Zero-Range Effective Field Theory for Resonant Wimp Dark Matter

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support from US Department of Energy

Zero-Range Effective Field Theory for Resonant Wino Dark Matter

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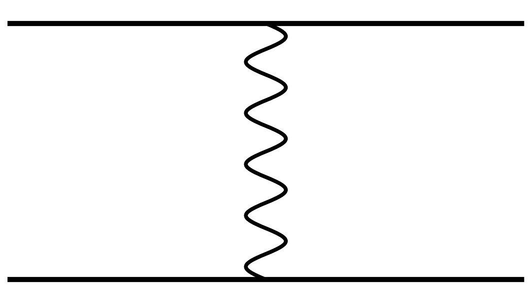
arXiv:1706.02253 I. Framework
arXiv:1708.07155 II. Coulomb resummation
arXiv:1712.nnnnn III. Annihilation effects

WIMP: Weakly Interacting Massive Particle

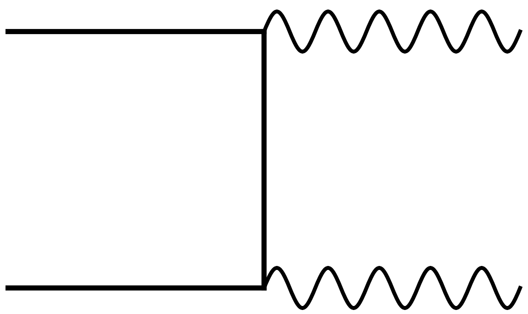
- massive particle: $M \gtrsim m_W \approx 100$ GeV
- weakly interacting: $\alpha_2 = 1/29.5$

wimp-wimp scattering



$\sigma_{el} \sim \alpha_2^2/m_W^2$

wimp-pair annihilation



$v\sigma_{ann} \sim \alpha_2^2/M^2$

neutral wimp: candidate for dark matter particle

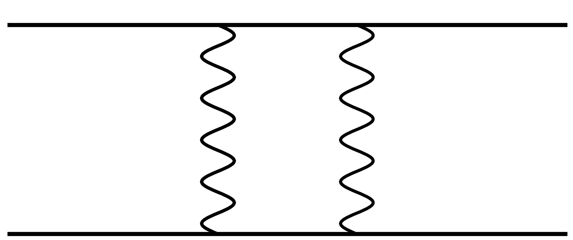
thermal relic abundance $\sim$ observed dark matter density

Wimps can have NonPerturbative Weak Interactions

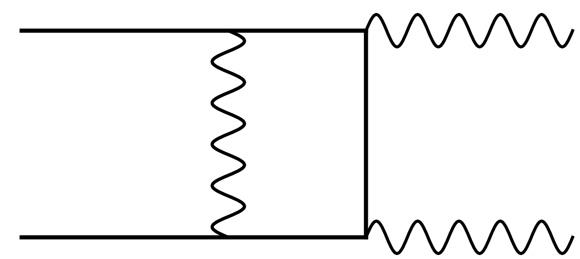
Hisano, Matsumoto, Nojiri 2002

loop diagrams can produce factors of $\alpha_2 M/m_W$

wimp-wimp scattering

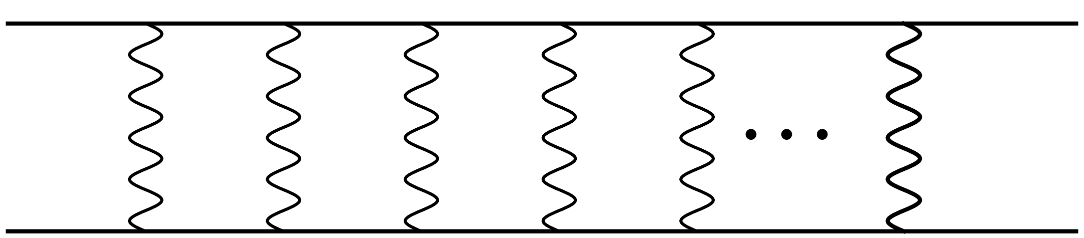


wimp-pair annihilation

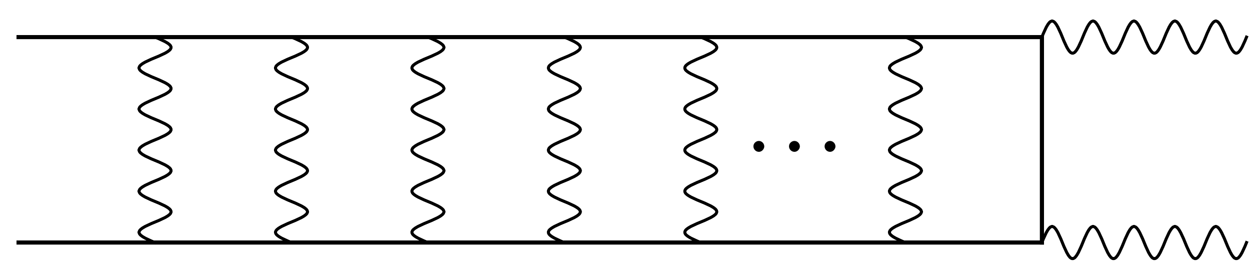


If $M \gtrsim m_W/\alpha_2 \approx$ few TeV,

ladder diagrams must be summed to all orders!



$\sigma_{\text{el}} \sim \alpha_2^2/m_W^2$



$v\sigma_{\text{ann}} \sim \alpha_2^4/m_W^2$

“Sommerfeld enhancement”

NonRelativistic Effective Field Theory

Hisano, Matsumoto, Nojiri 2002

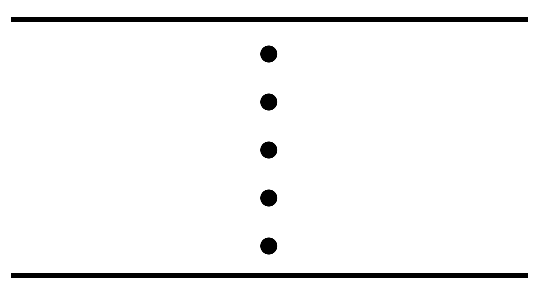
NREFT for wimps with momentum ≪ M

(velocity ≪ c)

weak interactions of wimps:

instantaneous nonlocal potential

(from exchange of $W^{\pm}$, Z^{0})



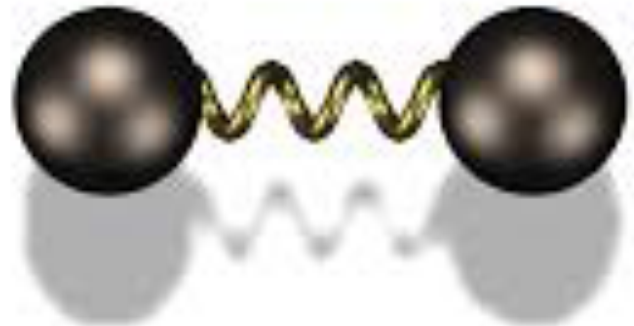
numerical results for 2-wimp problem

from solving Schroedinger equation

- wimp-wimp cross sections
- wimp-pair bound states
- wimp-pair annihilation rates ("Sommerfeld enhancement")

Wimps can have Resonant Weak Interactions

If M is large enough,
attractive weak interactions can produce
wimp-pair bound states!



Near critical values of M for bound state at threshold,
σ_{el} and $v\sigma_{ann}$ can be enhanced by orders of magnitude!

most dramatic enhancements: S-wave channel
if S-wave resonance is exactly at threshold
elastic cross section saturates unitarity bound
for velocity in range $\alpha_2^2 \ll v \ll \alpha_2$

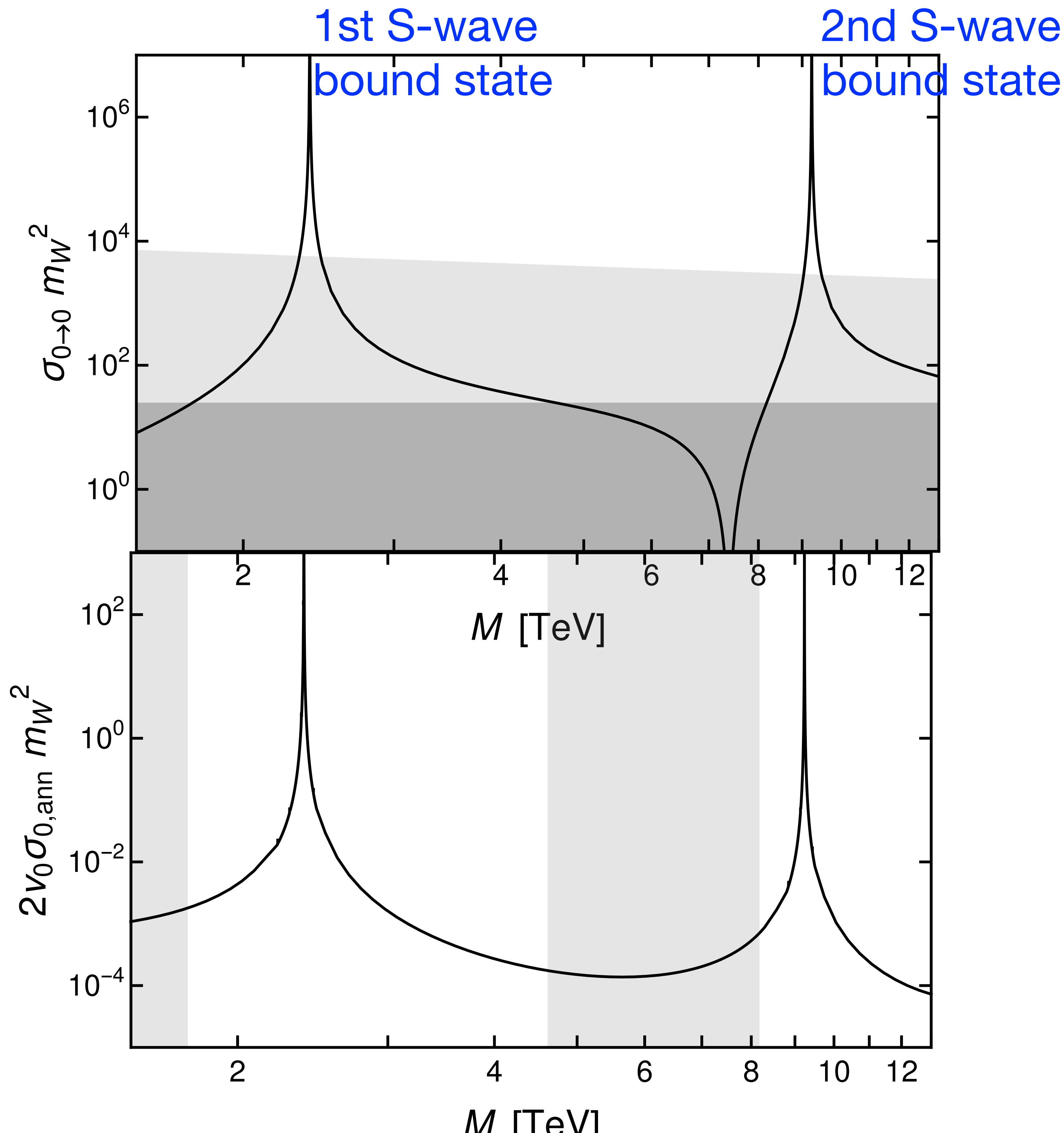
$\sigma_{el} \sim 1/M^2v^2$ $\quad\quad$ $v\sigma_{ann} \sim \alpha_2^2/M^2v^2$

enhancement factors as large as $1/\alpha_2^4 \approx 10^6$!

Resonant Reaction Rates of Wimps

wimp-wimp
elastic cross section

wimp-pair
annihilation rate



Zero-Range Effective Field Theory

Braaten, Johnson, and Zhang

arXiv:1706.02253, 1708.07155, 1712.nnnnn

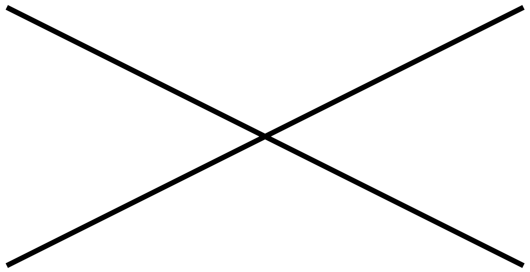
ZREFT

for wimps with S-wave resonance near threshold

(|scattering length| > $1/m_W$)

with momentum < m_W (velocity ≲ 10^{-3} c)

weak interactions of wimps: local



analytic results for 2-wimp problem

- wimp-wimp cross sections
- wimp-pair bound states
- wimp-pair annihilation rates (Sommerfeld enhancement)

Zero-Range Effective Field Theory

ZREFT

for wimps with S-wave resonance near threshold
with momentum < m_W

analytic results for 2-wimp problem

- wimp-wimp cross sections
- wimp-pair bound states
- wimp-pair annihilation rates (Sommerfeld enhancement)

simplifies numerical solution of 3-wimp problem

- 3-body recombination:
  wimp + wimp + wimp $\longrightarrow$ (bound state) + wimp

predictive model for Strongly Interacting Dark Matter
with dark matter mass ≈ 15 GeV

Zero-Range Effective Field Theory for Resonant Wimp Dark Matter

- Field Theories for nonrelativistic wimps
- Systematically improvable EFTs
- Zero-Range Effective Field Theory
  - I. Framework
  - II. Coulomb resummation
  - III. Annihilation effects
- ZREFT at LO
- Conclusion

Wino Dark Matter

MSSM in corner of parameter space where
neutral wino is LSP
only nearby SUSY partners are charged winos

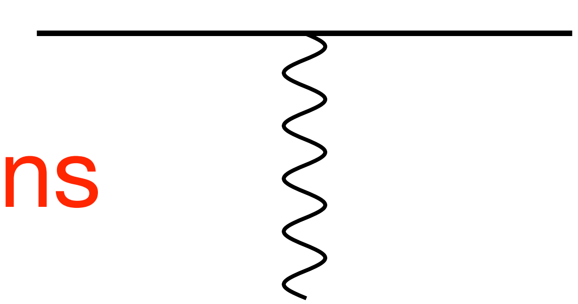
OR

Standard Model with additional *SU(2)* multiplet $\begin{pmatrix} w^+ \\ w^0 \\ w^- \end{pmatrix}$

neutral-wino mass: M ~ few TeV (adjustable)

charged-wino masses: $M+\delta$, δ = 170 MeV — electroweak radiative correction

wino interactions:
local couplings to electroweak gauge bosons
γ $W^{\pm}$ Z^0



Wino Momentum Scales

range over more than 5 orders of magnitude!

Fundamental FT

wino mass $M \sim$ 2 TeV

---

weak boson masses $m_W \sim$ 80 GeV NREFT

nonperturbative weak scale: $\alpha_2 M \sim$ 80 GeV

Bohr momentum: $\alpha M \sim$ 20 GeV ZREFT

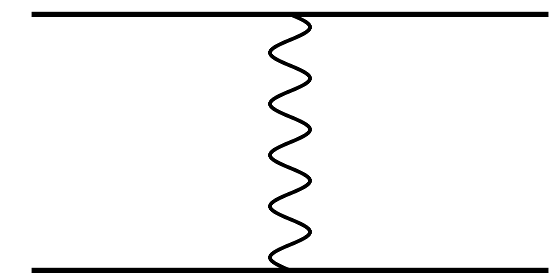
$w^0 w^0$-to-$w^+ w^-$ transition: $(2M\delta)^{1/2} \sim$ 30 GeV

inverse scattering length: $|\gamma_0| > \alpha_2^2 m_W \sim$ 100 MeV

Wino Weak Interactions

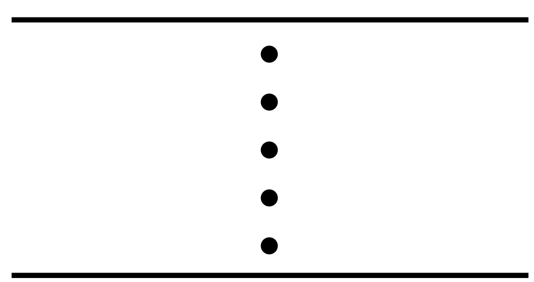
Fundamental Field Theory

local couplings to $W^{\pm}$, Z^{0}



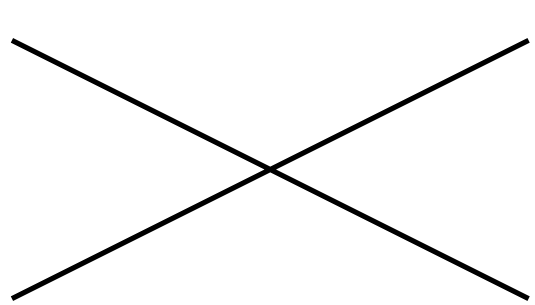
NREFT

instantaneous nonlocal potential
(from exchange of $W^{\pm}$, Z^{0})



ZREFT

local interactions between winos



NonRelativistic EFT for Winos

Hisano, Matsumoto, Nojiri 2002

		kinetic mass	rest energy
neutral wino	w^0	M	0
charged winos	$w^\pm$	M	δ = 170 MeV

coupled S-wave scattering channels

0: $w^0 w^0$

1: $w^+ w^-$

2×2 potential matrix in spin-singlet channel

from exchange of γ, $W^\pm$, Z^0

$$V(r) = -\frac{\alpha}{r}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} - \frac{\alpha_2}{r}\begin{pmatrix} 0 & \sqrt{2}\, e^{-m_W r} \\ \sqrt{2}\, e^{-m_W r} & c_w^2\, e^{-m_Z r} \end{pmatrix}$$

long-range Coulomb potential

weak potential with range $1/m_W$

solve Schrodinger equation <u>numerically</u>

wimp-wimp cross sections

wimp-pair bound state

NonRelativistic EFT for Winos

Hisano, Matsumoto, Nojiri 2002

coupled S-wave scattering channels: $w^0 w^0$, $w^+ w^-$

2×2 potential matrix in spin-singlet channel

from exchange of $V = \gamma, W^\pm, Z^0$

and annihilation into VV

$$V(r) = -\frac{\alpha}{r}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} - \frac{\alpha_2}{r}\begin{pmatrix} 0 & \sqrt{2}\,e^{-m_W r} \\ \sqrt{2}\,e^{-m_W r} & c_w^2\,e^{-m_Z r} \end{pmatrix}$$

$$-i\,\frac{\pi\alpha_2^2}{2M^2}\begin{pmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 3 \end{pmatrix}\delta^3(\vec{r})$$

imaginary zero-range annihilation potential

solve Schrodinger equation numerically

wimp-wimp cross sections

wimp-pair bound state

wimp-pair annihilation rates

Systematically Improvable EFT

- RG fixed point: scale invariant!
- scaling perturbations:

  operators with increasing scaling dimensions

EFT at LO

include all relevant and marginal operators

Systematic Improvements:

add irrelevant operators with …

NLO: … lowest scaling dimension

NNLO: … next lowest scaling dimension

⋮

RG fixed point is usually noninteracting fixed point

but it can be a nontrivial interacting fixed point

Systematically Improvable EFT

for nonrelativistic particles with one S-wave scattering channel

particle w: $E = p^2/2M$ scale invariant!

S-wave scattering channel: ww

T-matrix for total energy $E = k^2/M$:

$$T(E) = \frac{4\pi/M}{k\cot\delta(k) - ik}$$

short-range interaction $\Longrightarrow$ low-energy expansion

$T(E) = -1/a + r\, k^2/2 + \ldots$

a = scattering length

r = effective range

noninteracting fixed point

$T(E) = 0 \Longrightarrow$ no scattering

unitarity fixed point

$$T(E) = \frac{4\pi i/M}{k} \Longrightarrow$$

scattering saturates

S-wave unitarity bound:

$$\sigma(E) = \frac{4\pi/M}{E}$$

Systematically Improvable EFT

for nonrelativistic particles with two S-wave scattering channels

particles w^0: $E = p^2/2M$

$w^\pm$: $E = \delta + p^2/2M$

scale invariance

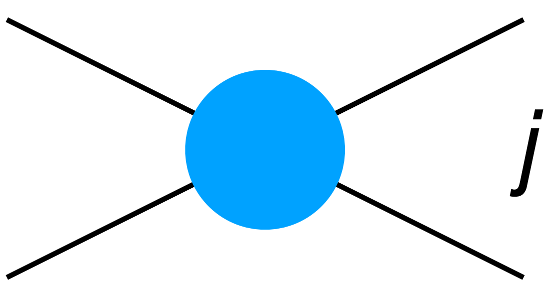
$\Longrightarrow \quad \delta = 0$

coupled S-wave scattering channels

0: $w^0 w^0$

1: $w^+ w^-$

T-matrix elements for total energy E:

$T_{ij}(E) \; = \; i$  j

RG fixed points?

Systematically Improvable EFT

for nonrelativistic particles with two S-wave scattering channels

RG fixed points

$\delta = 0$: no splitting between w^0 and $w^\pm$

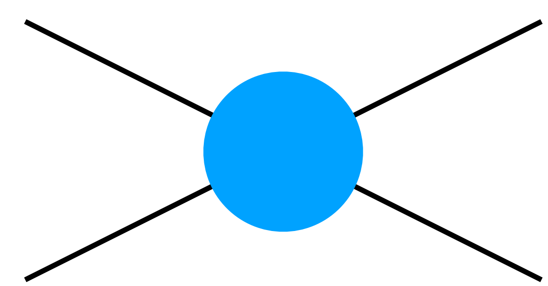
$\alpha = 0$: no electromagnetism

noninteracting fixed point

$$T(E) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \implies \text{no scattering}$$

2-channel unitarity fixed point

$$T(E) = \frac{4\pi i/M}{k} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



$\implies$ scattering saturates unitarity bound in both channels

Systematically Improvable EFT

for nonrelativistic particles with two S-wave scattering channels

noninteracting fixed point

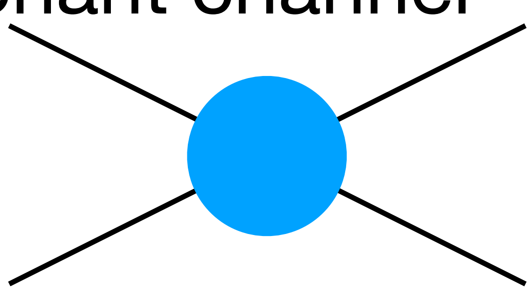
2-channel unitarity fixed point

single-channel unitarity fixed point

Lensky and Birse 2011 (coupled channels: ^{7}Li p, ^{7}Be n)

$$T(E) = \frac{4\pi i/M}{k} \begin{pmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{pmatrix}$$

$\Longrightarrow$ scattering saturates unitarity bound in resonant channel



$\cos\phi\, |w^0 w^0\rangle + \sin\phi\, | w^+ w^-\rangle$

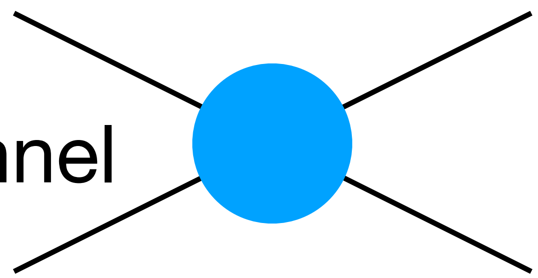
no scattering in orthogonal channel

$-\sin\phi\, |w^0 w^0\rangle + \cos\phi\, | w^+ w^-\rangle$

Systematically Improvable EFT

for nonrelativistic particles with two S-wave scattering channels

single-channel unitarity fixed point



scattering saturates unitarity bound in resonant channel
no scattering in orthogonal channel

scaling perturbations   Lensky & Birse 2011

relevant: mass splitting δ
scattering length a_u in resonance channel
marginal: mixing angle ϕ
irrelevant: scattering length a_v in orthogonal channel
effective range r_u in resonance channel
...

EFT at LO: kinematic parameters: M, δ
interaction parameters: a_u, ϕ

EFT at NLO: 2 additional interaction parameters a_v, r_u

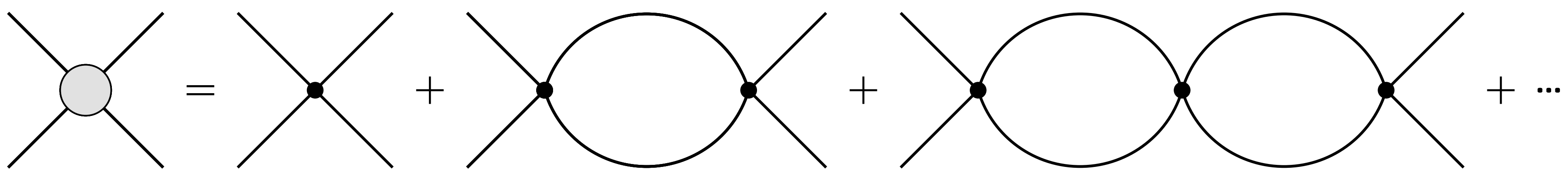
Zero-Range Effective Field Theory for Resonant Wino Dark Matter

I. Framework

arXiv:1706.02253

turn off electromagnetism: $\alpha = 0$

sum bubble diagrams



by solving Lippmann-Schwinger equations

ZREFT at LO

2 interaction parameters: a_u, ϕ

good predictions for wino-wino cross sections

except $w^+ w^- \longrightarrow w^+ w^-$

ZREFT at NLO

2 additional interaction parameters

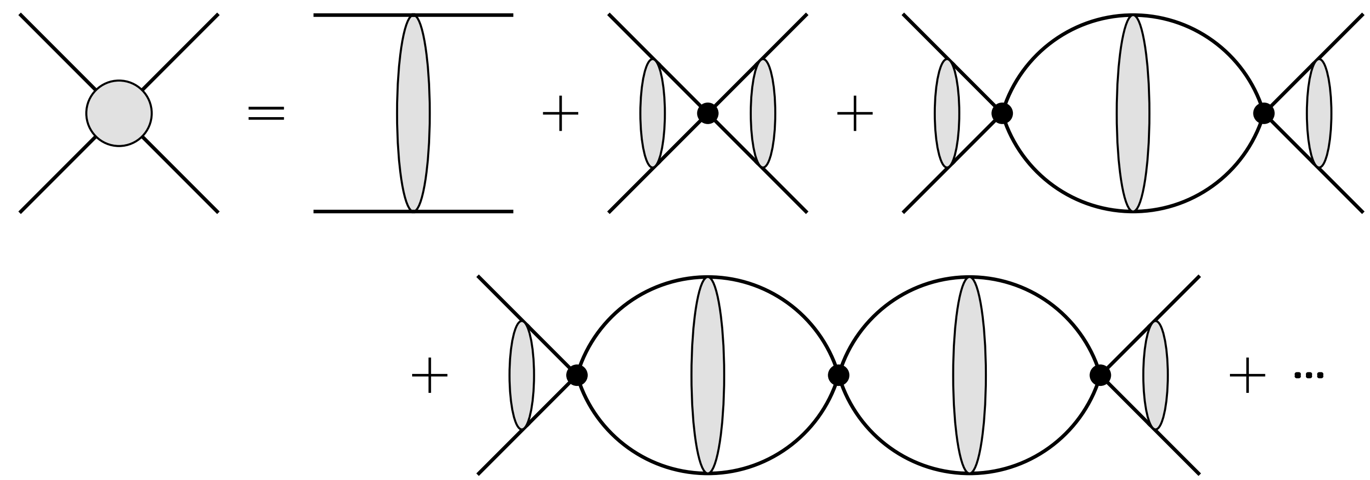
improved predictions for all wino-wino cross sections

Zero-Range Effective Field Theory for Resonant Wino Dark Matter

II. Coulomb resummation   arXiv:1708.07155

turn on electromagnetism: α =1/137

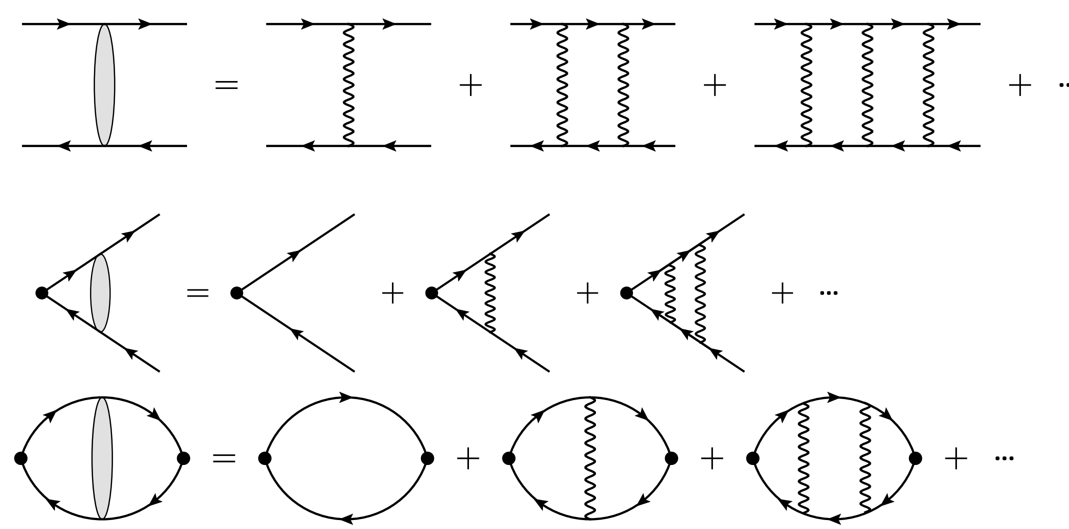
sum bubble diagrams



sum ladder diagrams with photons

Kong and Ravndal   1999

(EFT for p p-bar scattering)



ZREFT at LO

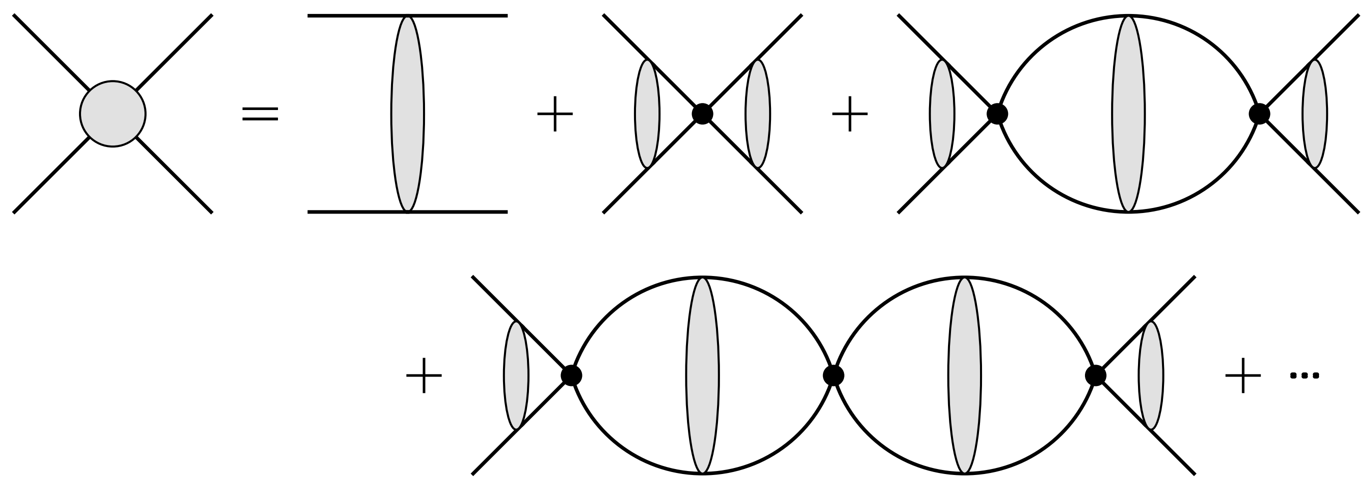
α=1/137 plus 2 real interaction parameters: a_u, ϕ

good predictions for wino-wino cross sections

Zero-Range Effective Field Theory for Resonant Wino Dark Matter

III. Annihilation effects arXiv:1712.nnnnn

electromagnetism: $\alpha = 1/137$
turn on annihilation into gauge boson pairs
(by analytically continuing real parameters to complex values)



ZREFT at LO

α=1/137 and 2 complex interaction parameters: a_U, ϕ
good predictions for wino-wino cross sections
wino-pair annihilation rates

Zero-Range Effective Field Theory for Resonant Winos

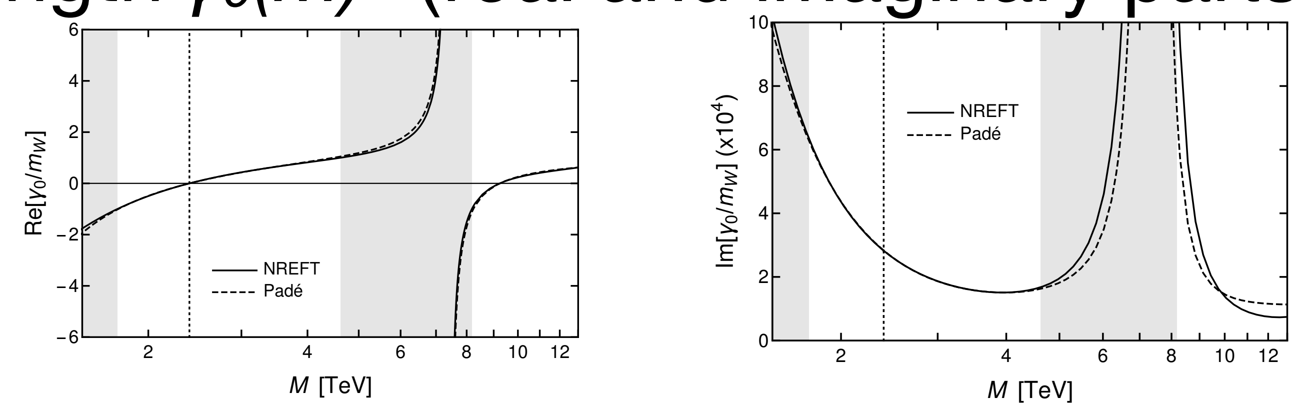
ZREFT at LO

interaction parameters: α = 1/137, a_u (complex), ϕ (complex)

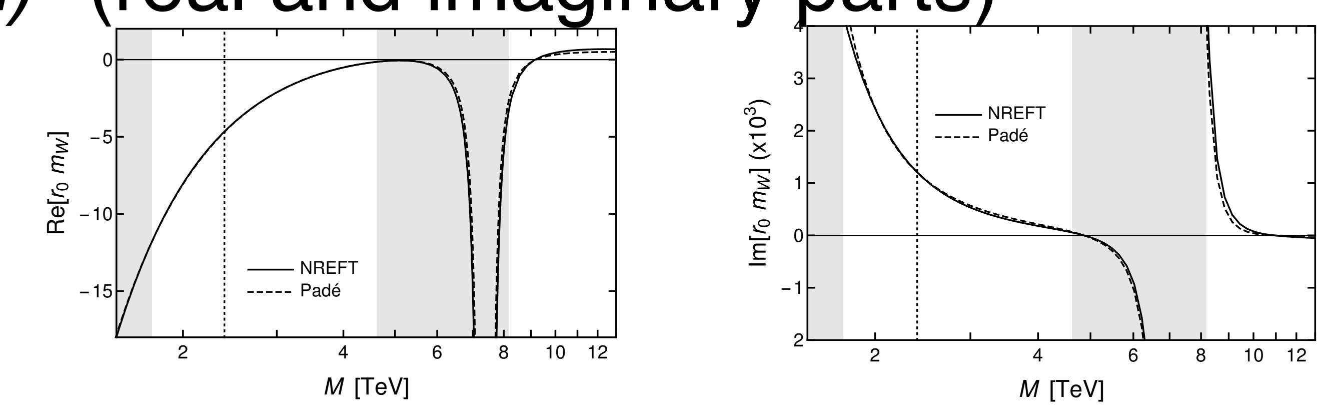
Matching with NREFT

determine a_u and ϕ as functions of M with δ = 170 MeV

w^0 inverse scattering length $\gamma_0(M)$ (real and imaginary parts)



w^0 effective range $r_0(M)$ (real and imaginary parts)



1st unitarity mass: $M_* = 2.39$ TeV

$\gamma_0(M_*) = 0 + (3.4 \times 10^{-4})\, i\, m_W$

$\tan \phi(M_*) = 0.88 - (1.4 \times 10^{-4})\, i \quad \phi(M_*) \approx 40^o$

Zero-Range Effective Field Theory for Resonant Winos

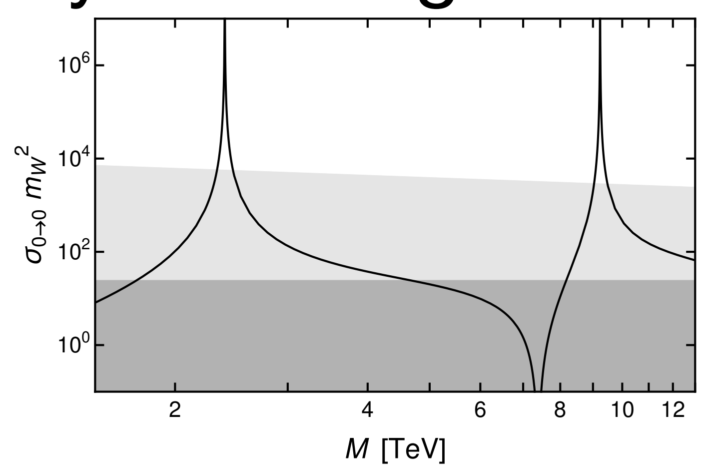
analytic predictions of ZREFT at LO

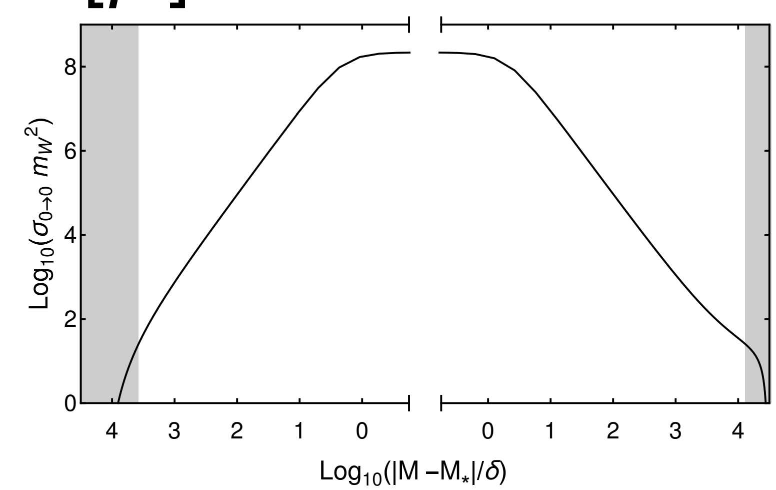
matching conditions: w^0 inverse scattering length $\gamma_0(M)$

w^0 effective range $r_0(M)$

w^0 elastic cross section at 0 energy as function of M

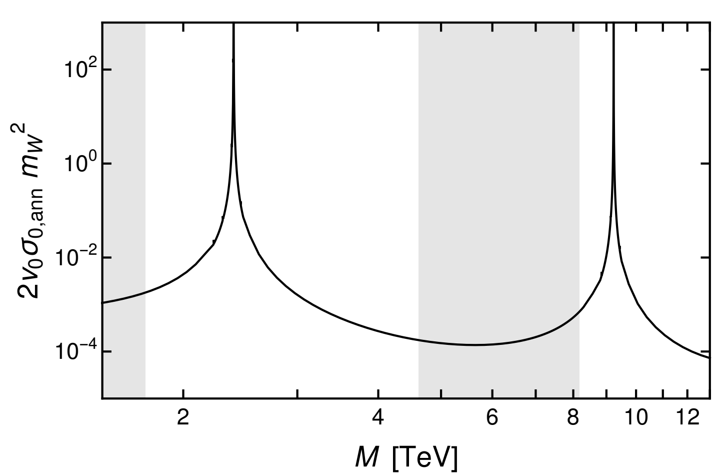
exact by matching condition for Re[γ_0]

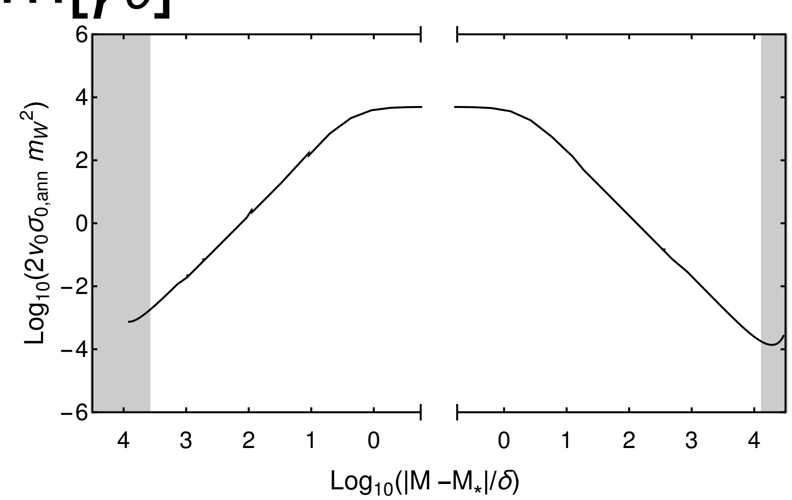




$w^0 w^0$ annihilation rate at 0 energy as function of M

exact by matching condition for Im[γ_0]





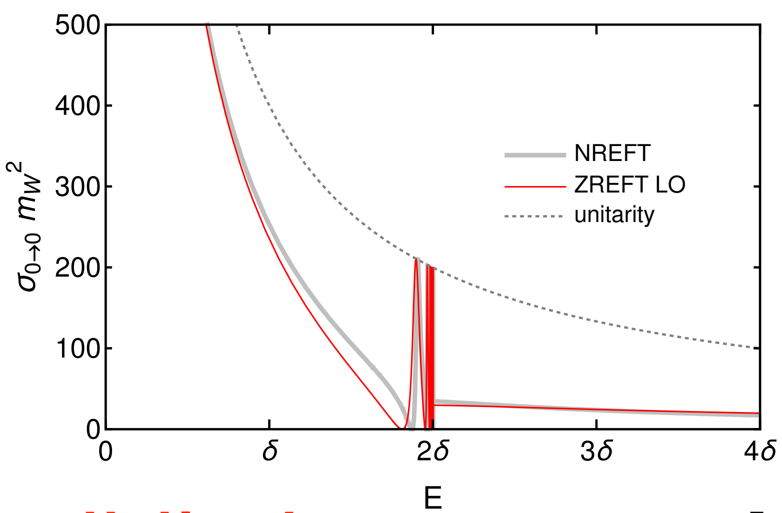
Zero-Range Effective Field Theory for Resonant Winos

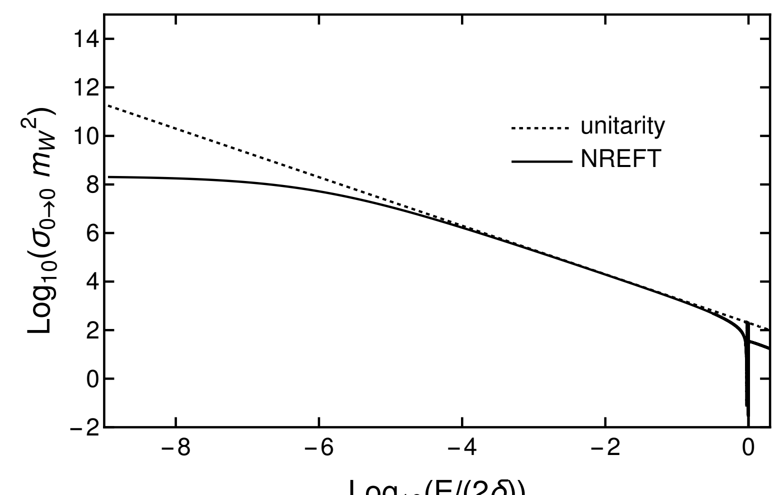
predictions of ZREFT at LO

matching conditions: w^0 inverse scattering length $\gamma_0(M)$

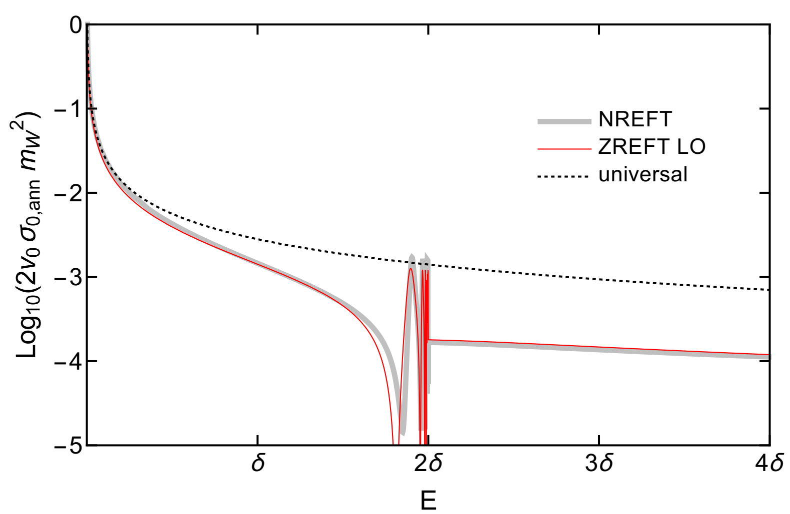
w^0 effective range $r_0(M)$

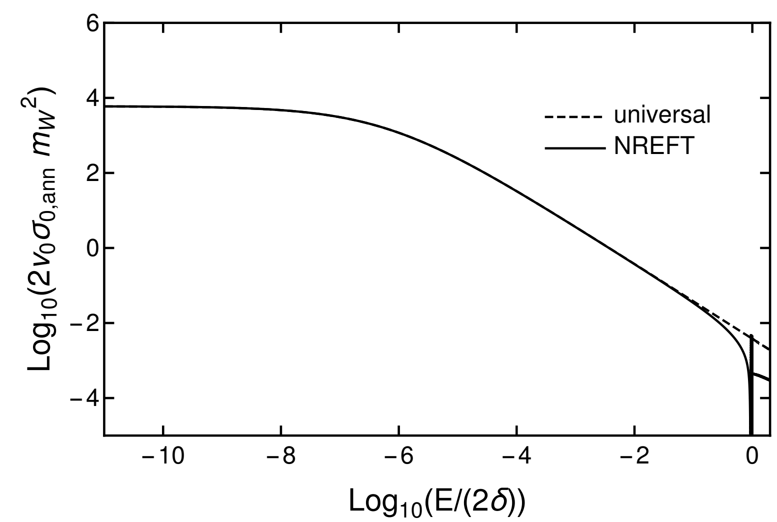
w^0 elastic cross section at $M_* = 2.39$ TeV as function of E





w^0w^0 annihilation rate at $M_* = 2.39$ TeV as function of E





Zero-Range Effective Field Theory for Resonant Winos

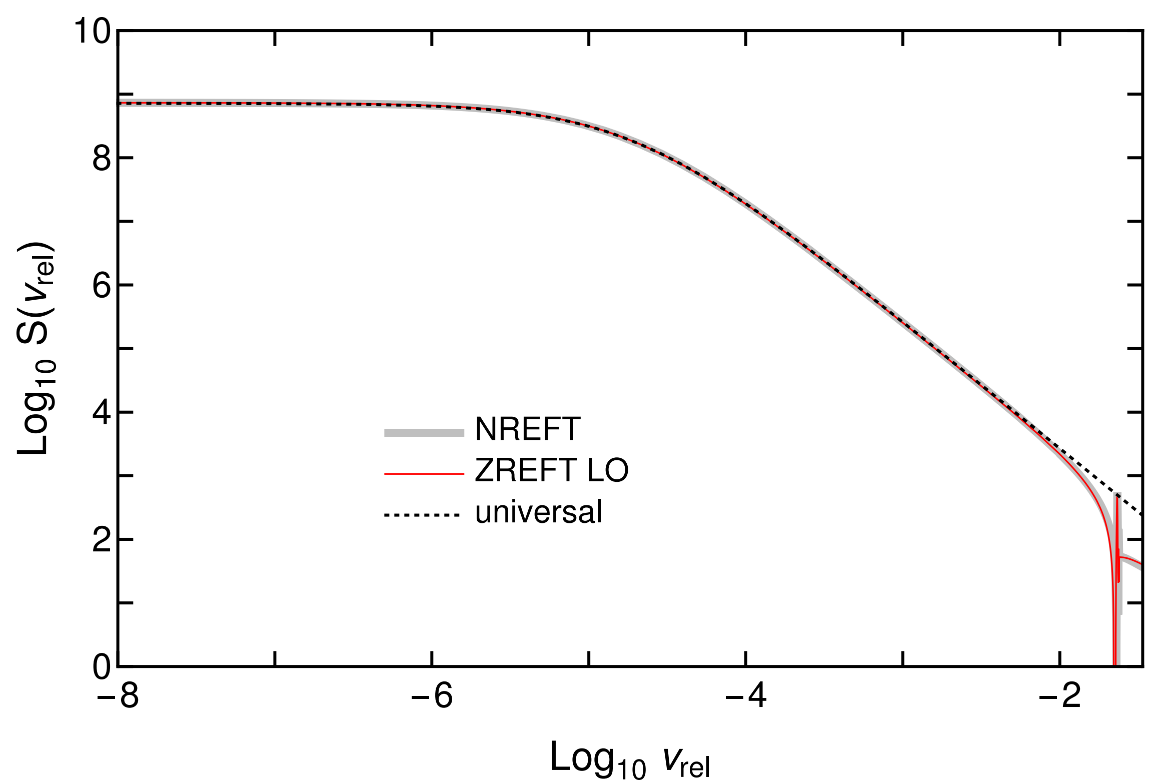
inclusive annihilation rate of $w^0 w^0$

leading order: $w^0 w^0 \rightarrow W^+ W^-$ $\quad v\sigma_{\rm ann}[w^0 w^0] = \dfrac{2\pi\alpha_2^2}{M^2}$

Sommerfeld enhancement factor at $E = Mv^2/4$

$$S(v) = \frac{8M}{\alpha_2^2}\frac{\mathrm{Im}[\gamma_0] - \mathrm{Im}[\tan^2\phi]\big(\mathrm{Re}[K_1(E)] - K_1(0)\big)}{\big|\gamma_0 - \tan^2\phi\,[K_1(E) - K_1(0)] + iMv/2\big|^2}$$

$$K_1(E) = \alpha M\left[\psi(z) + 1/z - \log(-z)\right] \qquad z = -\frac{\alpha}{\sqrt{8\delta/M - v^2 - i\epsilon}}$$



Zero-Range Effective Field Theory for Resonant Wino Dark Matter

systematically improvable effective field theory
based on scaling perturbations of
single-channel unitarity fixed point

ZREFT at LO

3 interaction parameters: α=1/137 and a_u, ϕ
convenient matching variables: $\gamma_0(M)$, $r_0(M)$

analytic predictions for

- wimp-wimp cross sections
- wimp-pair bound states
- wimp-pair annihilation rates (Sommerfeld enhancement)

unitarization of wimp-pair annihilation

ZREFT at NLO

2 additional interaction parameters

Zero-Range Effective Field Theory for Resonant Wino Dark Matter

analytic results for 2-wimp problem from ZREFT
more convenient for exploring the effects of resonances
than numerical results from NREFT

simplifies numerical solution of 3-wimp problem

- 3-body recombination:
  wimp + wimp + wimp $\rightarrow$ (bound state) + wimp

predictive model for Strongly Interacting Dark Matter
with dark matter mass ≈ 15 GeV

other dark matter applications: resonant Higgsinos

other high energy physics applications:
charm mesons and X(3872) resonance