On the Pseudo-axion in the Simplest Little Higgs Model

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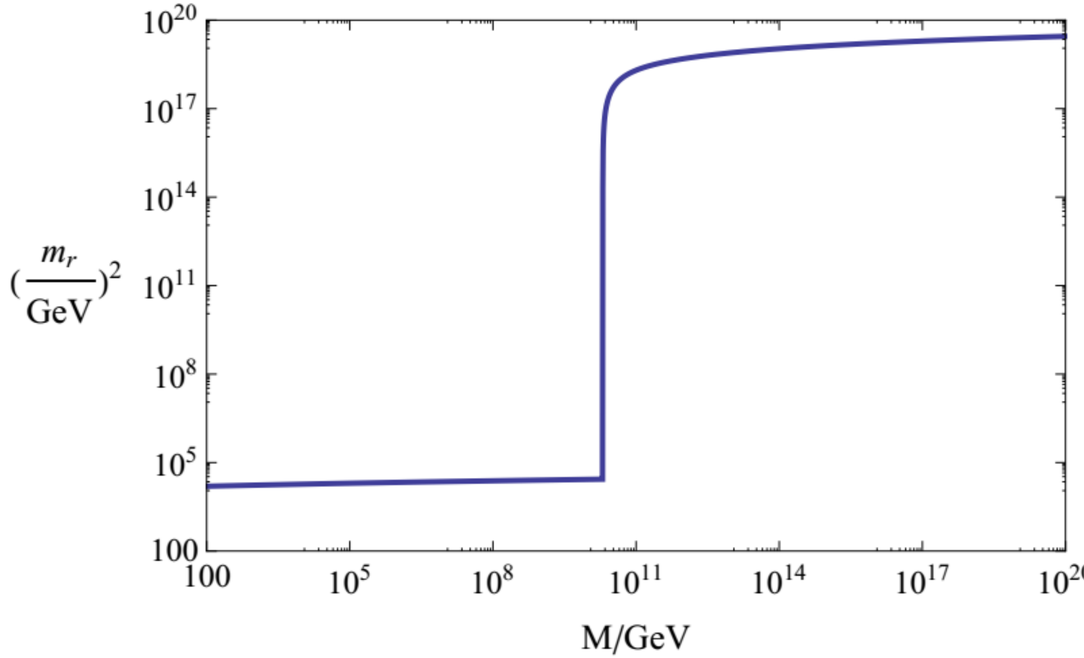
Outline

1. Naturalness
2. Simplest Little Higgs (SLH)
3. Pseudo-axion couplings revisited
4. Conclusion

Naturalness

R. Barbieri, 1309.3473



- The SM particle content is complete with the discovery of the 125 GeV Higgs boson.
- How could the SM remains stable against quantum corrections when embedded into a larger theory is still a mystery.

Naturalness

- One way to stabilize the SM is the Little Higgs mechanism, based on Collective Symmetry Breaking (CSB): Higgs is realized as a Goldstone boson of some spontaneous global symmetry breaking. The global symmetry is also explicitly broken in a collective manner.

	Coupling 1	Coupling 2	Explicit Breaking	Higgs Mass
Case 1	On	Off	No	No
Case 2	Off	On	No	No
Case 3	On	On	Yes	Yes

- With CSB, more operators are needed to renormalize the Higgs mass.

- Therefore the theory is more stable against quantum corrections.

Simplest Little Higgs

- Simplest Little Higgs (SLH): based on $SU(3)_L \times U(1)_X$ electroweak gauge group and the global symmetry breaking pattern

$$[SU(3)_1 \times U(1)_1] \times [SU(3)_2 \times U(1)_2] \to [SU(2)_1 \times U(1)_1] \times [SU(2)_2 \times U(1)_2]$$

nonlinearly realized through two scalar triplets. M. Schmaltz, JHEP 08(2004)056

- Nonlinearly realized scalar sector makes us get rid of the radial mode.
- Nonlinearly realized scalar sector could introduce additional complication for diagonalization of the bosonic sector.

Simplest Little Higgs

- Parametrization of the scalar triplets

F. del Aguila et al., JHEP 03(2011)080

10-8=2 degrees of freedom will ultimately be physical.

$$\Phi_1 = \exp\left(\frac{i\Theta'}{f}\right)\exp\left(\frac{it_\beta\Theta}{f}\right)\begin{pmatrix}0\\0\\fc_\beta\end{pmatrix} \qquad \Phi_2 = \exp\left(\frac{i\Theta'}{f}\right)\exp\left(-\frac{i\Theta}{ft_\beta}\right)\begin{pmatrix}0\\0\\fs_\beta\end{pmatrix}$$

η: pseudo-axion

$$\Theta = \frac{\eta}{\sqrt{2}} + \begin{pmatrix}\mathbf{0}_{2\times 2} & h\\ h^\dagger & 0\end{pmatrix}, \quad \Theta' = \frac{\zeta}{\sqrt{2}} + \begin{pmatrix}\mathbf{0}_{2\times 2} & k\\ k^\dagger & 0\end{pmatrix}$$

$$h = \begin{pmatrix}h^0\\h^-\end{pmatrix}, \quad h^0 = \frac{1}{\sqrt{2}}(v + H - i\chi)$$

$$k = \begin{pmatrix}k^0\\k^-\end{pmatrix}, \quad k^0 = \frac{1}{\sqrt{2}}(\sigma - i\omega)$$

CP-even scalars: H, σ

CP-odd scalars: η, ζ, χ, ω

$$s_\beta \equiv \sin\beta, c_\beta \equiv \cos\beta, t_\beta \equiv \tan\beta$$

Simplest Little Higgs

- Gauge kinetic terms for the scalar triplets (automatic CSB from gauge symmetry)

$$\mathcal{L}_{gk} = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2)$$

$$D_\mu = \partial_\mu - igA^a_\mu T^a + ig_x Q_x B^x_\mu, \quad g_x = \frac{g t_W}{\sqrt{1 - t_W^2/3}} \quad Q_x = -\tfrac{1}{3}$$

$$A^a_\mu T^a = \frac{A^3_\mu}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{A^8_\mu}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & W^+_\mu & Y^0_\mu \\ W^-_\mu & 0 & X^-_\mu \\ Y^{0\dagger}_\mu & X^+_\mu & 0 \end{pmatrix}$$

$$Y^0_\mu \equiv \frac{1}{\sqrt{2}}(Y_{R\mu} + iY_{I\mu}), \quad Y^{0\dagger}_\mu \equiv \frac{1}{\sqrt{2}}(Y_{R\mu} - iY_{I\mu})$$

First order gauge boson mixing

$$\begin{pmatrix} A^3 \\ A^8 \\ B_x \end{pmatrix} = \begin{pmatrix} 0 & c_W & -s_W \\ \sqrt{1 - \frac{t_W^2}{3}} & \frac{s_W t_W}{\sqrt{3}} & \frac{s_W}{\sqrt{3}} \\ -\frac{t_W}{\sqrt{3}} & s_W\sqrt{1 - \frac{t_W^2}{3}} & c_W\sqrt{1 - \frac{t_W^2}{3}} \end{pmatrix} \begin{pmatrix} Z' \\ Z \\ A \end{pmatrix}$$

Simplest Little Higgs

- Tree level scalar effective potential (Note: $\Phi_1^\dagger\Phi_1 = f^2 c_\beta^2, \Phi_2^\dagger\Phi_2 = f^2 s_\beta^2$)

$$V_0 = -\mu^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) + \lambda |\Phi_1^\dagger\Phi_2|^2$$

- One-loop corrected scalar effective potential at small field value

$$V_{0+1} = -\mu^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) + \bar{\lambda} |\Phi_1^\dagger\Phi_2|^2 + \Delta\hat{h}^4 \qquad \hat{h} \equiv (h^\dagger h)^{1/2}$$

- Pseudo-axion mass comes from the μ term.

Simplest Little Higgs

- The significance of the η particle:
  - In the literature, the ZHη coupling is proposed as a way to distinguish group structures of the Little Higgs models (simple group/product group). (However, this point will be challenged by our study.) W. Kilian, D. Rainwater & J. Reuter, PRD 71, 015008(2005), PRD 74, 095003(2006).
  - (On-going study) There is a mass relation connecting η mass and the top partner mass, which is a crucial test of the SLH mechanism. Also, η mass is related to the degree of fine-tuning in the SLH.

    K. Cheung, S-P. He, Y-n. Mao, **CZ**, Y. Zhou, to appear

Simplest Little Higgs

[2]There are other possible choices for the generator T_η that multiplies the η field, e.g., T^8 or diag(0, 0, 1). However, after EWSB these choices introduce kinetic mixing of the η with unphysical Goldstone bosons. Removing this mixing by appropriate field redefinitions is equivalent to choosing T_η proportional to the unit matrix.

W. Kilian, D. Rainwater & J. Reuter, PRD **71**, 015008 (2005)

- It is argued by Kilian et al. that choosing the η generator proportional to the unit matrix would remove kinetic mixing between η and unphysical Goldstones.
- We have checked this kinetic mixing by various methods and various people, but couldn't confirm the above statement.
- This kinetic mixing is crucial for η phenomenology, affecting all the η vertices.
- Note: Changing the η generator would amount to a field redefinition.

Simplest Little Higgs: Check on Scalar Kinetic Mixing

$\xi \equiv \frac{v}{f}$

Row & Column order: η, ζ, χ, ω

Note: $\gamma \equiv \frac{vt_\beta}{\sqrt{2}f}, \delta \equiv \frac{v}{\sqrt{2}ft_\beta}$

Exact to all orders in ξ

$$V = \begin{pmatrix} 1 & 0 & \frac{fs_{2\beta}}{2\sqrt{2}v}(c_{2\gamma}-c_{2\delta}) & -\frac{1}{2}s_{2\beta}(s_{2\gamma}+s_{2\delta}) \\ 0 & 1 & -\frac{\sqrt{2}f}{v}(s_\gamma^2 c_\beta^2 + s_\delta^2 s_\beta^2) & s_{2\delta}s_\beta^2 - s_{2\gamma}c_\beta^2 \\ \frac{fs_{2\beta}}{2\sqrt{2}v}(c_{2\gamma}-c_{2\delta}) & -\frac{\sqrt{2}f}{v}(s_\gamma^2 c_\beta^2 + s_\delta^2 s_\beta^2) & \frac{2f^2}{v^2}(s_\gamma^2 c_\beta^2 + s_\delta^2 s_\beta^2) & -\frac{f}{\sqrt{2}v}(s_{2\delta}s_\beta^2 - s_{2\gamma}c_\beta^2) \\ -\frac{1}{2}s_{2\beta}(s_{2\gamma}+s_{2\delta}) & s_{2\delta}s_\beta^2 - s_{2\gamma}c_\beta^2 & -\frac{f}{\sqrt{2}v}(s_{2\delta}s_\beta^2 - s_{2\gamma}c_\beta^2) & 1 \end{pmatrix}$$

Expansion to O(ξ³)

$$V = \begin{pmatrix} 1 & 0 & \frac{\sqrt{2}}{t_{2\beta}}\xi - \frac{7c_{2\beta}+c_{6\beta}}{6\sqrt{2}s_{2\beta}^3}\xi^3 & -\sqrt{2}\xi + \frac{5+3c_{4\beta}}{3\sqrt{2}s_{2\beta}^2}\xi^3 \\ 0 & 1 & -\frac{1}{\sqrt{2}}\xi + \frac{5+3c_{4\beta}}{12\sqrt{2}s_{2\beta}^2}\xi^3 & -\frac{2\sqrt{2}}{3t_{2\beta}}\xi^3 \\ \frac{\sqrt{2}}{t_{2\beta}}\xi - \frac{7c_{2\beta}+c_{6\beta}}{6\sqrt{2}s_{2\beta}^3}\xi^3 & -\frac{1}{\sqrt{2}}\xi + \frac{5+3c_{4\beta}}{12\sqrt{2}s_{2\beta}^2}\xi^3 & 1 - \frac{5+3c_{4\beta}}{12s_{2\beta}^2}\xi^2 & \frac{2}{3t_{2\beta}}\xi^2 \\ -\sqrt{2}\xi + \frac{5+3c_{4\beta}}{3\sqrt{2}s_{2\beta}^2}\xi^3 & -\frac{2\sqrt{2}}{3t_{2\beta}}\xi^3 & \frac{2}{3t_{2\beta}}\xi^2 & 1 \end{pmatrix} + \mathcal{O}(\xi^4)$$

Simplest Little Higgs: Check on Vector-Scalar Mixing

$$\xi \equiv \frac{v}{f}$$

Row order: Z, Z', Y_1

Column order: η, ζ, χ, ω

Note: $\gamma \equiv \frac{vt_\beta}{\sqrt{2}f}, \delta \equiv \frac{v}{\sqrt{2}ft_\beta}$

$$\rho \equiv \sqrt{\frac{1+2c_{2W}}{1+c_{2W}}}, \quad \kappa \equiv \frac{c_{2W}}{2c_W^2\sqrt{3-t_W^2}}$$

Exact to all orders in ξ

$$F = \begin{pmatrix} \frac{gfs_{2\beta}}{4\sqrt{2}c_W}(c_{2\gamma}-c_{2\delta}) & -\frac{gf}{\sqrt{2}c_W}(s_\delta^2 s_\beta^2 + s_\gamma^2 c_\beta^2) & \frac{gf^2}{vc_W}(s_\delta^2 s_\beta^2 + s_\gamma^2 c_\beta^2) & \frac{gf}{2\sqrt{2}c_W}(s_{2\gamma}c_\beta^2 - s_{2\delta}s_\beta^2) \\ \frac{\sqrt{3-t_W^2}\,gfs_{2\beta}}{4\sqrt{2}}(c_{2\gamma}-c_{2\delta}) & \frac{gf[1+(1+2c_{2W})(c_{2\gamma}c_\beta^2 + c_{2\delta}s_\beta^2)]}{2c_W^2\sqrt{6-2t_W^2}} & \frac{(1-t_W^2)gf^2}{v\sqrt{3-t_W^2}}(s_\delta^2 s_\beta^2 + s_\gamma^2 c_\beta^2) & -\frac{gf}{2\sqrt{6-2t_W^2}\,c_W^2}(s_{2\gamma}c_\beta^2 - s_{2\delta}s_\beta^2) \\ -\frac{gfs_{2\beta}}{2\sqrt{2}}(s_{2\gamma}+s_{2\delta}) & \frac{gf}{\sqrt{2}}(s_{2\delta}s_\beta^2 - s_{2\gamma}c_\beta^2) & -\frac{gf^2}{2v}(s_{2\delta}s_\beta^2 - s_{2\gamma}c_\beta^2) & \frac{gf}{\sqrt{2}} \end{pmatrix}$$

Expansion to O(ξ³)

$$F = gf \begin{pmatrix} \frac{1}{\sqrt{2}c_W t_{2\beta}}\xi^2 & -\frac{1}{2\sqrt{2}c_W}\xi^2 & \frac{1}{2c_W}\xi - \frac{5+3c_{4\beta}}{24c_W s_{2\beta}^2}\xi^3 & \frac{1}{3c_W t_{2\beta}}\xi^3 \\ \frac{\rho}{t_{2\beta}}\xi^2 & \frac{\sqrt{2}}{\sqrt{3-t_W^2}} - \frac{1+2c_{2W}}{2\sqrt{2}c_W^2\sqrt{3-t_W^2}}\xi^2 & \kappa\xi - \frac{\kappa(5+3c_{4\beta})}{12s_{2\beta}^2}\xi^3 & -\frac{1}{3c_W^2\sqrt{3-t_W^2}\,t_{2\beta}}\xi^3 \\ -\xi + \frac{5+3c_{4\beta}}{6s_{2\beta}^2}\xi^3 & -\frac{2}{3t_{2\beta}}\xi^3 & \frac{\sqrt{2}}{3t_{2\beta}}\xi^2 & \frac{1}{\sqrt{2}} \end{pmatrix} + \mathcal{O}(\xi^4)$$

Simplest Little Higgs

- **General formulation of the problem**

Consider a gauge field theory with

$$n_S \text{ real scalar fields } G_i, i = 1, 2, ..., n_S$$

$$n_M \text{ real massive gauge boson fields } Z_p^\mu, p = 1, 2, ..., n_M.$$

Suppose its classical Lagrangian contains

$$\mathcal{L}_{quad} \supset \frac{1}{2}V_{ij}(\partial_\mu G_i)(\partial^\mu G_j) + F_{pi}Z_p^\mu(\partial_\mu G_i) - \frac{1}{2}(\mathbb{M}_G^2)_{ij}G_iG_j + \frac{1}{2}(\mathbb{M}_V^2)_{pq}Z_{p\mu}Z_q^\mu$$

Question: How to diagonalize this system?

Simplest Little Higgs

$$\mathcal{L}_{quad} \supset \frac{1}{2}V_{ij}(\partial_\mu G_i)(\partial^\mu G_j) + F_{pi} Z_p^\mu (\partial_\mu G_i) - \frac{1}{2}(\mathbb{M}_G^2)_{ij} G_i G_j + \frac{1}{2}(\mathbb{M}_V^2)_{pq} Z_{p\mu} Z_q^\mu$$

1. Diagonalize V & M_G^2 by standard linear algebra method.
2. Diagonalize M_V^2 by standard linear algebra method.
3. Add gauge-fixing term in the form (the gauge fields below are in mass eigenstates) to eliminate vector-scalar mixing

$$\mathcal{L}_{gf} = -\sum_{p=1}^{n_M} \frac{1}{2\xi^p}(\partial_\mu \bar{Z}_p^\mu - \xi^p \bar{G}_p)$$

Note: It is important that massive gauge bosons eat their corresponding Goldstone bosons along the directions dictated by their mass eigenstates.

Simplest Little Higgs

η^m component contained in η, ζ, χ, ω: needed for derivation for all pseudo-axion couplings; can be expressed using only V & F.

$$\Upsilon \equiv \begin{pmatrix} \sqrt{(V^{-1})_{11}} \\ -\sqrt{(V^{-1})_{11}}\tilde{F}^{-1}\begin{pmatrix} F_{Z\eta} \\ F_{Z'\eta} \\ F_{Y\eta} \end{pmatrix} \end{pmatrix} \qquad \tilde{F} \equiv \begin{pmatrix} F_{Z\zeta} & F_{Z\chi} & F_{Z\omega} \\ F_{Z'\zeta} & F_{Z'\chi} & F_{Z'\omega} \\ F_{Y\zeta} & F_{Y\chi} & F_{Y\omega} \end{pmatrix}$$

Exact to all orders in ξ

$$\Upsilon = \begin{pmatrix} c_{\gamma+\delta}^{-1} \\ -c_{\gamma+\delta}^{-1}(s_\delta^2 t_\beta - s_\gamma^2 t_\beta^{-1}) \\ \frac{v}{\sqrt{2}f} c_{\gamma+\delta}^{-1}(c_{2\delta}t_\beta - c_{2\gamma}t_\beta^{-1}) \\ \frac{1}{2}c_{\gamma+\delta}^{-1}(s_{2\delta}t_\beta + s_{2\gamma}t_\beta^{-1}) \end{pmatrix}$$

$$\gamma \equiv \frac{vt_\beta}{\sqrt{2}f}, \delta \equiv \frac{v}{\sqrt{2}ft_\beta}$$

Expansion to O(ξ³) $\quad \xi \equiv \frac{v}{f}$

$$\Upsilon = \begin{pmatrix} 1 + \frac{1}{s_{2\beta}^2}\xi^2 + \mathcal{O}(\xi^4) \\ -\frac{1}{t_{2\beta}}\xi^2 + \mathcal{O}(\xi^4) \\ -\frac{\sqrt{2}}{t_{2\beta}}\xi - \frac{3-c_{4\beta}}{\sqrt{2}s_{2\beta}^2 t_{2\beta}}\xi^3 + \mathcal{O}(\xi^5) \\ \sqrt{2}\xi + \frac{3-c_{4\beta}}{3\sqrt{2}s_{2\beta}^2}\xi^3 + \mathcal{O}(\xi^5) \end{pmatrix}$$

Pseudo-axion Couplings Revisited

- The final results, for the coefficient of mass eigenstate antisymmetric ($Z^\mu(\eta\partial_\mu H - H\partial_\mu\eta)$) & symmetric $(Z^\mu(\eta\partial_\mu H + H\partial_\mu\eta))$ ZHη vertex, to $\mathcal{O}(\xi^3)$, are

$$c^{as}_{ZH\eta} = -\frac{g}{4\sqrt{2}c_W^3 t_{2\beta}}\xi^3 + \mathcal{O}(\xi^5)$$

$$c^{s}_{ZH\eta} = \frac{g}{\sqrt{2}c_W t_{2\beta}}\xi + \frac{g}{24\sqrt{2}c_W s_{2\beta}}\left[\frac{8}{s_{2\beta}t_{2\beta}} + 3c_{2\beta}\left(8 + \frac{6}{c_W^2} - \frac{1}{c_W^4}\right)\right]\xi^3 + \mathcal{O}(\xi^5)$$

- From our derivation, the antisymmetric vertex vanishes at O(ξ), which is different from the expression that has existed for a long time.

$$\mathcal{L}_{ZH\eta} = \frac{m_Z}{\sqrt{2}F}N_2 Z_\mu(\eta\partial^\mu H - H\partial^\mu\eta) \qquad N_2 = \frac{F_2^2 - F_1^2}{F_1 F_2}$$

W. Kilian, D. Rainwater & J. Reuter, PRD 71, 015008(2005), PRD 74, 095003(2006).

- The η phenomenology could thus be very different. (On-going study)

Pseudo-axion Couplings Revisited

- SLH lepton Yukawa terms

$$\mathcal{L}_Y \supset \mathrm{i}\lambda_N^m \bar{N}_{Rm} \Phi_2^\dagger L_m + \frac{\mathrm{i}\lambda_\ell^{mn}}{\Lambda} \bar{\ell}_{Rm} \epsilon_{ijk} \Phi_1^i \Phi_2^j L_n^k + \mathrm{h.c.}\,,$$

- With the exact expression for Upsilon vector, we obtain for the neutral sector

$$\varepsilon_{ijk} \Phi_1^i \Phi_2^j \supset -if \begin{pmatrix} 0 \\ f s_\beta c_\beta s_{\gamma+\delta} + \frac{1}{\sqrt{2}} c_{\gamma+\delta} H \\ 0 \end{pmatrix}$$

- Therefore the η coupling to a pair of SM leptons vanish to all order in ξ.

Conclusion

- Gauged nonlinear sigma model is often employed in natural BSM model building. In such cases, it is important to check whether the scalar kinetic terms are diagonalized canonically-normalized and whether there are 'unexpected' vector-scalar mixing.
- We checked the quadratic part of the SLH bosonic Lagrangian and found an important discrepancy compared to previous literature. The discrepancy is related to the pseudo-axion in the model, which is crucial for the test of the SLH mechanism and its fine-tuning.
- We applied an appropriate diagonalization procedure to find the mass eigenstate pseudo-axion component. The $ZH\eta$ coupling and η coupling to SM leptons are found to be very different from the previous literature, and are thus expected to affect its phenomenology significantly.

Thank you!