Neutrino models

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- Neutrino:

Since neutrinos are electrically neutral, there exist two types of masses; Dirac and Majorana.

This Majorana nature can be determined by measuring the signal of neutrinoless beta decay, although it is not detected yet…

I will focus on Majorana type of neutrino.

Majorana neutrinos

One of the renowned ideas is to induce the neutrino masses through Weinberg operator: $(LHLH)/\Lambda \Rightarrow v^2/\Lambda$ ($v \ll \Lambda$).

S. Weinberg, Phys. Rev. Lett. 43, 1566 (1979).

We have no way how theoretically to determine the scale and or structure of cut-off scale Λ.

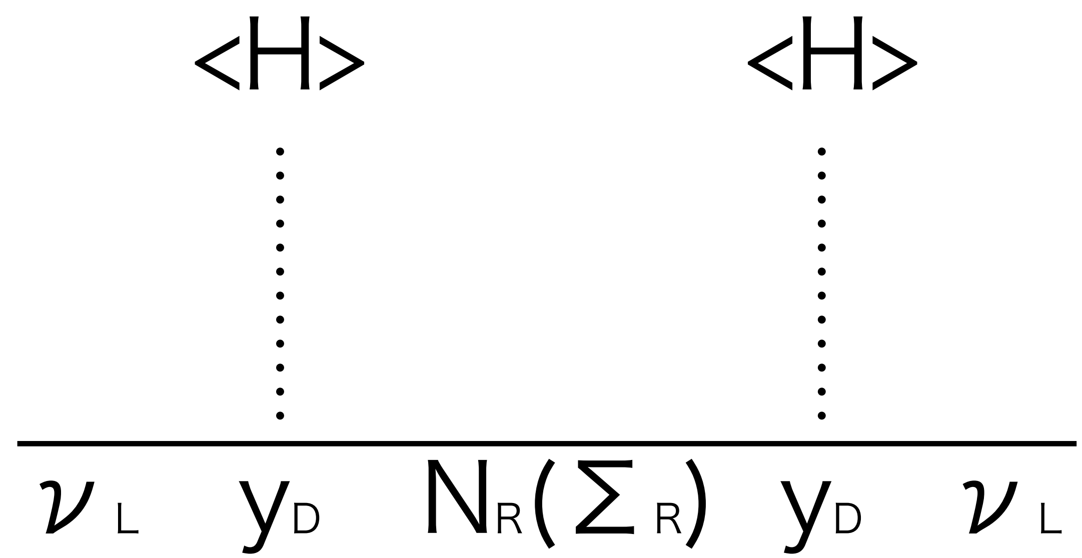
This question might lead to renormalizable theories.

Seesaw models

There are three representative types of models:

Mohapatra, Valle, Foot, Lew, He, Joshi, etc.

Type-1(3):



$$m_\nu \equiv -m_D^T M_N^{-1} m_D;\ D_\nu = U_{MNS}^T m_\nu U_{MNS}.$$

The neutrino mass matrix can be found by replacing Λ into heavy mass of $N_R(\Sigma_R)$.

Casas-Ibarra(CI) parametrization:

$$y_D = i\frac{\sqrt{2}}{v}\sqrt{M_N}\mathcal{O}\sqrt{D_\nu}U^\dagger_{MNS},$$

3 $N_R(\Sigma_R)$ case

$$\mathcal{O} = \begin{bmatrix} c_{\theta_{13}}c_{\theta_{12}} & c_{\theta_{13}}s_{\theta_{12}} & s_{\theta_{13}} \\ -c_{\theta_{23}}s_{\theta_{12}} - s_{\theta_{23}}s_{\theta_{13}}c_{\theta_{12}} & c_{\theta_{23}}c_{\theta_{12}} - s_{\theta_{23}}s_{\theta_{13}}s_{\theta_{12}} & s_{\theta_{23}}c_{\theta_{13}} \\ s_{\theta_{23}}s_{\theta_{12}} - c_{\theta_{23}}s_{\theta_{13}}c_{\theta_{12}} & -s_{\theta_{23}}c_{\theta_{12}} - c_{\theta_{23}}s_{\theta_{13}}s_{\theta_{12}} & c_{\theta_{23}}c_{\theta_{13}} \end{bmatrix},$$

2 $N_R(\Sigma_R)$ case

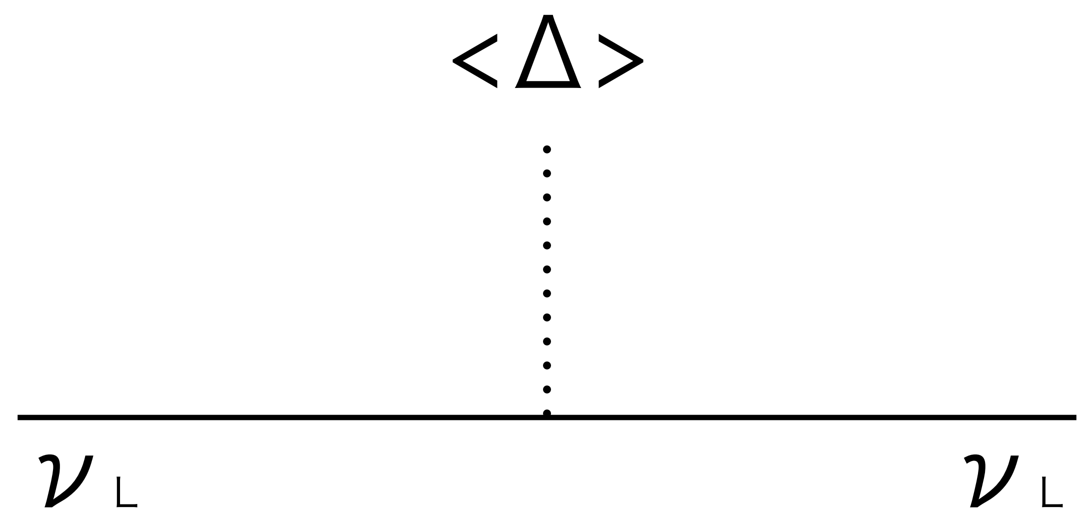
$$\mathcal{O} = \begin{bmatrix} 0 & 0 \\ \cos z & -\sin z \\ \pm\sin z & \pm\cos z \end{bmatrix}^T, \quad O = \begin{bmatrix} \cos z & -\sin z \\ \pm\sin z & \pm\cos z \\ 0 & 0 \end{bmatrix}^T,$$

NH IH

This is a useful parametrization not only to fit the current neutrino oscillation data but also to connect a theory and experiment.

Note: The mass of $N_R(\Sigma_R)$ has to be diagonal !

Type-2:



The neutrino mass matrix can be found by replacing v^2/Λ into VEV of Δ.

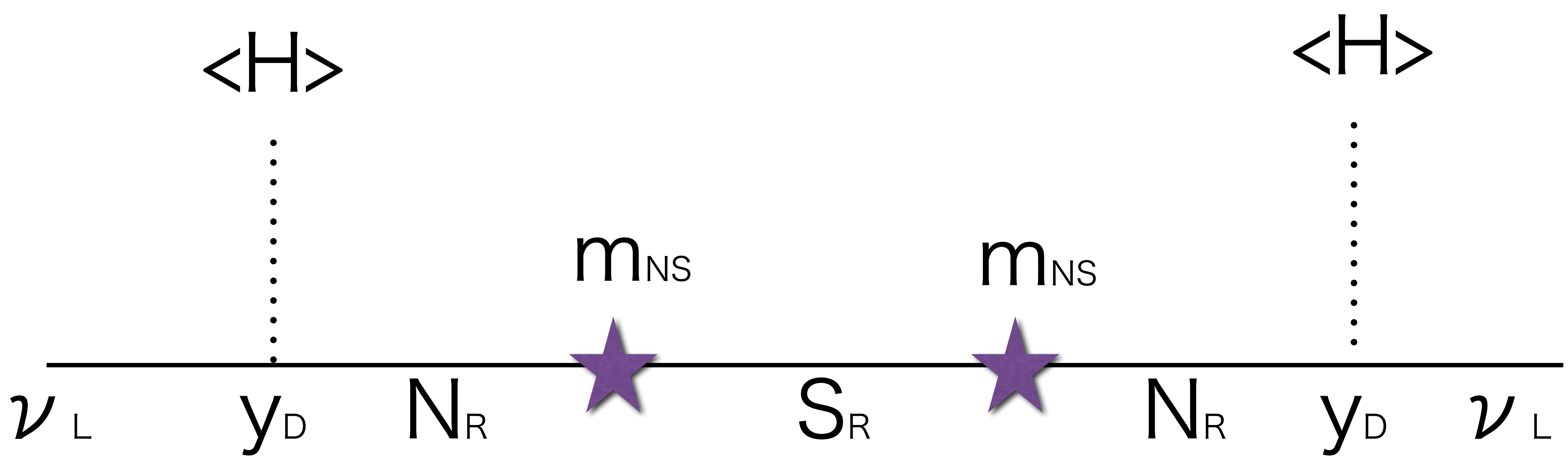
$$m_{\nu_{ab}} = \frac{y_{\Delta_{ab}}}{\sqrt{2}} v_\Delta.$$

CI parametrization: $y_\Delta = \frac{\sqrt{2}}{v_\Delta} U^*_{MNS} D_\nu U^\dagger_{MNS}.$

This is completely determined by the experimental values except VEV of Δ!

The other complicated but representative models introducing additional tiny Majorana fermions: S_R

1. Inverse seesaw:



Assuming $M_N=0$ and hierarchy: $\mu_S \ll y_D v < m_{NS}$,

$$m_\nu \approx m_D^T m_{NS}^{-1} \mu_S (m_{NS}^T)^{-1} m_D,$$

CI parametrization of Inverse seesaw

$$m_\nu \approx m_D^T \boxed{m_{NS}^{-1}\mu_S(m_{NS}^T)^{-1}} m_D,$$

$$= M_N^{-1}$$

Let us identify type-I like seesaw model by regarding the red part as inverse of M_{N}.

However since inverse of M_N is not diagonal but symmetric, one cannot apply the simplest CI !

One of the conventional solution is to apply Cholesky Factorization(CF) into M_N

$$M_N^{-1} \equiv L_N^T D_N^{-1} L_N$$

※ LN is an triangular matrix

Then neutrino mass matrix can be redefined by

$$m_\nu \approx m_D^T L_N^T D_N^{-1} L_N m_D \equiv M_D^T D_N^{-1} M_D$$

Thus one finds the CI parametrization just by the following replacements:

$$m_D \to M_D \text{ and } M_N \to D_N$$

Point: (DN , LN) can uniquely be fixed by components of MN, when DN is taken to be positive real.

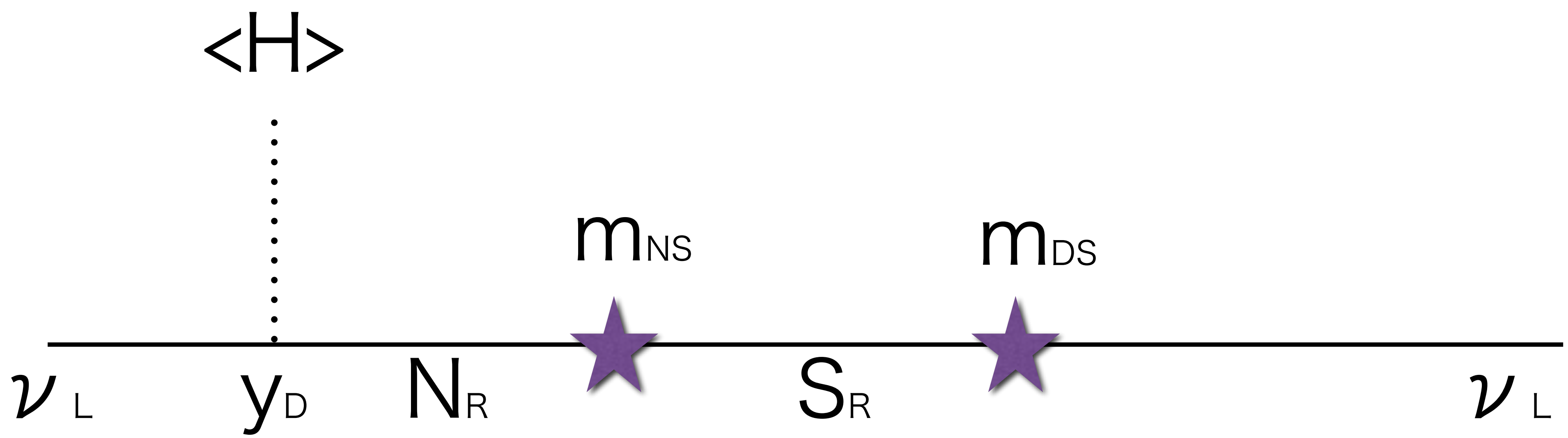
Generalized CI parameterization:

One can generalize 3xN right-handed matrix.

In this case, O is extended by 3 by N mixing matrix and its degrees of freedom is given by 3(N-2).

Note: O is not orthogonal matrix but just satisfies $OO^T=1_{3x3}$.

2. Linear seesaw:



Assuming M_N=0 and hierarchy: $m_{DS} \ll y_D v < m_{NS}$,

$$m_\nu \approx -m_{DS}^T (m_{NS}^T)^{-1} m_D - m_D^T (m_{NS})^{-1} m_{DS}$$

CI parametrization of linear seesaw is given by introducing anti-symmetric matrix A as:

$$m_D = -\frac{1}{2} m_{NS}^T (m_{DS}^T)^{-1} (U_{MNS}^* D_\nu U_{MNS}^\dagger + A).$$

where $$A + A^T = 0.$$

Careful points to applying CI!

If you have a specific texture of a Yukawa coupling related to the neutrino mass matrix (such as two-zero texture) or the undefined inverse matrix (due to the reduction of rank), applying “CI” is not recommended.

Textures

H. Fritzsch, Z. Xing, S. Zhou, etc,…

Reduced matrix of y_D

Juan Herrero-Garcia, Miguel Nebot, Nuria Rius, Arcadi Santamaria, etc,...

Radiatively induced neutrino mass models:

It provides a concrete structure of Λ at loop levels.
=> Theory can be within low energy scale(~TeV).

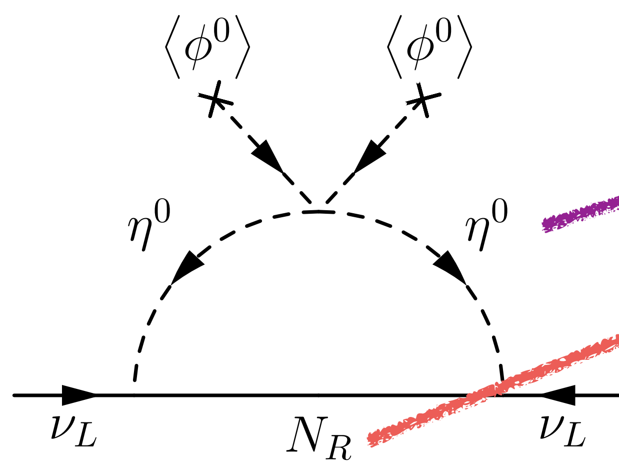
2. Dark matter candidate, muon anomalous magnetic dipole moment, origin of Baryon number asymmetry via leptogenesis, can be explained linking to the active neutrinos!

Note: One has to take care of Lepton flavor violations(LFVs), since they are always arisen from the related neutrino Yukawa couplings !!

Ma Model

SM+right-handed Neutrino +SU(2) doublet Scalar

	N^c	η
$SU(2)_L$	$\mathbf{1}$	$\mathbf{2}$
$U(1)_Y$	0	$1/2$
$\mathbb{Z}_2$	-1	-1



The lightest of N_R and or
η _{R/I} can be dark matter candidate!

Interacting term

$$\mathcal{L} = (y_\nu)_{\alpha i}\,\eta^\dagger \overline{\ell_\alpha} N_i^c + \text{h.c.} - \mathcal{V}(\phi,\eta) + \frac{1}{2} M_i N_i N_i + H.c.$$

※ y_ν is restricted by LFVs at one-loop level.

Scalar potential term

$$\mathcal{V}(\phi,\eta) = m_\phi^2|\phi|^2 + m_\eta^2|\eta|^2 + \frac{\lambda_1}{2}|\phi|^4 + \frac{\lambda_2}{2}|\eta|^4 + \lambda_3|\phi|^2|\eta|^2$$
$$+\lambda_4\left(\phi^\dagger\eta\right)\left(\eta^\dagger\phi\right) + \frac{\lambda_5}{2}\left[\left(\phi^\dagger\eta\right)^2 + \left(\eta^\dagger\phi\right)^2\right]$$

Summary

Along this line of ideas, a vast of literature has arisen,

since nonzero theta_13 has been measured (2012),

More than 110 ideas at one-loop level,

More than 40 ideas at two-loop level,

More than 30 ideas at three-loop level,

and a few ideas at four-loop level…

Another applications:

- Small mass SM fermions(e,u,d,muon) can be explained by introducing exotic charged fermions.
- Anomaly such as b->s,mu mu_bar could be explained by introducing leptoquarks with/without flavor dependent gauged symmetries.
- Several anomalies of indirect detections from cosmic-rays as a DM candidate could be explained.
- Collider signatures could be expected in the near future.

Thanks.