Hadron interactions in lattice QCD
- HAL QCD potential method -

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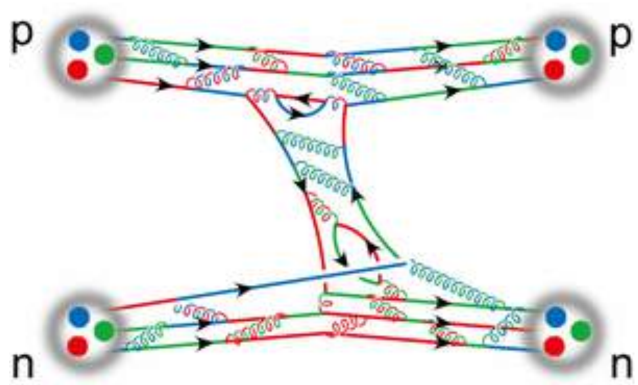
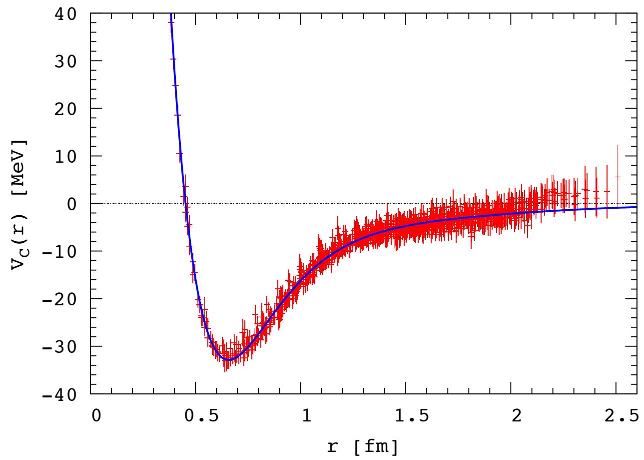
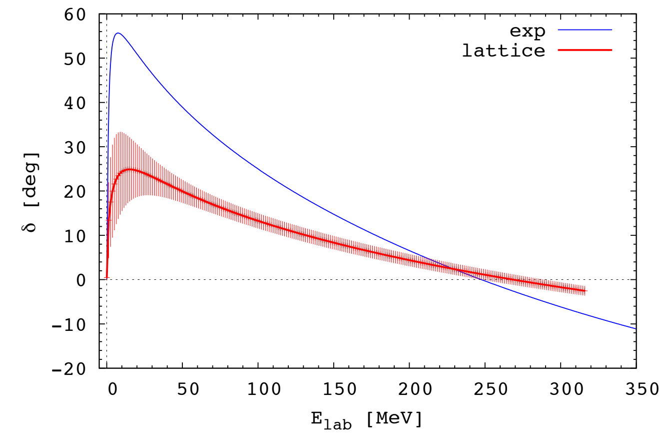
HAL QCD approach to Nuclear/Astro physics

Potentials from
lattice QCD

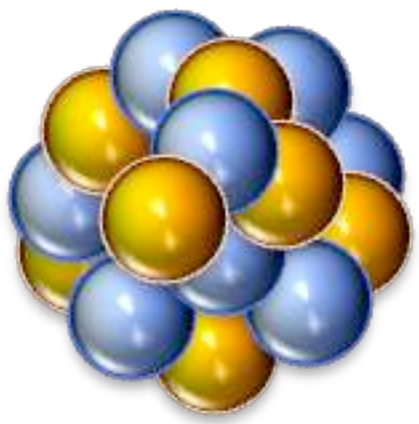


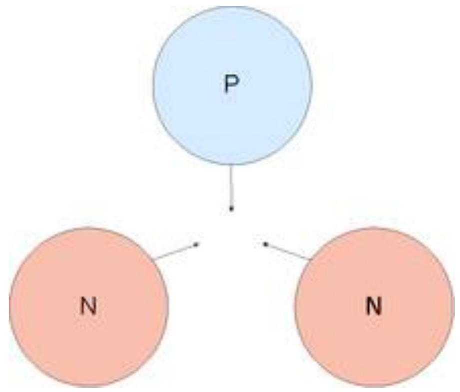




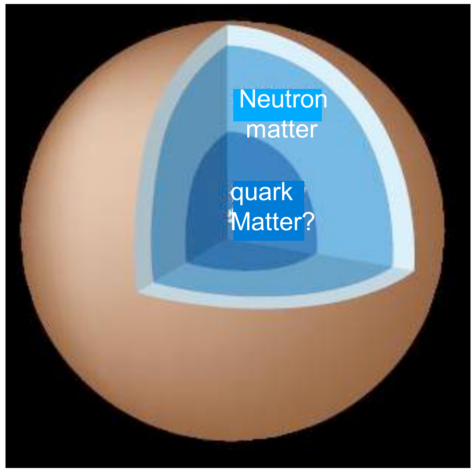
Nuclear Physics
with these potentials

parameters of EFT





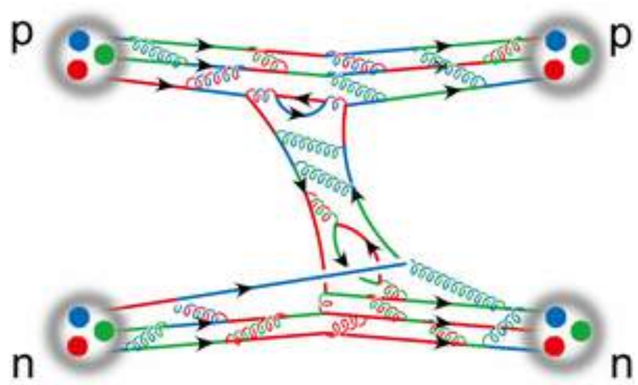
Neutron stars
Supernova explosion

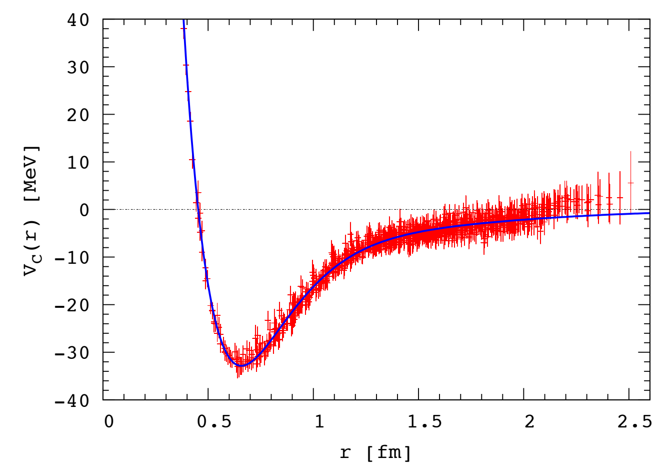


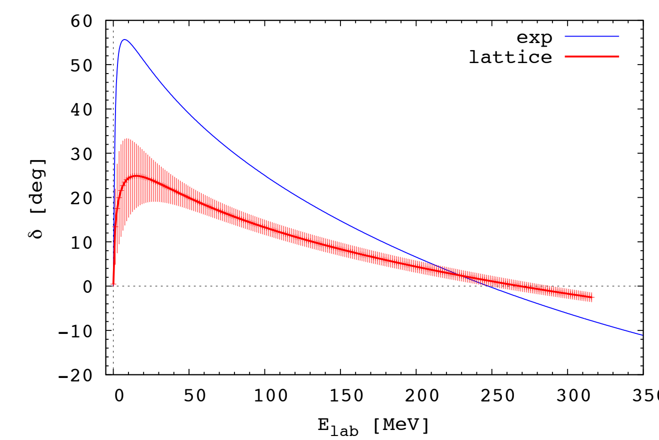
HAL QCD approach to Nuclear/Astro physics

Our task

Potentials from lattice QCD







Nuclear Physics
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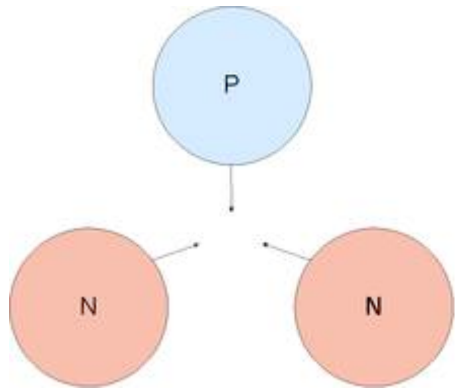
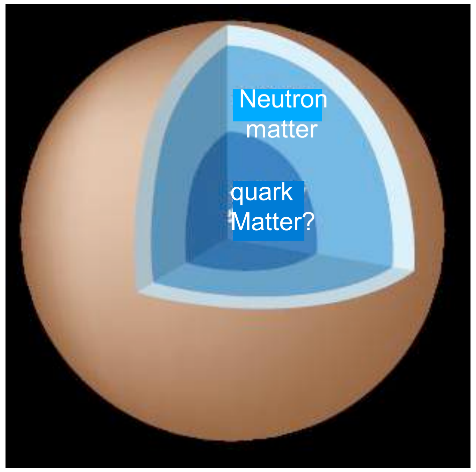
parameters of EFT



Neutron stars
Supernova explosion



Plan of my talk

I. HAL QCD method

II. Recent results (selected)

1.Dibaryons

2. Exotic hadron Zc(3900)

III. Discussions

I. HAL QCD method

References

N. Ishii, S. Aoki, T. Hatsuda, Phys. Rev. Lett. 99 (2007) 022001,
"The Nuclear Force from Lattice QCD"

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theor. Phys. 123 (2010) 89-128,
" Theoretical Foundation of the Nuclear Force in QCD and its applications to Central and Tensor Forces in Quenched Lattice QCD Simulations "

HAL QCD Collaboration (S. Aoki *et al.* ,), PTEP 2012 (2012) 01A105,
"Lattice QCD approach to Nuclear Physics"

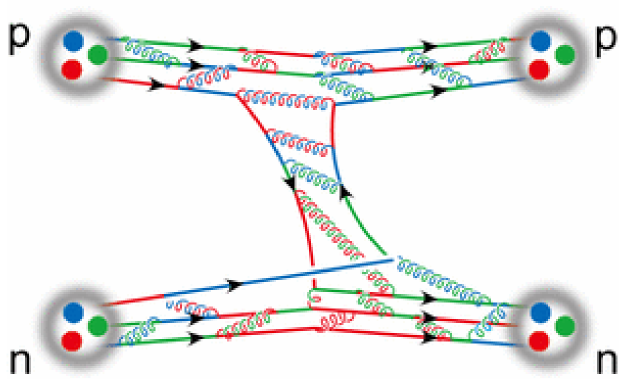
Our strategy in lattice QCD

consider (Equal-time) Nambu-Bethe-Salpeter (NBS) Wave function

$$\varphi_{\mathbf{k}}(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | NN, W_k \rangle$$

QCD eigen-state

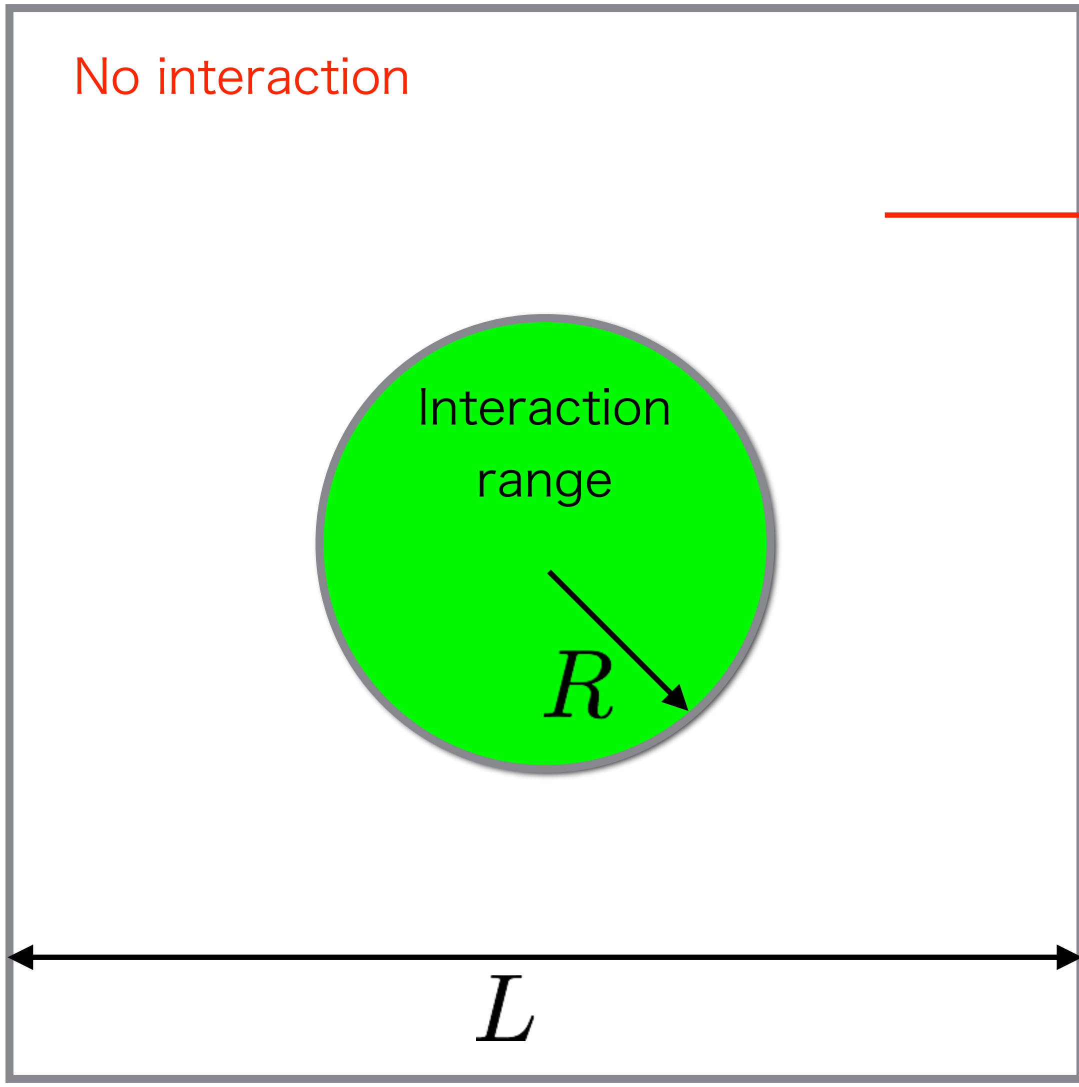
$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$: local operator "scheme"



energy $W_k = 2\sqrt{\mathbf{k}^2 + m_N^2} < W_{\rm th} = 2m_N + m_\pi$ Elastic threshold

$NN \rightarrow NN$ only elastic scattering

Main idea



Outside

$$\varphi_{\mathbf{k}}(\mathbf{r}) \simeq \sum_{l,m} C_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} Y_{ml}(\Omega_{\mathbf{r}})$$

scattering phase shift (phase of the S-matrix by unitarity) in QCD.

We consider the inside region and extract information of interactions there.

Potential

Non-local but energy-independent, defined from the NBS wave function

$$\left[\epsilon_k - H_0\right]\varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, \underline{U(\mathbf{x},\mathbf{y})}\varphi_{\mathbf{k}}(\mathbf{y})$$

$$\epsilon_k = \frac{\mathbf{k}^2}{2\mu} \qquad H_0 = \frac{-\nabla^2}{2\mu}$$

$$U(\mathbf{x},\mathbf{y}) \Longleftrightarrow V_{\mathbf{k}}(\mathbf{x}) = \frac{[\epsilon_k - H_0]\varphi_{\mathbf{k}}(\mathbf{x})}{\varphi_{\mathbf{k}}(\mathbf{x})}$$

By construction

potential $U(\mathbf{x},\mathbf{y})$ is faithful to QCD phase shift $\delta_l(k)$.

Note however that $U(\mathbf{x},\mathbf{y})$ is not unique.

(Formal) proof of "existence"

We can construct a non-local but energy-independent potential easily as

$$U(\mathbf{x},\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'}^{W_k, W_{k'} \le W_{\rm th}} \left[\epsilon_k - H_0\right] \varphi_{\mathbf{k}}(\mathbf{x}) \eta^{-1}_{\mathbf{k},\mathbf{k}'} \varphi^\dagger_{\mathbf{k}'}(\mathbf{y})$$

$\eta^{-1}_{\mathbf{k},\mathbf{k}'}$: inverse of $\eta_{\mathbf{k},\mathbf{k}'} = (\varphi_{\mathbf{k}}, \varphi_{\mathbf{k}'})$

inner product

For $^\forall W_{\mathbf{p}} < W_{\rm th}$

$$\int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_{\mathbf{p}}(\mathbf{y}) = \sum_{\mathbf{k},\mathbf{k}'} (\epsilon_k - H_0)\varphi_{\mathbf{k}}(\mathbf{x})\eta^{-1}_{\mathbf{k},\mathbf{k}'}\eta_{\mathbf{k}',\mathbf{p}} = (\epsilon_p - H_0)\varphi_{\mathbf{p}}(\mathbf{x})$$

Derivative expansion (approximation)

$$U(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\nabla)\delta^3(\mathbf{x}-\mathbf{y})$$

$$V(\mathbf{x},\nabla) = V_0(r) + V_\sigma(r)(\sigma_\mathbf{1}\cdot\sigma_\mathbf{2}) + V_T(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + O(\nabla^2)$$

LO LO LO NLO NNLO

$$S_{12} = \frac{3}{r^2}(\sigma_\mathbf{1}\cdot\mathbf{x})(\sigma_\mathbf{2}\cdot\mathbf{x}) - (\sigma_\mathbf{1}\cdot\sigma_\mathbf{2})$$

tensor operator spins

At LO we simply obtain

$$V_{\rm LO}(\mathbf{x}) = \frac{[\epsilon_k - H_0]\varphi_\mathbf{k}(\mathbf{x})}{\varphi_\mathbf{k}(\mathbf{x})}$$

phase shifts and binding energy below inelastic threshold

Advantage of potential method

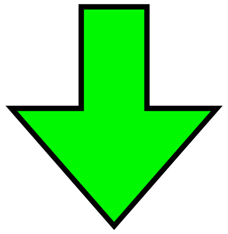
Direct method

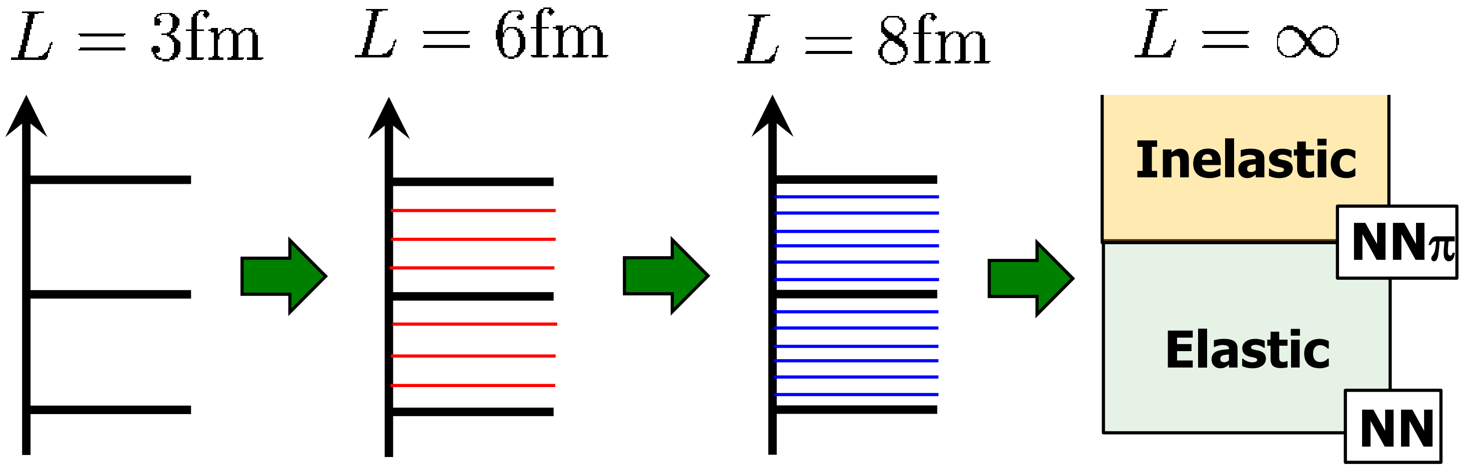
$$G_{NN}(t) = \langle N(t)N(t)\bar{N}(0)\bar{N}(0)\rangle = Z_0 e^{-E_0 t} + Z_1 e^{-E_1 t} + \cdots \to Z_0 e^{-E_0 t}, \quad t \to \infty$$

Excitation energy

finite volume effect for scattering state $\simeq \frac{1}{m_N}\frac{(2\pi)^2}{L^2}$

$E_1 - E_0 \simeq 50$ MeV at $L = 4$ fm





$t \gg 1/(E_1 - E_0) \simeq 4$ fm is needed to suppress excited states.

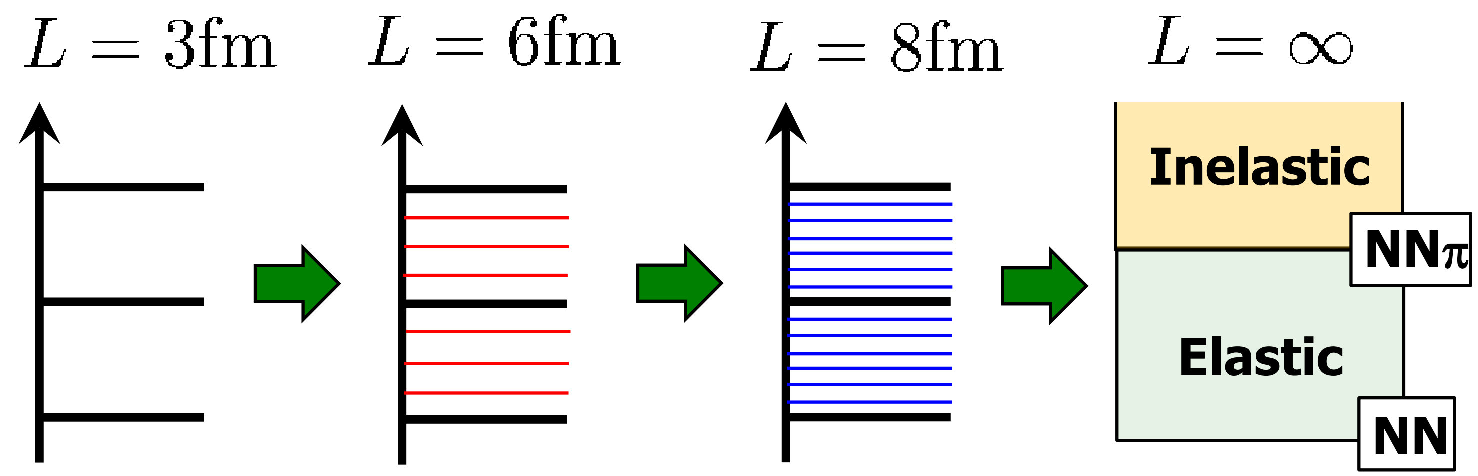
Large separation is impossible due to the bad signal-to-noise ratio.

$$\frac{S}{N} := \frac{\langle N^2(t)\bar{N}^2(t)\rangle}{\sqrt{\langle |N^2(t)\bar{N}^2(t)|^2\rangle}} \sim \exp[-(m_N - 3m_\pi/2)t]$$

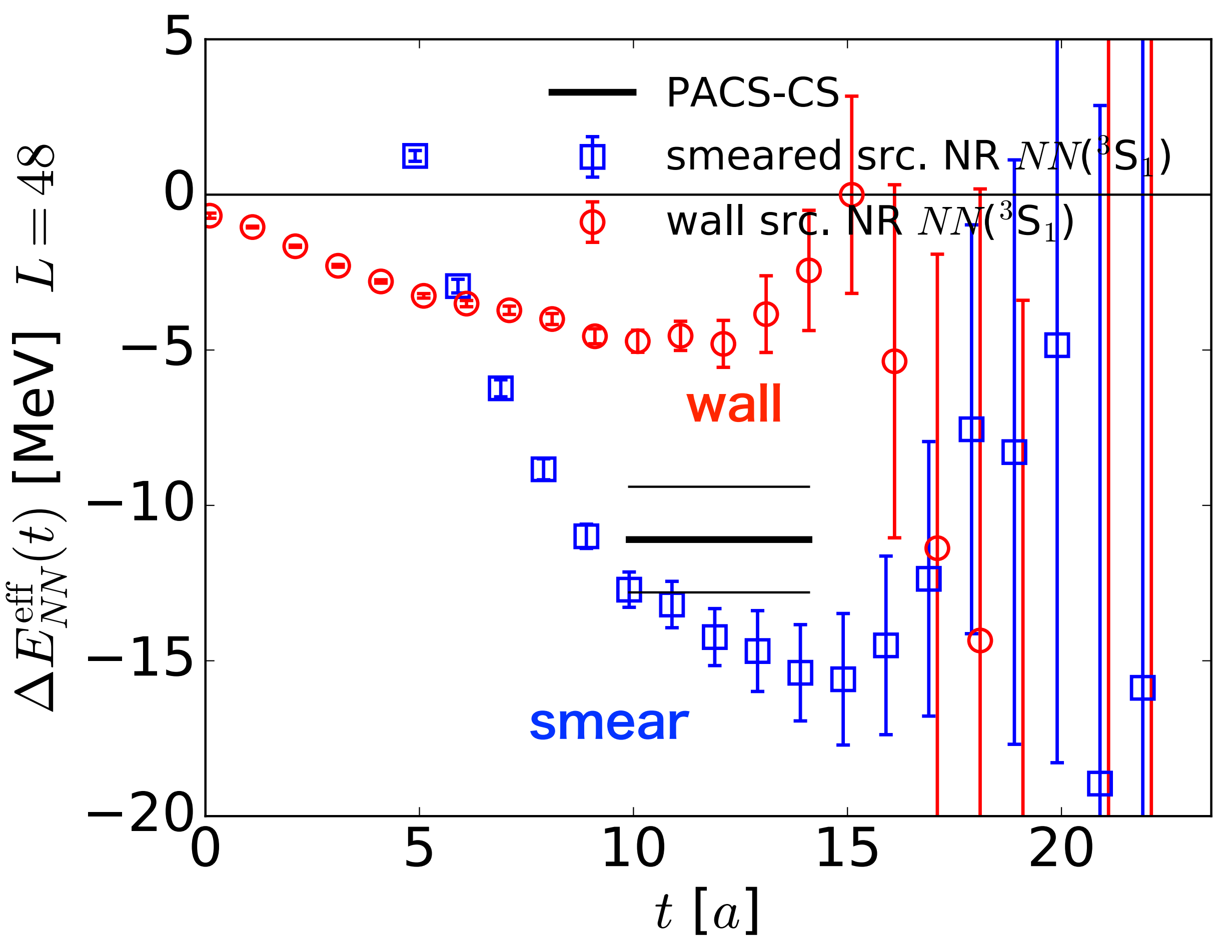
Failure of the direct method for baryons

Excited state contaminations

$$\Delta E = E - 2m_B$$

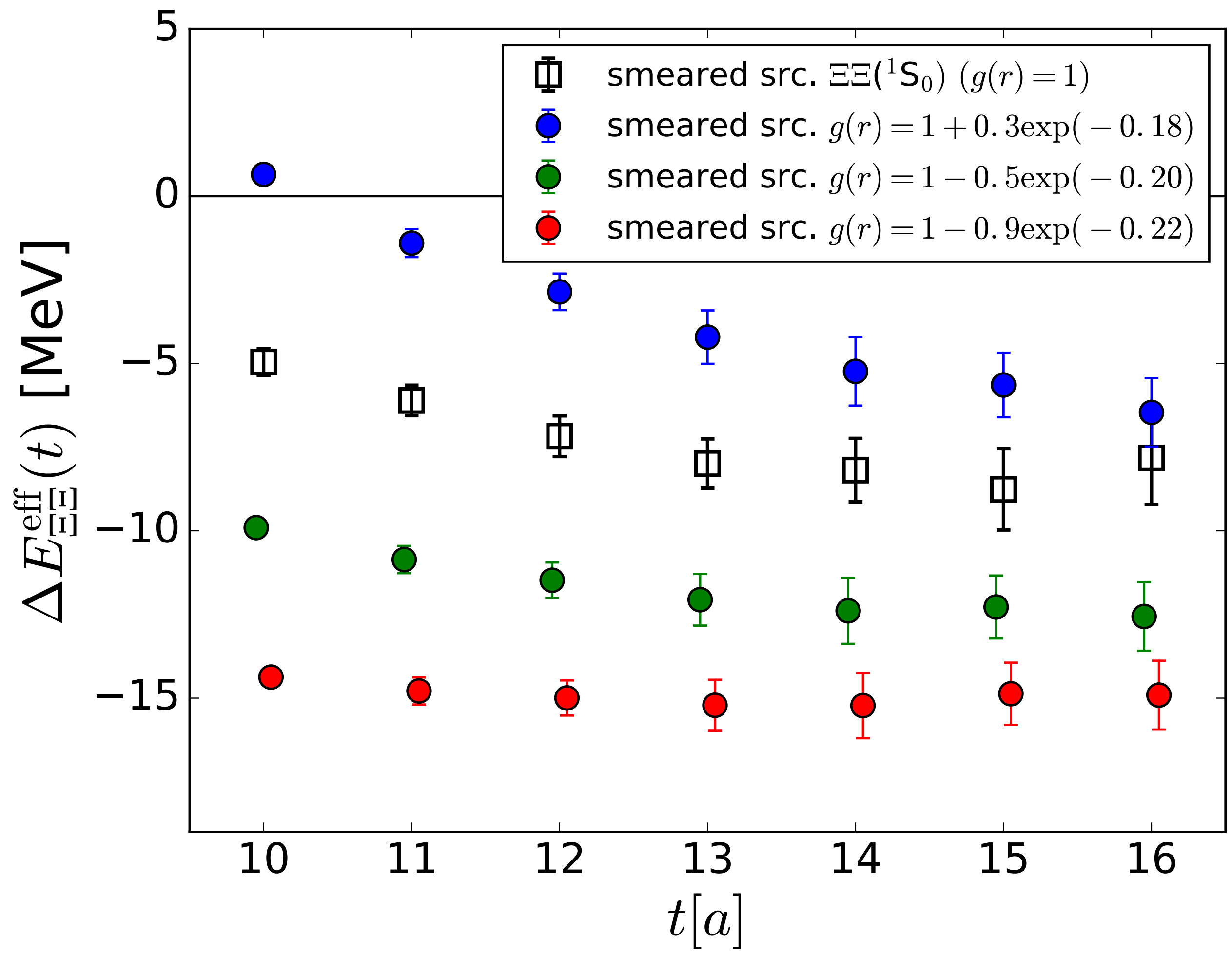


$NN(^3S_1)$



$\Xi\Xi(^1S_0)$

Smeared source



strong source operator dependence

strong sink operator dependence

Fake plateau problem

Mock-up data @ $m_\pi = 0.5$ GeV, $L = 4$ fm (setup of YIKU2012)

$$R(t) = e^{-\Delta E t}\left(1 + b\, e^{-\delta E_{\rm el} t} + c\, e^{-\delta E_{\rm inel} t}\right)$$

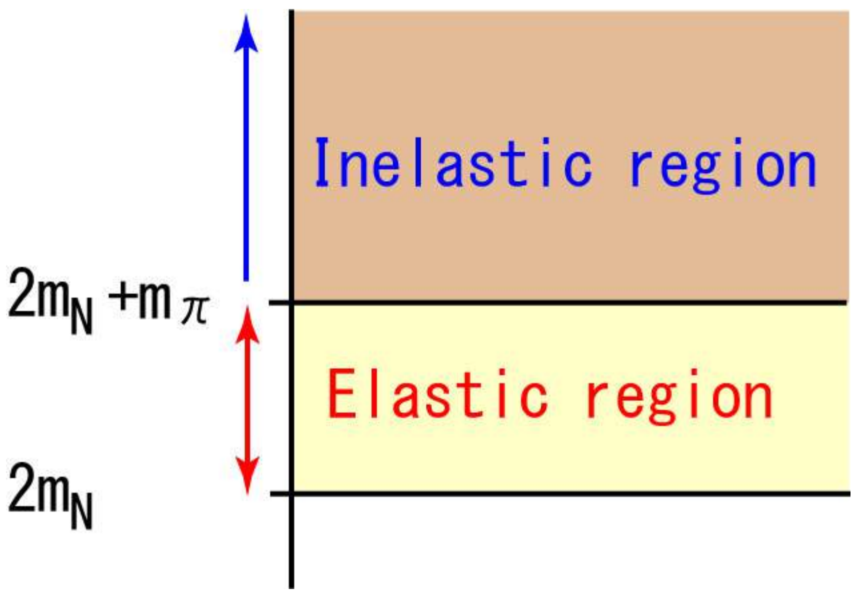
$\delta E_{\rm el} \propto \frac{1}{L^2}$ the lowest excitation energy of elastic scattering state

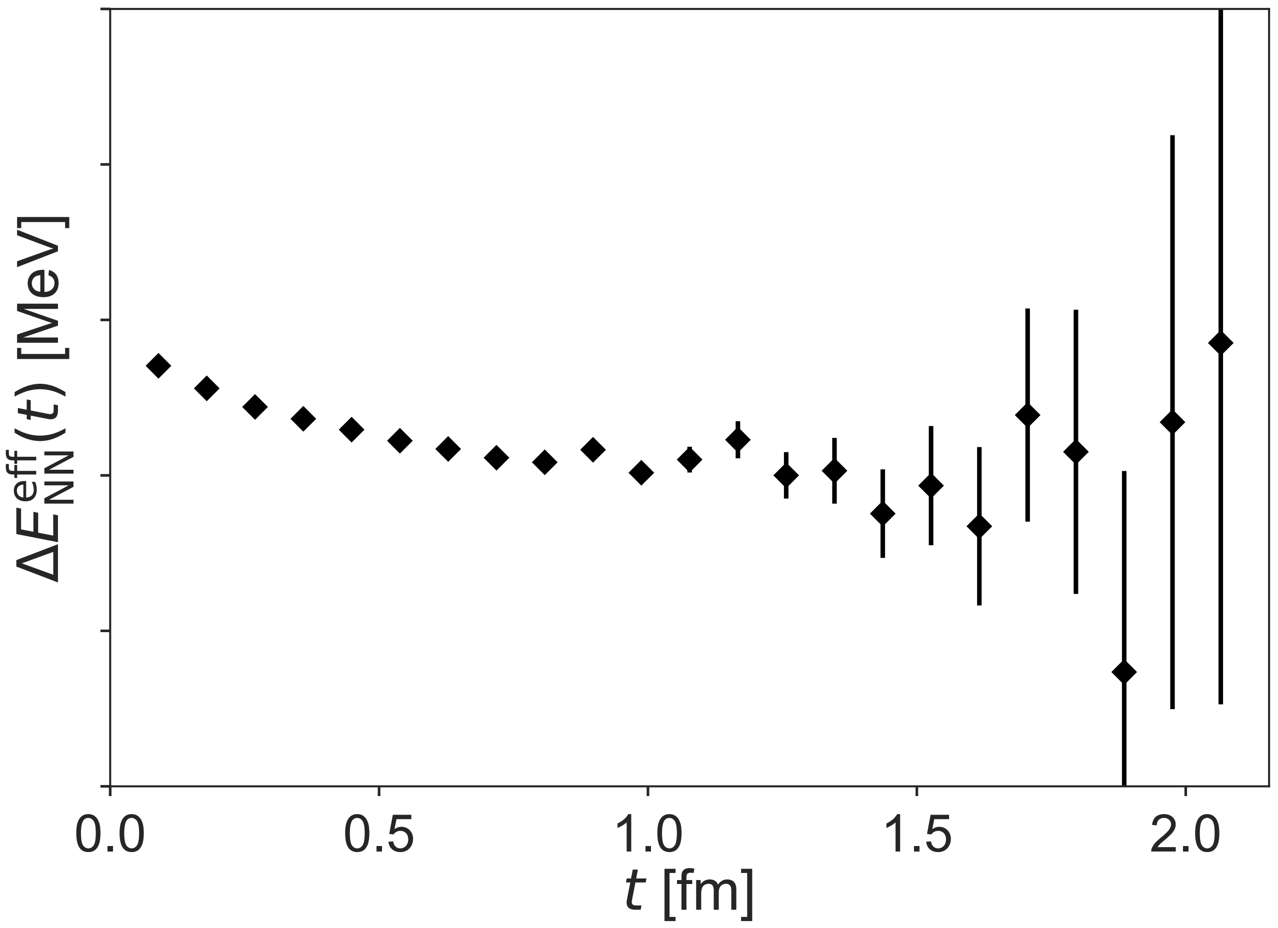
$\delta E_{\rm el} = 50$ MeV at $L \simeq 4$ fm

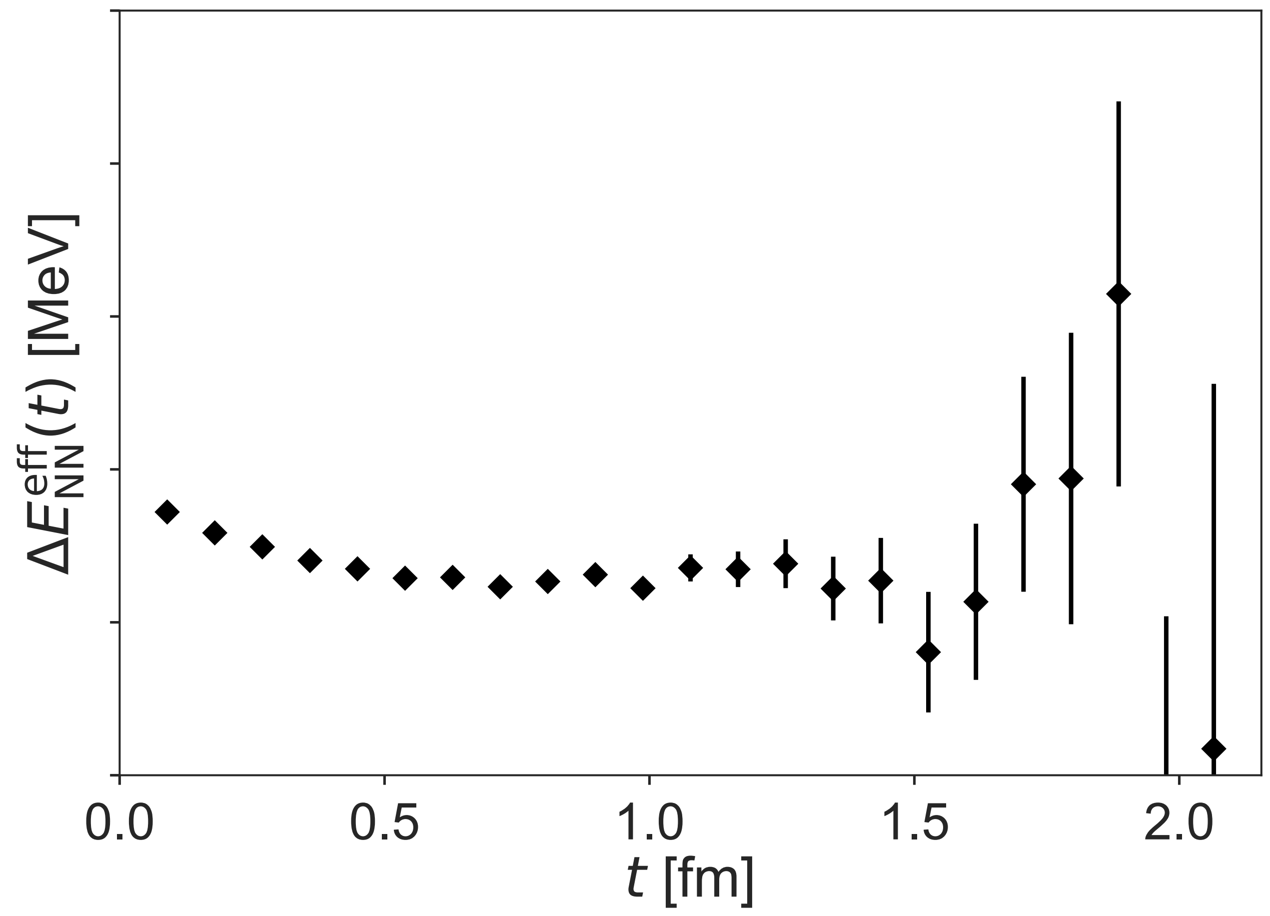
$b = \pm 0.1$ 10 % contamination $b = 0$ for a comparison

$\delta E_{\rm inel} = 500$ MeV the inelastic energy from heavy pions

$c = 0.01$ 1% contamination

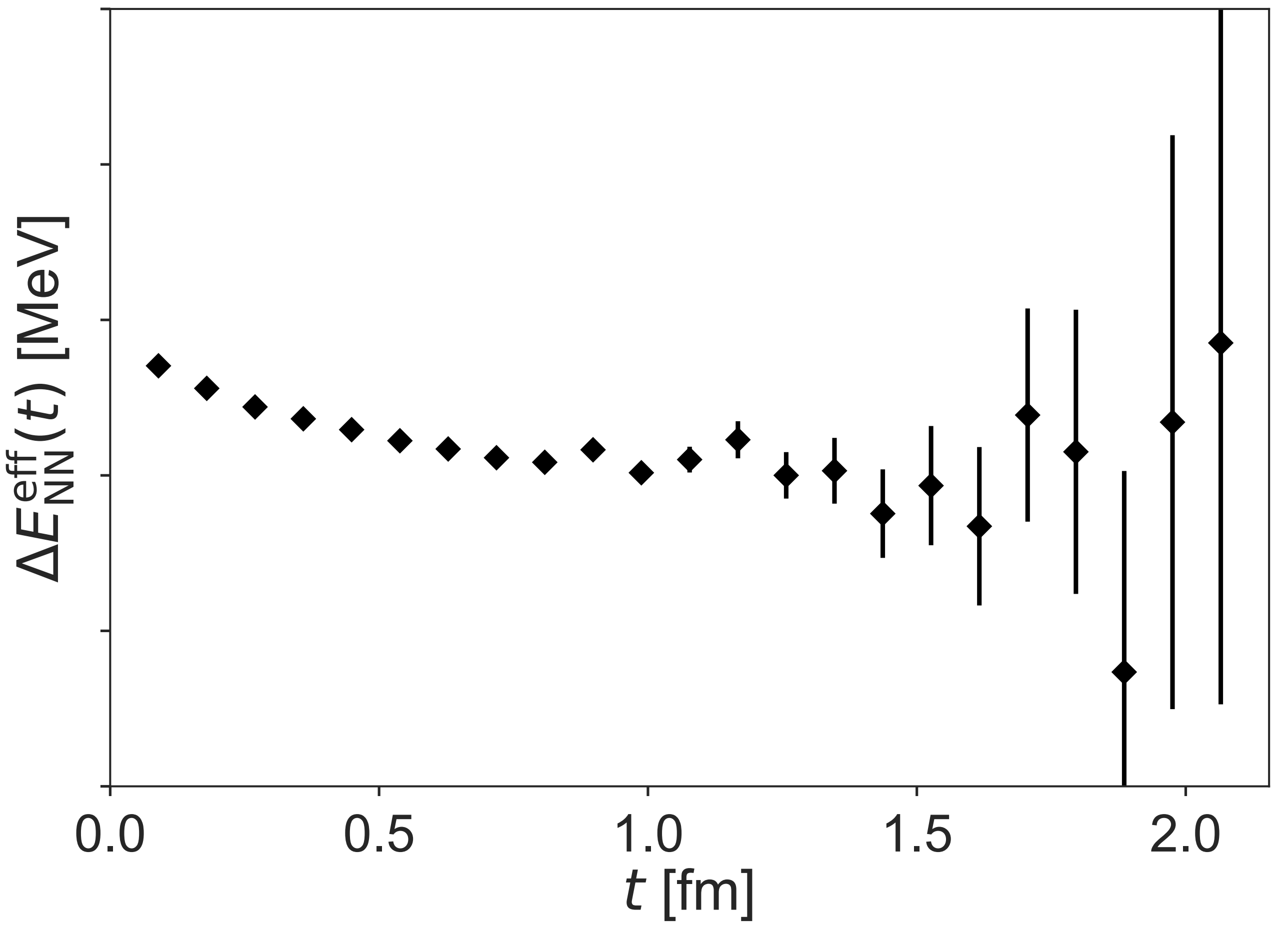


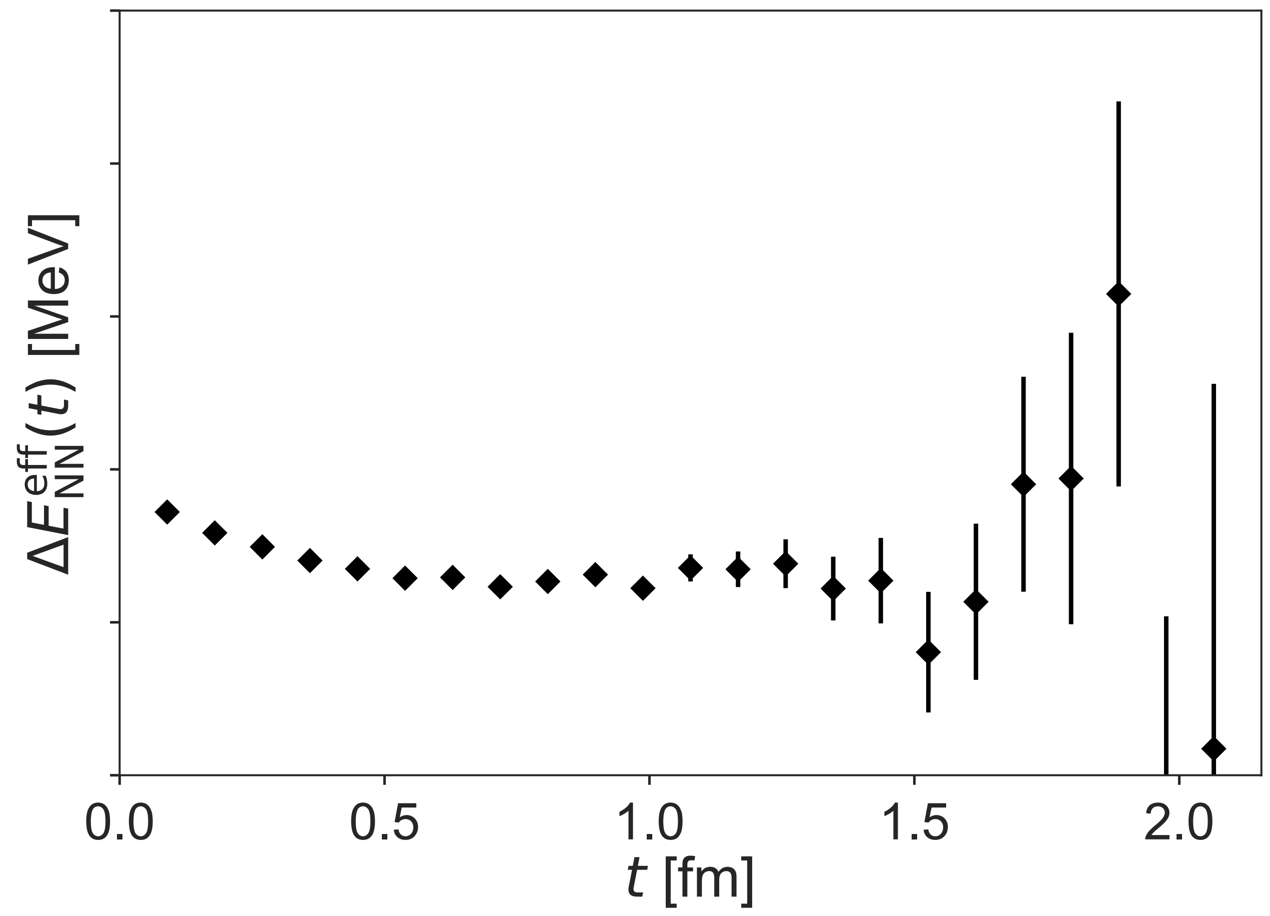




We can not tell which is the correct one.

increasing errors and fluctuations in time





We can not tell which is the correct one.

Improvement with potentials

Ishii et al. (HALQCD), PLB712(2012) 437

$$F(\mathbf{r}, t-t_0) = \langle 0|N(\mathbf{x}+\mathbf{r}, t)N(\mathbf{x}, t)\bar{\mathcal{J}}(t_0)|0\rangle$$

4-pt function

$$= \sum_n A_n \varphi^{W_n}(\mathbf{r}) e^{-W_n(t-t_0)}$$ + inelastic contributions

Normalized 4-pt function

$$R(\mathbf{r}, t) \equiv F(\mathbf{r}, t)/G_N(t)^2 = \sum_n A_n \varphi^{W_n}(\mathbf{r}) e^{-\Delta W_n t}$$

a sum of many NBS wave functions

$$\int d\mathbf{y}\, U(\mathbf{x}, \mathbf{y}) \varphi^{W_0}(\mathbf{y}) = (E_{W_0} - H_0)\varphi^{W_0}(\mathbf{x})$$

$$\int d\mathbf{y}\, U(\mathbf{x}, \mathbf{y}) \varphi^{W_1}(\mathbf{y}) = (E_{W_1} - H_0)\varphi^{W_1}(\mathbf{x})$$

$$\cdots$$

controlled by the same U

$$\left[\frac{\mathbf{k}_n^2}{m_N} - H_0\right]\varphi^{W_n}(\mathbf{r}) = U \cdot \varphi^{W_n}(\mathbf{r})$$

$$\Delta W_n = W_n - 2m_N = \frac{\mathbf{k}_n^2}{m_N} - \frac{(\Delta W_n)^2}{4m_N}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$-\frac{\partial}{\partial t} R(\mathbf{r}, t) = \left\{ H_0 + U - \frac{1}{4m_N}\frac{\partial^2}{\partial t^2} \right\} R(\mathbf{r}, t)$$

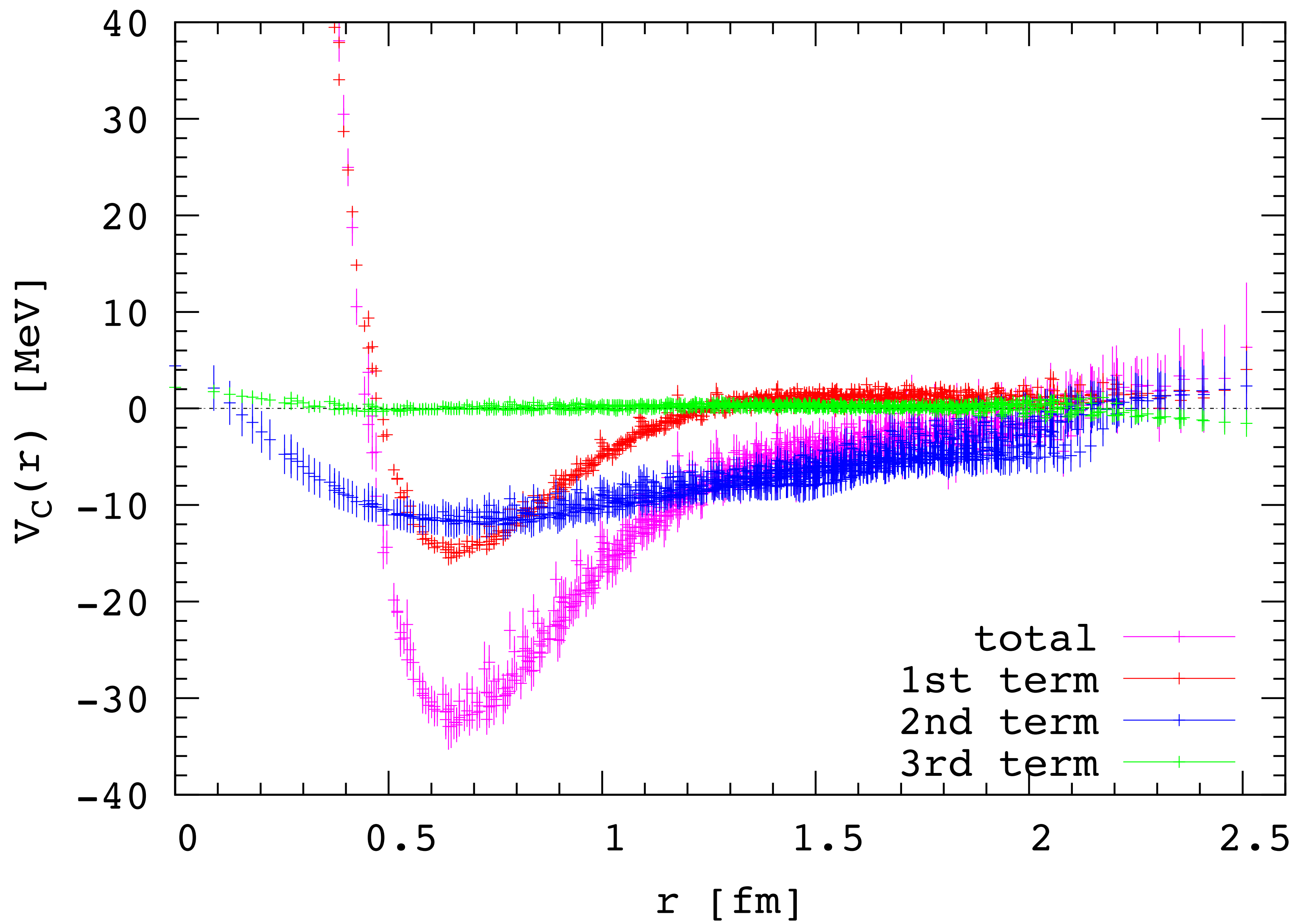
time corr. space corr. time corr.

$$\left\{-H_0 - \frac{\partial}{\partial t} + \frac{1}{4m_N}\frac{\partial^2}{\partial t^2}\right\} R(\mathbf{r},t) = \int d^3r'\, U(\mathbf{r},\mathbf{r}')R(\mathbf{r}',t) = V_C(\mathbf{r})R(\mathbf{r},t) + \cdots$$

1st 2nd 3rd

Potential

3rd term(relativistic correction) is negligible.



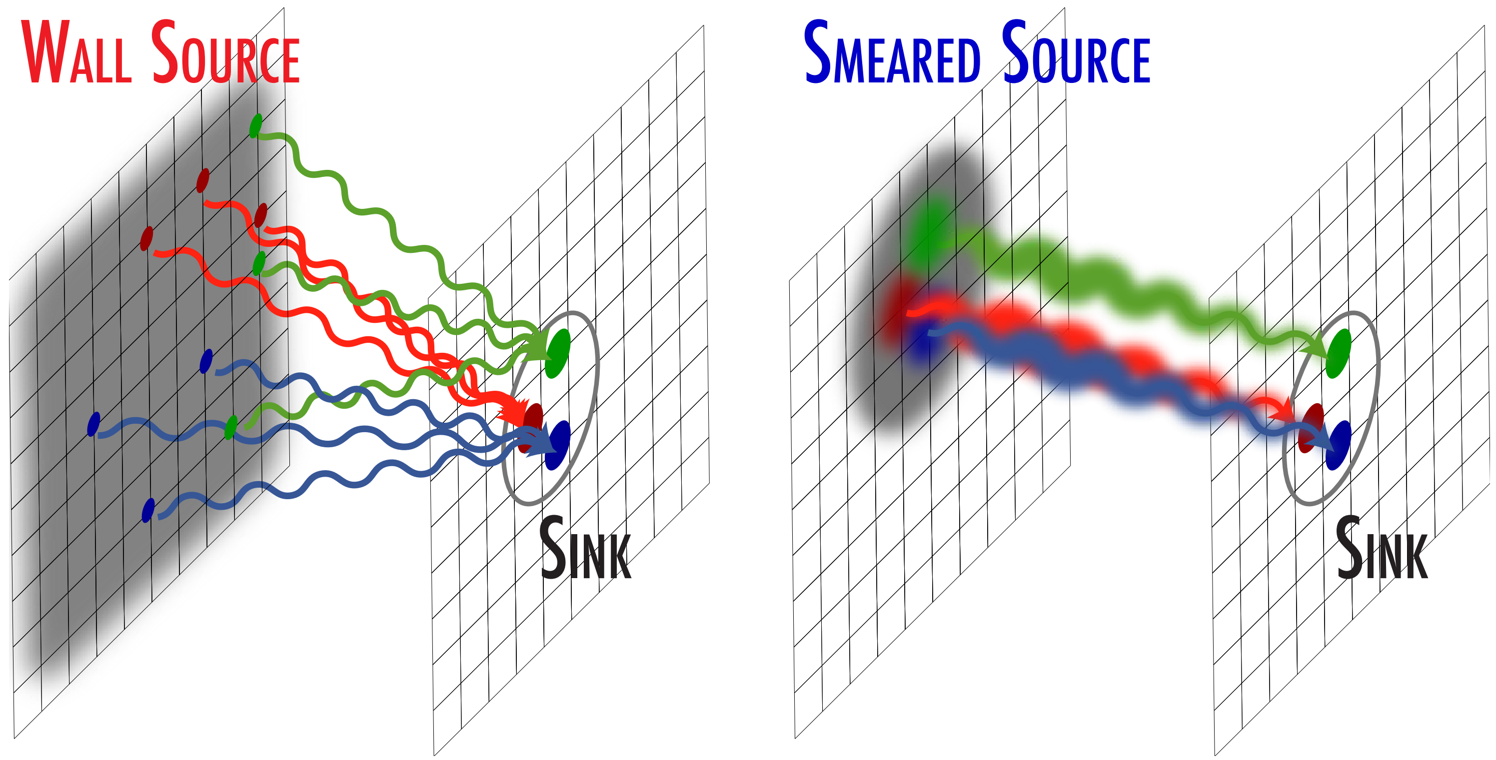
This method overcomes the difficulty of the direct method.

Systematics: derivative expansion

T. Iritani, Talk at Lat2016, arXiv1610.09779[hep-lat].
T. Iritani, Talk at Lat2017

Two source operators

quark wall source vs quark smeared source



$$\sum_{\mathbf{y}} q(\mathbf{y}, t_0)$$

$$\sum_{\mathbf{y}} e^{-B|\mathbf{x}_0-\mathbf{y}|} q(\mathbf{y}, t_0)$$

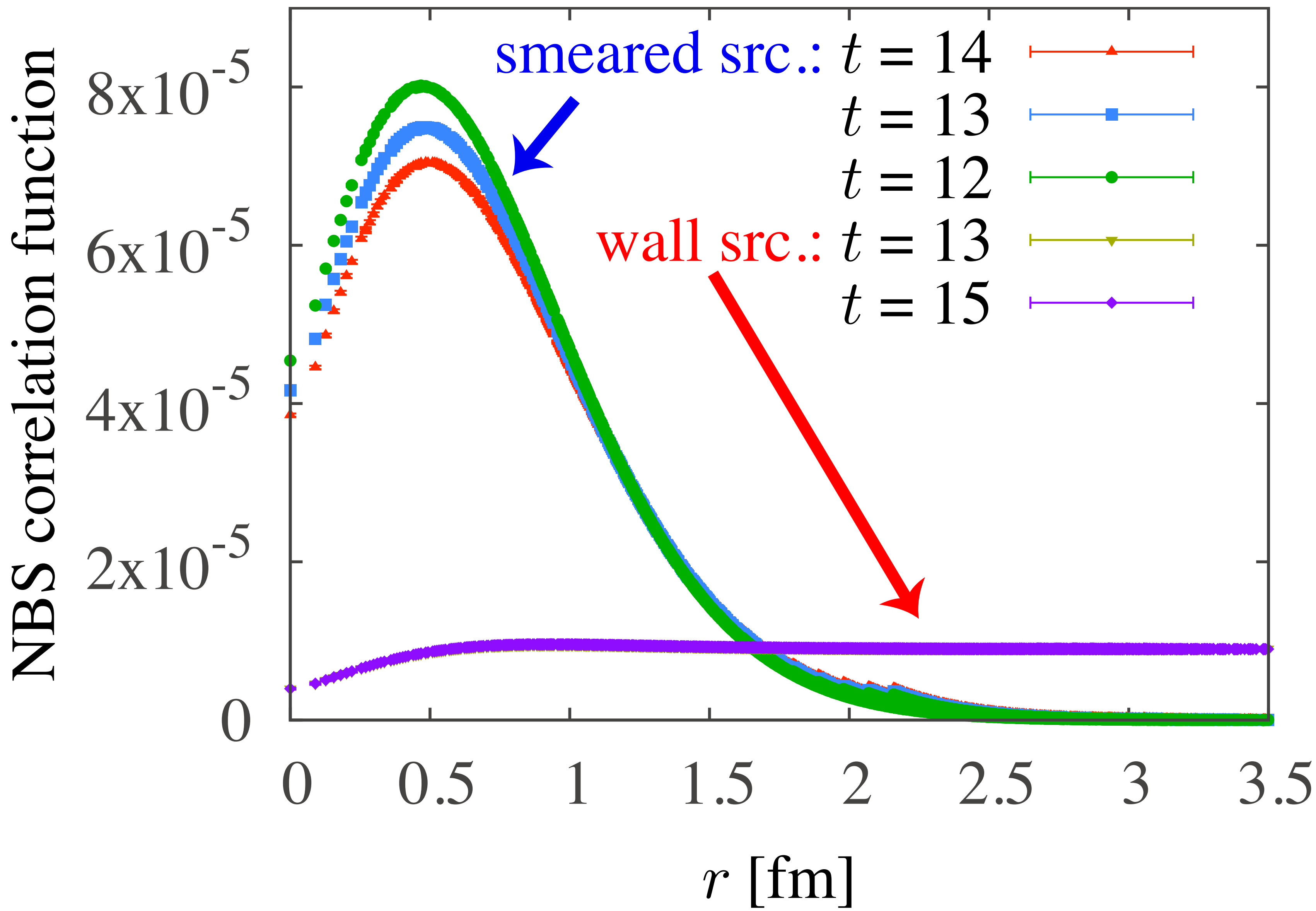
Lattice setup 2+1 flavor QCD

$$a = 0.09 \text{ fm } (a^{-1} = 2.2 \text{ GeV})$$

$$m_\pi = 0.51 \text{ GeV}, \; m_N = 1.32 \text{ GeV}, \; m_K = 0.62 \text{ GeV}, \; m_\Xi = 1.46 \text{ GeV}$$

NBS wave function

$\Xi\Xi(^1S_0)$



wall source very weak t-dependence

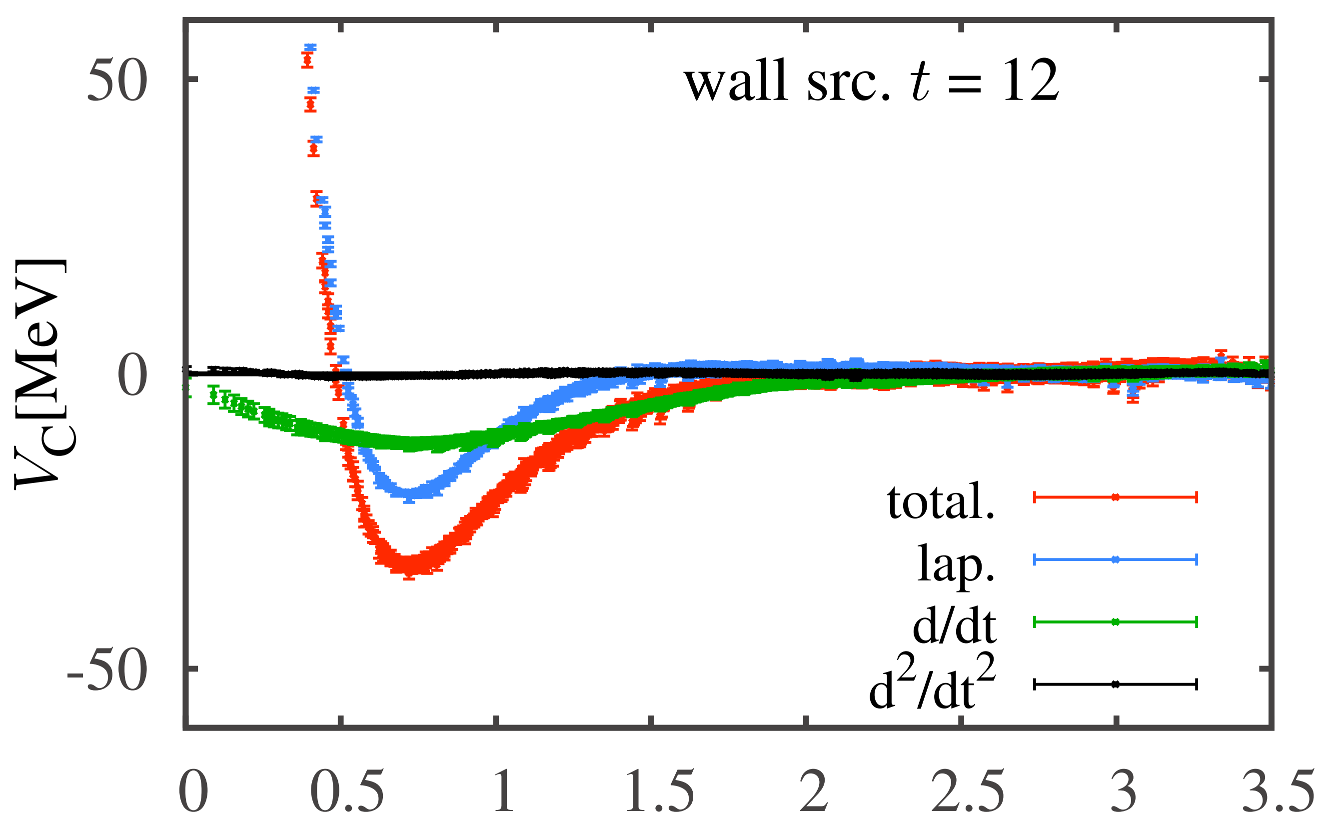
smeared source strong t-dependence

contributions from excited states

LO Potential

$$V_c(\mathbf{r}) = -\frac{H_0 R}{R} - \frac{(\partial/\partial t)R}{R} + \frac{(\partial/\partial t)^2 R}{4mR}$$

wall



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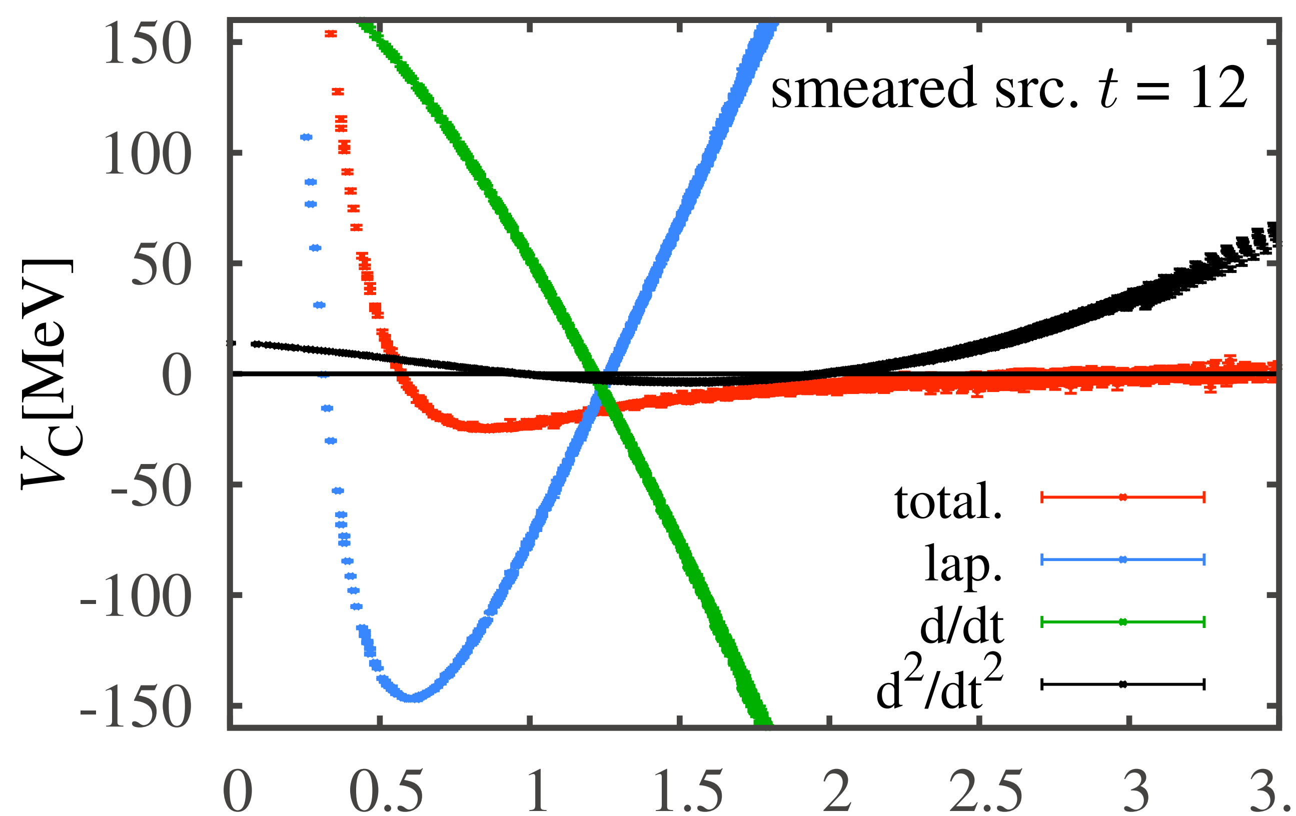
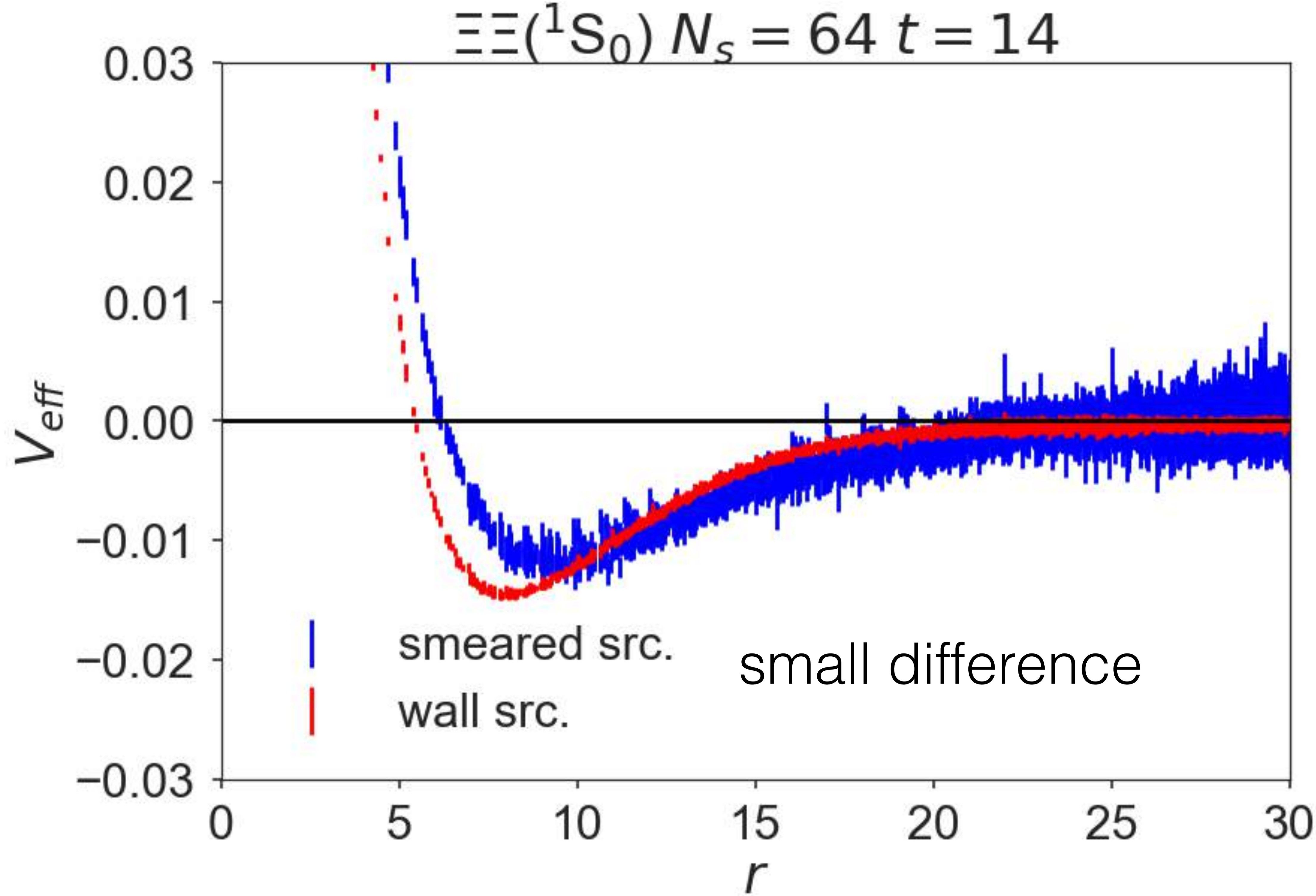


O(100) MeV cancellation

time-dependent HAL method works well



Small difference → **LO + NLO potential ?**

LO + NLO potential

$$V^X_{\text{eff}}(r,t) = \frac{1}{4m}\frac{\frac{\partial^2}{\partial t^2}R^X(r,t)}{R^X(r,t)} - \frac{\frac{\partial}{\partial t}R^X(r,t)}{R^X(r,t)} - \frac{H_0 R^X(r,t)}{R^X(r,t)}$$

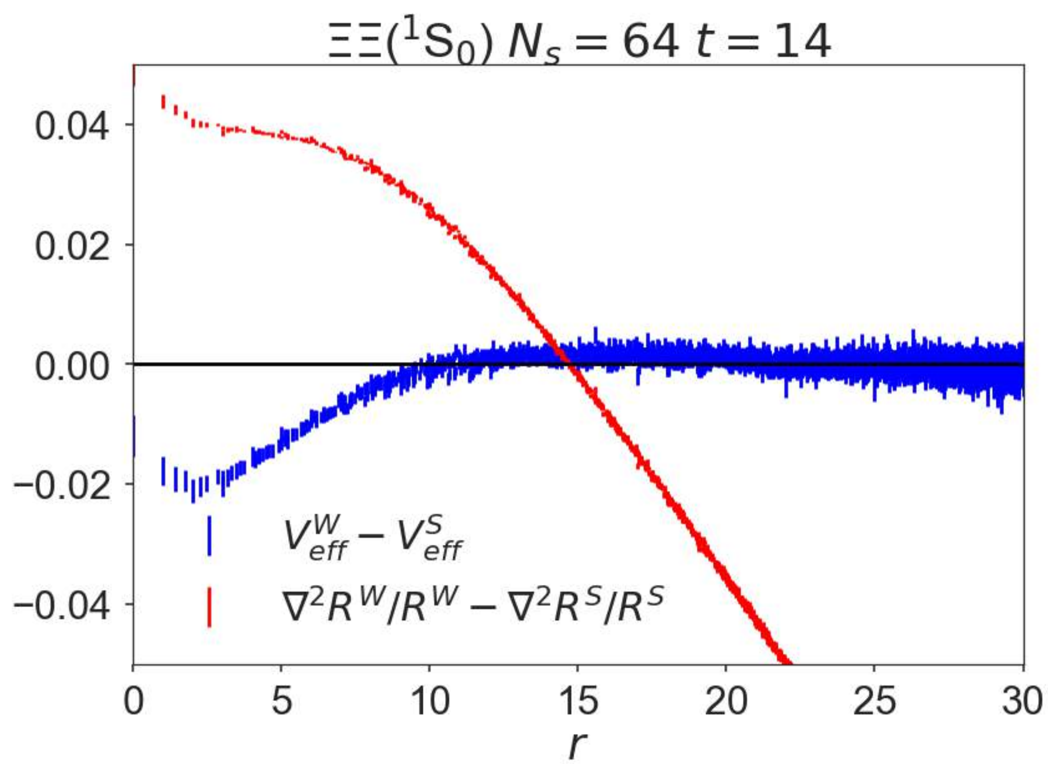
$$X = \text{Wall},\ \text{Smear}$$

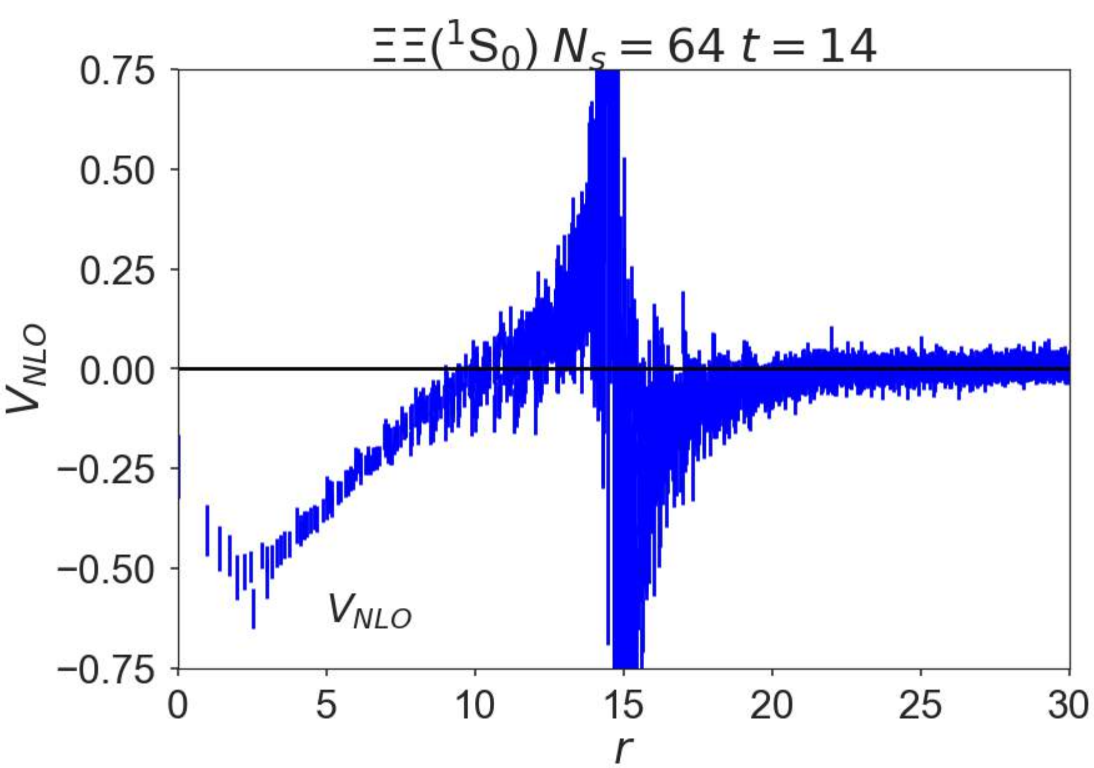
$$= \underline{V_{\text{LO}}(r)} + \underline{V_{\text{NLO}}(r)}\frac{\nabla^2 R^X(r,t)}{R^X(r,t)}$$

$$V_{\text{LO}}(r) = \frac{V^{\text{S}}_{\text{eff}}\nabla^2 R^{\text{W}}/R^{\text{W}} - V^{\text{W}}_{\text{eff}}\nabla^2 R^{\text{S}}/R^{\text{S}}}{\nabla^2 R^{\text{W}}/R^{\text{W}} - \nabla^2 R^{\text{S}}/R^{\text{S}}}(r,t)$$

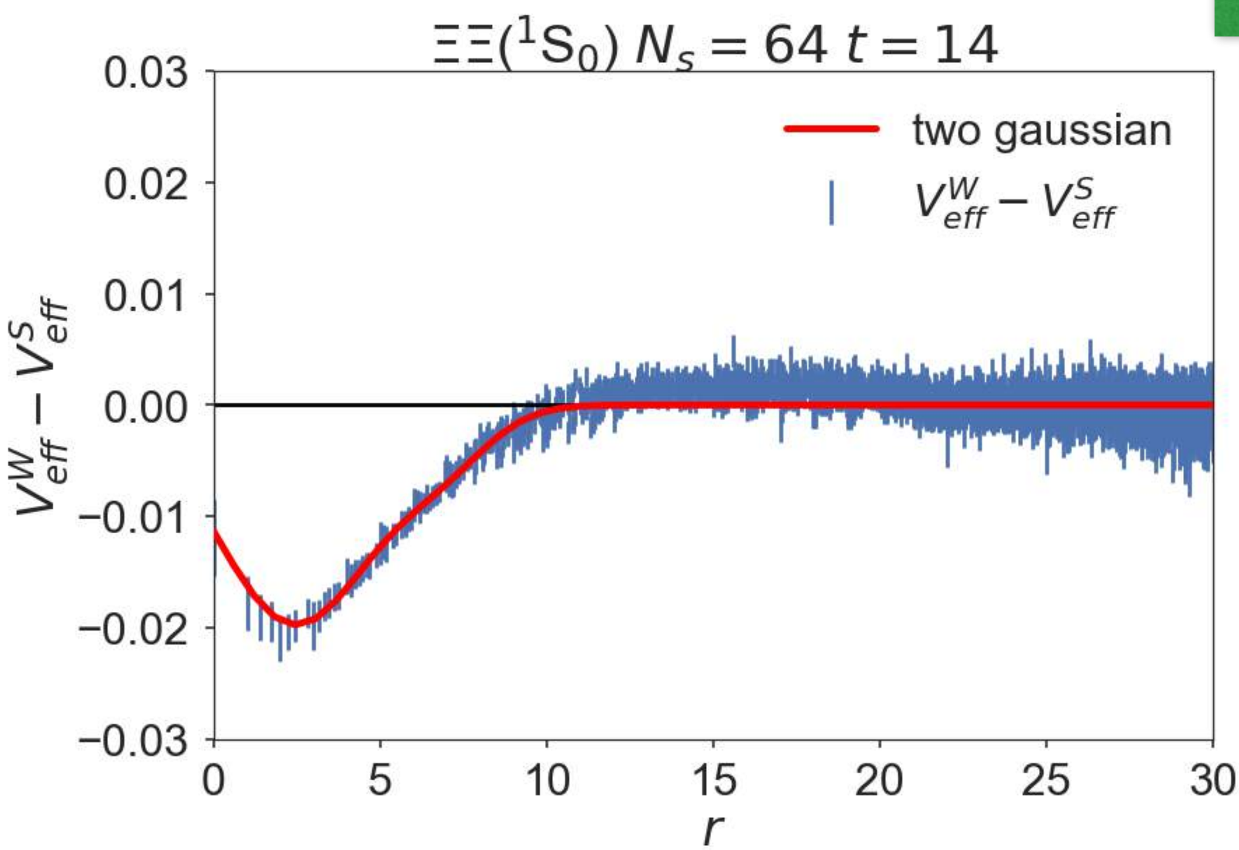
$$V_{\text{NLO}}(r) = \frac{V^{\text{W}}_{\text{eff}} - V^{\text{S}}_{\text{eff}}}{\nabla^2 R^{\text{W}}/R^{\text{W}} - \nabla^2 R^{\text{S}}/R^{\text{S}}}(r,t)$$

singular behavior



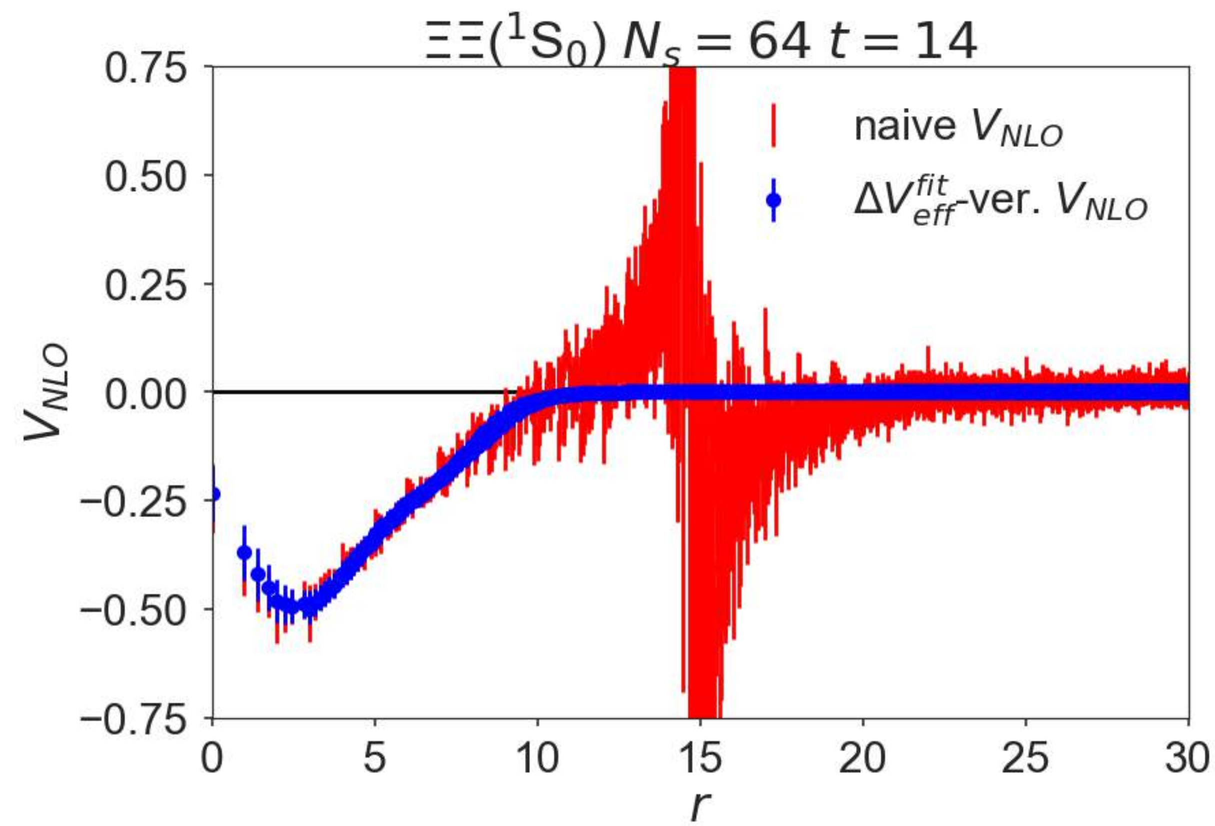


Fit of numerator

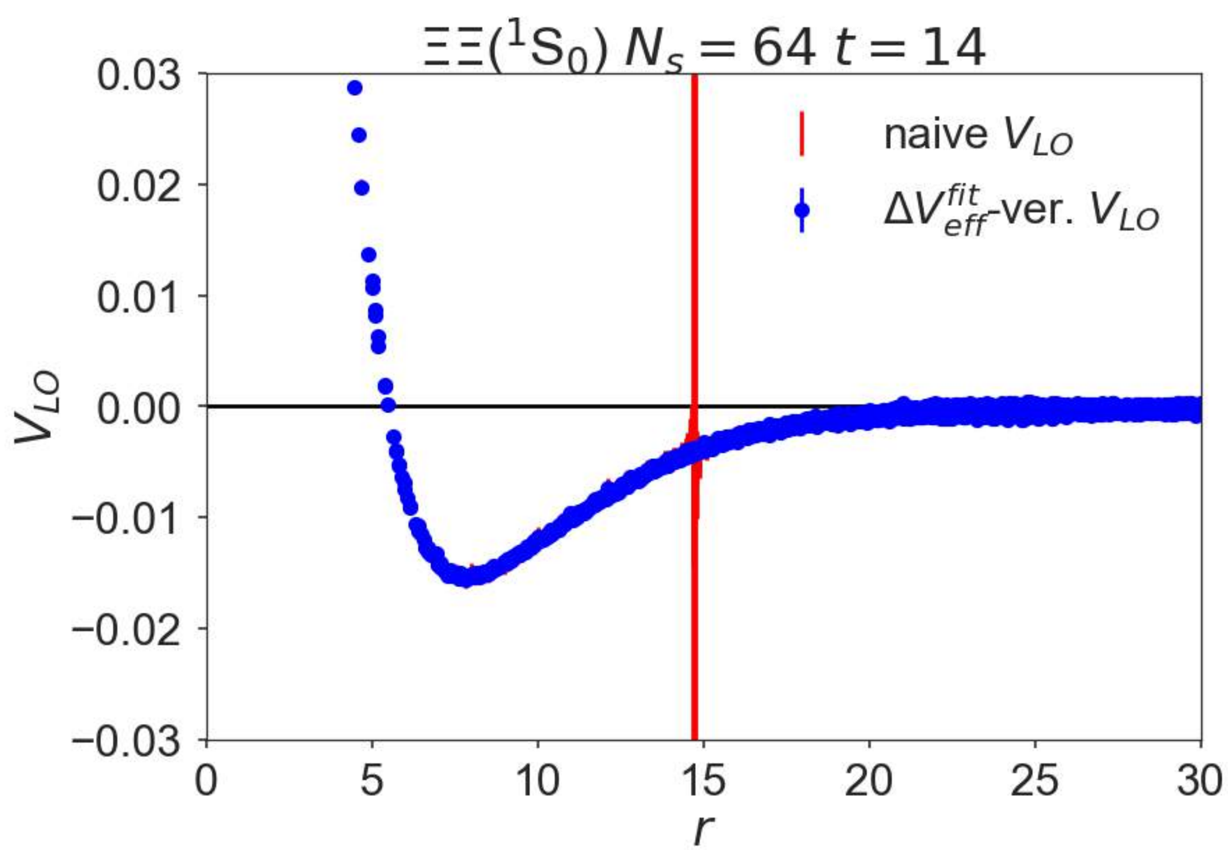


NLO potentail

$$V_{\rm NLO}(r) = \frac{(V^{\rm W}_{\rm eff} - V^{\rm S}_{\rm eff})^{\rm fit}}{\nabla^2 R^{\rm W}/R^{\rm W} - \nabla^2 R^{\rm S}/R^{\rm S}}(r,t)$$

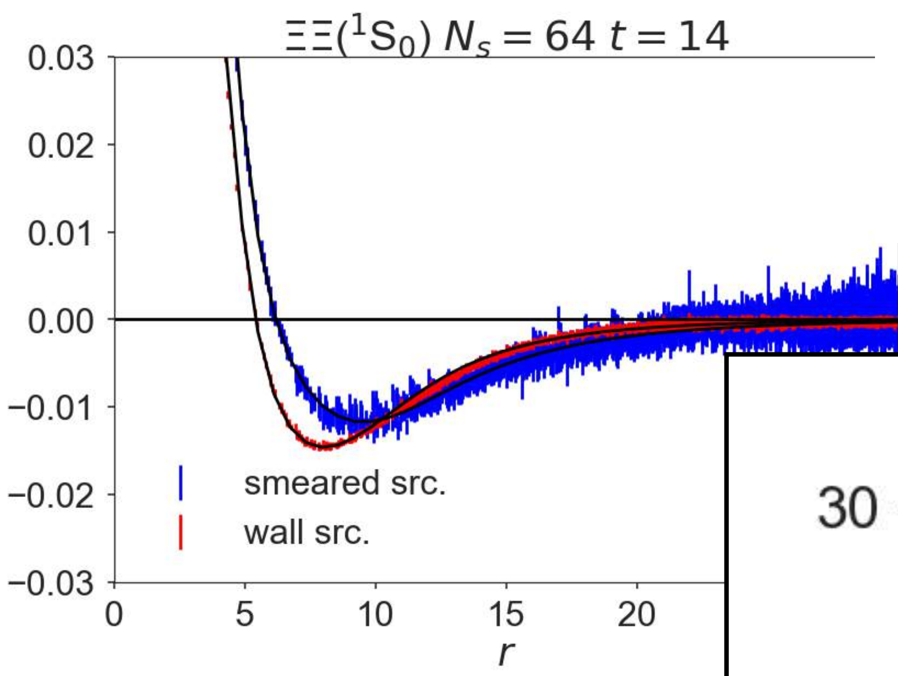


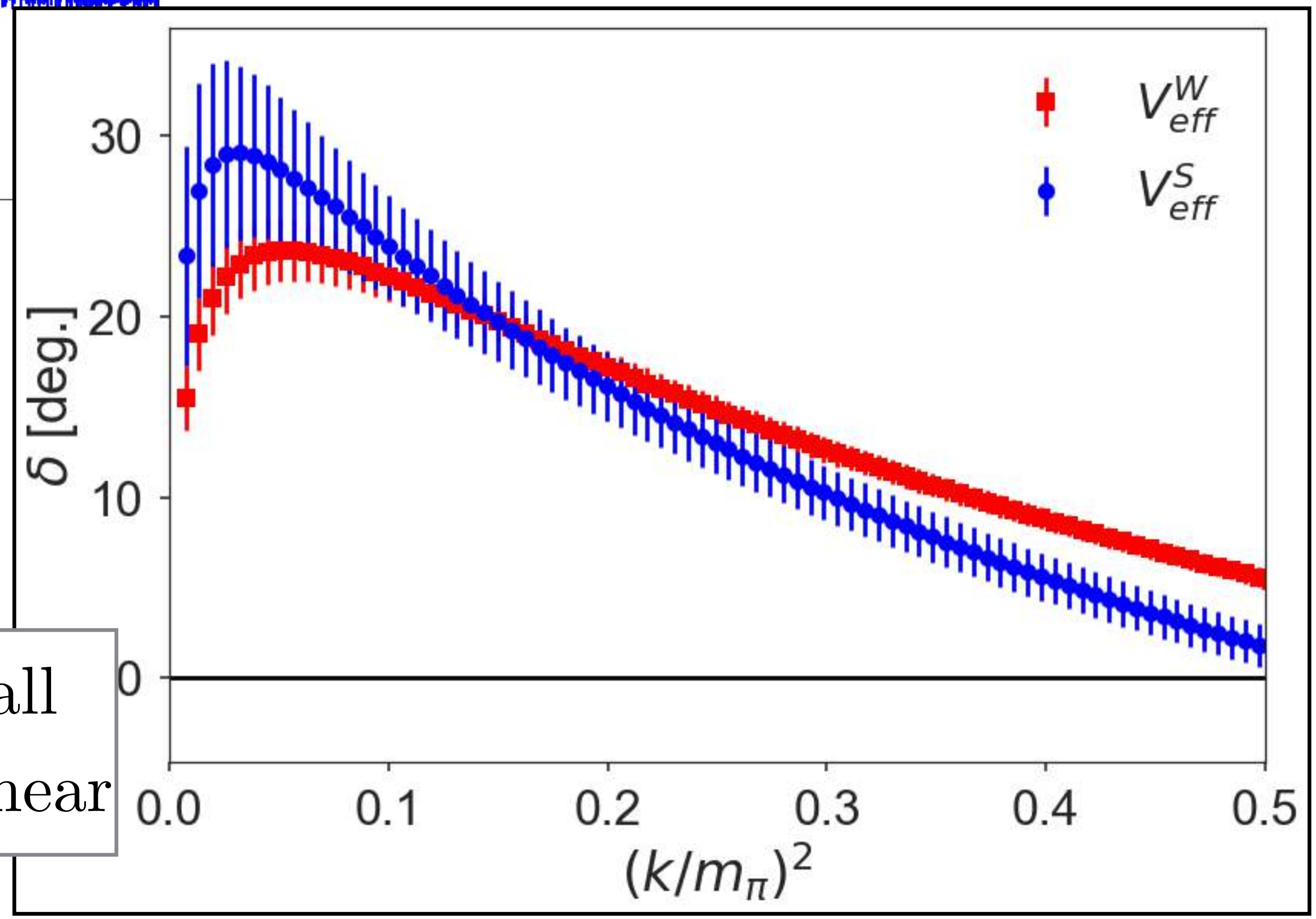
LO potentail



singular behaviors disappear.

Phase shift at LO analysis



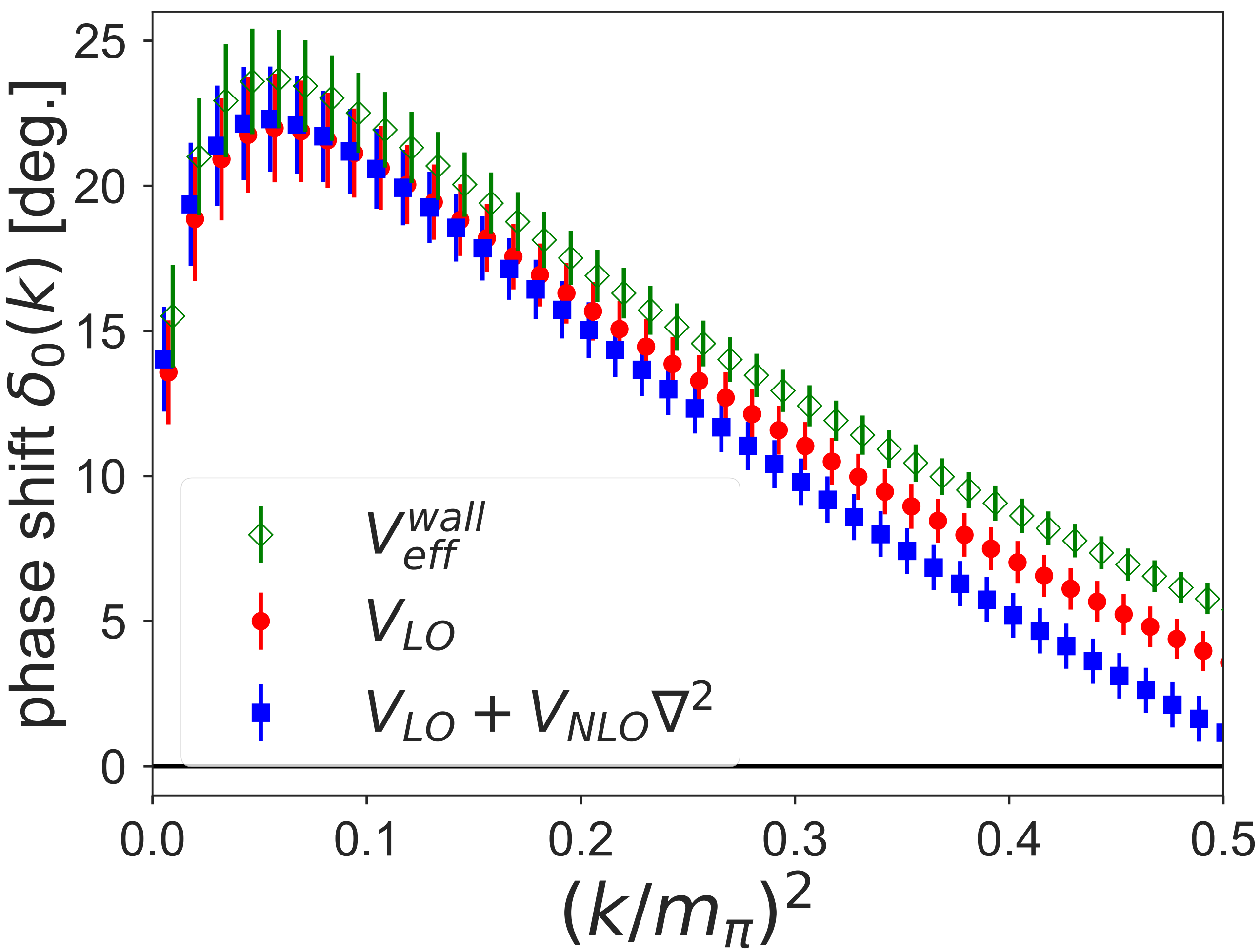


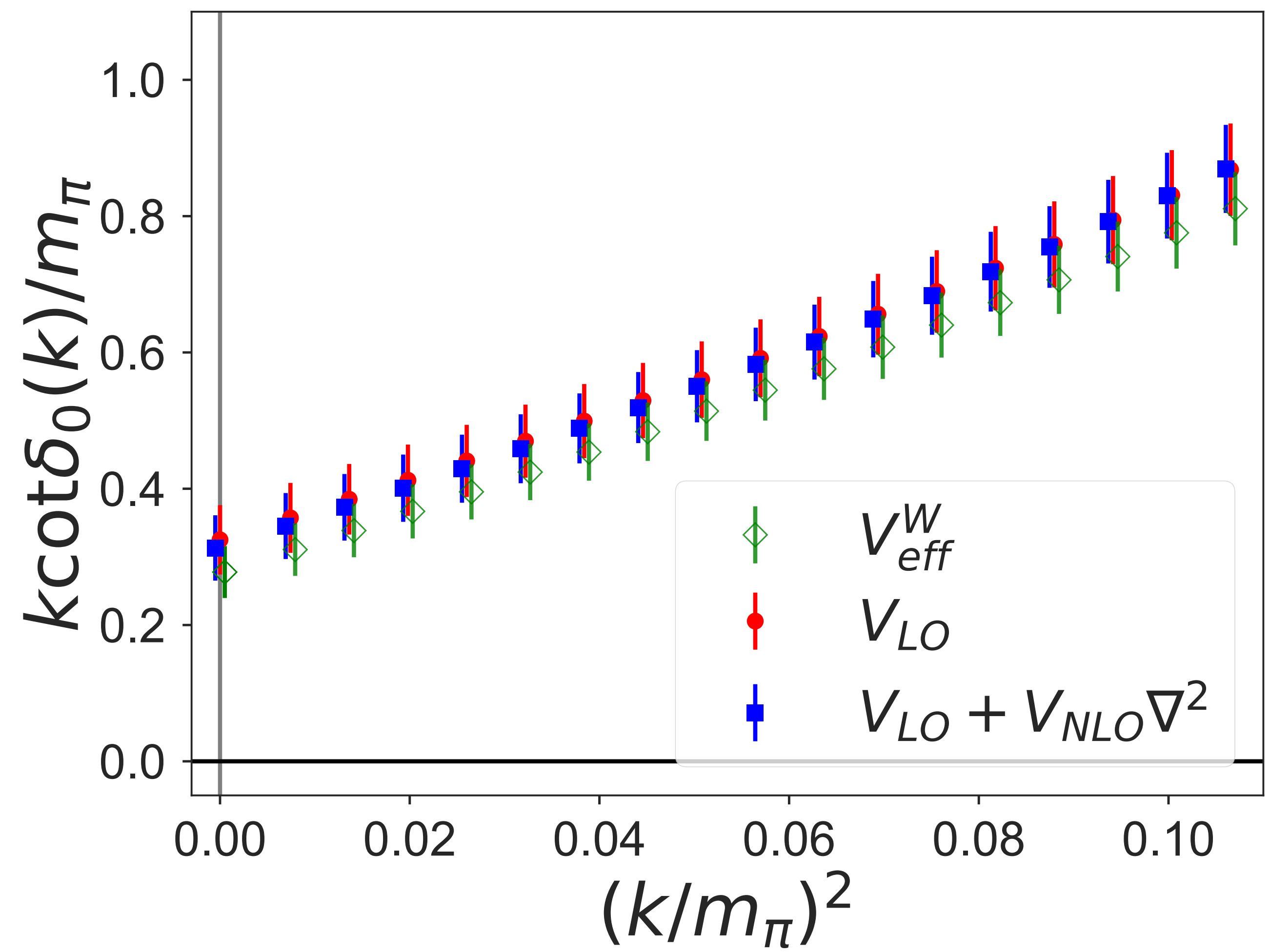
src. dep

$$a_0 m_\pi = \begin{cases} 3.7(3) & \text{wall} \\ 6.6(2) & \text{smear} \end{cases}$$

some source dependence is also observed in phase shifts.

Phase shift at NLO analysis





NLO correction appears at high momentum.

Derivative expansion works well at NLO.

LO approximation from the wall source also works rather well.

Convergence of velocity expansion

PTP 125 (2011)1225.

K. Murano, N. Ishii, S. Aoki, T. Hatsuda

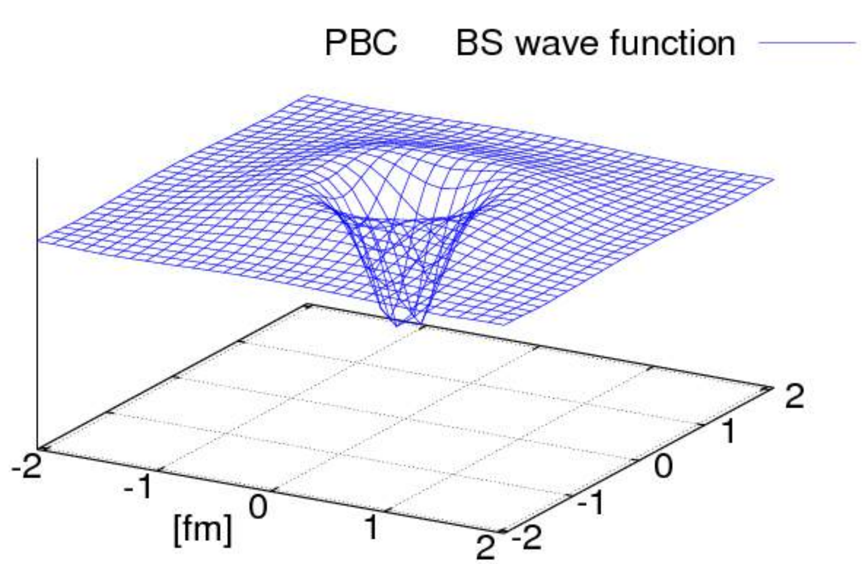
If the higher order terms are large, LO potentials determined from NBS wave functions at different energy become different.(cf. LOC of ChPT).

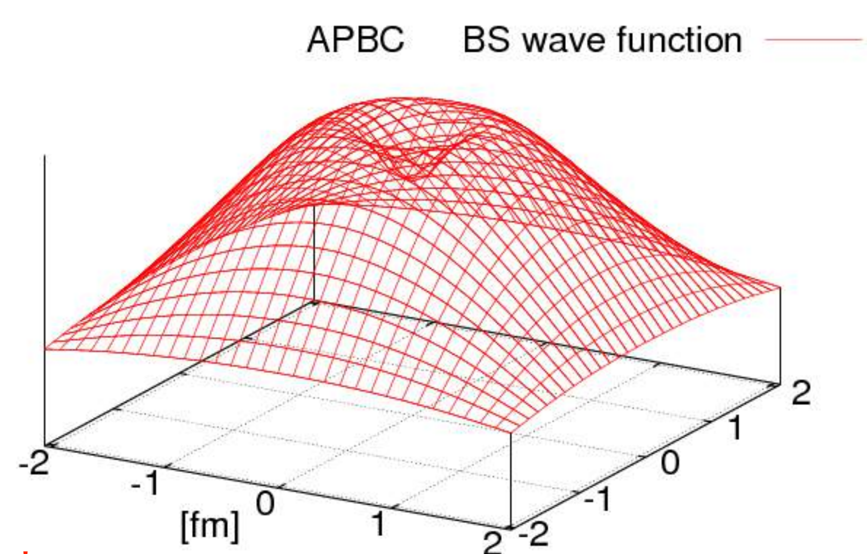
Numerical check in quenched QCD

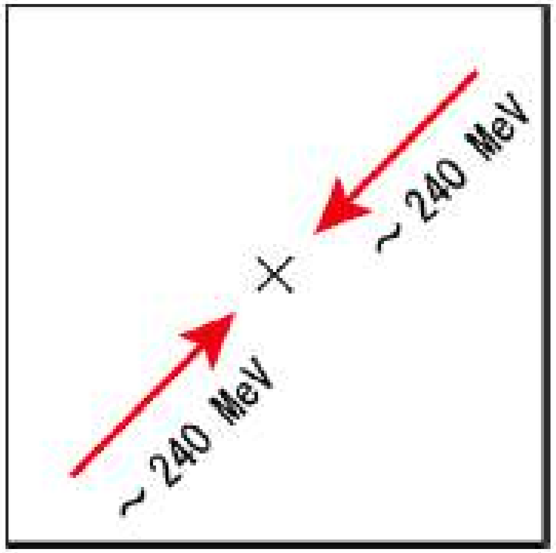
a=0.137fm, L=4.0 fm $m_\pi \simeq 0.53$ GeV

- PBC (E∼0 MeV)
- APBC (E∼46 MeV)

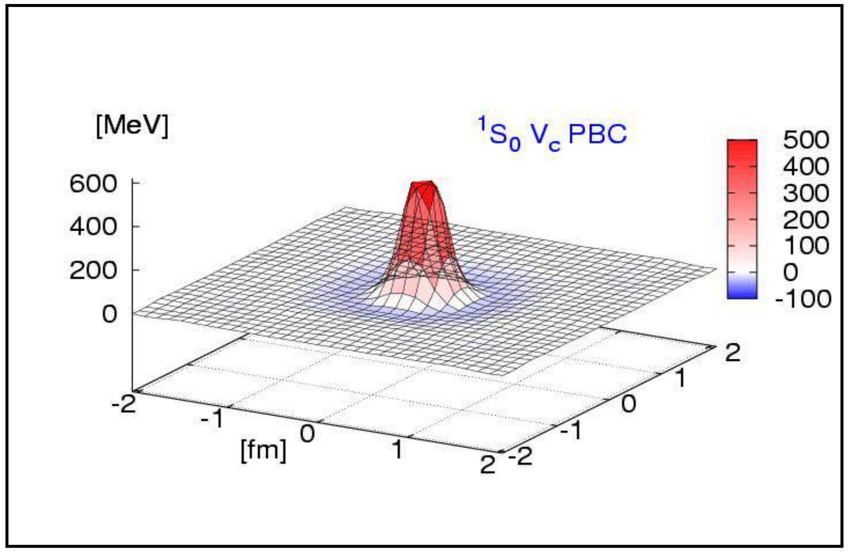
NBS wave functions

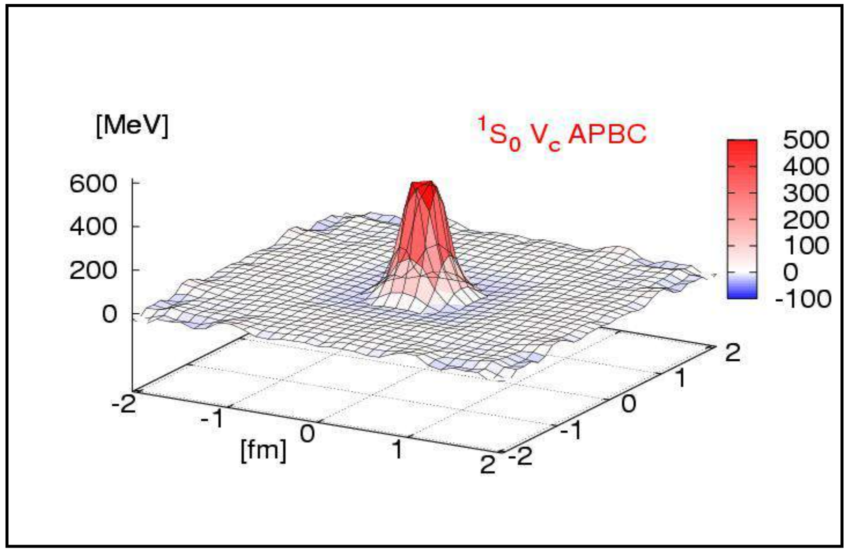


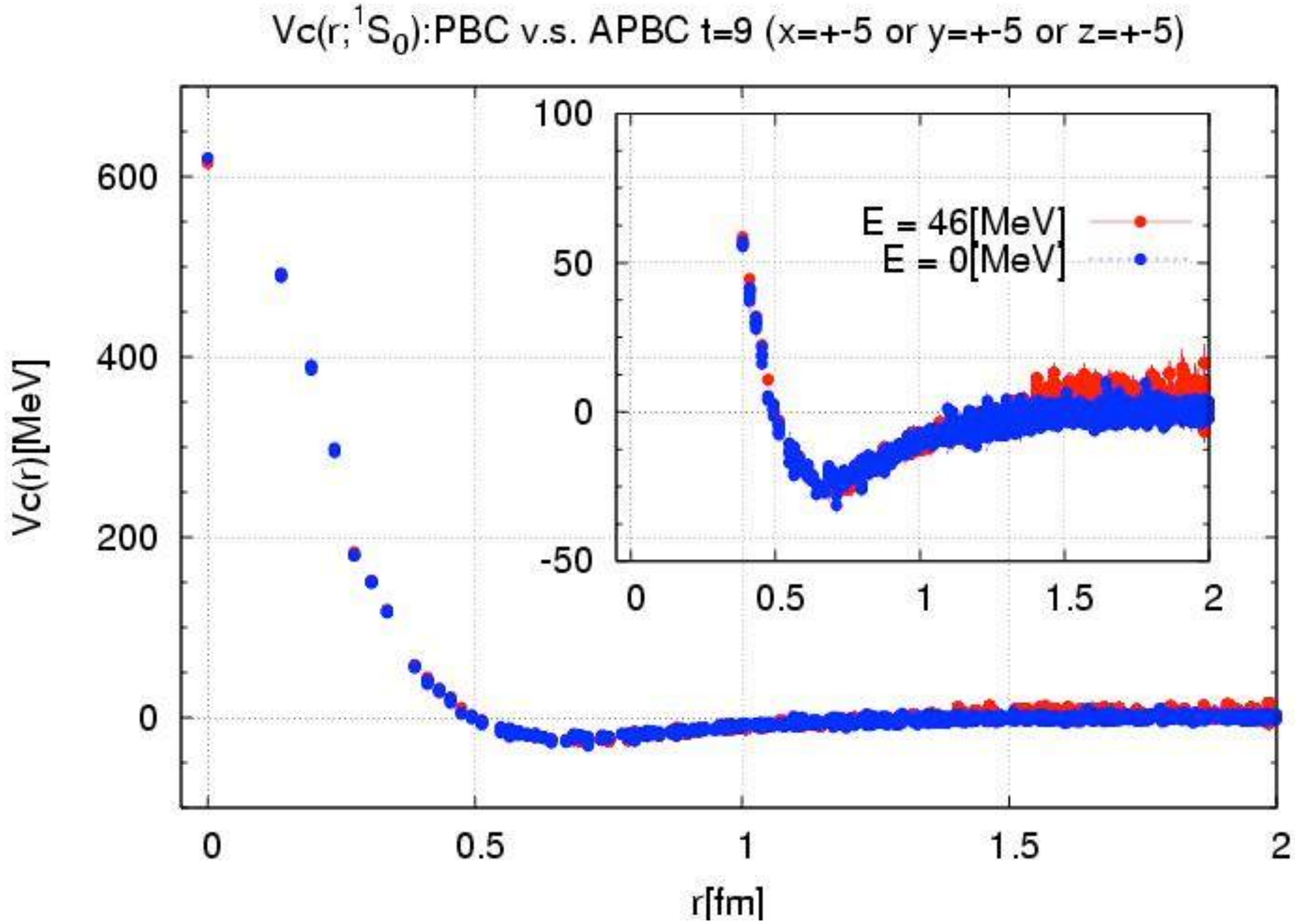


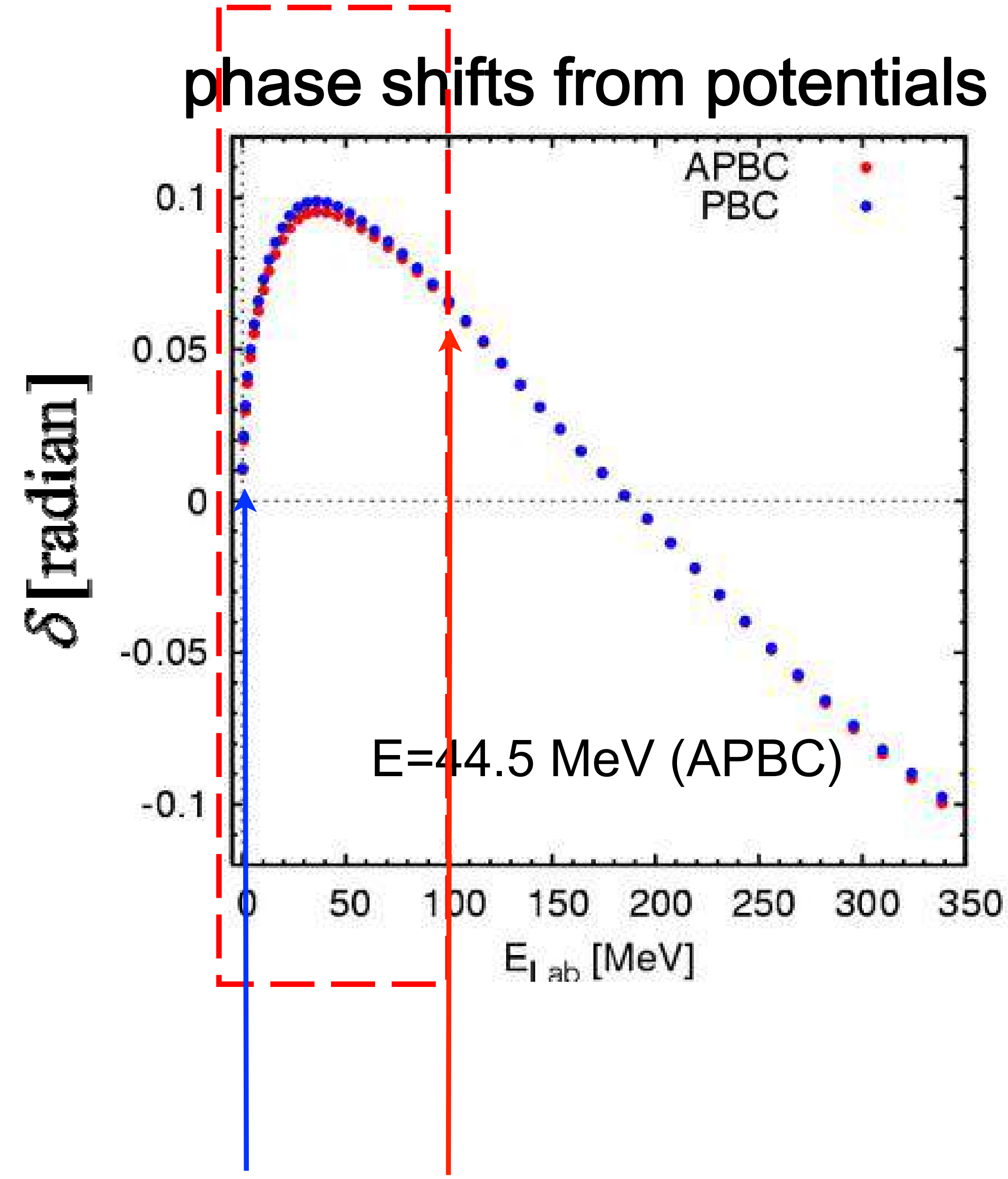


potentials









Higher order terms turn out to be very small at low energy in our scheme.

$\delta_0(\epsilon_k \simeq 0)$ exact

$\delta_0(\epsilon_k \simeq 92\mathrm{MeV})$ exact

$\delta_0(\epsilon_k \simeq 0)$ approx.

$\delta_0(\epsilon_k \simeq 92\mathrm{MeV})$ approx.

Note: convergence of the velocity expansion can be checked within this method.

(in contrast to convergence of ChPT, convergence of perturbative QCD)

Note: Old method.

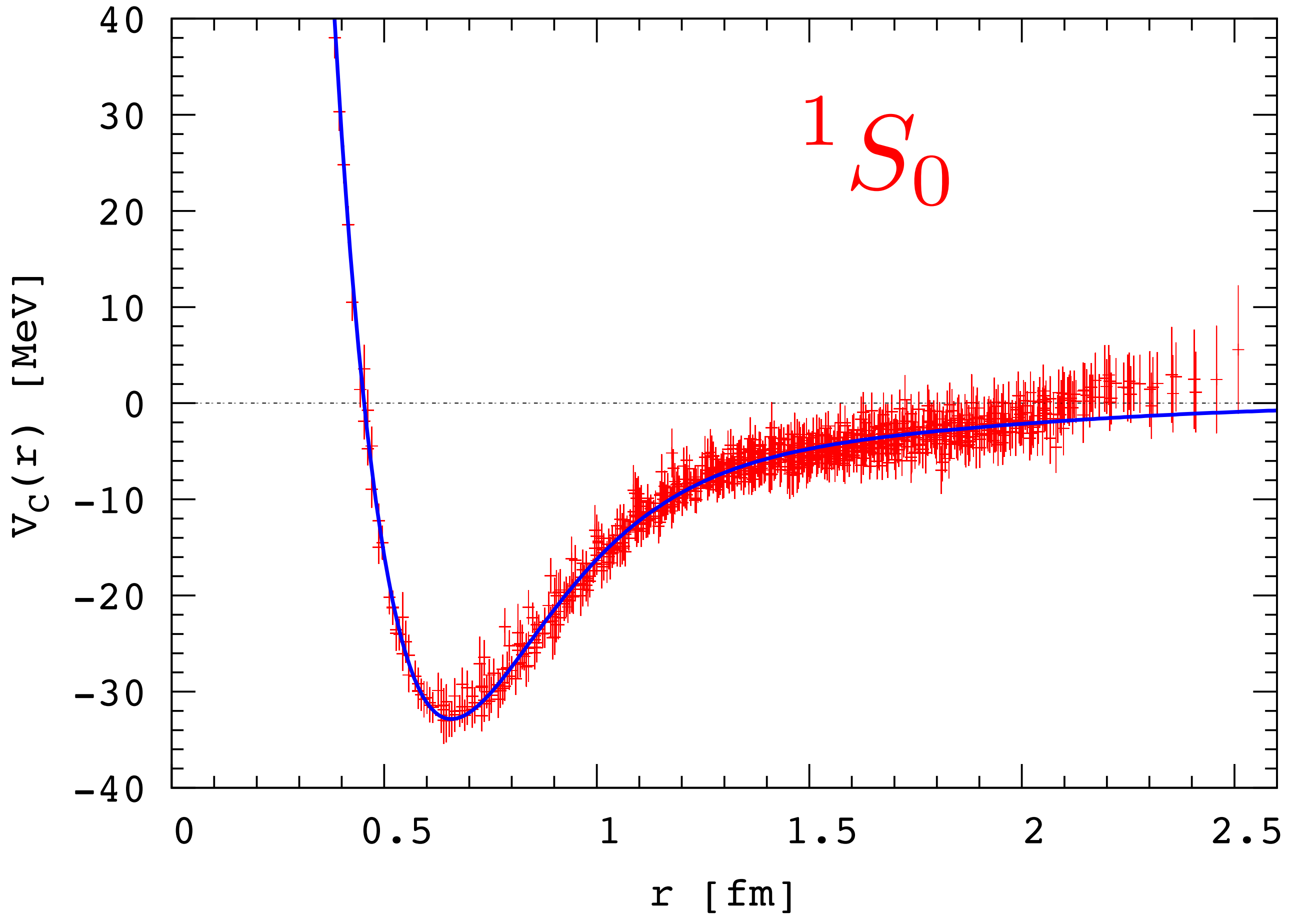
2+1 flavor QCD

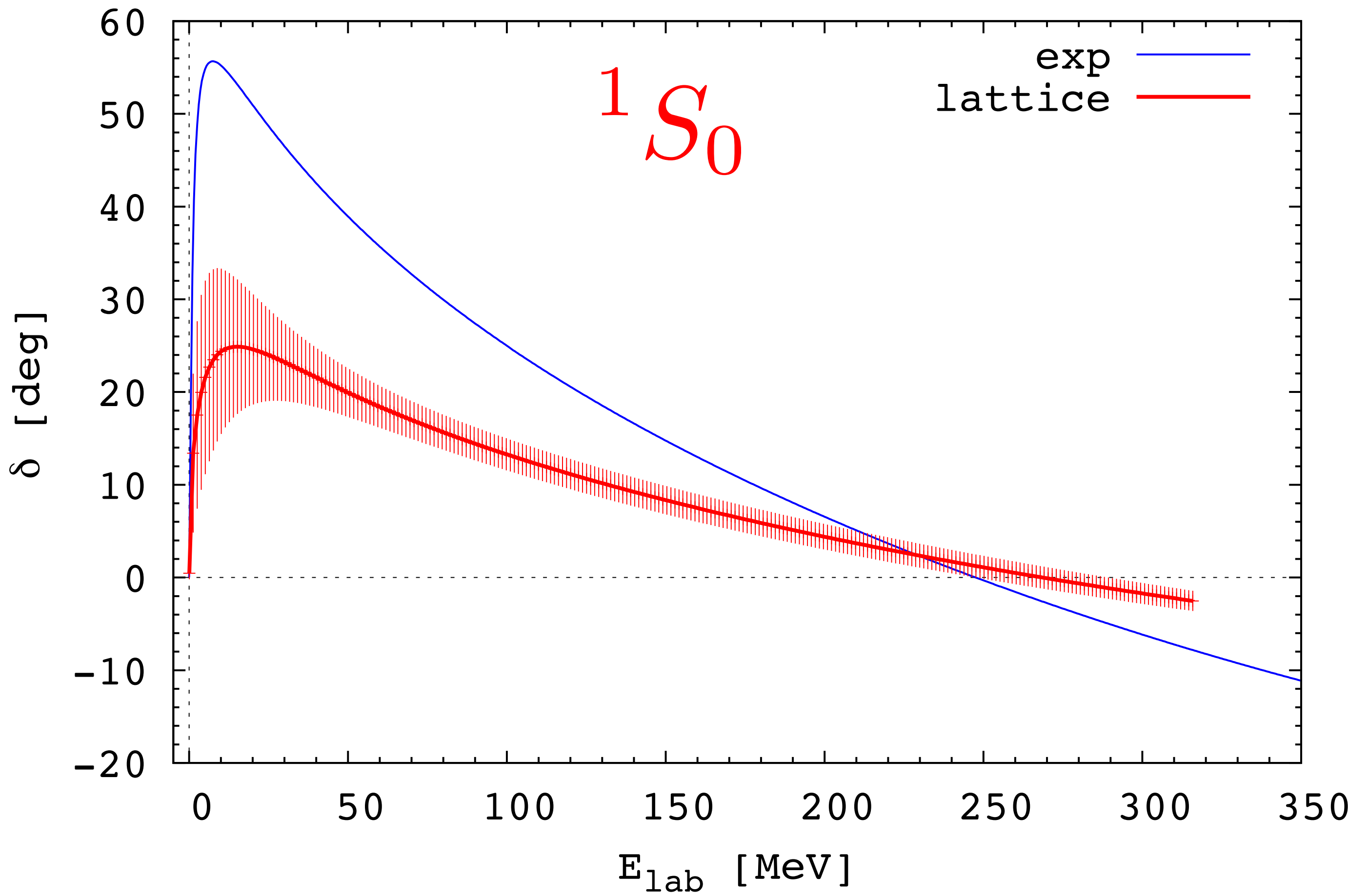
a=0.09fm, L=2.9fm

$m_\pi \simeq 700$ MeV

Ishii et al. (HALQCD), PLB712(2012) 437.

NN potential → phase shift





Qualitative features of NN potential are reproduced.

→ It has a reasonable shape. The strength is weaker due to the heavier quark mass.

No dineutron at heavier pion mass.

Frequently Asked Questions

[Q1] Scheme/Operator dependence of the potential

- The potential depends on the definition of the wave function, in particular, on the choice of the nucleon operator N(x). (Scheme-dependence)
  - local operator -> manifest causality
  - a similar example: running coupling is scheme-dependent
- Moreover, the potential itself is NOT a physical observable. Therefore it is NOT unique and is naturally scheme-dependent.
  - Observables: scattering phase shift of NN, binding energy of deuteron

QM: (wave function, potential) -> observables
QFT: (asymptotic field, vertex) -> observables
EFT: (choice of field, vertex) -> observables

- Is the scheme-dependent potential useful ? Yes !
  - useful to understand/describe physics
  - cf. running coupling: it is useful to understand the deep inelastic scattering data (asymptotic freedom)
- "good" scheme ?
  - good convergence of the perturbative expansion for the running coupling
  - good convergence of the velocity expansion for the potential ?
    - local operator is found to be "good" (see later)
    - completely local and energy-independent one is the best (Inverse scattering method)

[Q2] Energy dependence of the potential

Non-local & Energy-independent

$$[\epsilon_k - H_0]\,\varphi_{\mathbf{k}}(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_{\mathbf{k}}(\mathbf{y})$$

Local & Energy-dependent

$$[\epsilon_k - H_0]\,\varphi_{\mathbf{k}}(\mathbf{x}) = V_{\mathbf{k}}(\mathbf{x})\varphi_{\mathbf{k}}(\mathbf{x})$$

non-locality can be determined order by order in the velocity expansion (“scheme”)

$$V(\mathbf{x},\nabla) = V_0(r) + V_\sigma(r)(\sigma_{\mathbf{1}}\cdot\sigma_{\mathbf{2}}) + V_T(r)S_{12} + V_{\mathrm{LS}}(r)\mathbf{L}\cdot\mathbf{S} + \{V_D(r),\nabla^2\} + \cdots$$

[Q3] How good is the velocity expansion ?

Leading order $$V_C(r) = \frac{(\epsilon_k - H_0)\varphi_{\mathbf{k}}(\mathbf{x})}{\varphi_{\mathbf{k}}(\mathbf{x})}$$

The local potential obtained at given energy may depend on its energy.
If the energy dependence of the potential is weak, the LO potential is good.

numerical check in quenched QCD

II. Recent results (selected)

1. Dibaryons

Dibaryon = a bound state with baryon number B=2

Only deuteron is the dibaryon observed in Nature so far.

Other candidates.

H-dybaryon (uuddss)

Omega-Omega (ssssss)

N-Omega

Delta-Delta

Some of them are investigated in the potential method.

$\Omega\Omega$

(ssssss)

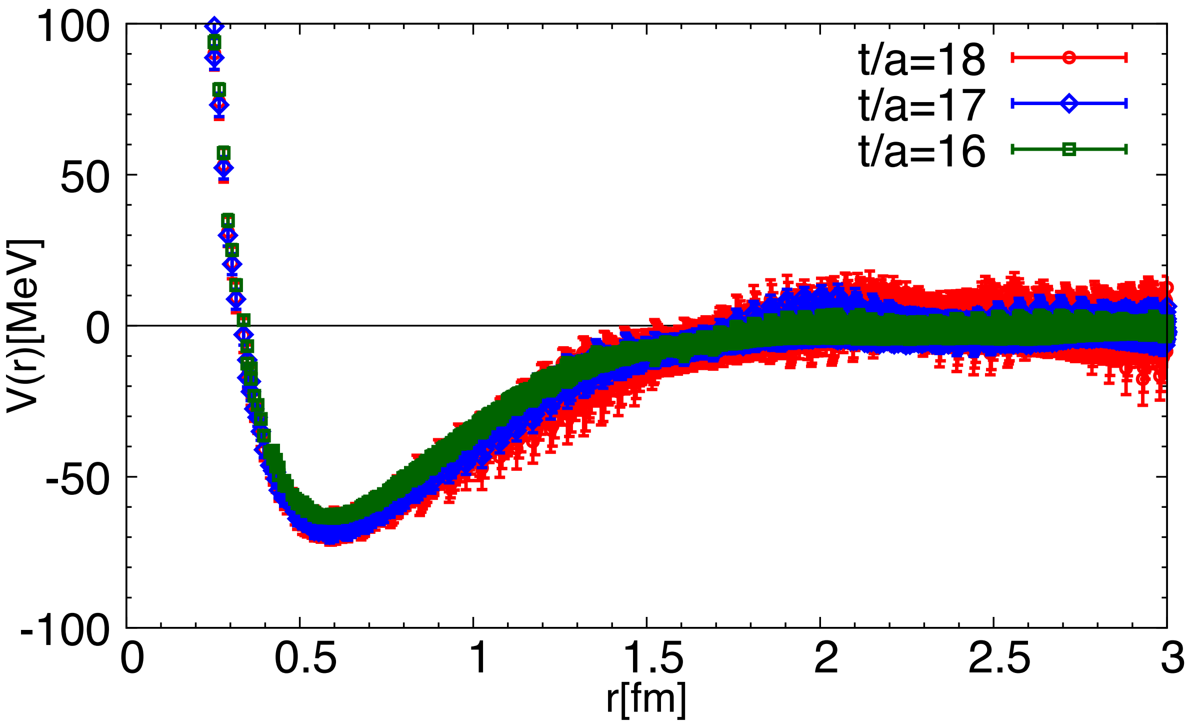
S. Gongyo et al. arXiv;1709.000654

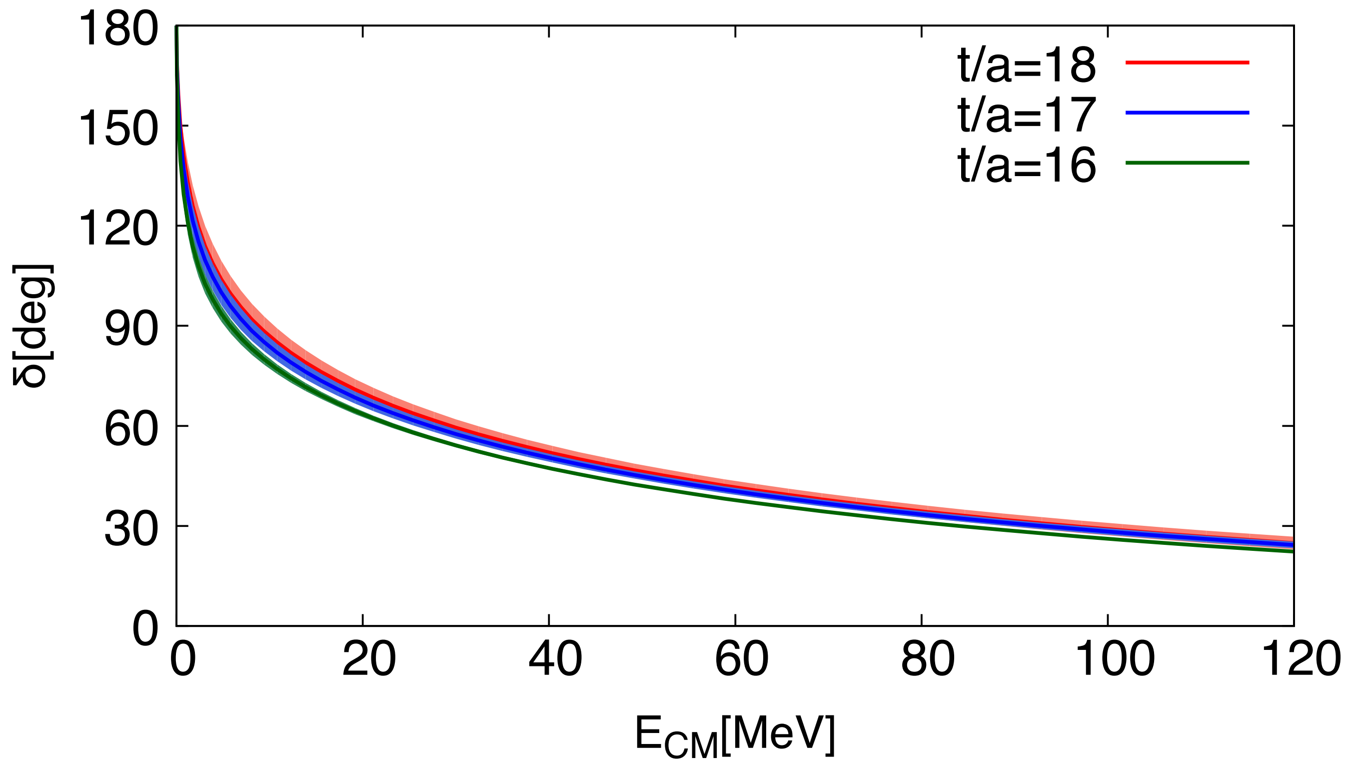
Lattice QCD at physical pion mass

2+1 flavor QCD, $m_\pi \simeq 145$ MeV, $a \simeq 0.085$ fm, $L \simeq 8$ fm

K-computer [10PFlops]

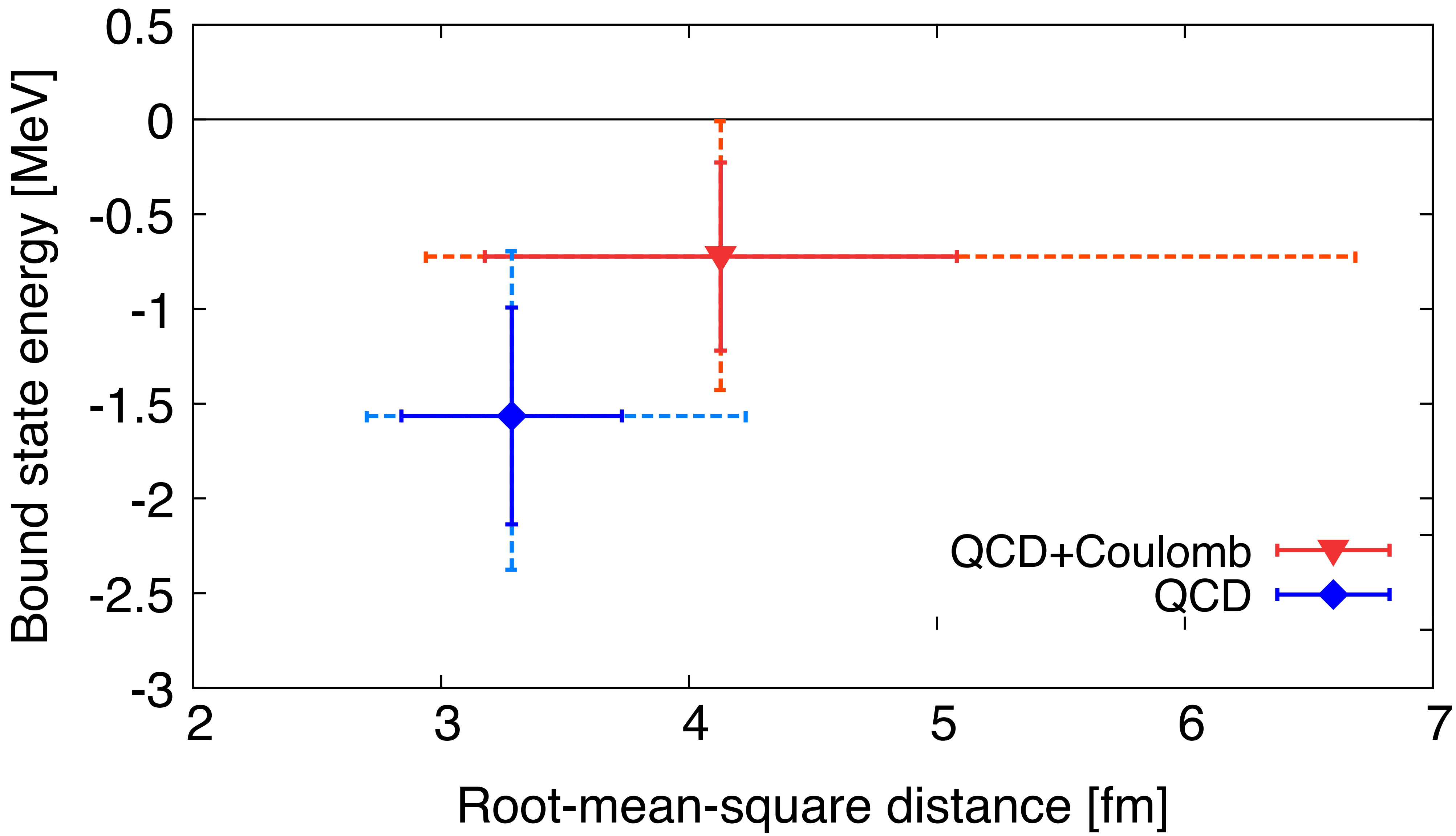
$\Omega\Omega$ potential → phase shift





Strong attraction → **Vicinity of bound/unbound (~ unitary limit)**

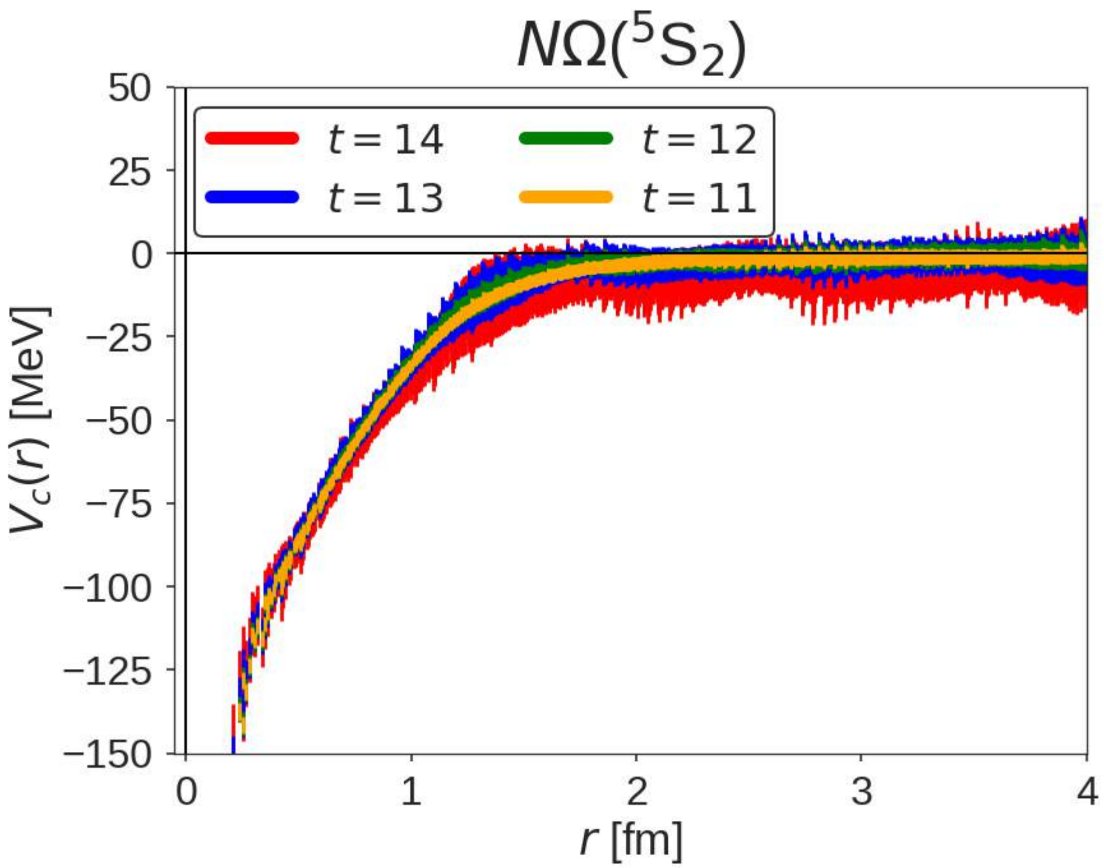
Binding energy



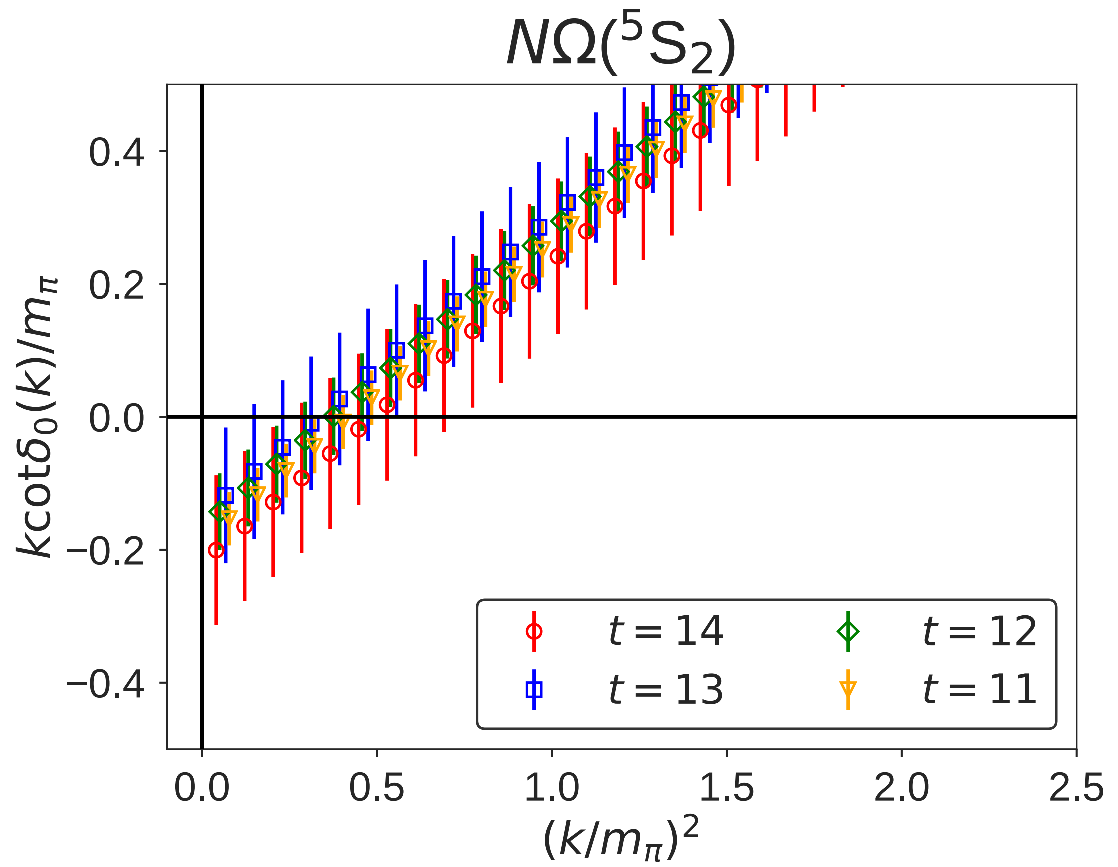
The most strange (sss sss) dibaryon ?

$N\Omega$

Potential



Phase shift



New dibaryon in Heavy Ion Collision ?

Preliminary !

Lattice QCD at physical pion mass

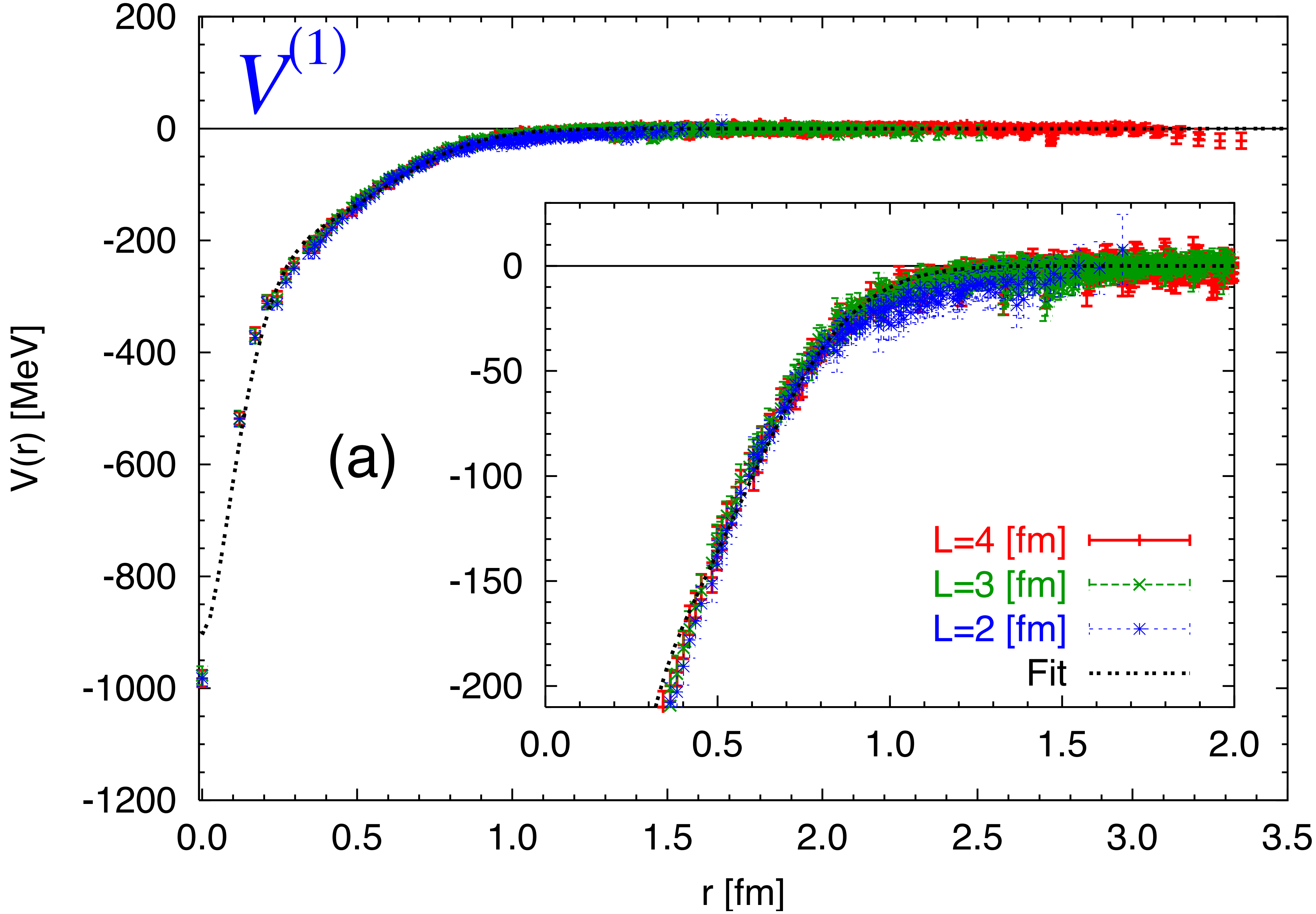
H-dibaryon

(uuddss)

flavor SU(3) singlet potential

Lattice QCD in the flavor SU(3) limit

Inoue *et al.* (HAL QCD Coll.), Progress of Theoretical Physics 124(2010)591

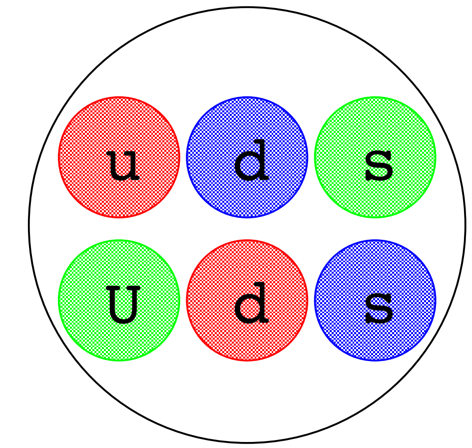


$a \simeq 0.12$ fm

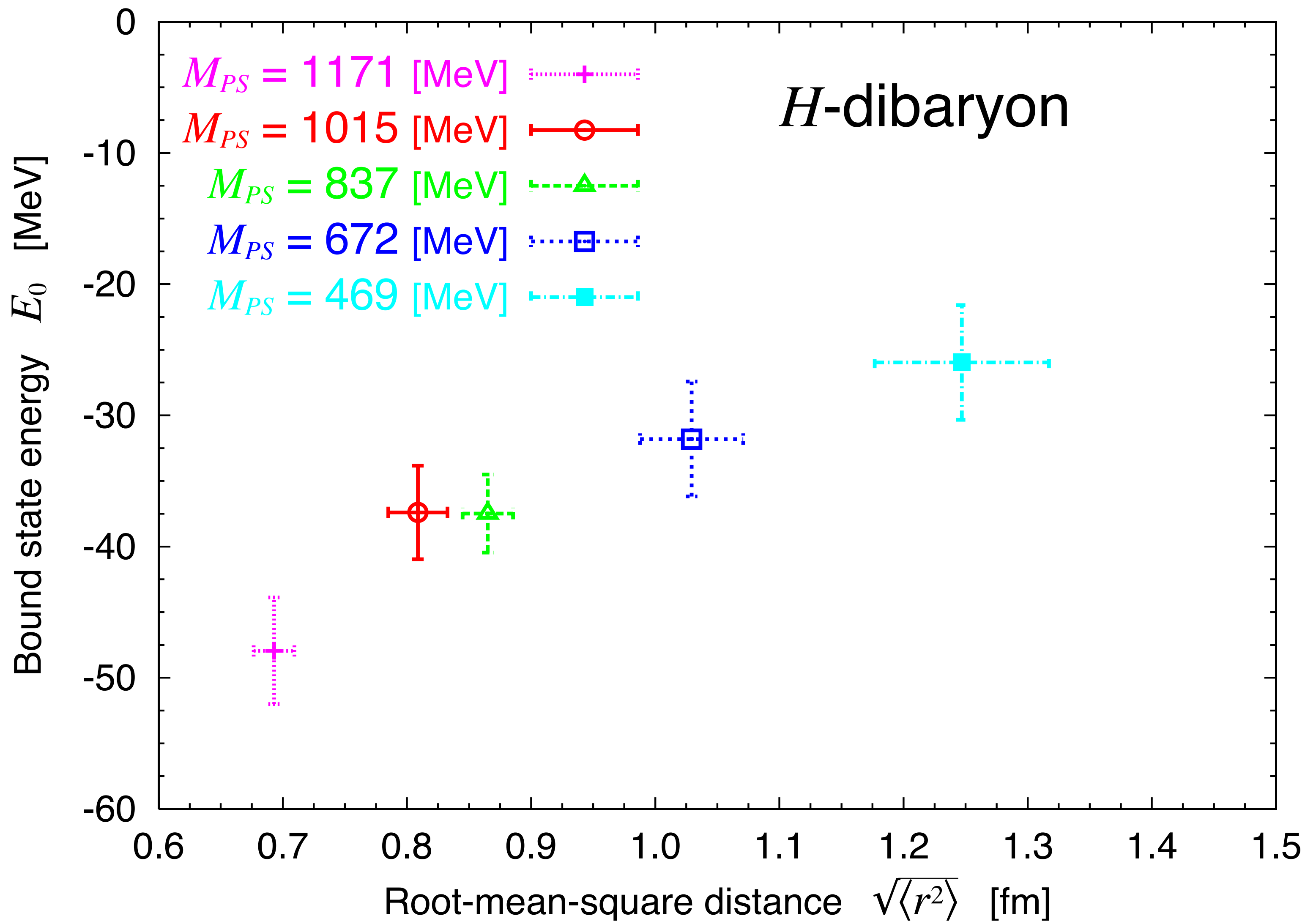
$m_\pi \simeq 470$ MeV

Force is attractive at all distances. Bound state ?

possibility of a bound state (H-dibaryon) $\Lambda\Lambda - N\Xi - \Sigma\Sigma$



Inoue et al. (HAL QCD Coll.), PRL106(2011)162002



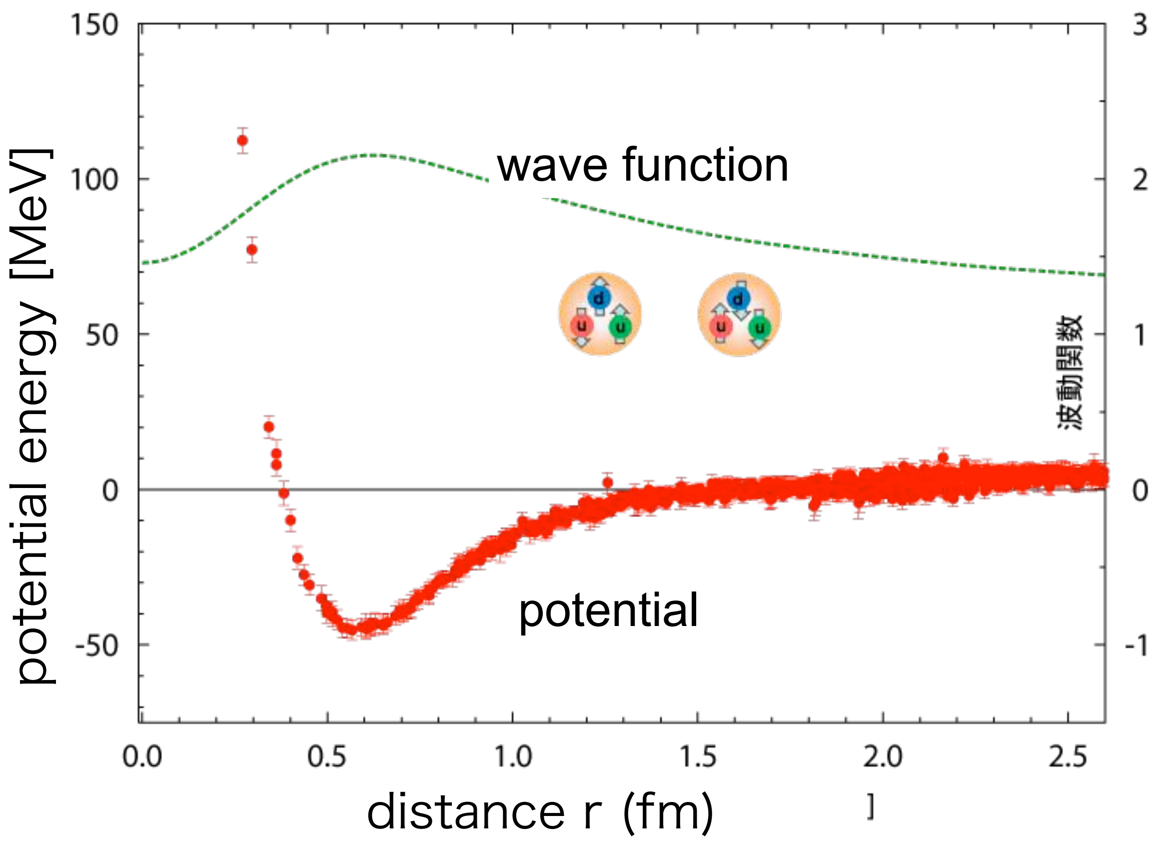
An H-dibaryon exists in the flavor SU(3) limit.

Binding energy = 25-50 MeV at this range of quark mass.

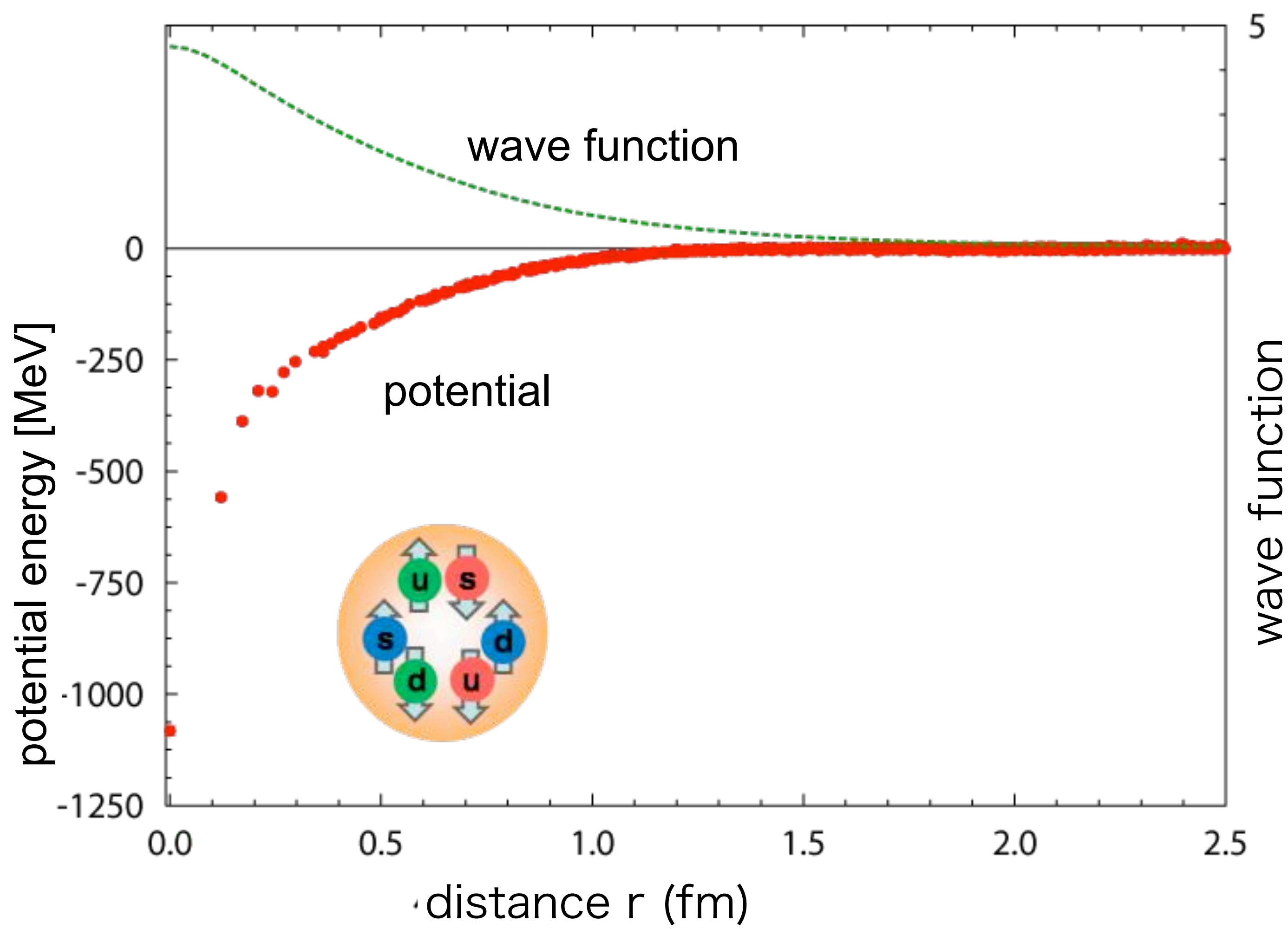
Real world ?

A mild quark mass dependence.

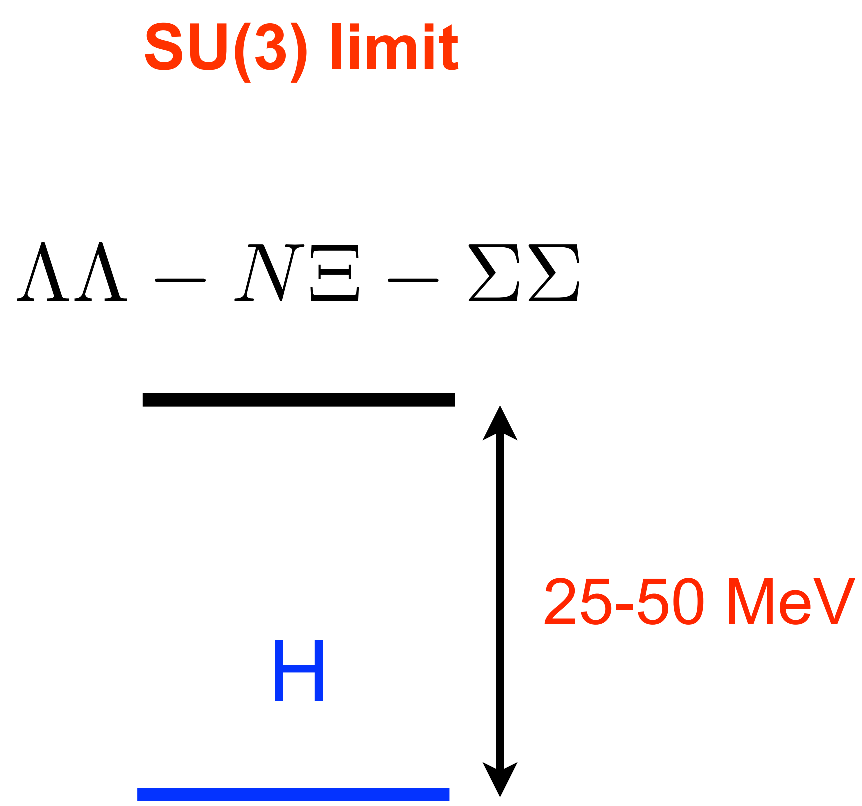
Deuteron

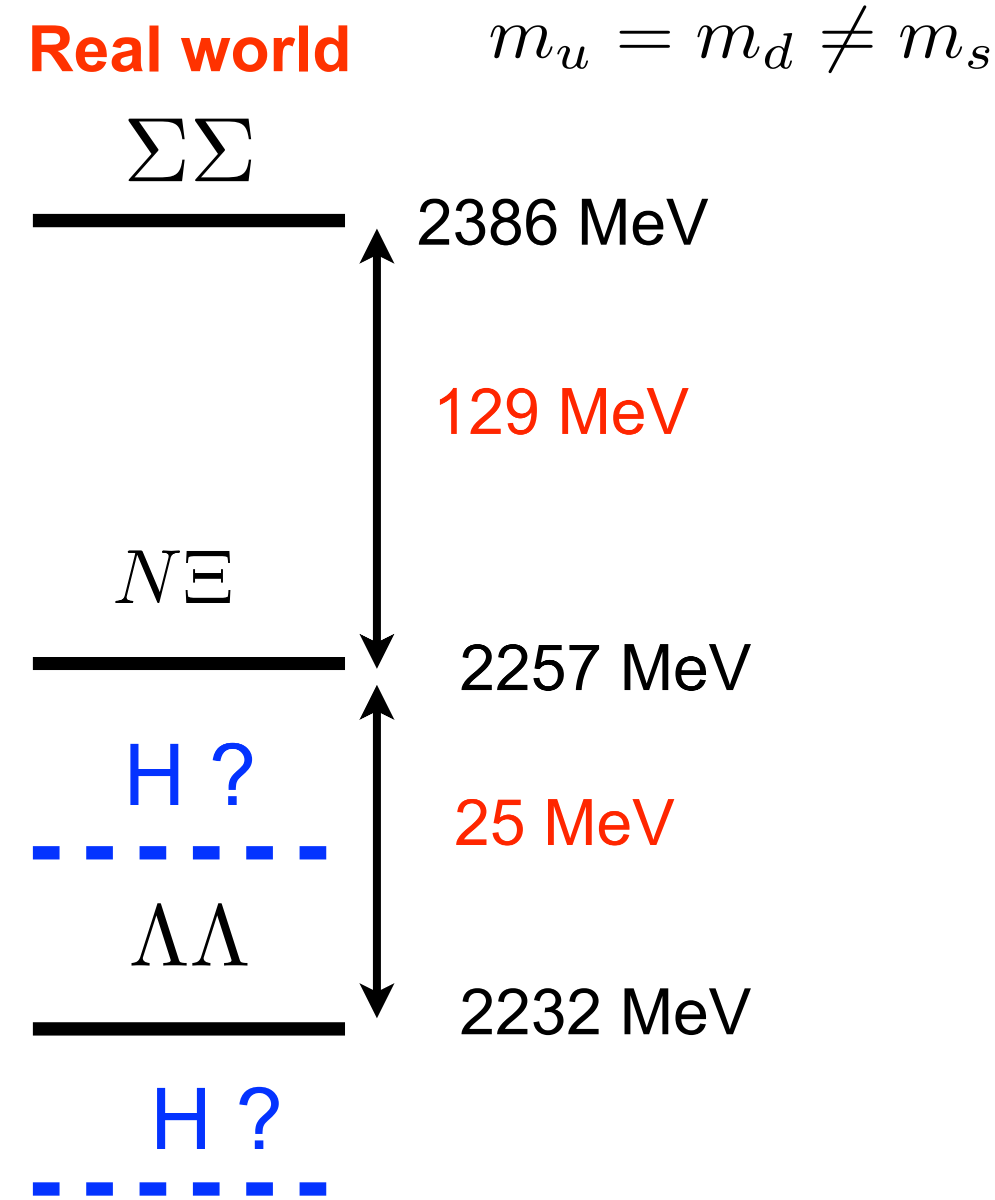


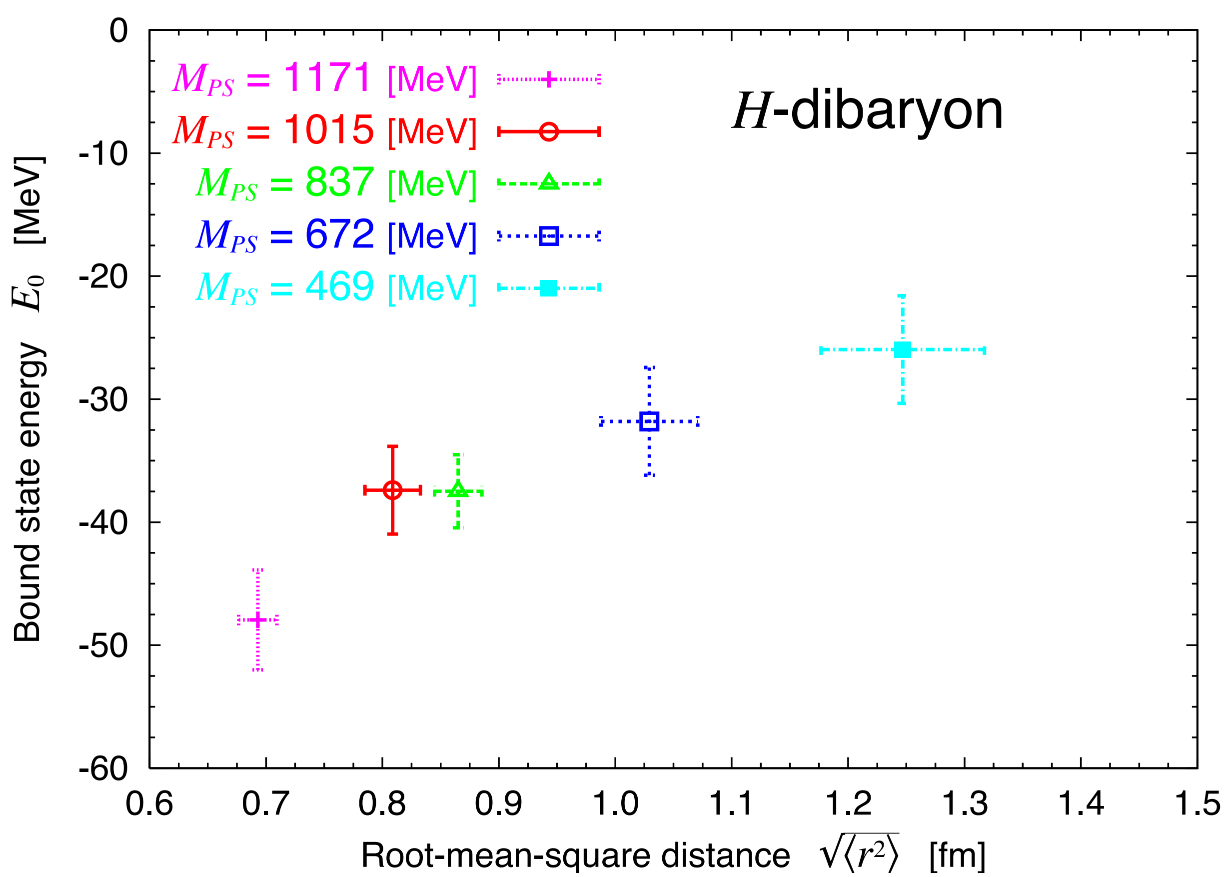
H-dibayon



H-dibaryon with the flavor SU(3) breaking







Preliminary results from HAL QCD Collaboration

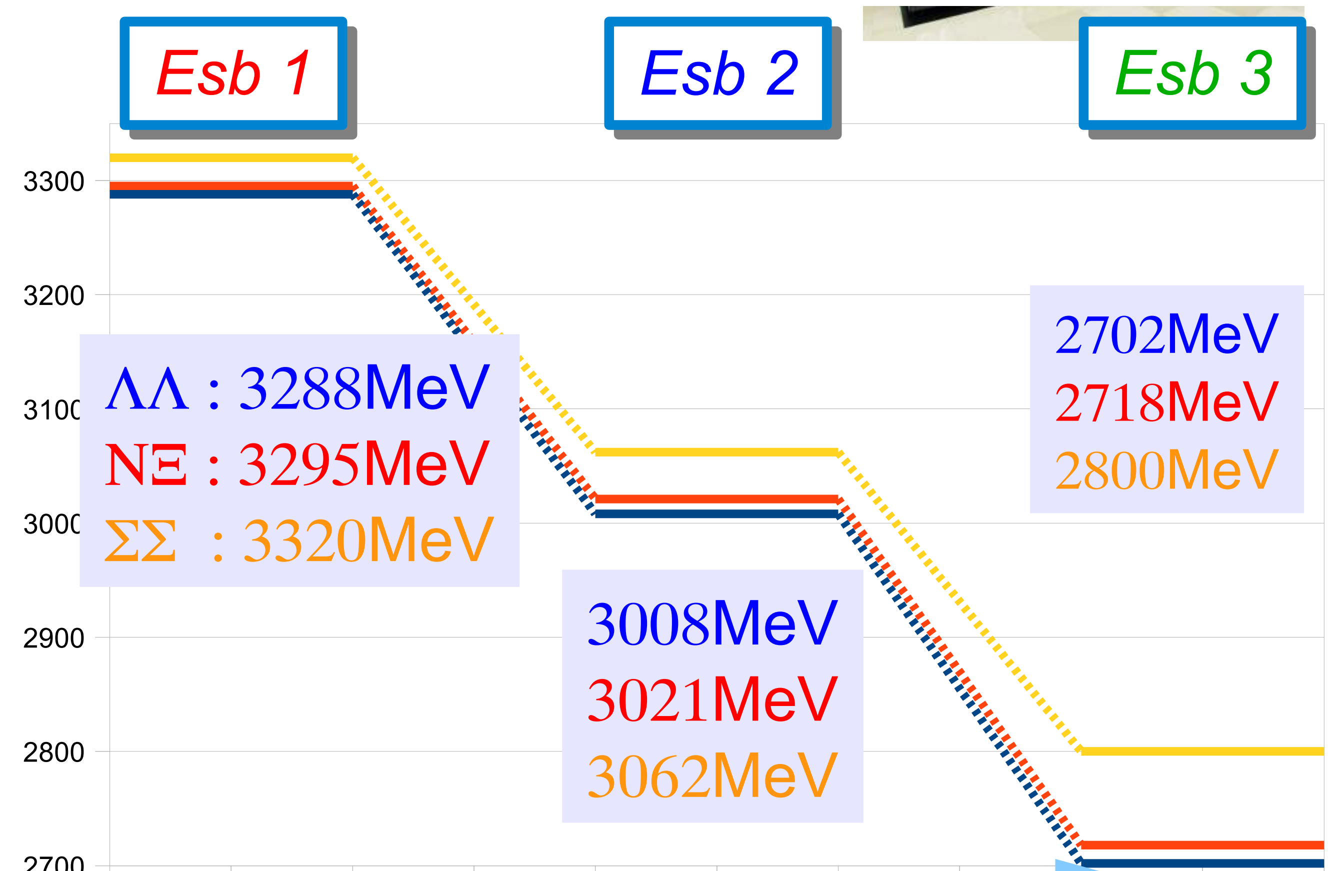
Sasaki for HAL QCD Collaboration

Gauge ensembles

In unit of MeV	Esb 1	Esb 2	Esb 3
π	701±1	570±2	411±2
K	789±1	713±2	635±2
m_π/m_K	0.89	0.80	0.65
N	1585±5	1411±12	1215±12
Λ	1644±5	1504±10	1351± 8
Σ	1660±4	1531±11	1400±10
Ξ	1710±5	1610± 9	1503± 7

u,d quark masses lighter

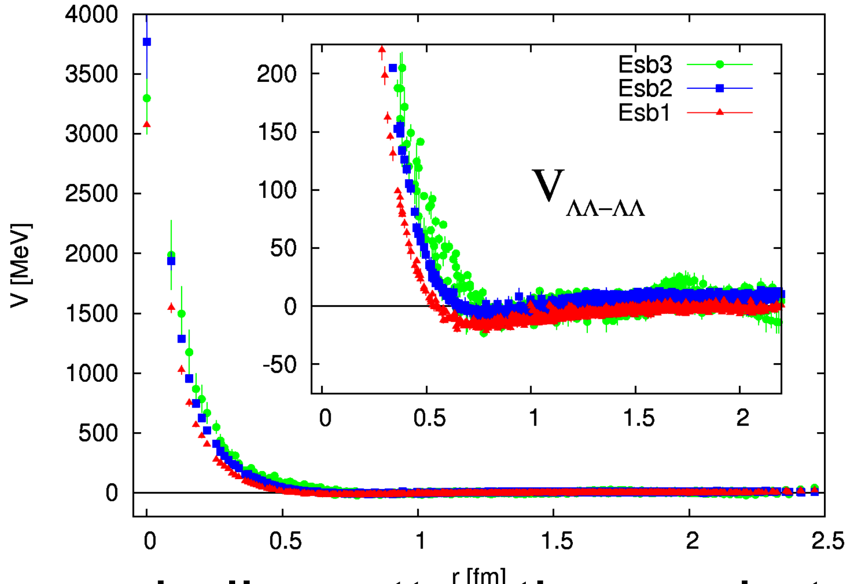
thresholds



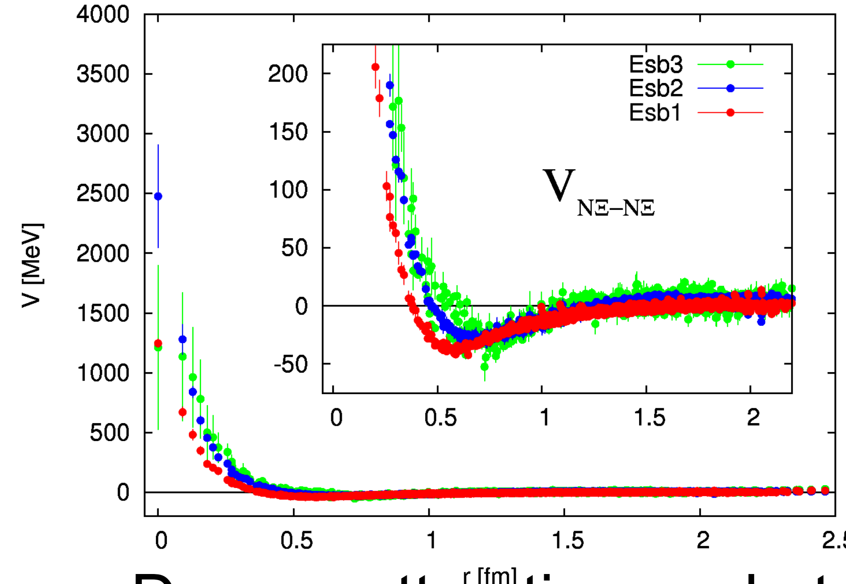
SU(3) breaking effects becomes larger

coupled channel 3x3 potentials

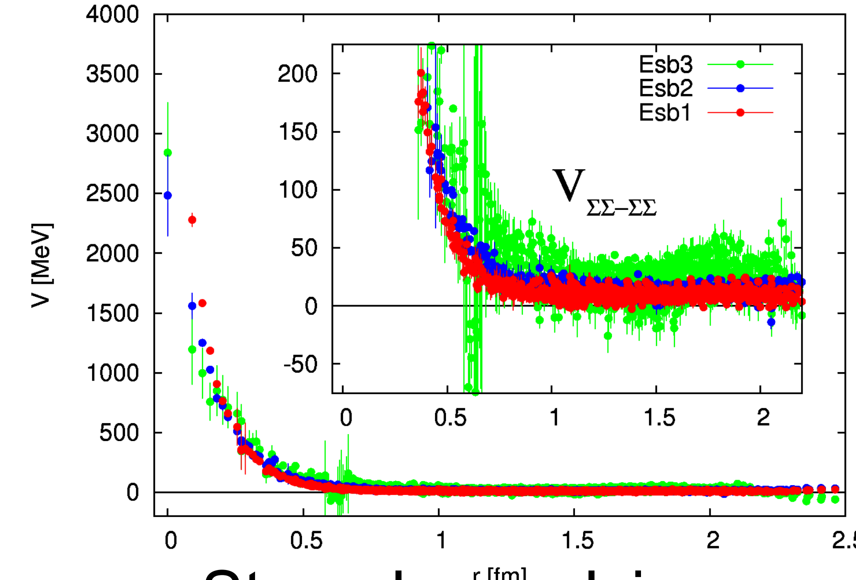
Diagonal elements



shallow attractive pocket



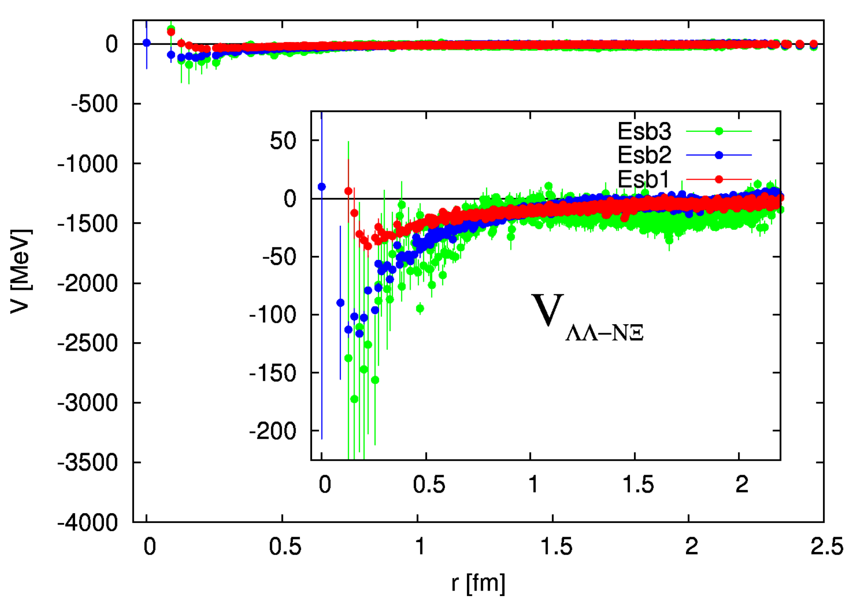
Deeper attractive pocket

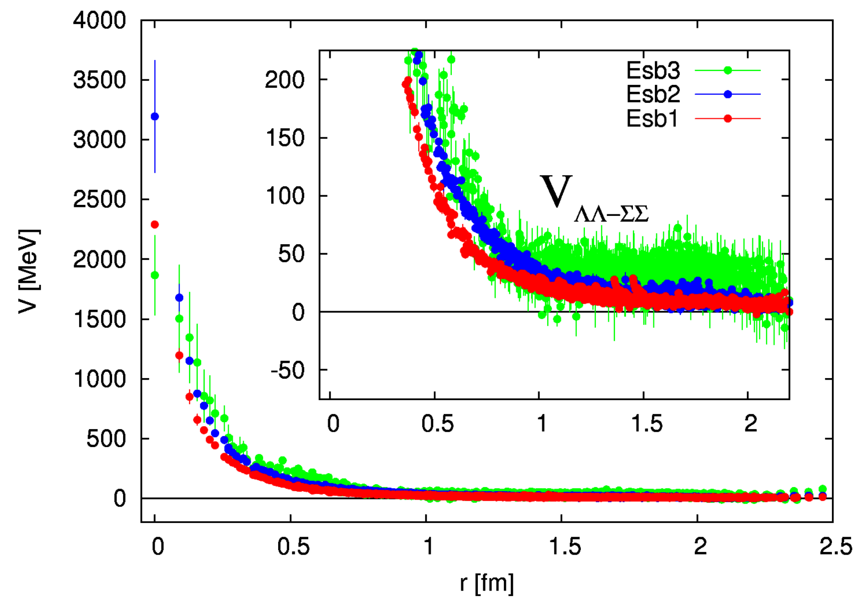


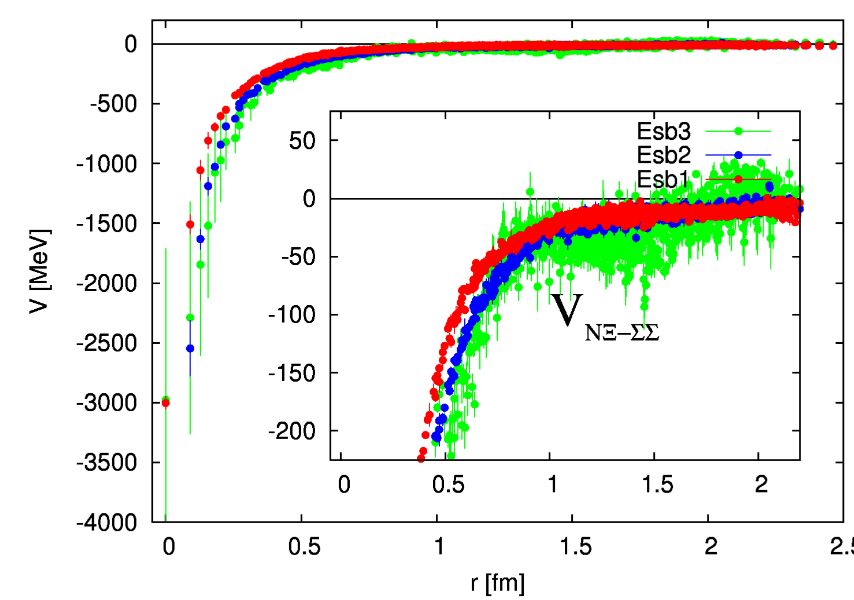
Strongly repulsive

All channels have repulsive core

Off-diagonal elements



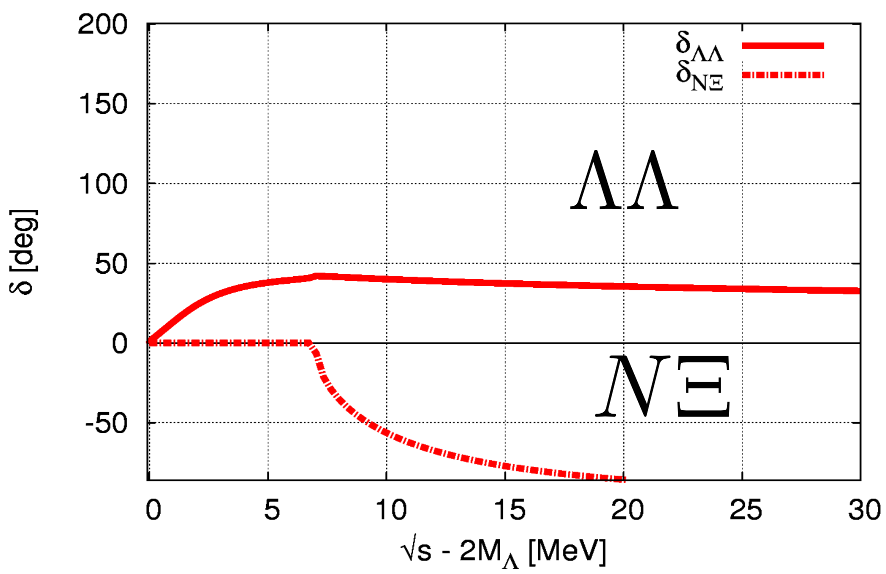




$\Lambda\Lambda$ and $N\Xi$ phase shift

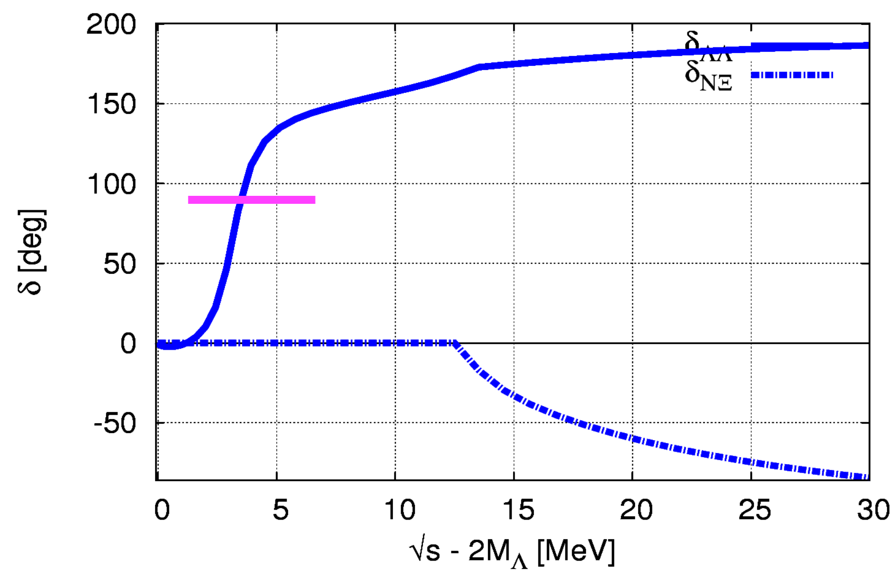
Preliminary !

Esb1 : m_π= 701 MeV



Bound H-dibaryon coupled to NΞ

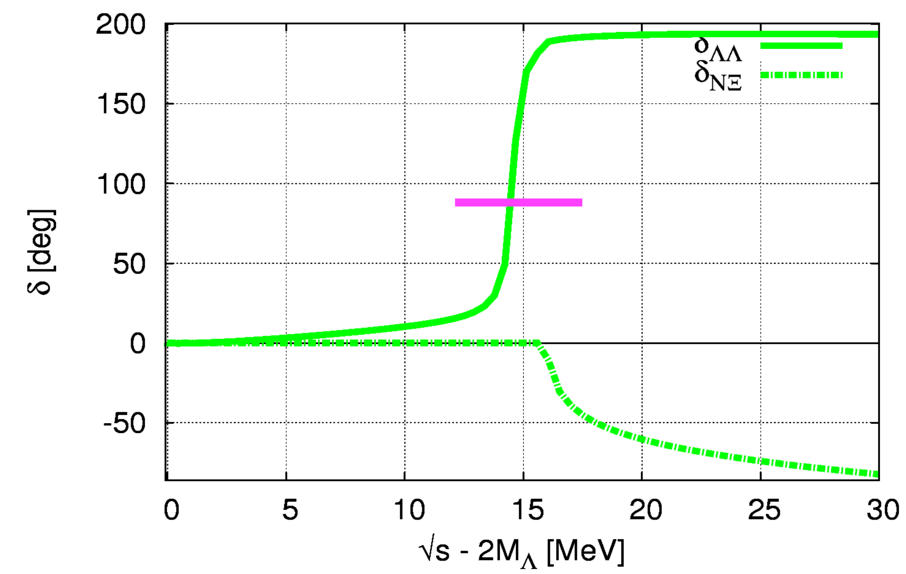
Esb2 : m_π= 570 MeV



H as ΛΛ resonance

H as bound NΞ

Esb3 : m_π= 411 MeV



H as ΛΛ resonance

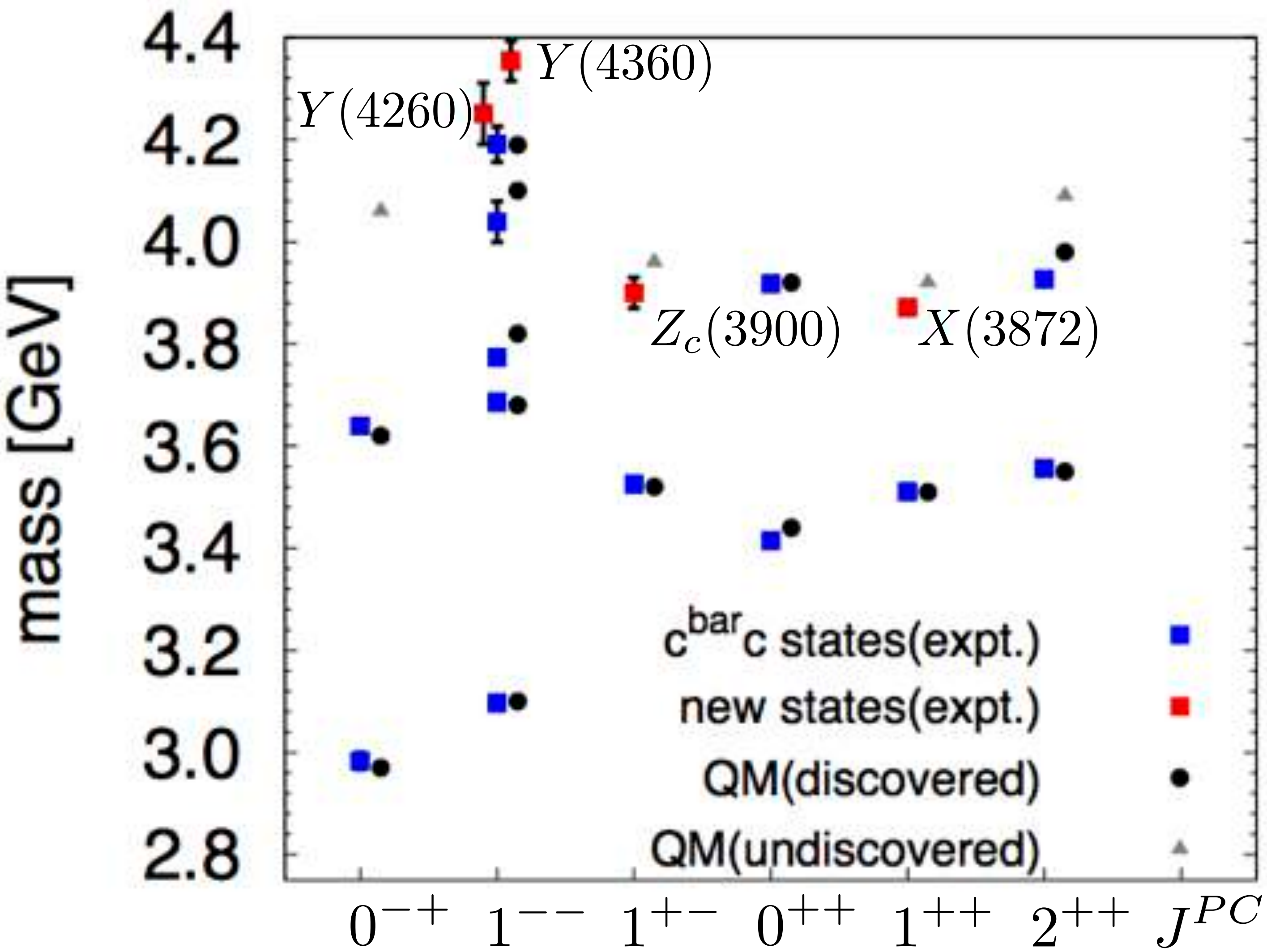
H as bound NΞ

This suggests that H-dibaryon becomes resonance at physical point.
Below or above NΞ ?
=> Simulation at physical point on K-computer

Physically, it is essential that H-dibaryon is a bound state in the flavor SU(3) limit.

2. Exotic hadron Zc(3900)

Charmonium(-like) spectrum



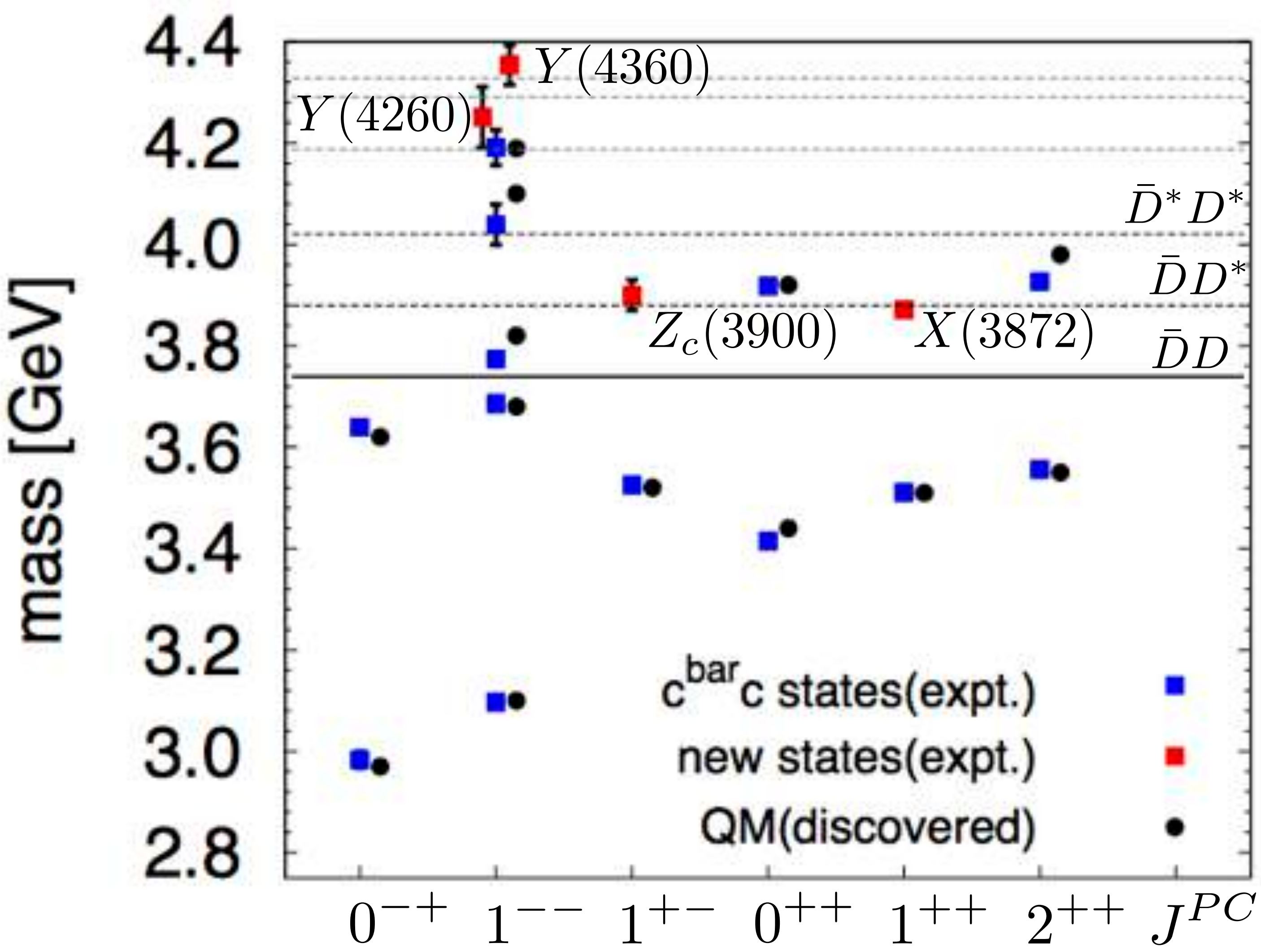
Quark model well describes observed mass spectra below 3.8 GeV.

Several states above 3.8 GeV are not discovered.

New (X,Y,Z) states, not predicted by QM, are experimentally observed.

Exotic ?

Charmonium(-like) spectrum



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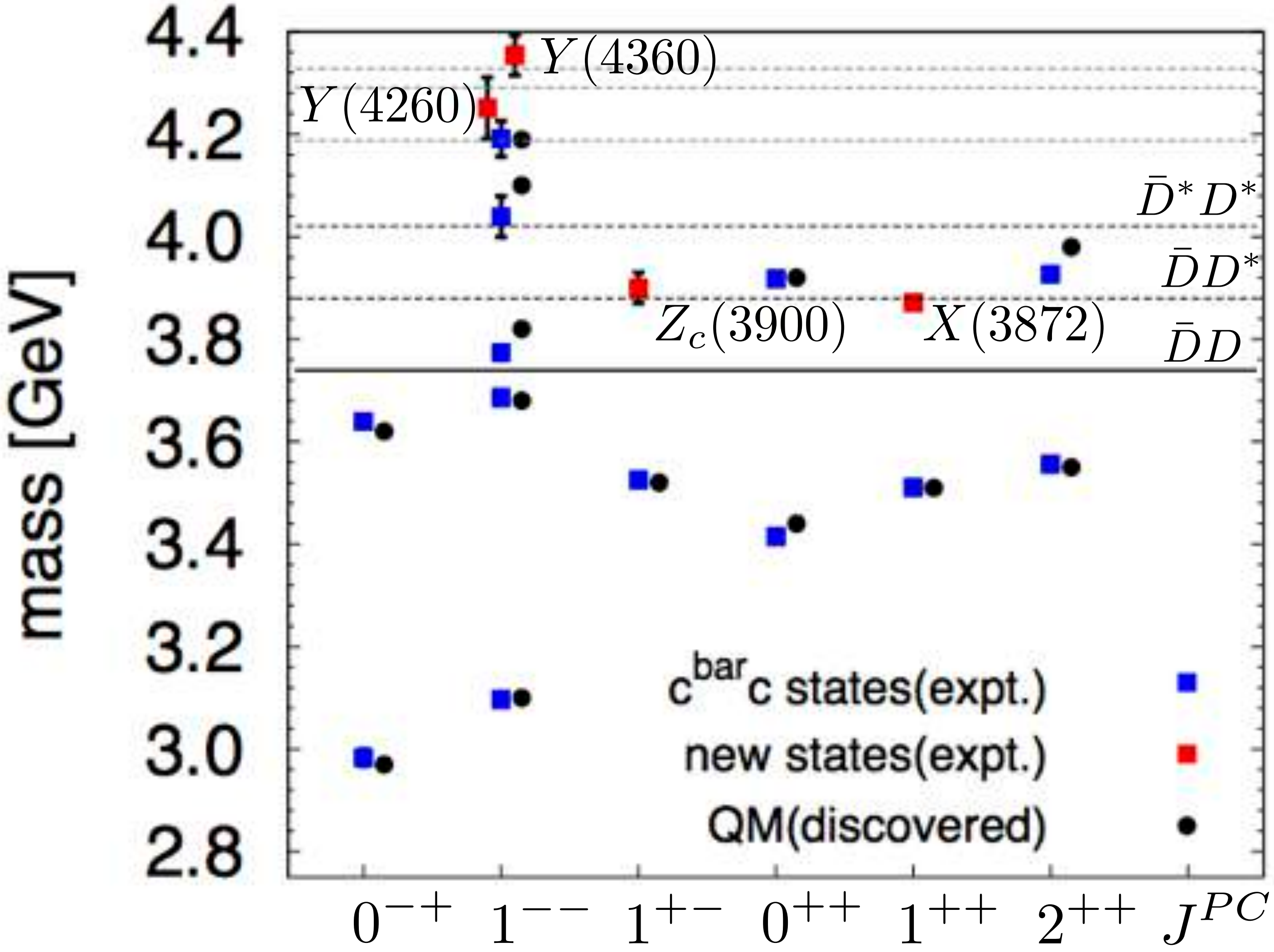
Charmonium(-like) spectrum



Quark model well describes observed mass spectra below 3.8 GeV.

Several states above 3.8 GeV are not discovered.

New (X,Y,Z) states, not predicted by QM, are experimentally observed.

Exotic ?

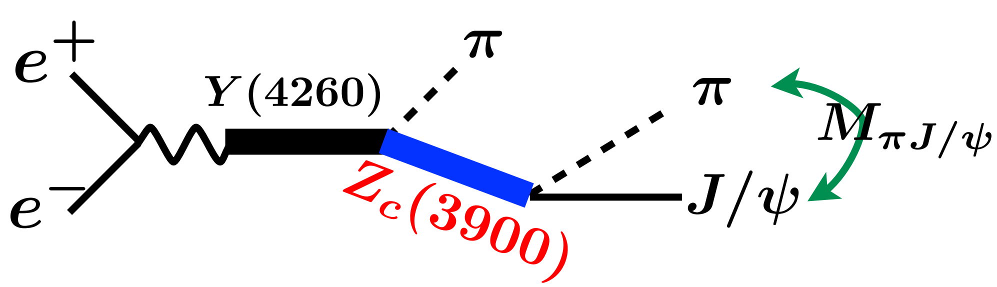
☆ All new states are found above 3.8 GeV

▶Lowest open charm threshold ($D^{bar}D$)のis 3.75 GeV

▶All new states embedded in two-meson continuum($D^{bar}D$, $D^{bar}D^*$, $D^{bar*}D^*$,⋯)

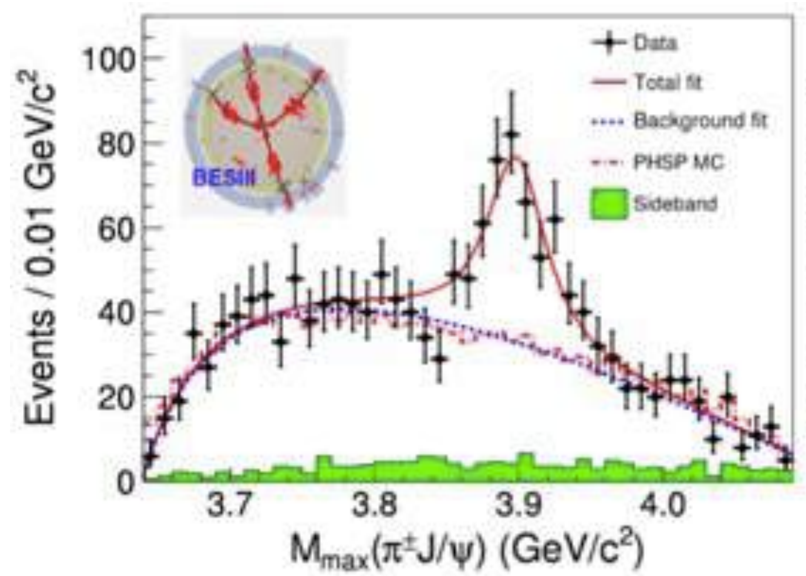
▶Channel coupling is a key to investigate X, Y, Z states

Tetra quark candidate $Z_c(3900)$

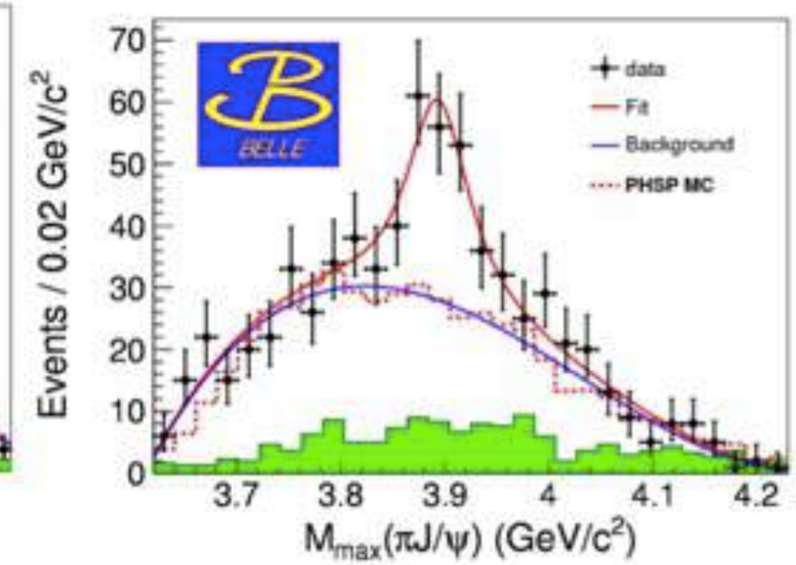


$$e^+ + e^- \to Y(4260) \to \pi + Z_c(3900)$$

BESIII Coll., PRL110 (2013).



Belle Coll., PRL110 (2013).

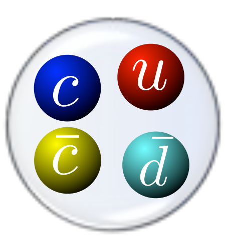


peak in $\pi^\pm J/\psi$ invariant mass (minimal quark content $c\bar{c}u\bar{d}$) terra quark candidate

$M + i\Gamma \sim 3900 + i60$ MeV (Breit-Wigner) just above $D^{bar}D^*$ threshold

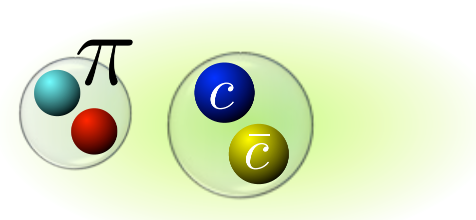
interpretations

tetraquark



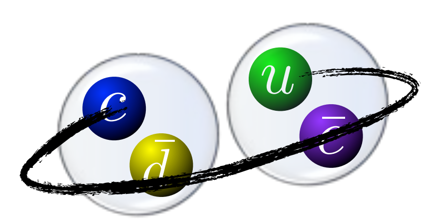
Maiani et al.('13)

J/ψ + π atom



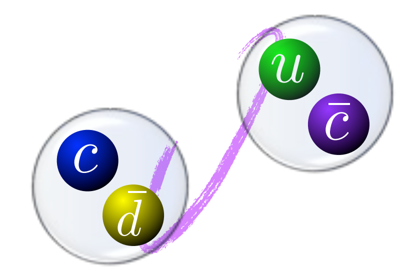
Voloshin('08)

$D^{bar}D^*$ molecule



Nieves et al.('11) + many others

$D^{bar}D^*$ threshold effect



Chen et al.('14), Swanson('15)

genuine state

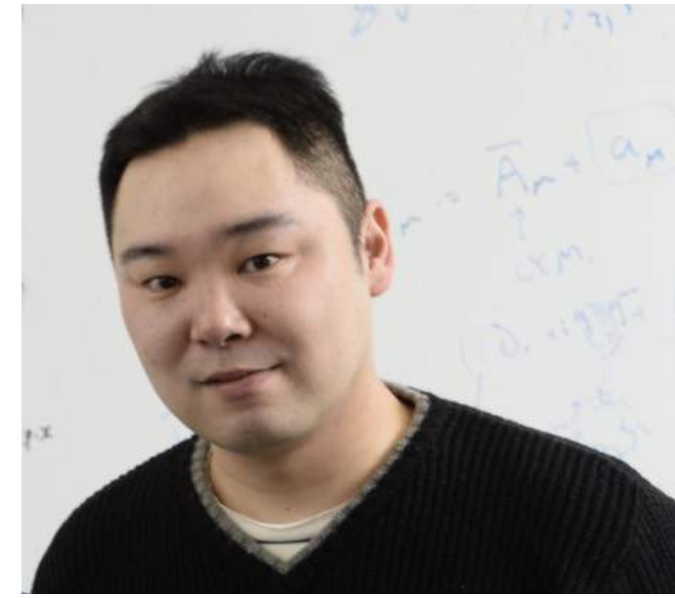
kinematical origin

coupled channel potential and $Z_c(3900)$

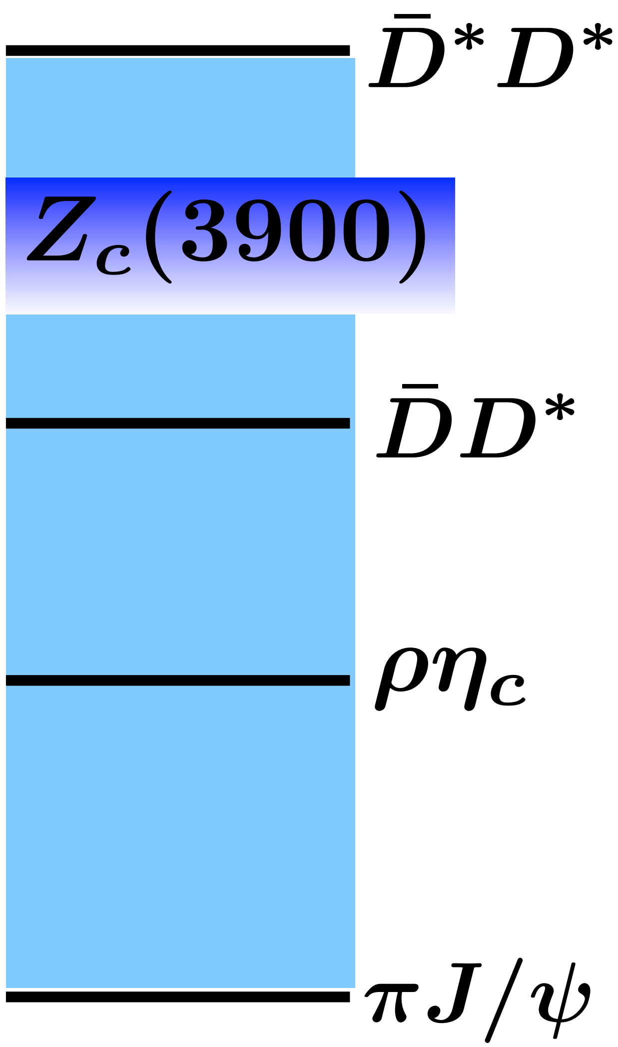
Y. Ikeda et al. [HAL QCD], PRL117, 24001 (2016)

coupled channel potential $V_{AB}(\vec{r})$

$A, B = \pi J/\psi, \rho\eta_c, \bar{D}D^*$



Yoichi Ikeda (Osaka U.)



$\bar{D} = \bar{c}u$ (spin 0) $D^* = \bar{d}c$ (spin 1)

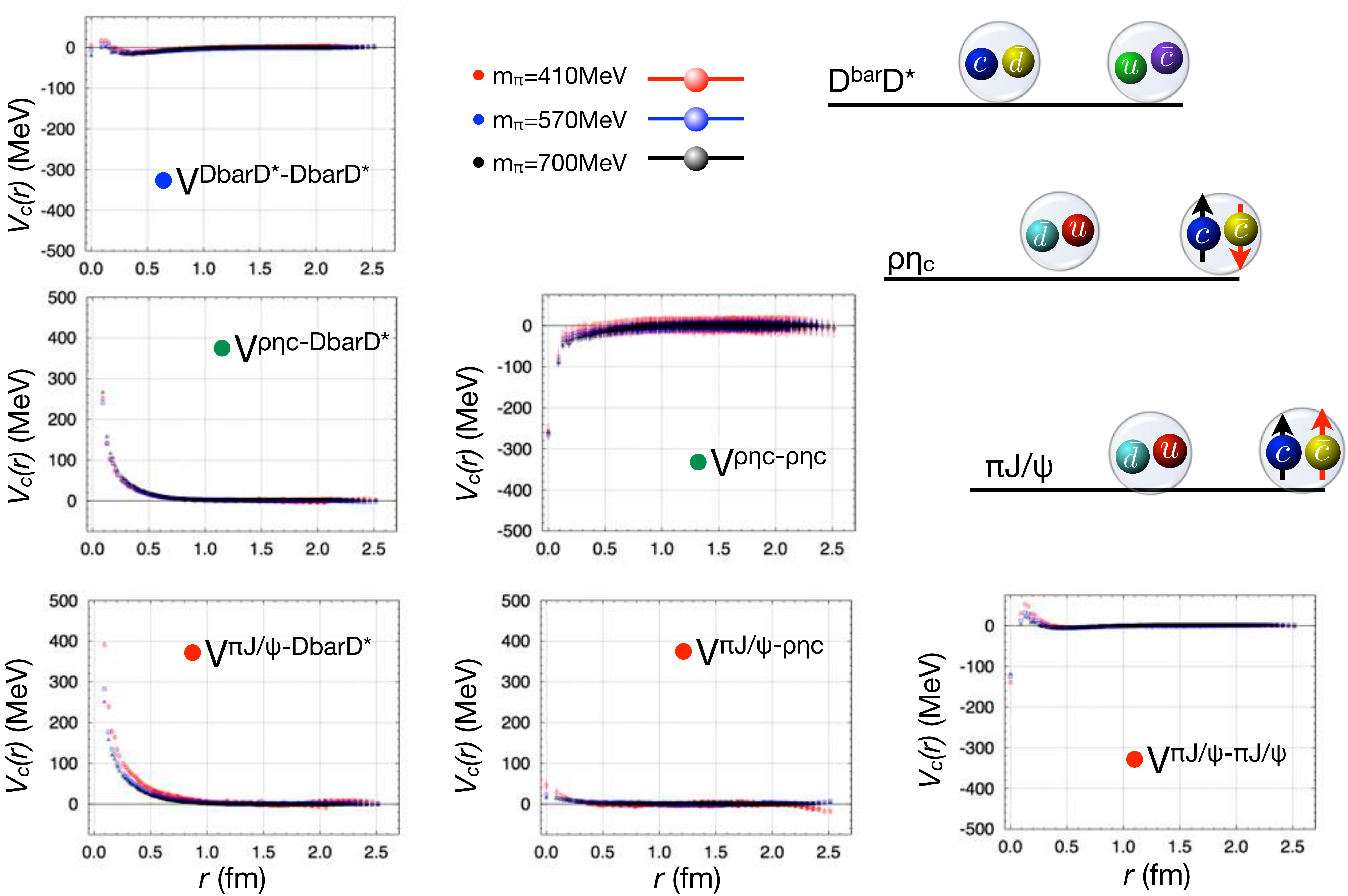
$\eta_c = \bar{c}c$ (spin 0) $\rho = \bar{d}u$ (spin 1)

$\pi = \bar{d}u$ (spin 0) $J/\psi = \bar{c}c$ (spin 1)

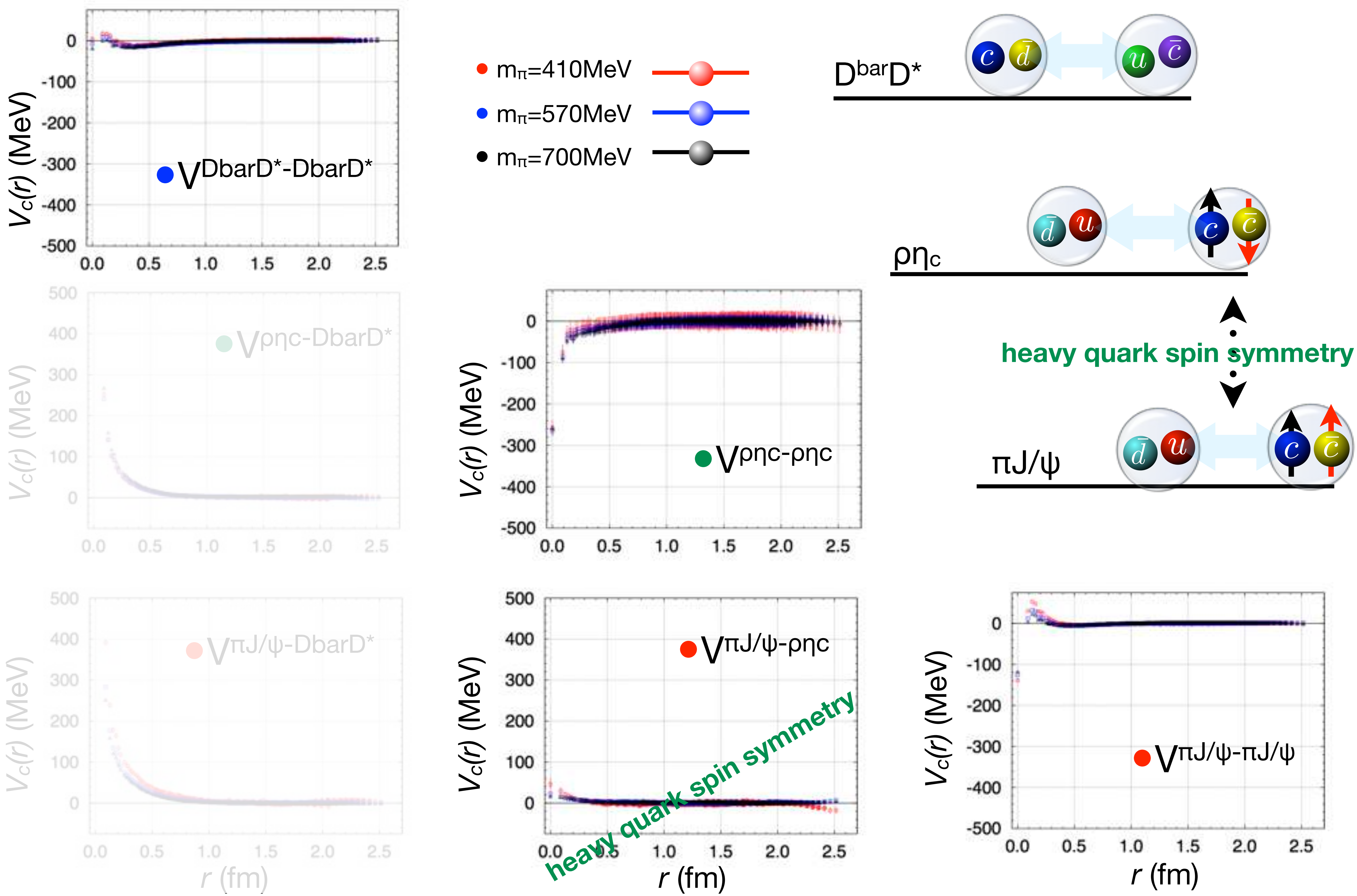
$a \simeq 0.09$ fm, $L \simeq 2.9$ fm

$m_\pi = 411, 572, 701$ MeV

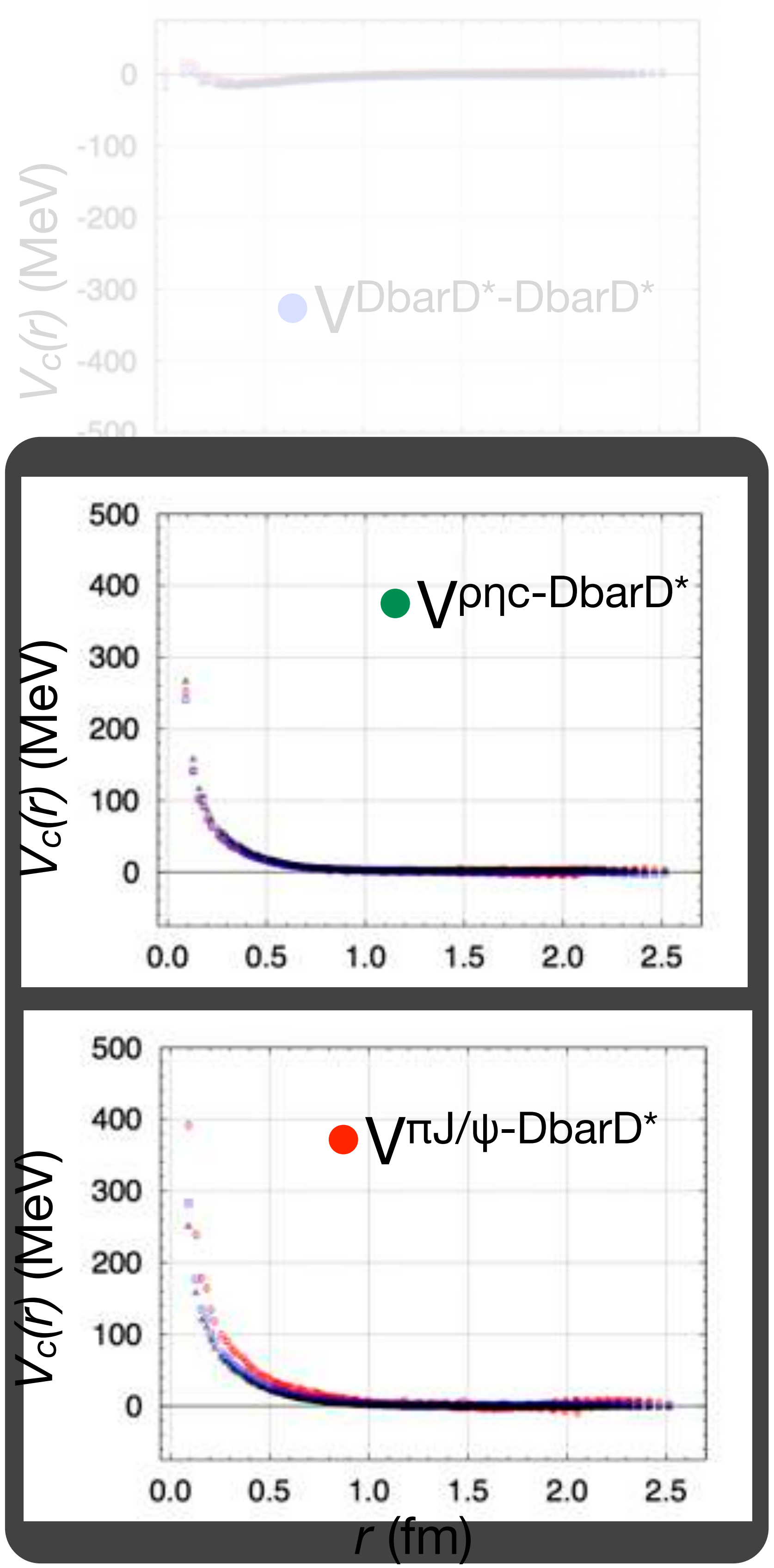
3×3 potential matrix $(\pi J/\psi - \rho\eta_c - \bar{D}D^*)$

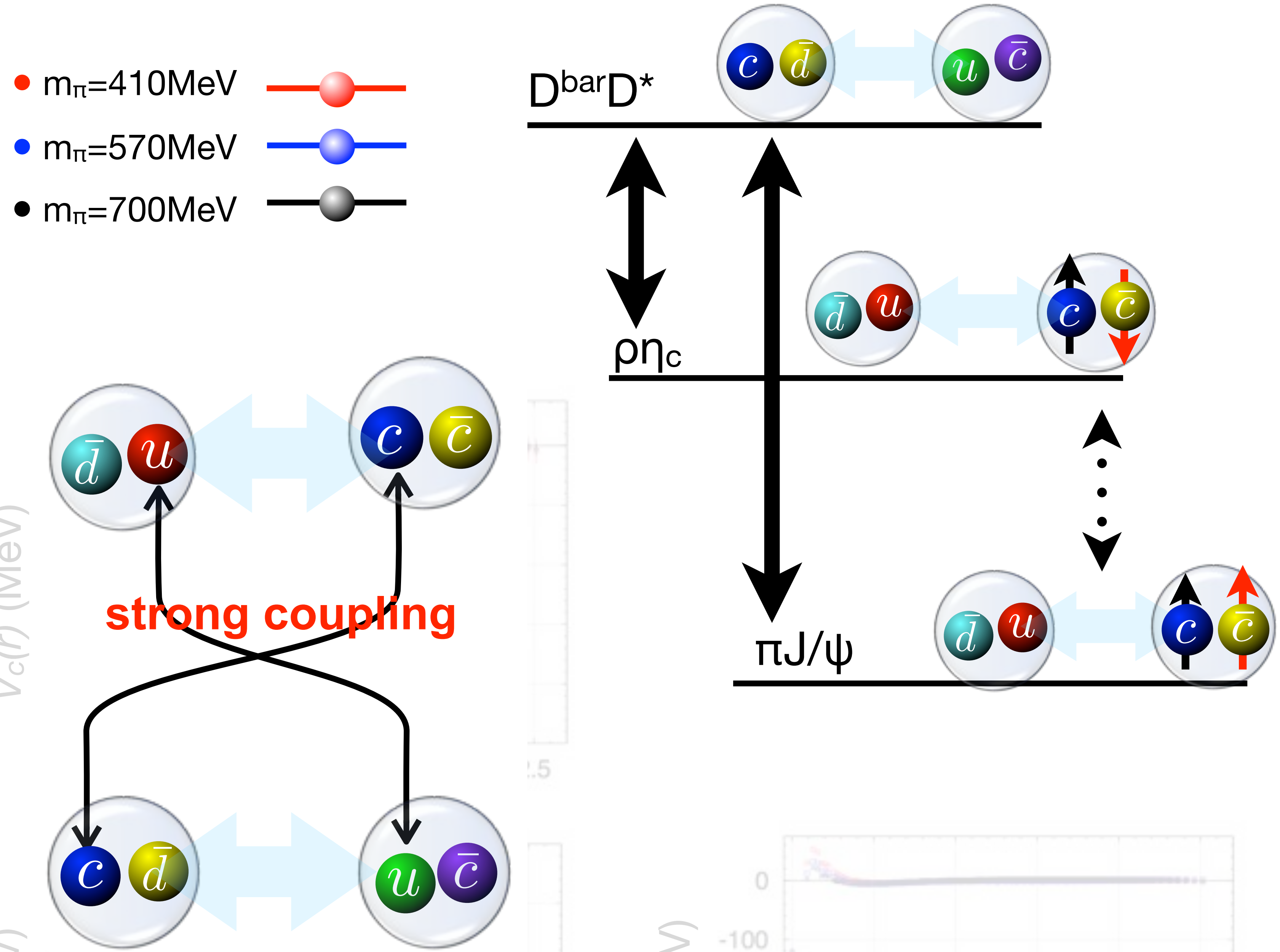


3×3 potential matrix $(\pi J/\psi - \rho\eta_c - \bar{D}D^*)$



3 × 3 potential matrix ($\pi J/\psi - \rho\eta_c - \bar{D}D^*$)





- strong $V^{\pi J/\psi,\mathrm{DbarD}^*}$ & $V^{\rho\eta c,\mathrm{DbarD}^*}$

➡ charm quark exchange process

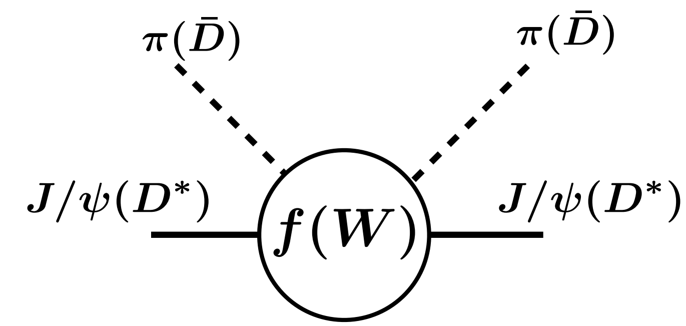
Structure of $Z_c(3900)$

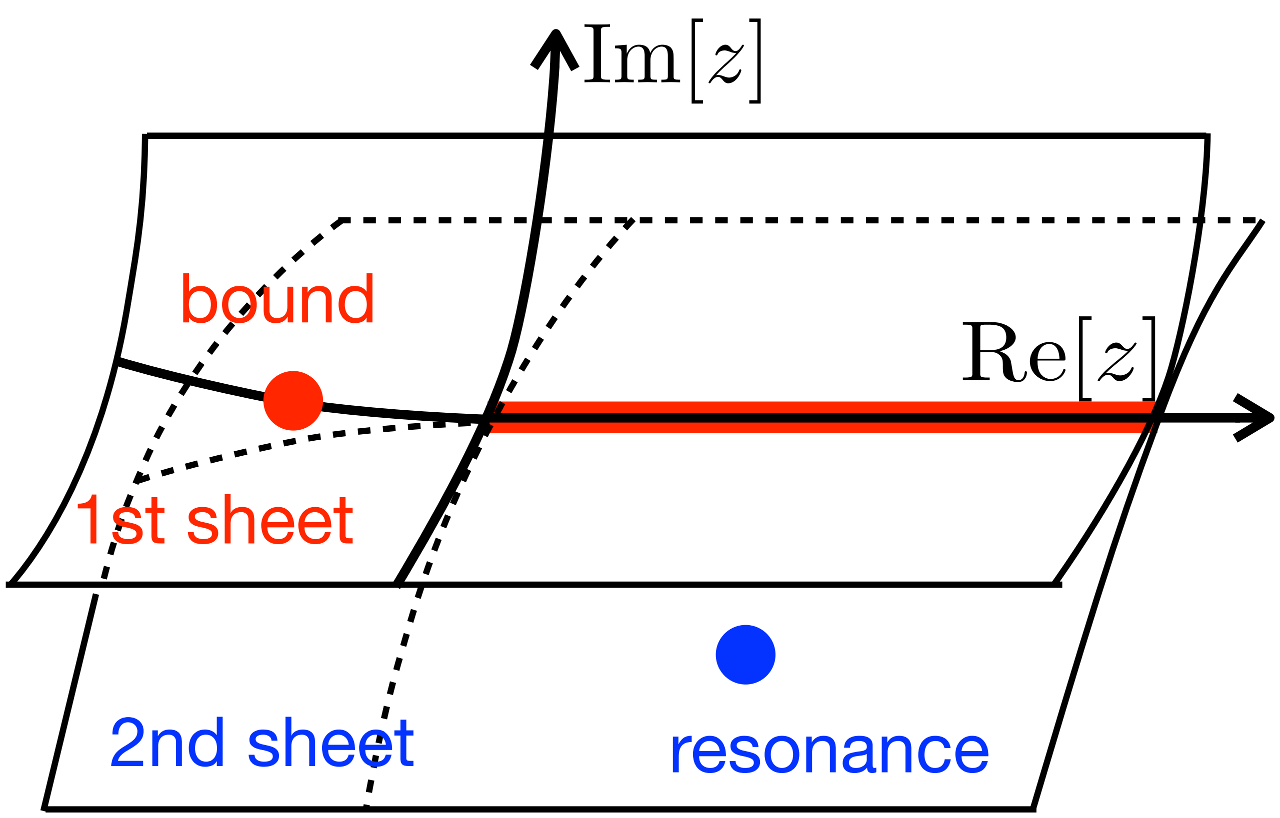
S-wave $\pi J/\psi - \rho\eta_c - \bar{D}D^*$ coupled channel scattering

1. invariant mass distribution of 2-body scattering

$$N_{\rm sc} \propto (\text{flax}) \cdot \sigma(W) \propto \text{Im} f(W)$$

2. pole position of S-matrix





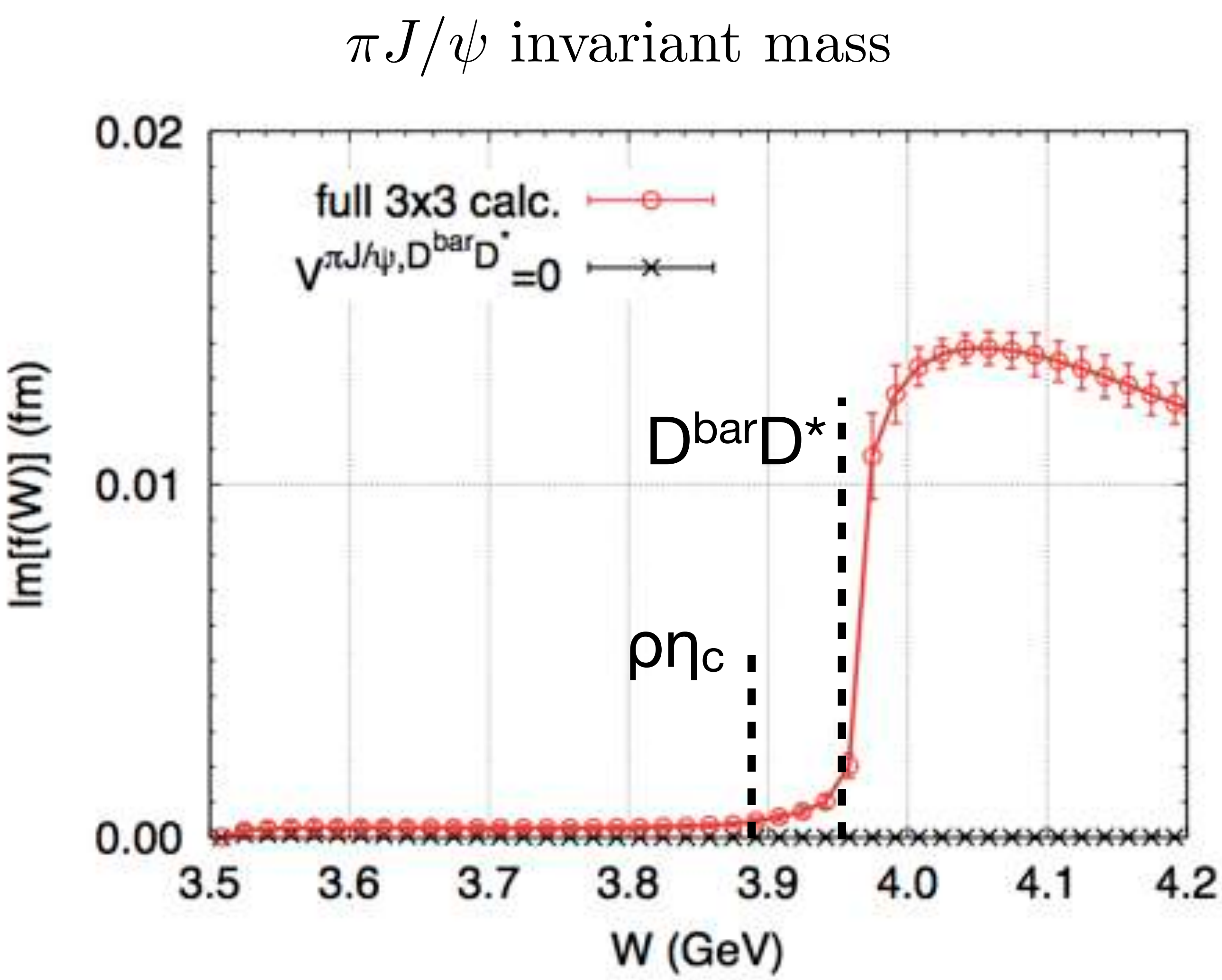
bound state (1st sheet)

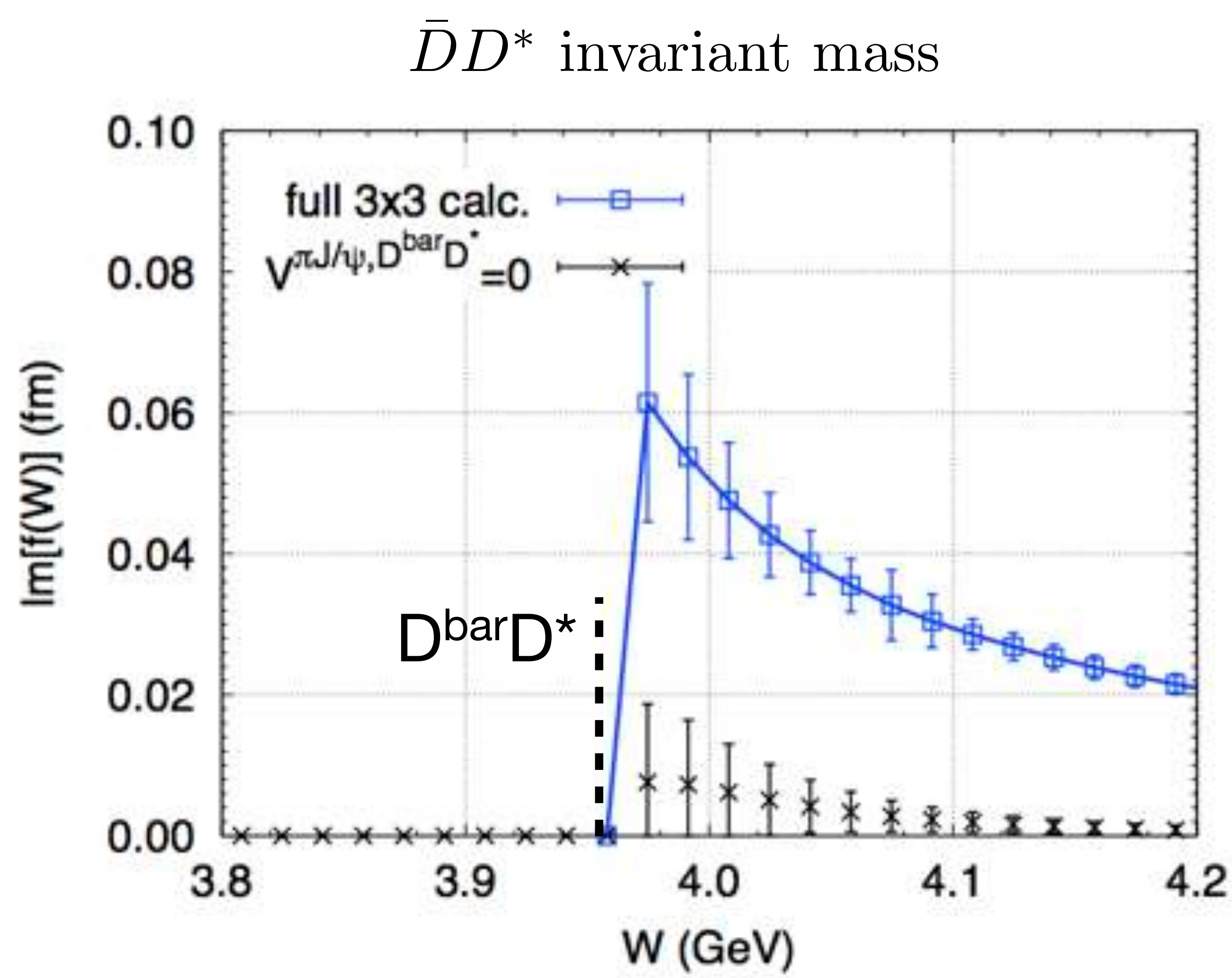
- pole position --> binding energy
- residue --> coupling to scattering state

resonance (2nd sheet)

- analytic continuation onto 2nd sheet
- pole position --> resonance energy
- residue --> coupling to scat. state, partial decay

Invariant mass distribution

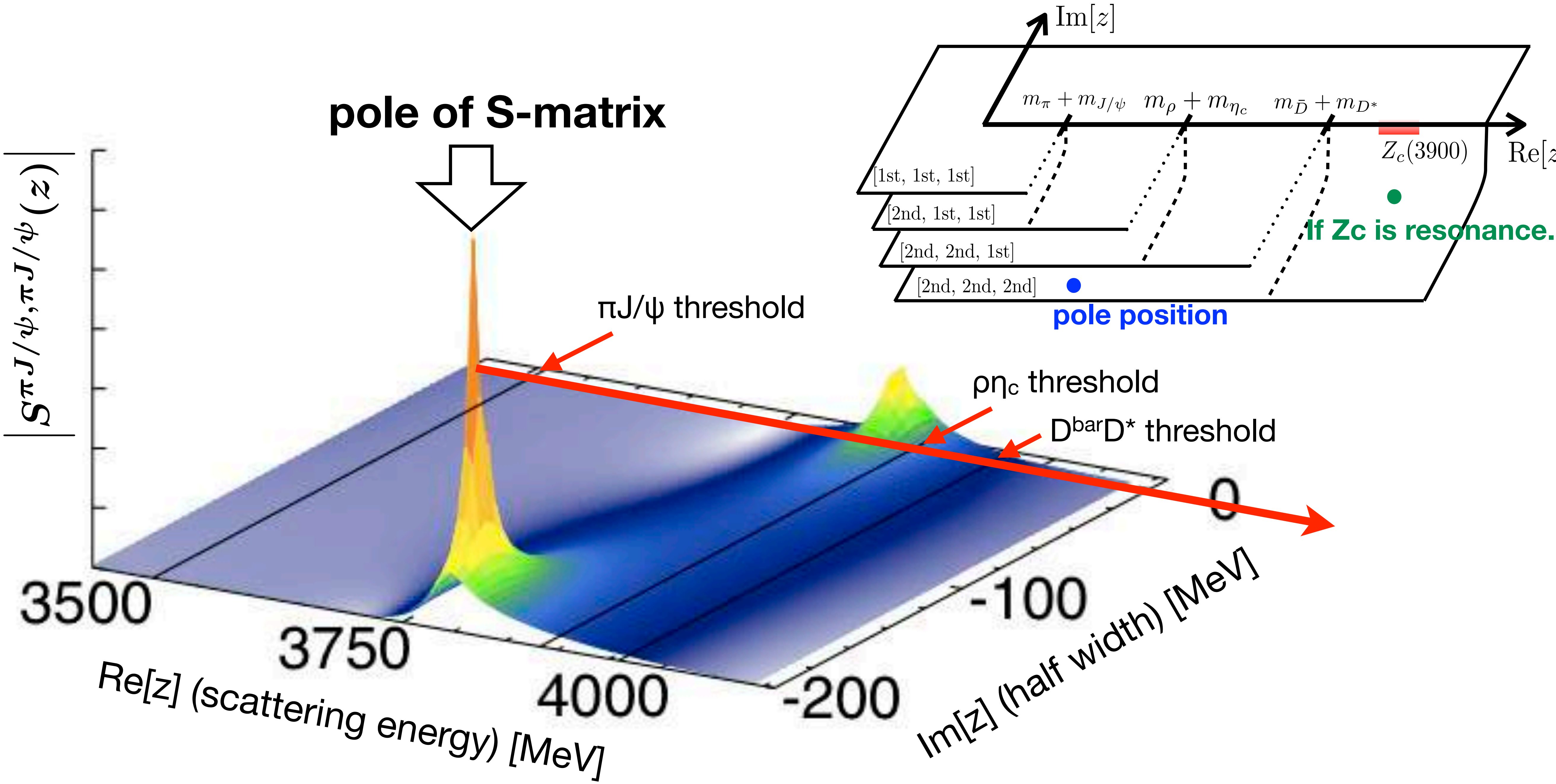




- ☆ Enhancement just above $D^{bar}D^*$ threshold in both amplitudes
  - effect of strong off-diagonal parts (black—>off-diagonal=0)
  - peak is not the Breit-Wigner shape
    - Is $Z_c(3900)$ conventional resonance ? —> pole of S-matrix

Pole of S-matrix

$(\pi J/\psi$:2nd, $\rho\eta_c$:2nd, $\bar{D}D^*$:2nd$)$



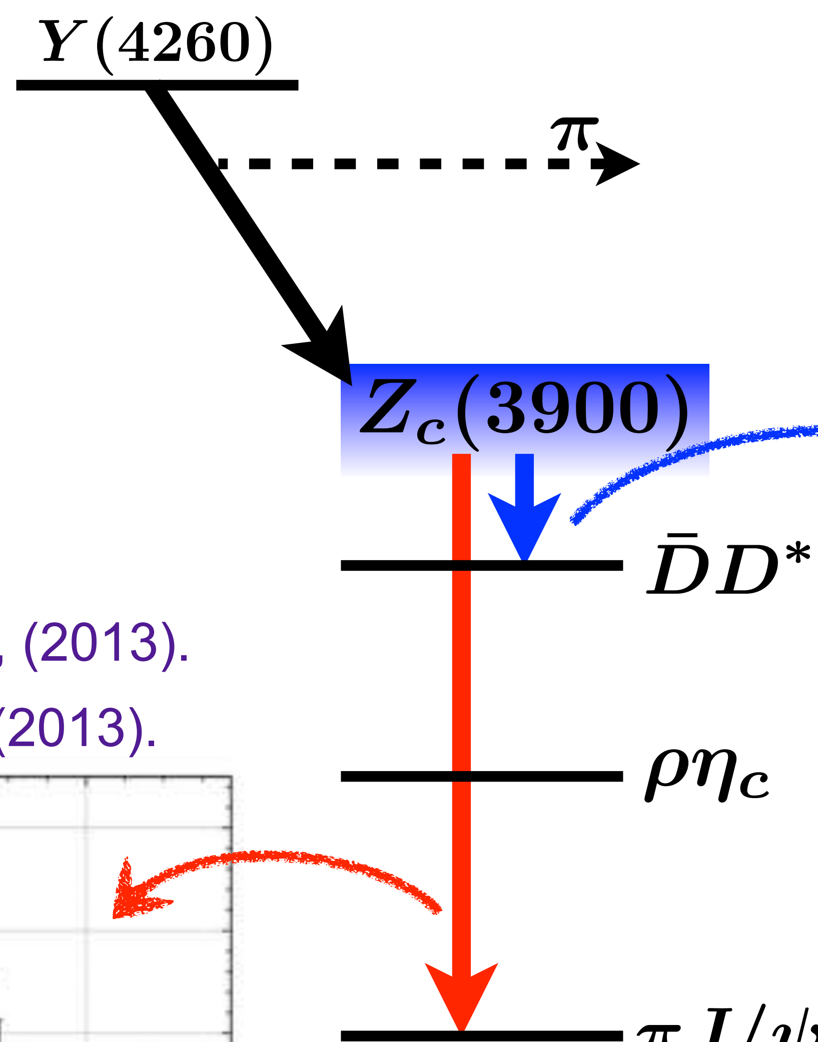
☆Pole corresponds to "virtual state"

☆ Pole contribution to scat. observable is small (far from scat. axis)

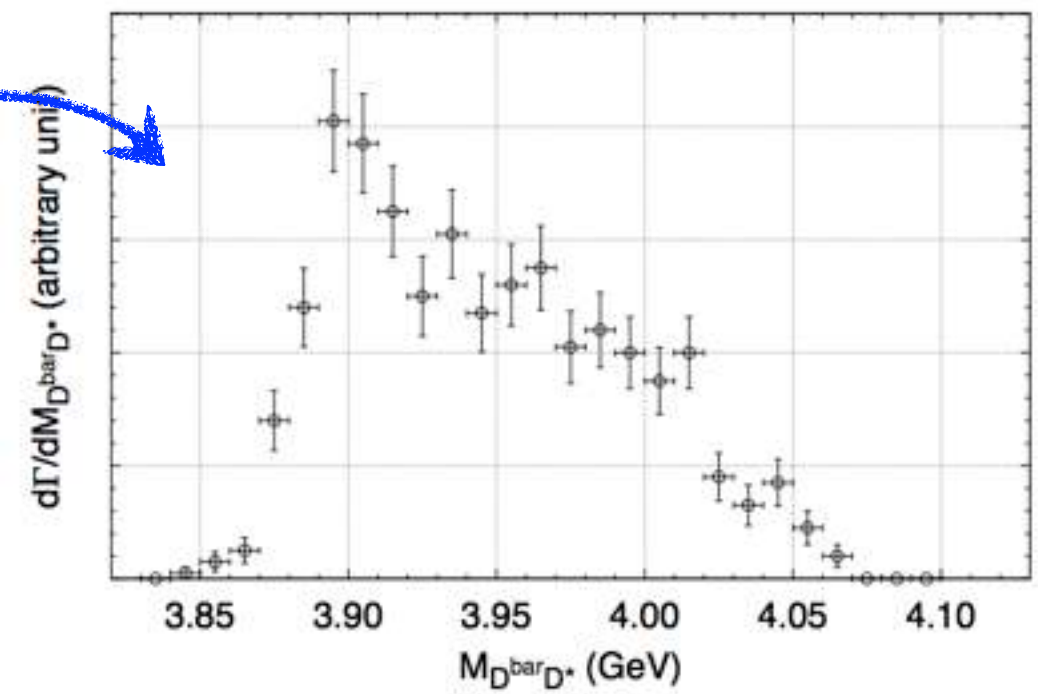
$Z_c(3900)$ is not a resonance but "threshold cusp" induced by strong channel couplings

Comparison with experimental data

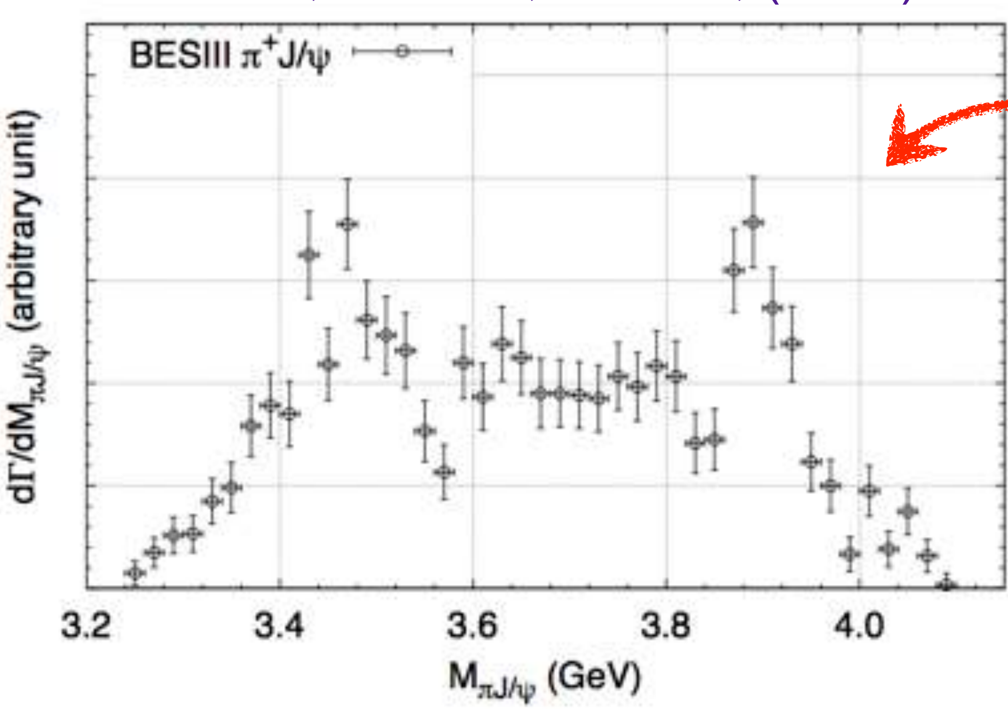
spectrum of Y(4260) 3-body decay

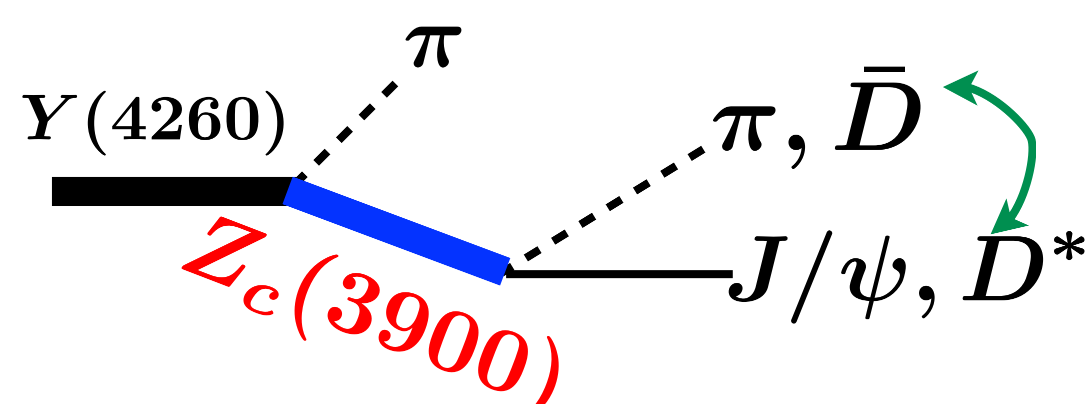


BESIII Coll., PRL112, 022001, (2014).
Wang (BESIII Coll.), MENU2016 talk



BESIII Coll., PRL110, 252001, (2013).
Belle Coll., PRL110, 252002, (2013).



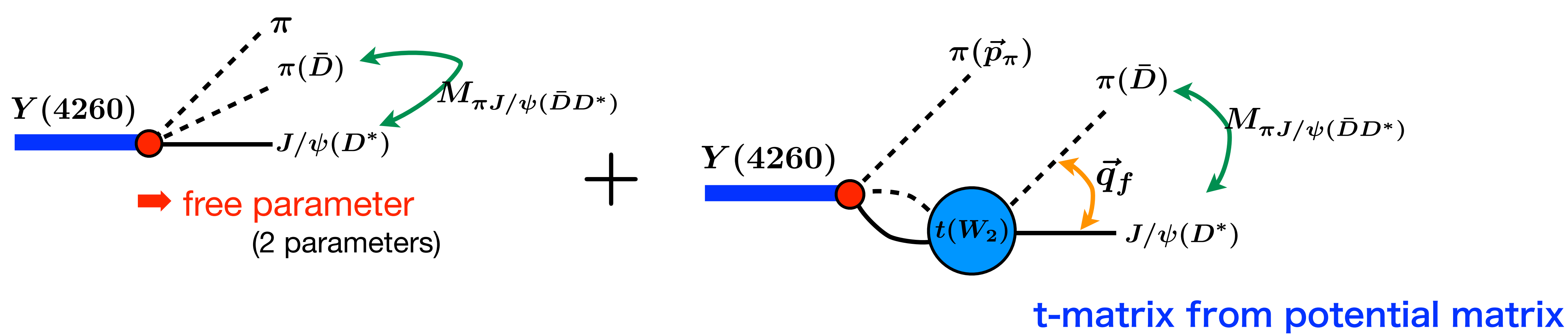


$$Y(4260) \to \pi\pi J/\psi, \pi\bar{D}D^*$$

$$d\Gamma_{Y\to\pi+f} = (2\pi)^4\delta(W_3 - E_\pi(\vec{p}_\pi) - E_f(\vec{q}_f))d^3p_\pi d^3q_f|T_{Y\to\pi+f}(\vec{p}_\pi,\vec{q}_f;W_3)|^2$$

3-body T-matrix

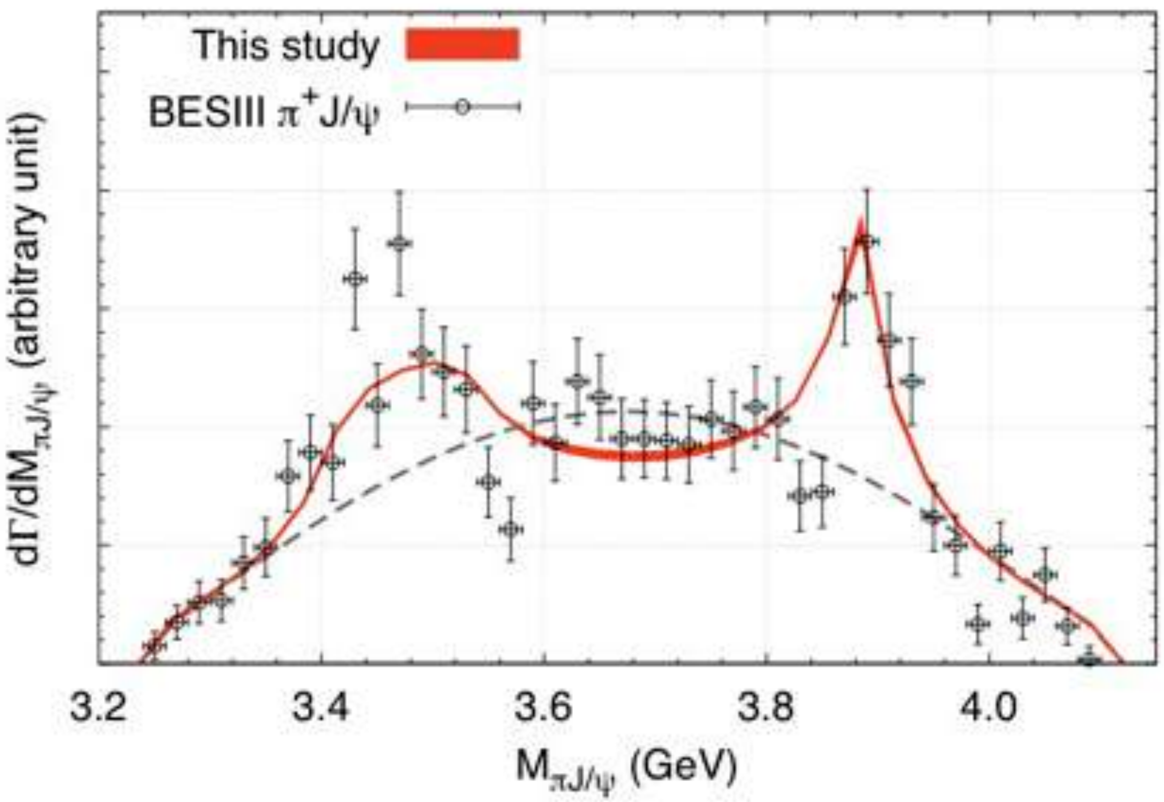
$$T_{Y\to\pi+f}(\vec{p}_\pi,\vec{q}_f;W_3) = \sum_{n=\pi J/\pi,\bar{D}D^*} C^{Y\to\pi+n}\left[\delta_{nf} + \int d^3q' \frac{t_{nf}(\vec{q'},\vec{q}_f,\vec{p}_\pi;W_3)}{W_3 - E_\pi(\vec{p}_\pi) - E_n(\vec{q'},\vec{p}_n) + i\epsilon}\right]$$



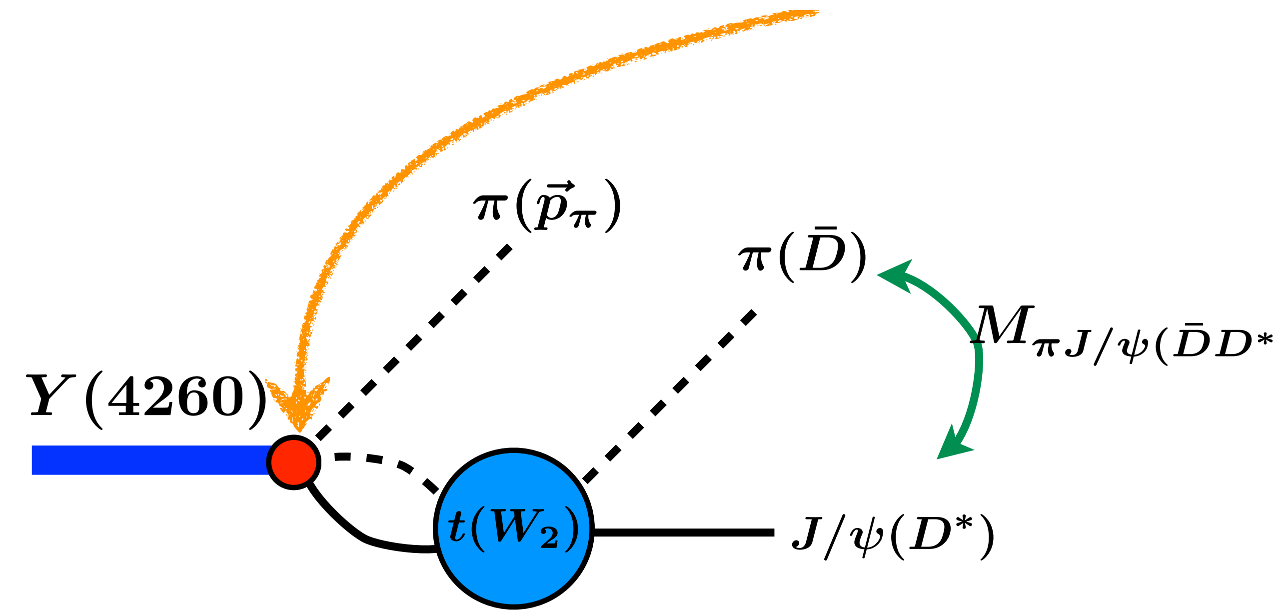
employ physical hadron masses for a comparison with experimental data

- potential matrix is used to calculate t-matrix in subsystem
- fix free parameters by fitting Y(4260) —>π π J/ψ experimental data

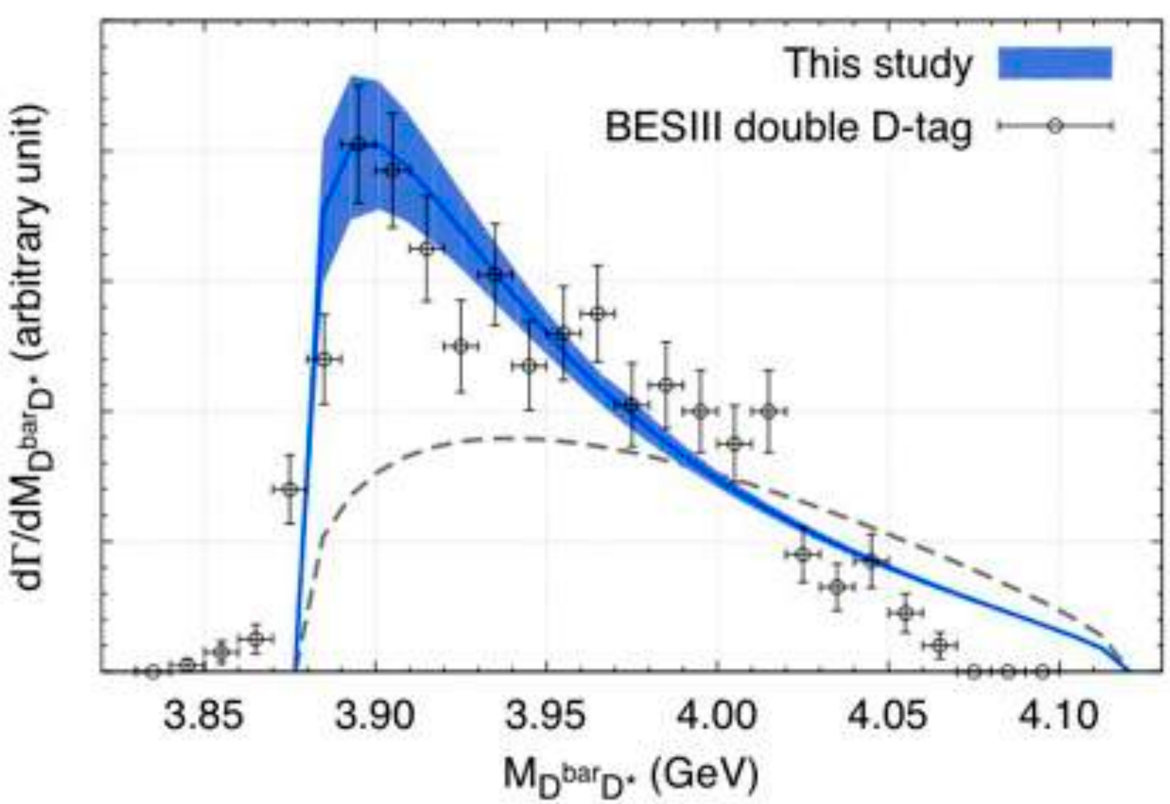
Invariant mass of 3-body decay



Data are well described by 2 parameters



Our coupled channel potential



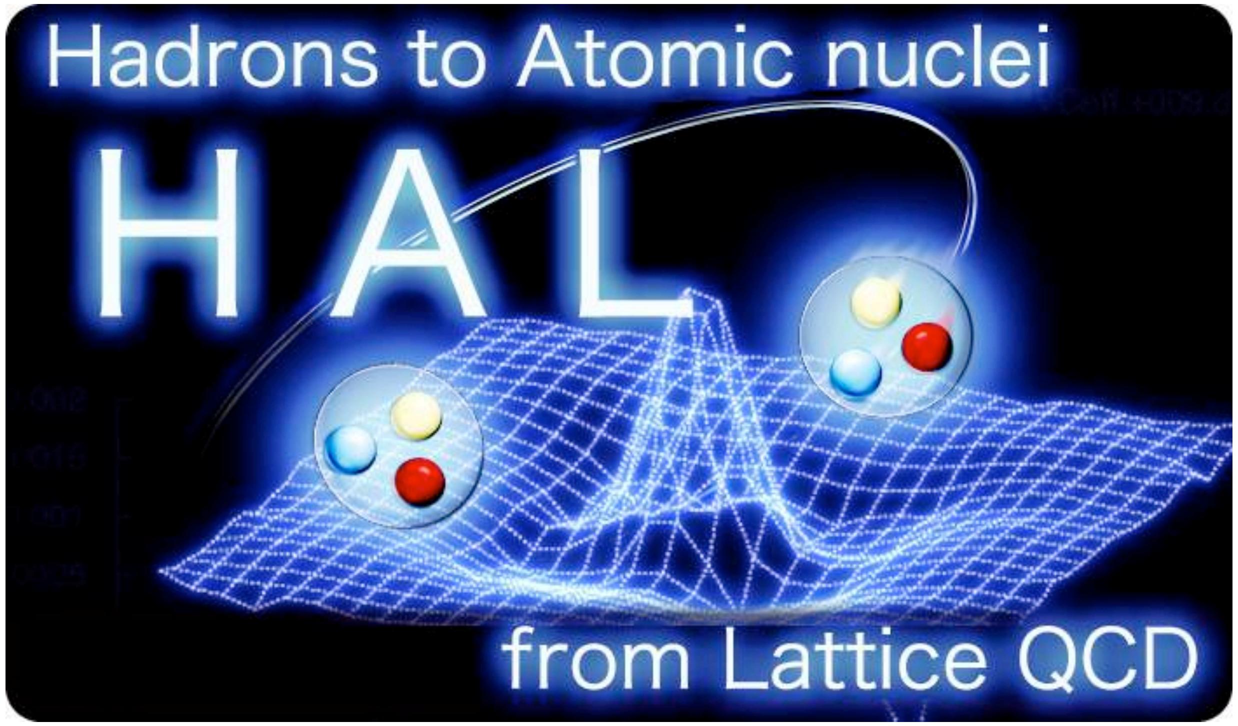
Without off-diagonal parts (dashed curves), peak structures are not reproduced.

$Z_c(3900)$ is threshold cusp caused by strong channel coupling

III. Discussion

- Currently the HAL QCD Potential method is the only way to investigate baryon interactions reliably in lattice QCD.
- Nuclear potentials, Hyperon potentials and more
- Omega-Omega: shallow bound state at physical point
- H-dibaryon: bound state at SU(3) limit, resonance with SU(3) breaking
    - physical point simulation is on-going on K-computer
- Application to exotic hadron: Zc(3900) is the threshold effect
- Other applications (rho & sigma resonances, heavy baryons, Tetra quark, Pena quark, 3 body forces and more)

HAL QCD Collaboration



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