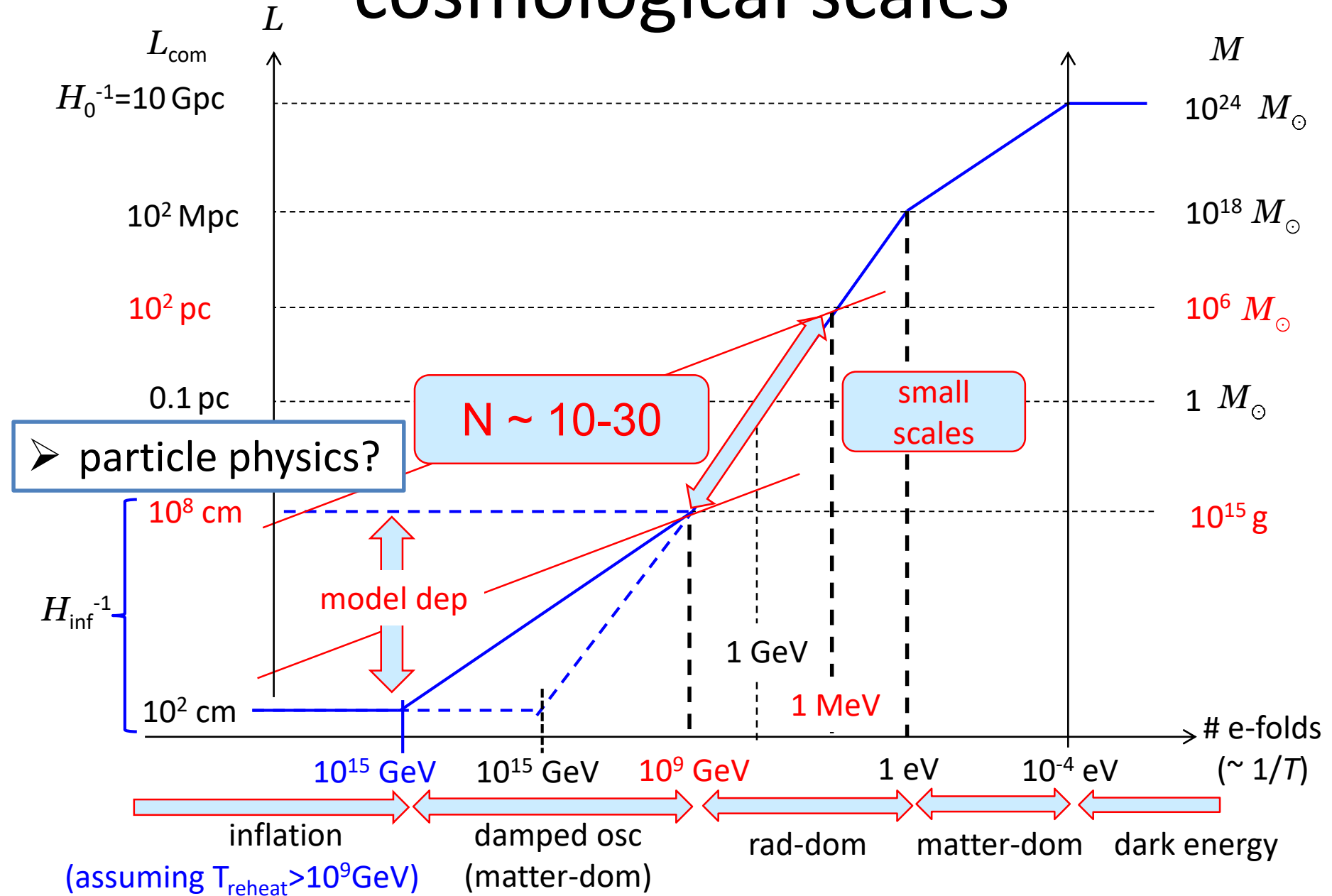
cosmological scales



cosmological scales



cosmological scales



- definition of inflation?

(conformal trans can realize any expansion law)

Domenech & MS '15

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$

$$d\tilde{s}^2 = \Omega^2(t)ds^2 \Rightarrow d\tilde{t} = \Omega(t)dt, \tilde{a}(\tilde{t}) = \Omega(t)a(t)$$

- initial condition before inflation, multiverse?
- successful reheating?
- non-linear effects, non-Gaussianities?
- gravitational waves at second order, PBHs?
- massive gravity?
-

Identification of Inflaton!

Inflation as the tool to explore Physics of the Early Universe

Era of

Observational/Experimental
Inflationary Cosmology!