Strongly-coupled Anisotropic Gauge Theories And Holography

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Based on works with: U. Gursoy(Utrecht Univ.) and J. Pedraza(Univ. of Amsterdam); arxiv:1708.05691

Talk given for: NCTS Annual Theory Meeting, Taiwan, December 6, 2017

Outline

Short talk: Briefly we have

- Strongly coupled anisotropic theory.
- How the theory looks like and how to obtain it?
  - ✓ 4d $SU(N)$ gauge theory in the large N_c-limit.
  - ✓ Its dynamics are affected by a scalar operator $\mathcal{O} \sim TrF^2$.
  - ✓ Anisotropy is introduced by another operator $\tilde{\mathcal{O}} \sim \theta(x_3) TrF \wedge F$ with a space dependent coupling.
  - ✓ On the gravity dual side we have a "backreacting" scalar field depending on spatial directions, the axion; and a non-trivial dilaton.
- Eventually the gravity dual theory is an Einstein-Axion-Dilaton theory in 5 dimensions with a non-trivial potential.
  - ✓ Solutions are RG flows:
    AdS in UV ⇒ Anisotropic (Hyperscaling Lifshitz-like) in IR.

Theory Evolution:

Non-Confining:

(Azeyanagi, Li, Takayanagi, 2009; Mateos, Trancanelli, 2011; Jain, Kundu, Sen, Sinha, Trivedi, 2015;...)

Confining:

(D.G., Gursoy, Pedraza, 2017)

Similar ideas in different context:

(Gaiotto, Witten 2008; Chu, Ho, 2006; Choi, Fernadez, Sugimoto 2017;...)

Instead of motivation:

- Such theories violate well known universal relations:
  - ✓ Shear Viscosity over entropy density ratio is parametrically lower than KSS bound: $1/4\pi$.
    (Rebhan,Steineder 2011; Jain, Samanta, Trivedy 2015; D.G., Gursoy, Pedraza, 2017;...)
  - ✓ Langevin Coefficient inequality for heavy quark motion in the anisotropic plasma $\kappa_L ><\kappa_T$.
    (Gursoy, Kiritsis, Mazzanti, Nitti 2010; D.G, Soltanpanahi, 2013a, 2013b)
  - ✓ ...

Especially interesting since several strongly coupled systems exhibit anisotropies.

The Anisotropic Theory

The generalized Einstein-Axion-Dilaton action with a potential for the dilaton and an arbitrary coupling between the axion and the dilaton:

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 + V(\phi) - \frac{1}{2}Z(\phi)(\partial\chi)^2\right].$$

The eoms read

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}Z(\phi)\partial_\mu\chi\partial_\nu\chi - \frac{1}{4}g_{\mu\nu}(\partial\phi)^2 - \frac{1}{4}g_{\mu\nu}Z(\partial\chi)^2 + \frac{1}{2}g_{\mu\nu}V(\phi)\ ,$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right) = \frac{1}{2}\partial_\phi Z(\phi)(\partial\chi)^2 - V'(\phi)\ ,$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\chi\right) = 0\ .$$

Where the functions

$$V(\phi) = 12\cosh(\sigma\phi) + \left(\frac{-m(\Delta)^2}{2} - 6\sigma^2\right)\phi^2, \qquad Z(\phi) = e^{2\gamma\phi},$$

((Gubser, Nellore), Pufu, Rocha 2008a,b)

Note: For $\sigma = 0, \gamma = 1, m(\Delta) = 0$ the action and the solution of eoms, are of IIB supergravity. *(Mateos, Trancanelli, 2011)*

A Solution: Schematically

- The background solution

$$ds^2 = \frac{e^{-\frac{1}{2}\phi(u)}}{u^2}\left(-\mathcal{F}\mathcal{B}\,dt^2 + dx_1^2 + dx_2^2 + \mathcal{H}dx_3^2 + \frac{du^2}{\mathcal{F}}\right),$$
$$\chi = \alpha x_3\,, \qquad \phi = \phi(u)\,,$$

- Note:
  The linear axion simplifies tremendously the system of equations!
- We solve the system of eoms to obtain the theory e.g. for $\Delta = 4$:
  Geometries that are AdS in UV flowing to Hyperscaling Lifshitz-like violation geometries in IR with arbitrary scalings:

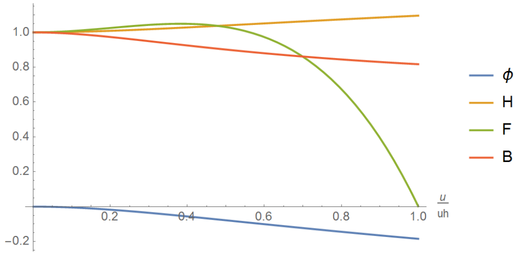
$$ds^2 = u^{-\frac{2\theta}{3}}\left(-u^{2z}\left(f(u)dt^2 + dx_{1,2}^2\right) + \tilde{\alpha}u^2 dx_3^2 + \frac{du^2}{f(u)u^2}\right)\,,$$

- Note: The values of (θ, z) depend on Axion-Dilaton coupling and Potential parameters (σ, γ). In supergravity there is a single solution $(\theta = 0, z = 3/2)$. *(Azeyanagi, Li, Takayanagi, 2009)*

Solution : The Full Flow

- Fixing (γ, σ) and α and u_h we get the metric flow from boundary to horizon:

$$ds^2 = \frac{e^{-\frac{1}{2}\phi(u)}}{u^2}\left(-\mathcal{F}\mathcal{B}\,dt^2 + dx_1^2 + dx_2^2 + \mathcal{H}dx_3^2 + \frac{du^2}{\mathcal{F}}\right),$$



- In sufficiently high temperatures, $T \gg \alpha$ the Einstein equations can be solved analytically; e.g. for $\gamma = 1, \sigma = 0$:

$$\mathcal{F}(u) = 1 - \frac{u^4}{u_h^4} + \alpha^2 \frac{1}{24u_h^2}\left[8u^2(u_h^2 - u^2) - 10u^4 \log 2 + (3u_h^4 + 7u^4)\log\left(1 + \frac{u^2}{u_h^2}\right)\right]$$

$$\mathcal{B}(u) = 1 - \alpha^2 \frac{u_h^2}{24}\left[\frac{10u^2}{u_h^2 + u^2} + \log\left(1 + \frac{u^2}{u_h^2}\right)\right], \quad \mathcal{H}(u) = \left(1 + \frac{u^2}{u_h^2}\right)^{\frac{\alpha^2 u_h^2}{4}}.$$

We have obtained the theories, are all of them physical and stable?

Local Thermodynamical Stability in CE and the request for physical theories (NEC) constrain the parameters:

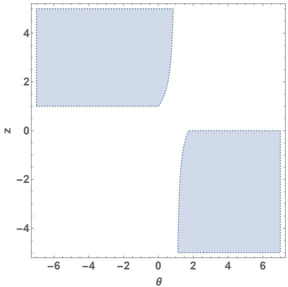
Three conditions that constrain (z, θ) and as a result (γ, σ).

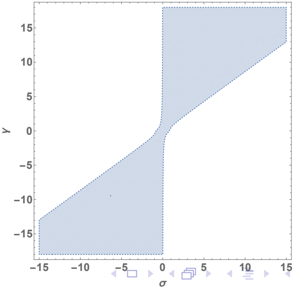
$$(z-1)(1-\theta+3z) \geq 0 \ , \qquad z = \frac{2+4\gamma^2-3\sigma^2}{2\gamma(2\gamma-3\sigma)} \ ,$$

$$\theta^2 - 3 + 3z(1-\theta) \geq 0 \ , \qquad \theta = \frac{3\sigma}{2\gamma} \ ,$$

$$1 - \theta + 2z \geq 0 \ .$$

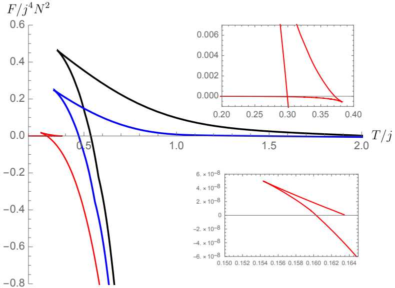
Translated to the following diagrams where the blue region is the acceptable for the theory parameters.





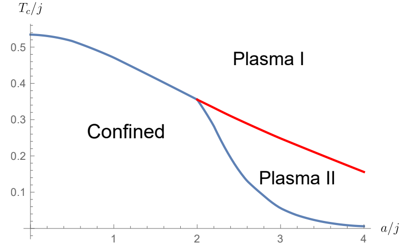
Confinement/Deconfinement Phase transitions

- The free energy of the theories vs the temperature T for different anisotropy (α/j=0,1,3):



- Horizontal Axis: Confining Phase.
- Upper Branch: Black hole A:Deconfining Plasma Phase.
- Lower Branch: Black hole B:Deconfining Plasma Phase.
- $\alpha/j \simeq 2$: A critical value above which a richer structure in the phase diagram exist.

- The Critical Temperature of the theories vs the anisotropy gives:



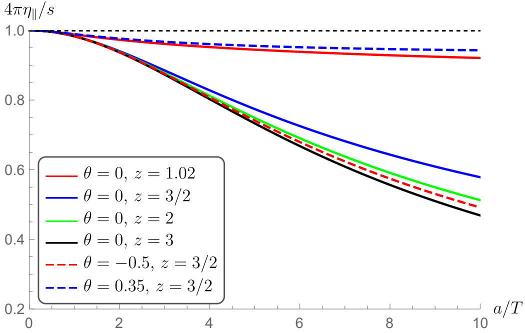
- The T_c is reduced in presence of anisotropies of the theory.

The Proposal

- The $Tc(\alpha)$ decrease with α, resembling the phenomenon of inverse magnetic catalysis where the confinement-deconfinement temperature decreases with the magnetic field B (where an anisotropy is introduced as in our plasma).
- No charged fermionic degrees of freedom in our case; our plasma is neutral.
- Our findings suggest that the anisotropy by itself could instead be the cause of lower T_c in presence of anisotropies.

η/s for our theory: Dependence on the Temperature.

The shear viscosity over entropy ratio for arbitrary (z, θ).



- The ratio depends on the temperature at $\alpha/T \gg 1$ as

$$\frac{\eta_{\|}}{s} \sim \left(\frac{T}{\tilde{\alpha}|1+3z-\theta|}\right)^{2-\frac{2}{z}} .$$

- The range of the temperature power is $[0, \infty)$.

Conclusions

- ✓ We have presented Confining Anisotropic theories and studied the Confinement/Deconfinement phase transitions.
- ✓ The theories are physical and stable for a wide range of parameters of the theory.
- ✓ The Confinement/Deconfinement phase transitions occur at lower critical Temperature as the anisotropy is increased!
- ✓ We suggest that the anisotropy by itself could instead be a cause of inverse magnetic catalysis.
- ✓ The shear viscosity over entropy density ratio, takes values parametrically lower than $1/4\pi$, and depends on the Temperature as $T^{2-2/z}$.
- ✓ The diffusion (butterfly velocity) of chaos occurs faster than isotropic systems.
- More studies needed on such theories...

Thank you!

Null Energy Condition

- The averaged radial acceleration between two null geodesics is

$$A_r = -4\pi T_{\mu\nu} N^\mu N^\nu \ ,$$

if it is negative the null geodesics observe a non-repulsive gravity on nearby particles along them.

- This imposes the Null Energy Condition

$$T_{\mu\nu} N^\mu N^\nu \geq 0 \ , \quad N^\mu N_\mu = 0 \ ,$$

leading to the following constrains:

- For the Lifshitz-like space $z \geq 1$.
- For the Hyperscaling violation anisotropic metric in 3+1-dim spacetime and anisotropic in 1-dim reads

$$(z-1)(1-\theta+3z) \geq 0 \ ,$$
$$\theta^2 - 3 + 3z(1-\theta) \geq 0 \ .$$

Additional conditions from thermodynamics?

Local Thermodynamic Stability

- The necessary and sufficient conditions for local thermodynamical stability in the canonical ensemble are

$$c_\alpha = T\left(\frac{\partial S}{\partial T}\right)_\alpha \geq 0 \ , \qquad \Phi' = \left(\frac{\partial \Phi}{\partial \alpha}\right)_T \geq 0$$

  c_α is the specific heat: increase of the temperature leads to increase of the entropy.
  Φ' is derivative of the potential: the system is stable under infinitesimal charge fluctuations.

- In the GCE these conditions should be equivalent of having no positive eigenvalues of the Hessian matrix of the entropy with respect to the thermodynamic variables. *(Gubser, Mitra 2001)*
- In the IR the positivity of the specific heat imposes

$$c_\alpha = 1 - \theta + 2z \geq 0$$