SPLITTING & SHOWERING IN THE EW SECTOR Univ. of Pittsburgh, PITT PACC Tao Han

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In collaboration with J.M. Chen & B. Tweedie, arXiv:1611.00788

LHC ROCKS!



W/Z, Higgs & top @ FCC_{hh}/SPPC



Snowmass QCD Working Group: 1310.5189

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REALITY IN HADRONIC COLLISIONS



Collinear splitting is one of the dominant phenomena.



 $d\sigma_{X,BC} \simeq d\sigma_{X,A} \times d\mathcal{P}_{A \to B+C}$ $E_B \approx z E_A, \quad E_C \approx \bar{z} E_A, \quad k_T \approx z \bar{z} E_A \theta_{BC}$

 $\frac{d\mathcal{P}_{A\to B+C}}{dz \, dk_T^2} \simeq \frac{1}{16\pi^2} \frac{z\bar{z} \, |\mathcal{M}^{(\text{split})}|^2}{(k_T^2 + \bar{z}m_B^2 + zm_C^2 - z\bar{z}m_A^2)^2}$ On the dimensional ground: $|\mathcal{M}_{split}|^2 \sim k_T^2 \text{ or } m^2$ **SPLITTING FUNCTIONS: QED Most familiar example in QED:** $\mathbf{f} \rightarrow \mathbf{f} \gamma$ $p_{\gamma/f}(z) = \frac{1+\bar{z}}{z}, \quad \bar{z} = 1-z.$

The familiar Weizsäcker-Williams approximation

 $P_{\gamma/f}(z) = \frac{\alpha}{2\pi} \frac{1+\bar{z}}{z} \ln \frac{Q^2}{m_c^2} .$



 $egin{array}{rll} \sigma(fa o f'X) &pprox \int dx \; dp_T^2 \; P_{\gamma/f}(x,p_T^2) \; \sigma(\gamma a o X), \ P_{\gamma/e}(x,p_T^2) &= \; rac{lpha}{2\pi} rac{1+(1-x)^2}{x} (rac{1}{p_T^2})|_{m_e}^E. \end{array}$

Note the infrared & collinear behavior.

SPLITTING FUNCTIONS: QCD Most common in hadronic collisions: q, q $P_{gq}(z) = \frac{1 + \bar{z}^2}{2}, \quad P_{gg}(z) = \frac{(1 - z\bar{z})^2}{2\bar{z}}, \quad P_{qg}(z) = \frac{z^2 + \bar{z}^2}{2}.$ ISR, parton distribution & (DGLAP) evolution: $f_B(z,\mu^2) = \sum_{A} \int_z^1 \frac{d\xi}{\xi} f_A(\xi) \int_{m^2}^{\mu^2} d\mathcal{P}_{A\to B+C}(z/\xi,k_T^2).$ $f_{B}(z,\mu^{2}) = \sum_{A} \int_{z}^{1} \frac{d\xi}{\xi} f_{A}(\xi) \int_{m^{2}}^{\mu^{2}} d\mathcal{P}_{A \to B+C}(z/\xi,k_{T}^{2}).$ $\frac{\partial f_{B}(z,\mu^{2})}{\partial \mu^{2}} = \sum_{A} \int_{z}^{1} \frac{d\xi}{\xi} \frac{d\mathcal{P}_{A \to B+C}(z/\xi,\mu^{2})}{dz \, dk_{T}^{2}} f_{A}(\xi,\mu^{2}).$ New $\int_{m^{2}}^{10^{12}} \int_{m^{2}}^{10^{12}} \int_{m^{2}}^{10^{1$ FSR, parton showers: $\Delta_A(t) = \exp\left[-\sum_{P} \int_{t_0}^t \int dz P_{A \to BC}(z)\right],$ 10³ 10^{2} $p_{_{T}}$ [GeV] $f_A(x,t) = \Delta_A(t)f_A(x,t_0) + \int_{t_0}^t \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} P_{A \to BC}(z) f_A(x/z,t')$ Very important formulation for LHC physics!

EW SPLITTING FUNCTIONS Motivations:

- We have marched into the territory where $E >> M_W$ where EW symmetry can be restored.
- Conceptually different from QCD: Λ_{QCD} vs vev: EW sector remains perturbative.
- New degrees of freedom: the Higgs sector / Longitudinal vector bosons
- Clear understanding of the "Equivalence theorem".
- Most sensitive to new physics above the EW scale.

SPLITTING FUNCTIONS: EW Start from the unbroken phase – all massless. $\mathcal{L}_{SU(2) \times U(1)} = \mathcal{L}_{gauge} + \mathcal{L}_{\phi} + \mathcal{L}_{f} + \mathcal{L}_{Yuk}$ Chiral fermions: \mathbf{f}_{s} , gauge bosons: $\mathbf{B}, \mathbf{W}^{0}, \mathbf{W}^{\pm}$; $\mathbf{\&}^{H} = \begin{pmatrix} H^{+} \\ H^{0} \end{pmatrix} = \begin{pmatrix} \phi^{+} \\ \frac{1}{\sqrt{2}}(h - i\phi^{0}) \end{pmatrix}$ Fermion splitting:

he scalar part of the Lagrangian is

$$\mathcal{L}_{\phi} = (D^{\mu}\phi)^{\dagger} D_{\mu} \phi - (\phi) \qquad D_{\mu}\phi = \left(\overrightarrow{\partial_{\mu} \pm ig} \frac{\tau^{i}}{2} W_{\mu}^{i} + \frac{ig'}{2} B_{\mu} \right) \phi,$$

$$V(\phi) = +\mu^{2} \phi^{\dagger} \phi \pm \lambda(\phi^{\dagger} \phi)^{2} + \overline{z}^{2} \\ \phi = \frac{1}{\sqrt{2}} e^{i\sum \varepsilon^{i}L} \left(\begin{array}{c} \psi^{\dagger} \phi \right)^{2} + \overline{z}^{2} \\ W_{T} \neq s^{H} \end{array} \right) [BW]_{T}^{0} f_{s} \qquad \frac{1}{8\pi^{2} k_{T}^{2}} \left(\begin{array}{c} z \\ \overline{2} \end{array} \right) \\ H^{0(*)} \psi_{f,s} = \sigma \phi^{\pm} f'_{-s} \\ \phi = \frac{1}{\sqrt{2}} e^{i\sum \varepsilon^{i}L} \left(\begin{array}{c} \psi^{\dagger} \phi \right)^{2} + \overline{z}^{2} \\ W_{T} \neq s^{H} \end{array} \right) [BW]_{T}^{0} f_{s} \qquad H^{0(*)} \psi_{f,s} = \sigma \phi^{\pm} f'_{-s} \\ f_{s=L,R} \\ \phi \rightarrow \phi' = e^{-i\sum \varepsilon^{4}L} \phi = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ \psi + H \end{array} \right)^{2} \left(\begin{array}{c} \psi^{\dagger} \phi \right)^{2} \\ \psi = (-\mu^{2}/\lambda)^{1/2} \\ Infrared & collinear \\ \varepsilon^{\phi} = (D^{\mu}\phi)^{\dagger} D_{\mu}\phi - (\phi) \\ singularities \\ W^{\mu} W^{\mu} W_{\mu}^{-} \left(\begin{array}{c} 1 + \frac{1}{\psi} \end{array} \right)^{2} + \begin{array}{c} helicity \\ M^{2}Z^{\mu}Z_{\mu}} \\ M^{2}Z^{\mu}Z_{\mu} \\ \psi = (1 + \frac{1}{\psi}) \\ \psi = (1 + \frac{1}{\psi})^{2} \\ W^{\mu} W^{\mu} W_{\mu}^{-} \left(\begin{array}{c} 1 + \frac{1}{\psi} \end{array} \right)^{2} + \begin{array}{c} M^{2}Z^{\mu}Z_{\mu} \\ M^{2}Z^{\mu}Z_{\mu} \\ \psi = 0 \\ \psi = 0$$

SPLITTING FUNCTIONS: EW SM in the unbroken phase Gauge boson splitting:





Infrared & collinear singularities (a charge source, similar to P_{gq}) Collinear, similar to (P_{qg})

EW Symmetry breaking:

The scalar part of the Lagrangian is

The Higgs: $\mathcal{L}_{\phi} = (D^{\mu}\phi)^{\dagger}D_{\mu}\phi - V(\phi) \quad D_{\mu}\phi = \left(\partial_{\mu} + ig\frac{\tau^{i}}{2}W^{i}_{\mu} + \frac{ig'}{2}B_{\mu}\right)\phi,$

$$V(\phi) = +\mu^2 \phi^{\dagger} \phi + \lambda (\phi^{\dagger} \phi)^2.$$

$$\phi = \frac{1}{\sqrt{2}} e^{\sum \xi^i L^i} \begin{pmatrix} 0\\ \nu + H \end{pmatrix}}$$

$$-\nu$$

$$\psi \phi$$

Unitary gauge: $\phi \to \phi' = e^{-i\sum \xi^i L^i} \phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0\\ \nu + H \end{pmatrix}$ $\nu = \left(-\mu^2/\lambda\right)^{1/2}$

$$\begin{split} \mathcal{L}_{\phi} &= (D^{\mu}\phi)^{\dagger} D_{\mu}\phi - V(\phi) \\ &= M_{W}^{2} W^{\mu+} W_{\mu}^{-} \left(1 + \frac{H}{\nu}\right)^{2} + \frac{1}{2} M_{Z}^{2} Z^{\mu} Z_{\mu} \left(1 + \frac{H}{\nu}\right)^{2} \\ &+ \frac{1}{2} \left(\partial_{\mu} H\right)^{2} - V(\phi). \end{split}$$

Goldstone-boson Equivalence Theorem: Lee, Quigg, Thacker (1977); Chanowitz & Gailard (1984)

At high energies E>>Mw, the longitudinally polarized gauge bosons behave like the corresponding Goldstone bosons. (They remember their origin!)

"Scalarization" to implement the Goldstoneboson Equivalence Theorem (GET):

$$\epsilon(k)_L^{\mu} = \frac{E}{m_W}(\beta_W, \hat{k}) \approx \frac{k^{\mu}}{m_W}$$

[Excersise: Try the decay $H \rightarrow W_L^+ W_L^-$ via a coupling $-g^{\mu\nu}$ and derive the equivalent $H\phi^+\phi^-$ vertex.]

(a). Unitarity at higher energies:



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Appelquist & Chanowitz (1987).





(b). Puzzle of massless fermion radiation V_L contributions dominant at high energies: $\epsilon(k)_L^{\mu} = \frac{E}{m_W}(\beta_W, \hat{k}) \approx \frac{k^{\mu}}{m_W}$ Then, massless fermion splitting $f \rightarrow f V_L$ would be zero, in accordance with GET for $f \rightarrow f \phi \quad (v_f = 0).$

GET ignored the EWSB effects at the order M_W/E

Corrections to GET 1st example: "Effective W-Approximation" S. Dawson (1985); G. Kane et al. (1984); Chanowitz & Gailard (1984) At colliding energies $E >> M_W$, $P_{q \to qV_T} = (g_V^2 + g_A^2) \frac{\alpha_2}{2\pi} \frac{1 + (1 - x)^2}{x} \ln \frac{Q^2}{\Lambda^2}$ $P_{q \to qV_L} = (g_V^2 + g_A^2) \frac{\alpha_2}{\pi} \frac{1 - x}{x}$ • $f \rightarrow f W_{I}$, $f Z_{I}$ do not vanish & non-log! Vector boson fusion observed at the LHC

WW, $ZZ \rightarrow h \& W^+W^+$ scattering

There are characteristically new channels in the broken phase: **"Ultra collinear":** $k_T^2 > m_W^2$, it shuts off; $k_T^2 < m_W^2$, flattens out! The DPFs for W_L thus don't run at leading log: Bjorken scaling restored (higher-twist effects)!



"GOLDSTONE EQUIVALENCE GAUGE" (GEG)

 $\epsilon(k)_L^{\mu} = \frac{E}{m_W}(\beta_W, \hat{k}) = \frac{k^{\mu}}{m_W} - \frac{m_W}{E + |\vec{k}|}n^{\mu}, \quad n^{\mu} = (1, -\hat{k}).$

1st term leads to GET $\rightarrow \phi$, well behaved; 2nd term captures EWSB ~ A_{μ}^{μ} , well behaved

Separate them out by a special gauge choice:

$$\mathcal{L}_{\text{fix}} = -\frac{1}{2\xi} \Big[n(k) \cdot W(k) \Big] \Big[n(k) \cdot W(-k) \Big] \qquad (\xi \to 0)$$

$$n^{0}(k) \equiv 1, \quad \vec{n}(k) \equiv -\frac{k^{0}}{|k^{0}|} \frac{\vec{k}}{|\vec{k}|},$$

$$\epsilon^{\mu}_{\text{long}}(k) \to \frac{\sqrt{|k^{2}|}}{n(k) \cdot k} n^{\mu}(k) \stackrel{\text{on-shell}}{\to} \frac{m_{W}}{E + |\vec{k}|} \left(-1, \hat{k} \right).$$

GEG:

 Non-covariant but physical gauge: Hybrid of Coulomb & light-cone gauge \rightarrow rotational and collinear boost invariance. • ε_I: Time-like component is removed: $-\xi \rightarrow 0$ eliminates the V_L collinear propagation - Goldstones remain as dynamical degrees of freedom. • $\epsilon_{\rm L} \sim M_{\rm W}/E$ as the remainder vector field: Gauge – Goldstone boson mixing exists. • Measures the departure from GET, keeps track of EWSB effect: As the "Higher-twist" power-counting by $v^2/s! \rightarrow \text{direct analogues to QCD } \Lambda^2/Q^2!$



Helicity conserving: Non-zero for massless f

Helicity flipping: ~m_f

SPLITTING IN THE BROKEN GAUGE							
Vector b	oson V _L	is of IR. \sim	$\frac{v^2}{2} \frac{v^2}{\tilde{k}_T^4} \left(\frac{1}{z}\right)$	$\frac{118}{k_L^2} \frac{v^2}{k_T^2} \frac{dk_T^2}{k_T^2} \sim (1 - \frac{v^2}{Q^2})$			
	$\rightarrow W_L^{\pm} \gamma_T$	$W_L^{\pm} Z_T$	$Z_L W_T^{\pm}$	$W_L^+ W_T^-$ or $W_L^- W_T^+$			
W_T^{\pm}	$e^2 g_2^2 \bar{z}^3 = \frac{1}{4}$	$c_W^2 g_2^4 \bar{z} \left((1 + \bar{z}) + t_W^2 z \right)^2$	$\frac{1}{4}g_2^4 \bar{z}(1+\bar{z})^2$	0			
γ_T	0	0	0	$e^2 g_2^2 ar{z}$			
Z_T	0	0	0	$\frac{1}{4}c_W^2 g_2^4 \bar{z} \left((1+\bar{z}) - t_W^2 z \right)^2$			
$[\gamma Z]_T$	0	0	0	$\frac{1}{2}c_W eg_2^3 \bar{z} \left((1+\bar{z}) - t_W^2 z \right)$			
h&fha	ve no IR VT	$\begin{array}{c} & & & & & & \\ & & & & \\ \hline 1 & & \\ 1 & & \\ \hline 1 & & & \\ 1$	$ \frac{\frac{1}{16\pi^2} \frac{v^2}{\tilde{k}_T^4}}{\frac{f_s \bar{f}_s^{(\prime)}}{f_s \bar{f}_s^{(\prime)}}} $ $ \frac{\frac{1}{2} g_V^2 \left(Q_{f_s}^V y_{f^{(\prime)}} z + \frac{1}{2} g_V^2 y_{f^{(\prime)}} z + \frac{1}{2} g_$	$\overline{Q_{f-s}^{V}y_{f}\bar{z}}^{2}$			
	$[\gamma Z]_T$		$\frac{1}{2}eg_Z y_f^2 Q_f^{\prime} \left(Q_{f_s}^2 z \right)$	$+Q_{f-s}^2\bar{z}$			



SPLITTING PROBABILITIES: gauge couplings

Process	$\approx \mathcal{P}(E)$ (leading-log term)	$\mathcal{P}(1 \mathrm{TeV})$	$\mathcal{P}(10 \mathrm{TeV})$
$q \to V_T q^{(\prime)}$ (CL+IR)	$- (3 \times 10^{-3}) \left[\log \frac{E}{m_W} \right]^2$	1.6%	7%
$q \to V_L q^{(\prime)} (\text{UC+IR})$	$(2 \times 10^{-3}) \log \frac{\ddot{E}}{m_W}$	0.4%	1.1%
$t_R \to W_L^+ b_L (CL)$	$(8 \times 10^{-3}) \log \frac{E}{m_W}$	2.5%	4%
$t_R \to W_T^+ b_L (\text{UC})$	(6×10^{-3})	0.6%	0.6%
$V_T \to V_T V_T$ (CL+IR)	$- (0.015) \left[\log \frac{E}{m_W} \right]^2$	7%	34%
$V_T \to V_L V_T (\text{UC+IR})$	$(0.014) \log \frac{E}{m_W}$	2.7%	7%
$V_T \to f\bar{f}$ (CL)	$(0.02)\log\frac{E''}{m_W}$	5%	10%
$V_L \to V_T h (CL+IR)$	$- (2 \times 10^{-3}) \left[\log \frac{E}{m_W} \right]^2$	0.8%	4%
$V_L \rightarrow V_L h (\text{UC+IR})$	$\left(2 \times 10^{-3}\right) \log \frac{\dot{E}}{m_W}$	0.5%	1%

- Non-Abelian gauge splitting larger than fermion splitting!
- Collinear splittings larger than perturbative radiation!
- Ultra-collinear splittings small

EW SHOWER IMPLEMENTATION: SUDAKOV FORMALISM

- On-shell massive particle: Breit-Wigner resonance
- Sequential showering with "back-reactions":

 $A^* \to B^*C \to (DE)C$

 $\frac{d\mathcal{P}(B^* \to DE)}{dz_{DE} dk_{T,DE}^2} \times \left(\frac{d\mathcal{P}(A^* \to B^*C)/dz^* dk_T^{2*}}{d\mathcal{P}(A^* \to BC)/dz \, dk_T^2} \cdot \left|\det\left[\frac{dz^* dk_T^{2*}}{dz \, dk_T^2}\right]\right|\right)$

• Mixed state evolution:

In QED/QCD, it is sub-leading, after color/spin averaging. But in chiral EW theory, need "density matrix" treatment:

$$\rho_{ij} \propto \mathcal{M}_i^{(\mathrm{prod})*} \mathcal{M}_j^{(\mathrm{prod})}$$

$$d\mathcal{P} = \frac{\rho_{ij} \ d\mathcal{P}_{ji}}{\mathrm{tr}[\rho]} \qquad \left[\frac{d\mathcal{P}_{A\to B+C}}{dz \ dk_T^2}\right]_{ij} \simeq \frac{1}{16\pi^2} \ \frac{1}{z\overline{z}} \ \mathcal{M}_k^{(\mathrm{split})*} \mathcal{D}_{ki}^* \mathcal{D}_{jl} \mathcal{M}_l^{(\mathrm{split})}$$

MULTI GAUGE-BOSON PRODUCTION



Each W costs you a factor of ~ 1/100 (EW coupling) Logarithmic enhanced production: each W costs ~ 1/10



COHERENCE IN SHOWERING



Coherent treatment important (almost) a pure W_3^0 exchange

(almost) a pure B⁰ exchange

HIGGS IN SHOWERING



Ultra-collinear behavior

NEW PHYSICS WITH ENERGETIC MULTI TOPS/GAUGE-BOSONS

SUSY examples: $\tilde{b}\tilde{b}^* \rightarrow t\chi^- \bar{t}\chi^+$, $\tilde{t}W^- \tilde{t}^*W^+ \rightarrow 4W^{\pm} b\bar{b}$. Heavy quark examples: *TT*', *BB*', ...

Energetic W^{\pm} , Z, H, t as new radiation sources from heavy W', Z' decays, subsequent showering.

An Example: W^+ ' Shower $W_L^{+'} \to t\bar{b}, t\bar{t}(W^-), b\bar{b}(W^+), b\bar{t}(W^+W^+).$



With W/Z showers, all t/b iso-spin components exist.





Lumi(W⁺_TW⁻_T) similar size to lumi(tt) @ 10 TeV; Lumi(W⁺_TW⁻_T) ~ Lumi(W[±] γ), Electro=weak Lumi(W⁺_LW⁻_L) 100 times smaller: "untra-collinear" Lumi(100/14) increased by 30 – 3000 for 500 GeV - 4 TeV!

CONCLUSIONS (D/QED

- With the discovery of the Higgs boson, we have a consistent QM, relativistic theory up to high scales, where is $\Lambda \sim 4 \pi v$?
- EW sector presents rich physics:
- Perturbative cutoff via SSB
- Longitudinals/scalars
- Chirality
- Yukawa showers
- Neutral boson interference
- Weak isospin self-averaging
- EW splitting/showering will become an increasingly important part at higher energies.
- It may serve as a tool for future discovery.