Measuring properties of a Heavy Higgs boson in the $H \to ZZ \to 4l$ decay

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Story of the Higgs Boson

- The Higgs boson is the key to understanding the origin of mass - our existence. The origin of mass is ultimately connected to Electroweak Symmetry Breaking.
- Before EWSB the whole universe is filled by a Higgs field and every particle is massless.
- When the Higgs field develops a VEV the EWSB happens and masses developed.
- Higgs boson is the evidence that EWSB did happen.
- The question is "this Higgs boson, that is it? What else?"

Motivations

- The SM with one Higgs doublet is not natural. Another heavy scalar boson can appear soon.
- LHC searched for H -> ZZ -> 4l and there are some 2-3 sigma here and there.
- The decay H ->ZZ->4l involves a number of angles that one can investigate the CP properties of the boson.

Interactions of HZZ

$$
\begin{aligned}
i\mathcal{M}^{H\to ZZ} &\equiv i\frac{gM_W}{c_W^2}\,\Gamma^{ZZ}_{\mu\nu}\epsilon_1^{*\mu}\epsilon_2^{*\nu} \\
&= i\frac{gM_W}{c_W^2}\Big\{g_{_{HZZ}}\,\epsilon_1^*\cdot\epsilon_2^* + S_H^{ZZ}(s)\left[\frac{-2k_1\cdot k_2}{s}\,\epsilon_1^*\cdot\epsilon_2^* \;+\; \frac{2}{s}\,k_1\cdot\epsilon_2^*\,k_2\cdot\epsilon_1^*\right] \\
&\qquad + P_H^{ZZ}(s)\,\frac{2}{s}\,\langle\epsilon_1^*\epsilon_2^*k_1k_2\rangle\Big\}
\end{aligned}
$$

The first term comes from $\mathcal{L} = \frac{gM_W}{2c_W^2}\,g_{_{HZZ}}\,Z_\mu Z^\mu H$

The second and third term come from higher-order or from genuine dim-6 operators. They can be complex if developed non-vanishing absorptive part.

Helicity Amplitude

$$H \quad \to \; Z(k_1,\epsilon_1)Z(k_2,\epsilon_2) \; \to f_1(p_1,\sigma_1)\bar{f}_1(\bar{p}_1,\bar{\sigma}_1) \;\; f_2(p_2,\sigma_2)\bar{f}_2(\bar{p}_2,\bar{\sigma}_2).$$

$$\begin{aligned}
i\mathcal{M}_{\sigma_1\bar{\sigma}_1:\sigma_2\bar{\sigma}_2} &= \left(i\frac{gM_W}{c_W^2}\Gamma^{ZZ}_{\mu\nu}\right) \; \frac{-i\left(g^{\mu\rho}-\frac{k_1^\mu k_1^\rho}{M_Z^2}\right)}{k_1^2-M_Z^2+iM_Z\Gamma_Z} \; \frac{-i\left(g^{\nu\sigma}-\frac{k_2^\nu k_2^\sigma}{M_Z^2}\right)}{k_2^2-M_Z^2+iM_Z\Gamma_Z} \\
&\times \left[-i\frac{g}{c_W} \sum_{A=L,R} \bar{u}(p_1,\sigma_1)\gamma_\rho(v_{f_1}-Aa_{f_1})P_A v(\bar{p}_1,\bar{\sigma}_1)\right] \\
&\times \left[-i\frac{g}{c_W} \sum_{B=L,R} \bar{u}(p_2,\sigma_2)\gamma_\sigma(v_{f_2}-Ba_{f_2})P_B v(\bar{p}_2,\bar{\sigma}_2)\right] \\
&= i\sum_{\lambda_1,\lambda_2} \mathcal{M}^{H\to ZZ}_{\lambda_1\lambda_2} \; \frac{1}{k_1^2-M_Z^2+iM_Z\Gamma_Z} \; \frac{1}{k_2^2-M_Z^2+iM_Z\Gamma_Z} \; \mathcal{M}^{Z\to f_1\bar{f}_1}_{\lambda_1:\sigma_1\bar{\sigma}_1}\mathcal{M}^{Z\to f_2\bar{f}_2}_{\lambda_2:\sigma_2\bar{\sigma}_2}
\end{aligned}$$

The helicity amplitude for the decay $H \rightarrow Z(k_1, \epsilon_1) Z(k_2, \epsilon_2)$ in the rest frame of H by

$$\mathcal{M}^{H\to ZZ}_{\lambda_1\lambda_2} = \frac{gM_W}{c_W^2} \langle\lambda_1\rangle \, \delta_{\lambda_1\lambda_2}$$

with the reduced amplitudes $\langle\lambda_1\rangle$ defined by

$$\langle + \rangle \equiv g_{_{HZZ}} + (1-\alpha_1-\alpha_2)\, S_H^{ZZ} \; - i\lambda^{1/2}(1,\alpha_1,\alpha_2)\, P_H^{ZZ} \,,$$

$$\langle - \rangle \equiv g_{_{HZZ}} + (1-\alpha_1-\alpha_2)\, S_H^{ZZ} \; + i\lambda^{1/2}(1,\alpha_1,\alpha_2)\, P_H^{ZZ} \,,$$

$$\langle 0 \rangle \equiv g_{_{HZZ}} \left(\frac{1-\alpha_1-\alpha_2}{2\sqrt{\alpha_1\alpha_2}} \right) - 2\sqrt{\alpha_1\alpha_2}\, S_H^{ZZ} \,,$$

$$\alpha_i = k_i^2/M_H^2.$$

The longitudinal amplitude <0> is enhanced by a factor $M_H^2/2\, M_Z^2$ in large M_H limit.

helicity amplitude for the decay $Z(k,\epsilon(k,\lambda)) \to f(p,\sigma)\bar{f}(\bar{p},\bar{\sigma})$ is

$$\mathcal{M}^{Z\to f\bar{f}}_{\lambda:\sigma\bar{\sigma}} = \begin{cases} -\frac{g}{c_W}\Big[\sqrt{2}m_f v_f\ \lambda\sigma e^{-i(\sigma-\lambda)\phi}\ s_\theta\ \delta_{\sigma\bar{\sigma}} \\ \qquad +\frac{\sqrt{k^2}}{\sqrt{2}}(v_f-\sigma\beta_f a_f)(\lambda c_\theta+\sigma)\ e^{i\lambda\phi}\ \delta_{\sigma-\bar{\sigma}}\Big] & \text{for } \lambda=\pm \\ -\frac{g}{c_W}\Big[2m_f v_f\ e^{-i\sigma\phi}\ (-\sigma c_\theta)\ \delta_{\sigma\bar{\sigma}}+\sqrt{k^2}(v_f-\sigma\beta_f a_f)s_\theta\ \delta_{\sigma-\bar{\sigma}}\Big] & \text{for } \lambda=0 \end{cases}$$

Combining all sub-amplitudes

$$\mathcal{M}_{\sigma_1\bar{\sigma}_1:\sigma_2\bar{\sigma}_2} = \frac{gM_W}{2c_W^2}\left(\frac{g}{c_W}\right)^2 \frac{\sqrt{k_1^2}}{k_1^2-M_Z^2+iM_Z\Gamma_Z}\ \frac{\sqrt{k_2^2}}{k_2^2-M_Z^2+iM_Z\Gamma_Z}$$
$$\times\ (v_{f_1}-\sigma_1 a_{f_1})(v_{f_2}-\sigma_2 a_{f_2})$$
$$\times\ \Big[\langle+\rangle(c_{\theta_1}+\sigma_1)(c_{\theta_2}+\sigma_2)e^{i(\phi_1+\phi_2)}+\langle-\rangle(-c_{\theta_1}+\sigma_1)(-c_{\theta_2}+\sigma_2)e^{-i(\phi_1+\phi_2)}$$
$$+2\langle 0\rangle s_{\theta_1}s_{\theta_2}\Big]\,\delta_{\sigma_1-\bar{\sigma}_1}\delta_{\sigma_2-\bar{\sigma}_2}\,.$$

The amplitude squared can be written as a sum of independent combinations of angular variables:

$$\sum_{\sigma_1,\bar{\sigma}_1,\sigma_2,\bar{\sigma}_2} |\mathcal{M}_{\sigma_1\bar{\sigma}_1:\sigma_2\bar{\sigma}_2}|^2 = \left(\frac{gM_W}{c_W^2}\right)^2 \left(\frac{g}{c_W}\right)^4 \frac{k_1^2}{(k_1^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}\,\frac{k_2^2}{(k_2^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}$$

$$\times\,(v_{f_1}^2+a_{f_1}^2)(v_{f_2}^2+a_{f_2}^2)\,\frac{128\pi}{9}\sum_{i=1}^{9} C_i\; f_i(\theta_1,\theta_2,\Phi) \qquad (10)$$

$$f_1(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left[(1+c_{\theta_1}^2)(1+c_{\theta_2}^2)+4\eta_1\eta_2 c_{\theta_1}c_{\theta_2}\right],$$

$$f_2(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left\{-2\left[\eta_1 c_{\theta_1}(1+c_{\theta_2}^2)+\eta_2 c_{\theta_2}(1+c_{\theta_1}^2)\right]\right\},$$

$$f_3(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left[4s_{\theta_1}^2 s_{\theta_2}^2\right],$$

$$f_4(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left[4(c_{\theta_1}c_{\theta_2}+\eta_1\eta_2)s_{\theta_1}s_{\theta_2}c_{\Phi}\right],$$

$$f_5(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left[-4(c_{\theta_1}c_{\theta_2}+\eta_1\eta_2)s_{\theta_1}s_{\theta_2}s_{\Phi}\right],$$

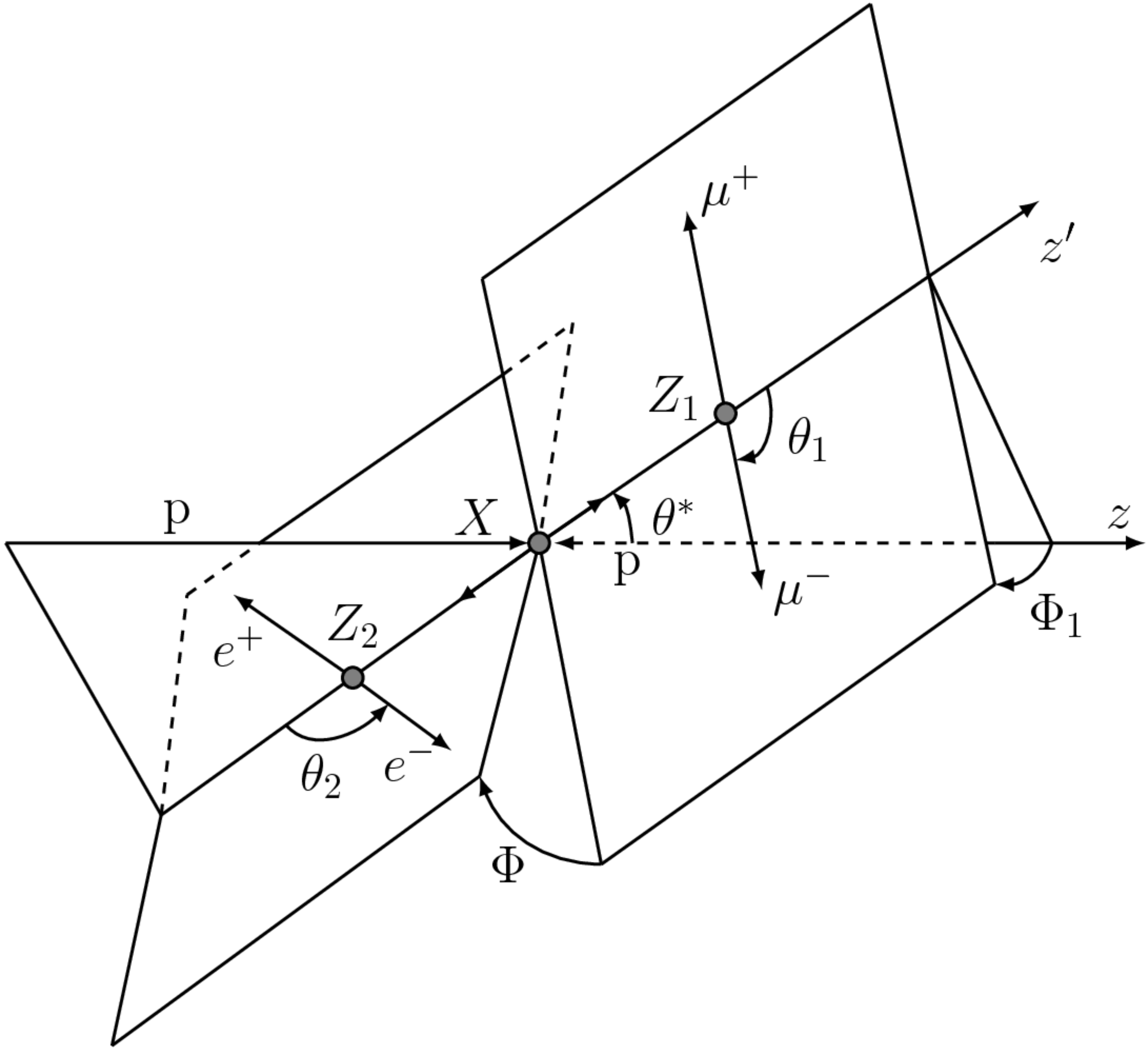
$$f_6(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left[-4(\eta_1 c_{\theta_2}+\eta_2 c_{\theta_1})s_{\theta_1}s_{\theta_2}c_{\Phi}\right],$$

$$f_7(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left[4(\eta_1 c_{\theta_2}+\eta_2 c_{\theta_1})s_{\theta_1}s_{\theta_2}s_{\Phi}\right],$$

$$f_8(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left[s_{\theta_1}^2 s_{\theta_2}^2 c_{2\Phi}\right],$$

$$f_9(\theta_1,\theta_2,\Phi) = \frac{9}{128\pi}\left[-s_{\theta_1}^2 s_{\theta_2}^2 s_{2\Phi}\right].$$

with $\Phi = \phi_1+\phi_2$ and $\eta_i = 2v_{f_i}a_{f_i}/(v_{f_i}^2+a_{f_i}^2)$.



μ^+
z'
Z_1
θ_1
p
X
θ^*
z
p
μ^-
Φ_1
Z_2
e^+
θ_2
e^-
Φ

Also, the 9 angular coefficients C_{1-9}, which are combinations of the reduced helicity amplitudes $\langle + \rangle$, $\langle - \rangle$, and $\langle 0 \rangle$, are defined as

$$
\begin{aligned}
&C_1 \equiv |\langle + \rangle|^2 + |\langle - \rangle|^2 \,, \quad C_2 \equiv |\langle + \rangle|^2 - |\langle - \rangle|^2 \,, \quad C_3 \equiv |\langle 0 \rangle|^2 \,, \\
&C_4 \equiv \Re\mathrm{e}\,[\langle + \rangle\langle 0 \rangle^* + \langle - \rangle\langle 0 \rangle^*] \,, \quad C_5 \equiv \Im\mathrm{m}\,[\langle + \rangle\langle 0 \rangle^* - \langle - \rangle\langle 0 \rangle^*] \,, \\
&C_6 \equiv \Re\mathrm{e}\,[\langle + \rangle\langle 0 \rangle^* - \langle - \rangle\langle 0 \rangle^*] \,, \quad C_7 \equiv \Im\mathrm{m}\,[\langle + \rangle\langle 0 \rangle^* + \langle - \rangle\langle 0 \rangle^*] \,, \\
&C_8 \equiv 2\Re\mathrm{e}\,[\langle + \rangle\langle - \rangle^*] \,, \quad C_9 \equiv 2\Im\mathrm{m}\,[\langle + \rangle\langle - \rangle^*] \,. \qquad (12)
\end{aligned}
$$

Under CP and CPT~ the reduced amplitudes transform like

$$
\langle \lambda \rangle \overset{\mathrm{CP}}{\leftrightarrow} \langle -\lambda \rangle \,, \qquad \langle \lambda \rangle \overset{\mathrm{CP\widetilde{T}}}{\leftrightarrow} \langle -\lambda \rangle^* \,.
$$

* C_2, C_5, C_6, C_9 are CP-odd and nonzero when g_{HZZ}/S^{ZZ}_H and P^{ZZ}_H exist.

* C_2, C_6, C_7 are CPT~ odd and nonzero when induced by the absorptive parts of S^{ZZ}_H and/or P^{ZZ}_H

The partial decay width of the process $H \to ZZ \to 2\ell_1 2\ell_2$ is given by

$$d\Gamma = \frac{1}{2M_H}\left(\sum_{\sigma_1,\bar\sigma_1,\sigma_2,\bar\sigma_2} \left|\mathcal{M}_{\sigma_1\bar\sigma_1:\sigma_2\bar\sigma_2}\right|^2\right) d\Phi_4$$

$$= \frac{1}{2^{13}\,\pi^6\,M_H}\,\lambda^{1/2}(1, k_1^2/M_H^2, k_2^2/M_H^2)\sqrt{k_1^2}\sqrt{k_2^2}$$

$$\times\left(\sum_{\sigma_1,\bar\sigma_1,\sigma_2,\bar\sigma_2} \left|\mathcal{M}_{\sigma_1\bar\sigma_1:\sigma_2\bar\sigma_2}\right|^2\right)\; d\sqrt{k_1^2}\,d\sqrt{k_2^2}\,dc_{\theta_1}\,dc_{\theta_2}\,d\Phi\,.$$

After integrating over $\sqrt{k_1^2}$ and $\sqrt{k_2^2}$, we obtain

$$\frac{1}{\Gamma}\frac{\mathrm{d}\Gamma}{\mathrm{d}c_{\theta_1}\mathrm{d}c_{\theta_2}\mathrm{d}\Phi} = \sum_{i=1}^{9}\,\overline{R}_i f_i(\theta_1,\theta_2,\Phi)$$

with the 9 angular observables defined by

$$\overline{R}_i \equiv \frac{w_i\overline{C}_i}{w_1\overline{C}_1 + w_3\overline{C}_3}.$$

$$\overline{C}_i = C_i(k_1^2 = M_Z^2, k_2^2 = M_Z^2)$$

All w_i = 1 for on shell Z's. We shall take the NWA.

We can integrate any 2 of the angles θ_1, θ_2, and Φ to obtain 1-dim angular distributions

$$\frac{1}{\Gamma}\frac{\mathrm{d}\Gamma}{\mathrm{d}c_{\theta_{1,2}}} = \frac{3}{8}\overline{R}_1\left(1+c^2_{\theta_{1,2}}\right) - \frac{3\eta_{1,2}}{4}\overline{R}_2\ c_{\theta_{1,2}} + \frac{3}{4}\overline{R}_3\left(1-c^2_{\theta_{1,2}}\right)\ ,$$

$$\frac{1}{\Gamma}\frac{\mathrm{d}\Gamma}{\mathrm{d}\Phi} = \frac{1}{2\pi} + \frac{9\pi\eta_1\eta_2}{128}\left(\overline{R}_4\ c_\Phi - \overline{R}_5\ s_\Phi\right) + \frac{1}{8\pi}\left(\overline{R}_8\ c_{2\Phi} - \overline{R}_9\ s_{2\Phi}\right)$$

$$\Gamma = \frac{1}{2^6 3^2 \pi^5 M_H}\left(\frac{gM_W}{c_W^2}\right)^2\left(\frac{g}{c_W}\right)^4\left(v^2_{f_1}+a^2_{f_1}\right)\ \left(v^2_{f_2}+a^2_{f_2}\right)\ \left(w_1\overline{C}_1 + w_3\overline{C}_3\right)\ \mathcal{F}$$

- * Only $C_{1,2,3}$ contribute to cos-theta$_{1,2}$ distributions. When S^{ZZ}_H and P^{ZZ}_H are real, C_2=0.
- * $R_{6,7}$ never appear in 1-dim distributions. We need 2-dim distributions, e.g., c_{θ_1}-Φ and c_{θ_2}-Φ distributions.

* The angular observables $R_{1,2,3}$ can be obtained from fitting to cos-theta$_{1,2}$ distributions.

* $R_{4,5,8,9}$ can be obtained from Fourier analysis or fitting to Ø distribution.

* A non-vanishing R_2 signals may imply new particles of mass < $M_H/2$, such that develops absorptive part for $S^{ZZ}{}_H$ and $P^{ZZ}{}_H$.
* Measurements of R's cannot determine the absolute size of $S^{ZZ}{}_H$, $P^{ZZ}{}_H$ and g_{HZZ}.

* We need to measure $C_1 + C_3$ in the partial width

$$\Gamma = 2.78 \times 10^{-4} \; (w_1 \overline{C}_1 + w_3 \overline{C}_3) \text{ GeV}$$

$$= \Gamma^H_{\rm tot} \; B(H \to ZZ \to 2\ell_1 2\ell_2) \simeq \Gamma^H_{\rm tot} \; B(H \to ZZ) \; [B(Z \to \ell\ell)]^2$$

Numerical Analysis

- We used M_H=260 GeV
- B(H->ZZ, Zh, hh) are comparable, B(H->tt) = 0.
- S^{ZZ}_H and P^{ZZ}_H are real. Only 3 input parameters of couplings.

	g_{HZZ}	S_H^{ZZ}	P_H^{ZZ}	$\frac{\overline{C}_1[++]}{10^{-2}}$	$\frac{\overline{C}_2[--]}{10^{-2}}$	$\frac{\overline{C}_3[++]}{10^{-2}}$	$\frac{\overline{C}_4[++]}{10^{-2}}$	$\frac{\overline{C}_5[-+]}{10^{-2}}$	$\frac{\overline{C}_6[--]}{10^{-2}}$	$\frac{\overline{C}_7[+-]}{10^{-2}}$	$\frac{\overline{C}_8[++]}{10^{-2}}$	$\frac{\overline{C}_9[-+]}{10^{-2}}$
S1	0.1	0	0	2.00	0.00	9.39	−6.13	0.00	0.00	0.00	2.00	0.00
S2	0	0.1	0	1.14	0.00	0.0605	−0.371	0.00	0.00	0.00	1.14	0.00
S3	0	0	0.1	1.02	0.00	0.00	0.00	0.00	0.00	0.00	−1.02	0.00
S4	0	0.1	0.1	2.15	0.00	0.0605	−0.371	0.351	0.00	0.00	0.121	−2.15
S5	0	0.1	−0.1	2.15	0.00	0.0605	−0.371	−0.351	0.00	0.00	0.121	2.15
S6	0.032	0.1	0.1	1.39	0.00	0.540	0.638	−1.05	0.00	0.00	−0.639	−1.24

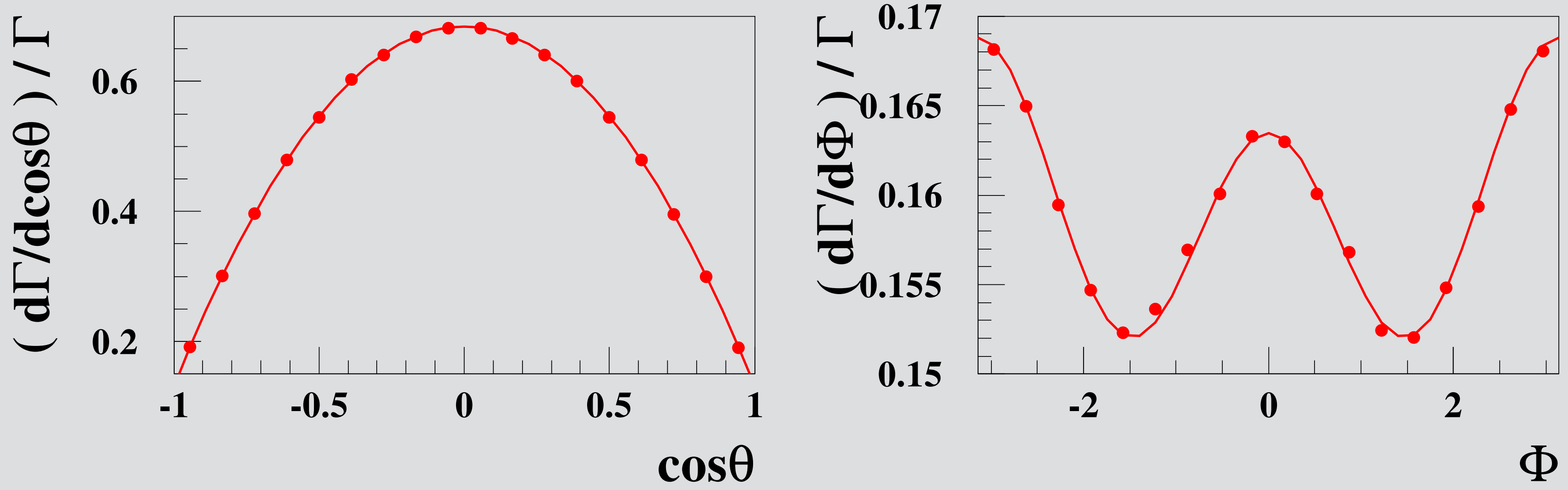
* $S_{1,2,3}$ are CP conserving because one coupling is nonzero.
* $S_{4,5}$ are CP violating.
* S_6 : parameters are chosen s.t. they contribute equally to amplitude squared. <O> is enhanced for large M_H.

Non-vanishing R's

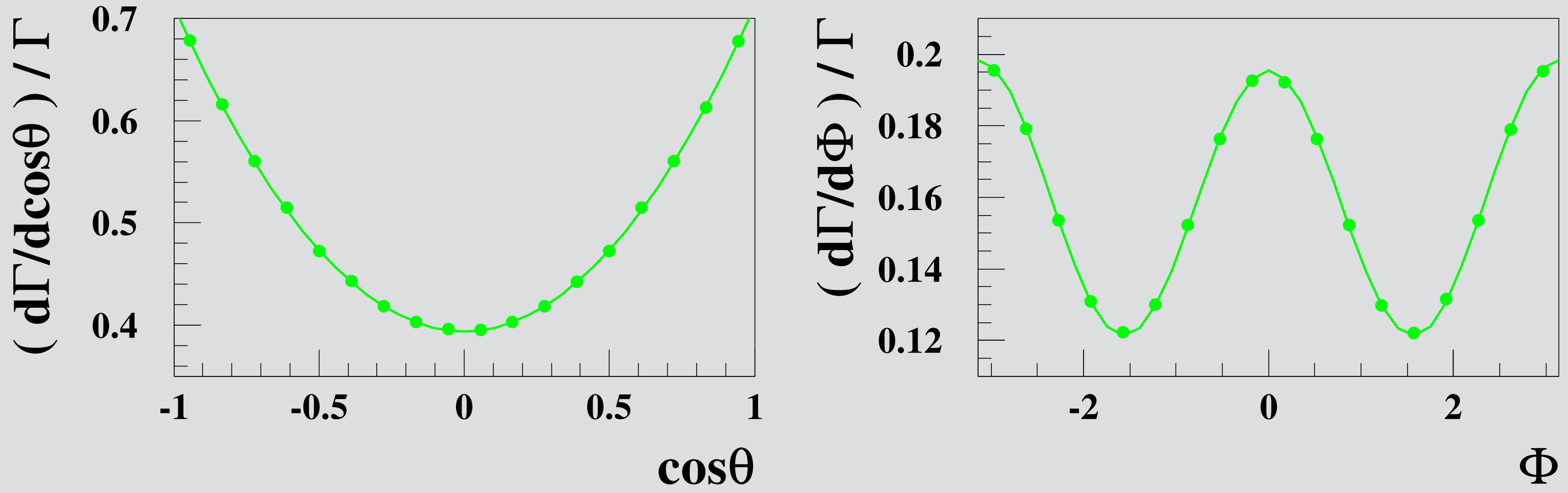
	g_{HZZ}	S_H^{ZZ}	P_H^{ZZ}	$\overline{R}_1[++]$	$\overline{R}_3[++]$	$\overline{R}_4[++]$	$\overline{R}_5[-+]$	$\overline{R}_8[++]$	$\overline{R}_9[-+]$	$(\overline{C}_1+\overline{C}_3)[++]\times 10^2$
S1	0.1	0	0	0.176	0.824	-0.538	0.00	0.176	0.00	11.4
S2	0	0.1	0	0.950	0.0505	-0.310	0.00	0.950	0.00	1.20
S3	0	0	0.1	1.00	0.00	0.00	0.00	-1.00	0.00	1.02
S4	0	0.1	0.1	0.973	0.0273	-0.168	0.158	0.0547	-0.971	2.21
S5	0	0.1	-0.1	0.973	0.0273	-0.168	-0.158	0.0547	0.971	2.21
S6	0.032	0.1	0.1	0.721	0.280	0.330	-0.542	-0.331	-0.640	1.93

CP Conserving

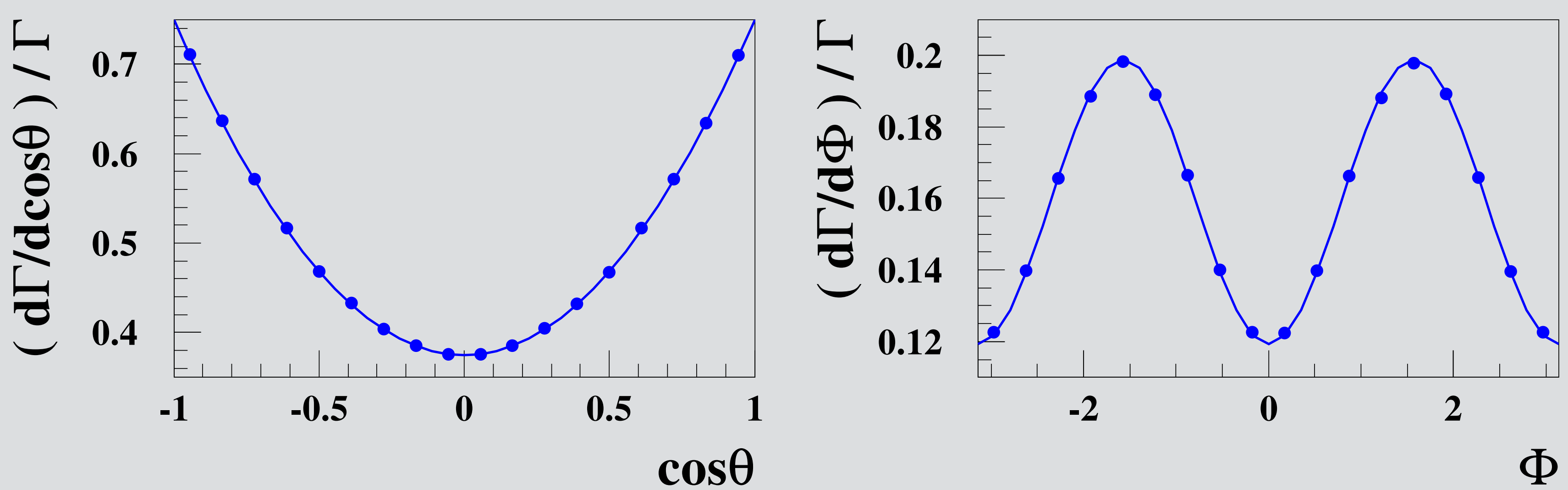
S_1 (0.1,0,0)



S_2 (0,0.1,0)



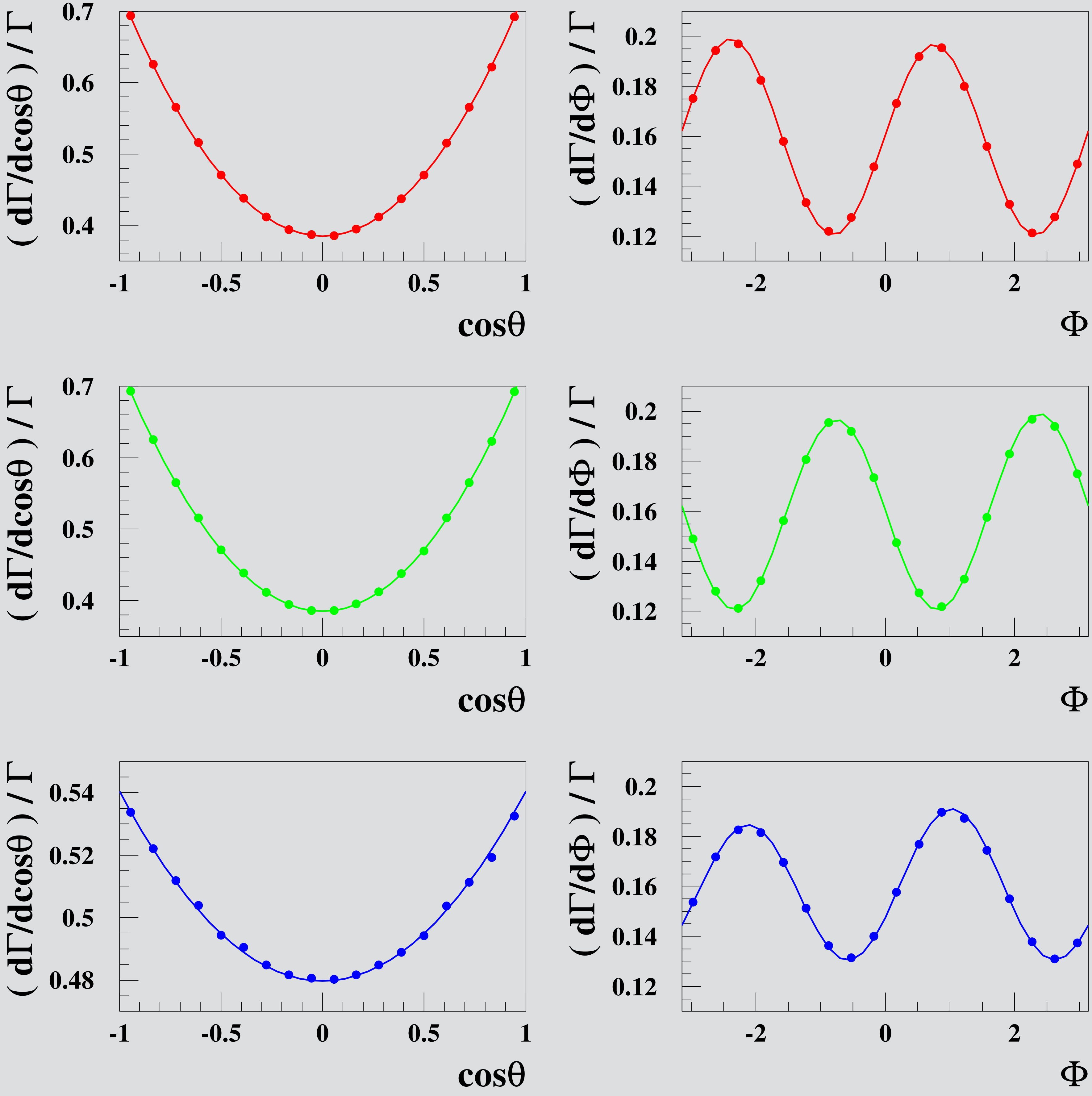
S_3 (0,0,0.1)



S_4 (0,0.1,0.1)

S_5 (0,0.1,-0.1)

S_6 (0.032,0.1,0.1)



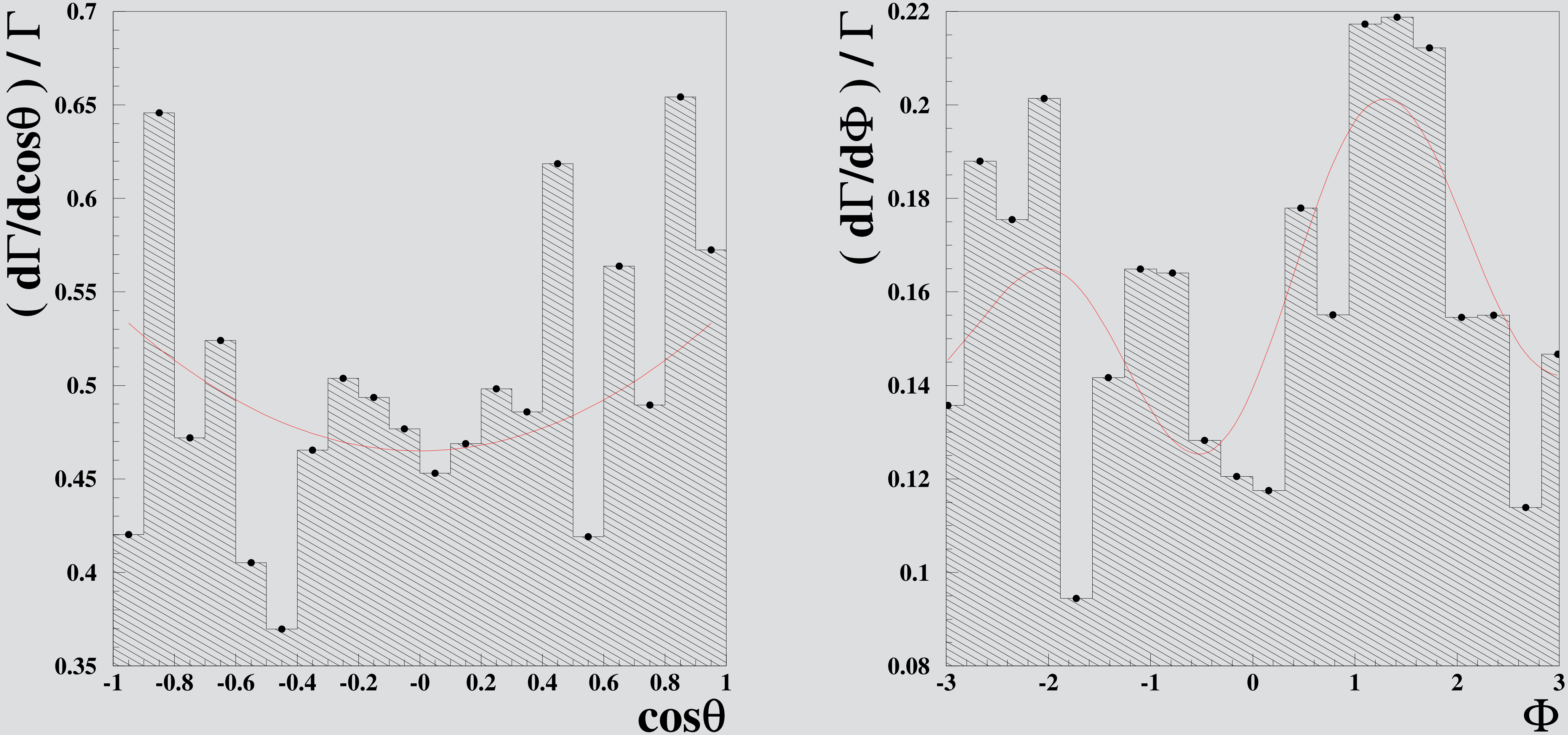
Illustrate the measurement in S_6

$$(g_{HZZ}, S_H^{ZZ}, P_H^{ZZ}) = (0.032, 0.1, 0.1)$$

Current upper limit: $\sigma(gg \to H) \cdot B(H \to ZZ) \lesssim 0.1\,\mathrm{pb}$ for a $260\,\mathrm{GeV}$

$$\sigma(gg \to H) \cdot B(H \to ZZ) \cdot 4[B(Z \to \ell\ell)]^2 \cdot \epsilon_{4\ell} \cdot \mathcal{L} \simeq 10^3$$

We take 10^3events in the following, and angular resolution of cos-theta = 0.1, Phi=0.1π.



S6	$\overline{R}_1[++]$	$\overline{R}_3[++]$	$\overline{R}_4[++]$	$\overline{R}_5[-+]$	$\overline{R}_8[++]$	$\overline{R}_9[-+]$	$(\overline{C}_1+\overline{C}_3)[++]\times 10^2$
Input	0.721	0.280	0.330	-0.542	-0.331	-0.640	1.93
Output (center value)	0.721	0.260	-0.339	-4.07	-0.452	-0.387	1.93
Output (parabolic error)	±0.037	±0.034	±1.37	±1.45	±0.17	±0.18	±0.386

correlation for $\overline{R}_1$ and $\overline{R}_3$ is $\rho = -0.813$.

Fit to the original inputs g_{HZZ}, S^{ZZ}_H, P^{ZZ}_H

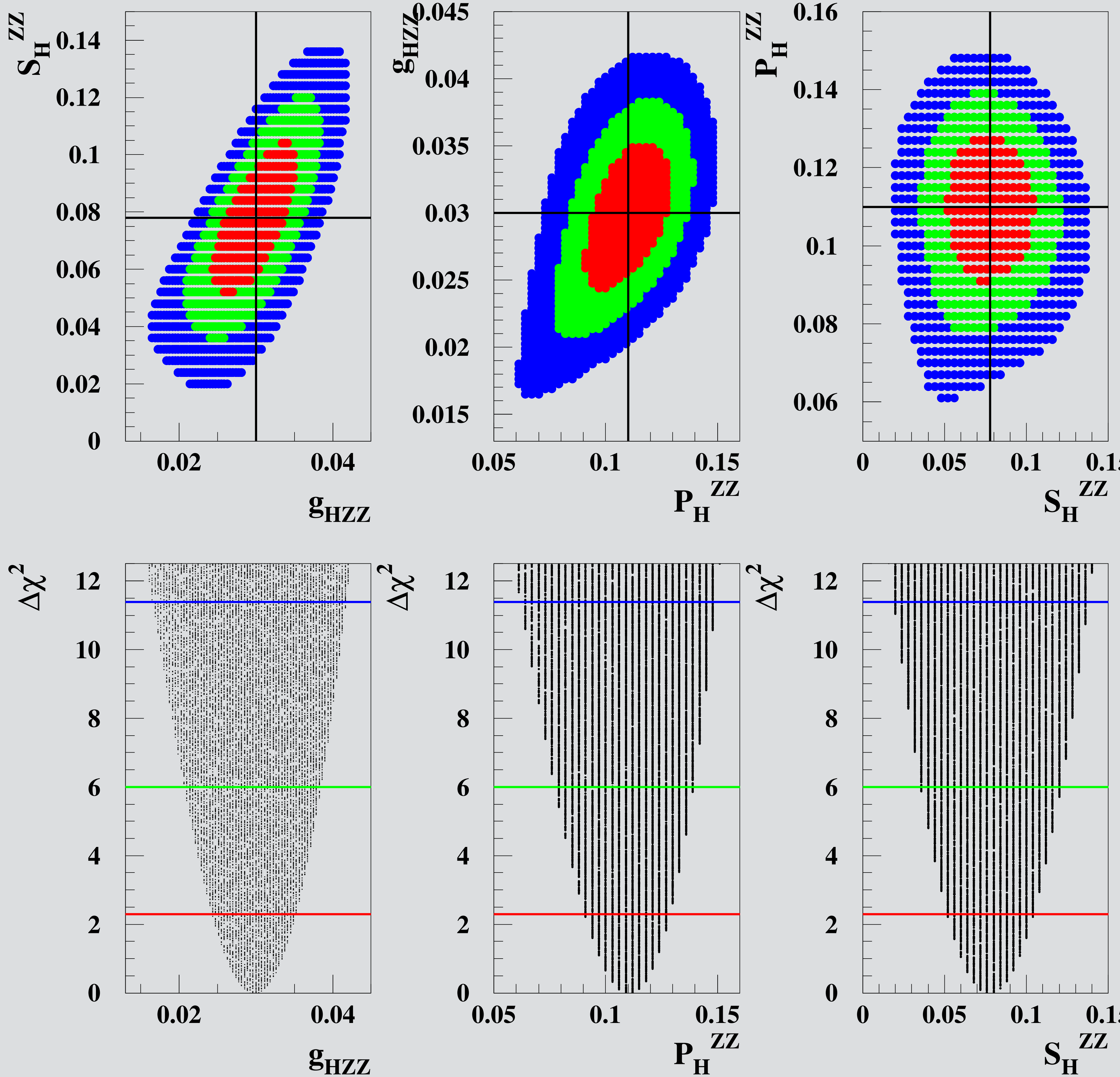
Best Fit

g_{HZZ}=0.030±0.0035

S^{ZZ}_H =0.0780±0.017

P^{ZZ}_H =0.11±0.013

12-20% error for 10^3 events



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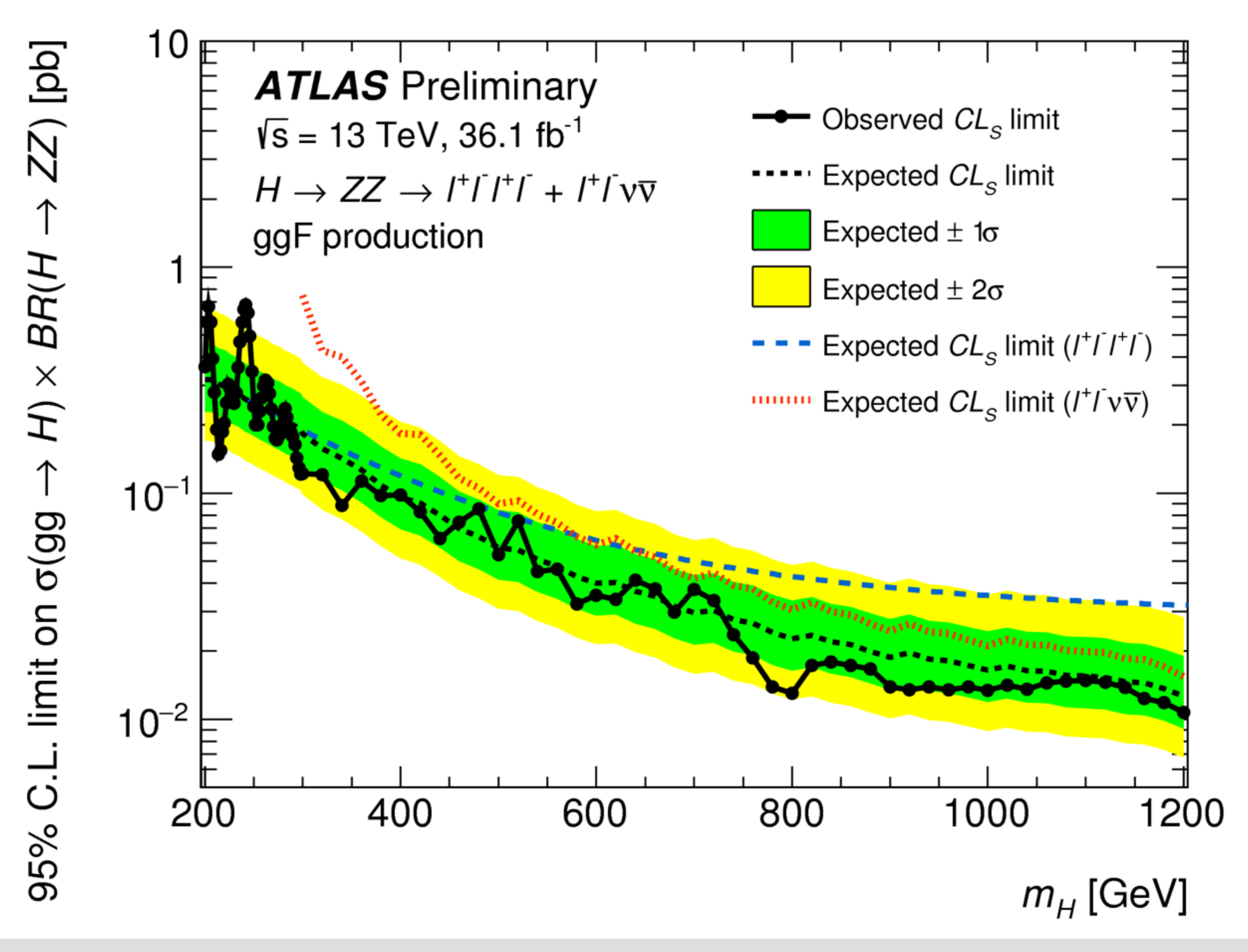
The upper limit around 240-270 GeV is 0.2-0.4 pb, corresponding to about 3000events at HL-LHC.

The uncertainty decreases to 10%.



Summary

- The angular distributions of $theta_{1,2}$ and Phi can be analysed and fitted to the observables R_i's and then to Higgs couplings.

- With 10^3 H -> ZZ -> 4l events one can determine the couplings g_{HZZ}, S^{ZZ}_H and P^{ZZ}_H with 12-20% uncertainty. With 3000 events uncertainty goes down to 10%.

- Appearance of C_2 signals new particles with mass $< M_H/2$ running in the loop.

- One can also extend to 2-dim distributions to analyse other observables $R_{6,7}$.