Non-Gaussian and loop effects of inflationary correlation functions in BRST formalism

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Introduction

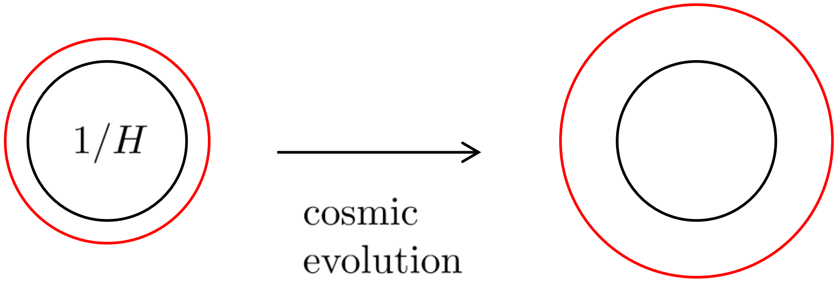
- In dS space, the increase of the degrees of freedom at the super-horizon scale gives rise to a secular growing term to the propagator of a massless and minimally coupled scalar field

$$\langle \phi_0^2(x) \rangle \simeq \int_{k=k_0}^{k=Ha} \frac{d^3k}{(2\pi)^3} \frac{H^2}{2k^3} = \frac{H^2}{4\pi^2} \log(a/a_0)$$

ϕ_0: free field

$a = e^{Ht}, \ a_0 \equiv k_0/H$

In terms of physical coordinates $a\mathbf{x}$,



H: Hubble parameter

a/k_0: size of universe

Introduction

- In interacting field theories with the scalar field, physical quantities acquire IR logarithmic corrections through internal propagators

E.g., in ϕ^4 theory,

$$\langle V(\phi(x))\rangle = \langle \frac{\lambda}{4!}\phi^4(x)\rangle \simeq H^4 \sum_{n=1}^{\infty} c_n \lambda^n \log^{2n}(a/a_0)$$

c_n: const. coefs.

Quantum IR effects grow with cosmic evolution

Today's talk ⊙ Such IR effects appear also in inflation theories because of the accelerated expansion of the universe and the existence of the light modes

$$\langle\zeta\zeta\rangle,\ \langle\gamma_{ij}\gamma_{kl}\rangle \longrightarrow \left(\frac{H^2}{M_{\rm pl}^2}\right)^n \log^n(a/a_0)$$

Problem

- In a conventional approach, we fix the gauge completely to express the action in terms of physical modes e.g. $g_{ij} = a^2 e^{2\zeta} \delta_{ik} (e^{\gamma})^k{}_j,\ \partial_i \gamma^i{}_j = \gamma^i{}_i = 0$
- The quantization scheme is highly nonlocal and the covariance is not manifest ⇒ Not suitable for evaluation of quantum effects

What we did

- As another quantization scheme, we adopted a BRST formalism where locality and covariance at the sub-horizon scale are manifest
- For a consistency check, we calculated the non-Gaussianity at the tree level by using the BRST formalism
- At the one-loop level, we evaluated IR logarithmic corrections to the slow-roll parameters. These IR effects are investigated on the sub-horizon dynamics, which determines the cosmological perturbations

BRST formalism

$$S = \int \sqrt{-g} d^4x \, [\frac{M_{\rm pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu(\hat{\varphi}+\varphi) \partial_\nu(\hat{\varphi}+\varphi) - V(\hat{\varphi}+\varphi)]$$

$$\epsilon \equiv \frac{1}{2}(\frac{M_{\rm pl}\hat{V}'}{\hat{V}})^2 \ll 1, \quad \eta \equiv \frac{M_{\rm pl}^2 \hat{V}''}{\hat{V}} \ll 1$$

$$g_{\mu\nu} = a^2 e^{2\kappa\omega} \eta_{\mu\rho} (e^{\kappa h})^\rho{}_\nu, \quad h_\mu{}^\mu = 0 \qquad\qquad \kappa \equiv \sqrt{2}/M_{\rm pl}$$

We add the gauge fixing term to the Lagrangian

$$\mathcal{L}_{\rm GF} = -\frac{1}{2} \sqrt{-\hat{g}} \; \hat{g}^{\mu\nu} F_\mu F_\nu,$$

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$$F_\mu = \partial_\rho h_\mu{}^\rho - 2\partial_\mu \omega + 2Hah_\mu{}^0 + 4Ha\delta_\mu{}^0 \omega - 2\sqrt{\epsilon} Ha \delta_\mu{}^0 \varphi$$

The Faddeev–Popov ghost term is

$$\mathcal{L}_{\rm FP} = -\sqrt{-\hat{g}} \; \hat{g}^{\mu\nu} \bar{b}_\mu \delta F_\nu \big|_{\delta x^\mu \to b^\mu}$$

List of propagators

Light modes

$$\langle \tilde{h}^{ij}(x)\tilde{h}^{kl}(x')\rangle = (\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk} - \frac{2}{3}\delta^{ij}\delta^{kl}) \times \langle \phi(x)\phi(x')\rangle|_{m^2=0}$$

$$\langle X(x)X(x')\rangle = -\langle \phi(x)\phi(x')\rangle|_{m^2=0}$$

$$\langle b^i(x)\bar{b}^j(x')\rangle = \delta^{ij}\langle \phi(x)\phi(x')\rangle|_{m^2=0}$$

$$\langle \varphi(x)\varphi(x')\rangle = \langle \phi(x)\phi(x')\rangle|_{m^2=(3\eta-6\epsilon)H^2}$$

Heavy modes

$$\langle h^{0i}(x)h^{0j}(x')\rangle = -\delta^{ij}\langle \phi(x)\phi(x')\rangle|_{m^2=2H^2}$$

$$\langle Y(x)Y(x')\rangle = \langle \phi(x)\phi(x')\rangle|_{m^2=2H^2}$$

$$\langle b^0(x)\bar{b}^0(x')\rangle = -\langle \phi(x)\phi(x')\rangle|_{m^2=2H^2}$$

Heavy modes themselves do not induce secular growths

$$\tilde{h}^{ij} \equiv h^{ij} - \frac{1}{3}\delta^{ij}h^{00}, \quad \tilde{h}^{ij} = \tilde{h}^{ij}_S + \tilde{h}^{ij}_V + \tilde{h}^{ij}_T$$

$$X \equiv -\frac{1}{\sqrt{3}}h^{00} + 2\sqrt{3}\omega, \quad Y \equiv h^{00} - 2\omega + \sqrt{\epsilon}\varphi$$

Relation between gauges

	Spatially-flat gauge		Comoving gauge		BRST gauge
Scalar field	φ	$\longrightarrow$ δt	0	$\longleftarrow$ δt	φ
Curvature perturbation	0		$\zeta\delta_{ij}$		$\zeta\delta_{ij}$
Tensor mode	γ_{ij}		γ_{ij}		γ_{ij}
Vector mode	0		0	$\longleftarrow$	$\frac{\partial_i}{\sqrt{\partial_k^2}}V_j + \frac{\partial_j}{\sqrt{\partial_k^2}}V_i$
				δx^i	
Longtitudinal mode	0		0	$\longleftarrow$	$\frac{\partial_i\partial_j}{\partial_k^2}S$

Non-Gaussianity

- Up to the second order, the gauge invariant quantities are

$$\zeta = \zeta^B - \frac{1}{\sqrt{2\epsilon}}\frac{\varphi}{M_{\rm pl}} + \frac{\epsilon - \frac{1}{2}\eta}{2\epsilon}\frac{\varphi^2}{M_{\rm pl}^2}$$

← δt

$$- \partial_k(\zeta^B - \frac{1}{\sqrt{2\epsilon}}\frac{\varphi}{M_{\rm pl}})(\frac{\sqrt{2}}{\sqrt{\partial_l^2}}\frac{V_k}{M_{\rm pl}} + \sqrt{\frac{3}{2}}\frac{\partial_k}{\partial_l^2}\frac{S}{M_{\rm pl}}),$$

← δx^k

$$\gamma_{ij} = \gamma_{ij}^B - \partial_k\gamma_{ij}^B(\frac{\sqrt{2}}{\sqrt{\partial_l^2}}\frac{V_k}{M_{\rm pl}} + \sqrt{\frac{3}{2}}\frac{\partial_k}{\partial_l^2}\frac{S}{M_{\rm pl}})$$

- Just by expanding the action, we can obtain the relevant vertices:

$$S_{\zeta\zeta\zeta} = \frac{1}{2}\frac{\sqrt{2}}{M_{\rm pl}}\int d^4x\ a^2\big[-2\omega\partial_\mu\varphi\partial^\mu\varphi + h^{00}\partial_0\varphi\partial_0\varphi + (\frac{1}{3}\delta^{ij}h^{00} + \tilde{h}_S^{ij})\partial_i\varphi\partial_j\varphi\big],$$

$$S_{\zeta\gamma\gamma} = \frac{1}{2}\frac{\sqrt{2}}{M_{\rm pl}}\int d^4x\ a^2\big[-\omega\partial_\mu\tilde{h}_{kl}^T\partial^\mu\tilde{h}_{kl}^T + \frac{1}{2}h^{00}\partial_0\tilde{h}_{kl}^T\partial_0\tilde{h}_{kl}^T + \frac{1}{2}(\frac{1}{3}\delta^{ij}h^{00} + \tilde{h}_S^{ij})\partial_i\tilde{h}_{kl}^T\partial_j\tilde{h}_{kl}^T\big]$$

Non-Gaussianity

Considering the contribution from the nonlinear terms of the gauge invariant quantities and that from the cubic vertices,

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\cdot\frac{H^6}{(2k_1^3)(2k_2^3)(2k_3^3)}$$
$$\times\frac{1}{2\epsilon H^2 M_{\rm pl}^4}\Big[\Big\{-\frac{1}{2}\sum_i k_i^3+\frac{1}{2}\sum_{i\neq j}k_i k_j^2+\frac{4\sum_{i<j}k_i^2k_j^2}{\sum_k k_k}\Big\}+\frac{2(\epsilon-\frac{1}{2}\eta)}{\epsilon}\sum_i k_i^3\Big],$$

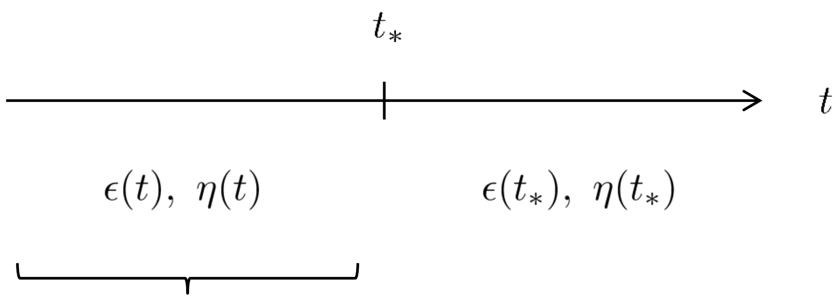
$$\langle \zeta_{\mathbf{k}_1}\gamma^{s_2}_{\mathbf{k}_2}\gamma^{s_3}_{\mathbf{k}_3}\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\cdot\epsilon^{s_2}_{ij}(\mathbf{k}_2)\epsilon^{s_3}_{ij}(\mathbf{k}_3)\cdot\frac{H^6}{(2k_1^3)(2k_2^3)(2k_3^3)}$$
$$\times\frac{1}{H^2M_{\rm pl}^4}\Big\{-\frac{1}{2}k_1^3+\frac{1}{2}k_1(k_2^2+k_3^2)+\frac{4k_2^2k_3^2}{k_1+k_2+k_3}\Big\}$$

Our construction in the BRST formalism reproduces the identical non-Gaussianity with the comoving gauge

We can make a consistency check more easily for $\langle\zeta\zeta\gamma\rangle$ and $\langle\gamma\gamma\gamma\rangle$

IR effects on slow-roll parameters

- At the one-loop level, we evaluate IR logarithmic corrections to the slow-roll parameters. These IR effects are investigated on the sub-horizon dynamics



$H(t_*)a(t_*) = k$

How soft fluctuations influence the time evolutions of ϵ, η before horizon crossing?

- We can construct quantum corrections to $\epsilon = \frac{1}{2}(\frac{M_{\rm pl}\hat{V}'}{\hat{V}})^2, \quad \eta = \frac{M_{\rm pl}^2\hat{V}''}{\hat{V}}$

  by evaluating quantum correction to each operator of the action:

$$\frac{M_{\rm pl}^2}{2}\sqrt{-\hat{g}}\hat{R}, \quad -\sqrt{-\hat{g}}\hat{V}, \quad -\frac{1}{2}\sqrt{-\hat{g}}\hat{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi, \quad -\sqrt{-\hat{g}}\hat{V}'\varphi, \quad -\frac{1}{2}\sqrt{-\hat{g}}\hat{V}''\varphi^2$$

IR effects on slow-roll parameters

- Although the kinetic term does not have any coupling, the wave function renormalization contributes to the one, two-point operators as

$$-\frac{1}{2}\sqrt{-\hat{g}}\hat{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\{1+\frac{1}{2}\frac{\kappa^2H^2}{4\pi^2}\log(a/a_0)\}$$

$$\varphi\to Z\varphi,\quad Z\simeq 1-\frac{1}{4}\frac{\kappa^2H^2}{4\pi^2}\log(a/a_0)$$

$$\Rightarrow\quad -\sqrt{-\hat{g}}\hat{V}'Z\varphi,\quad -\frac{1}{2}\sqrt{-\hat{g}}\hat{V}''Z^2\varphi^2$$

- Including the wave function renormalization, the IR effect on each operator is evaluated as

$$\frac{M_{\rm pl}^2}{2}\sqrt{-\hat{g}}\hat{R}\{1-\frac{3}{4}\frac{\kappa^2H^2}{4\pi^2}\log(a/a_0)\},\quad -\sqrt{-\hat{g}}\hat{V}\{1-\frac{3}{2}\frac{\kappa^2H^2}{4\pi^2}\log(a/a_0)\},$$

$$-\sqrt{-\hat{g}}\hat{V}'\varphi\{1-\frac{11}{32}\frac{\kappa^2H^2}{4\pi^2}\log(a/a_0)\},\quad -\frac{1}{2}\sqrt{-\hat{g}}\hat{V}''\varphi^2\{1-\frac{3}{4}\frac{\kappa^2H^2}{4\pi^2}\log(a/a_0)\}$$

IR effects on slow-roll parameters

At the one-loop level, ϵ acquires a secular growth while η does not

$$\frac{\delta\epsilon}{\epsilon} = \frac{25}{16} \cdot \frac{\kappa^2 H^2}{4\pi^2} \log(a/a_0),$$

$$\frac{\delta\eta}{\eta} = 0 \cdot \frac{\kappa^2 H^2}{4\pi^2} \log(a/a_0)$$

$$\Rightarrow \qquad \frac{\dot{\epsilon}}{H\epsilon} = -2\eta + 4\epsilon + \frac{25}{16}\frac{\kappa^2 H^2}{4\pi^2}$$

If ϵ, η are vanishingly small at the begining due to the shift symmetry, ϵ grows with time while η remains small

In this scenario, we obtain an inflation model with the linear potential

$$V = 3H^2 M_{\rm pl}^2 \big(1 - \sqrt{2\epsilon}\frac{\varphi}{M_{\rm pl}}\big)$$

Summary

- To evaluate quantum IR effects in inflation theories more practically, we adopt a BRST gauge which has advantages over the comoving gauge in terms of locality and covariance at the sub-horizon scale
- At the tree level, our construction in the BRST formalism reproduces the identical non-Gaussianity with the comoving gauge
- At the one-loop level, we investigated the IR effects on the dynamics at the sub-horizon scale. We found that ε acquires an additional time dependence while η does not
- If ε, η are vanishingly small at the beginning due to the shift symmetry, the quantum mechanism may lead to an inflation model with a linear potential

Future directions

- We need to confirm that the cancellation of the IR effect on η hold true, in other gauge choices and at higher loop levels

- A nonperturbative approach for IR effects from gravity is necessary to evaluate eventual contributions to physical quantities
  - $\Rightarrow$ The resummation formula of the leading IR effects should be extended in cases with derivative interactions, gauge degrees and tensor structure

- It is also interesting to investigate IR effects in other inflation theories, e.g. inflation theories with noncanonical kinetic terms

Backup slides
(for evaluation of IR effects)

Contribution to background

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The contribution to the cosmological constant operator is evaluated as

$$
\begin{aligned}
& -\sqrt{-\hat{g}}\hat{V}\langle e^{4\kappa\omega}\rangle \\
\simeq & -\sqrt{-\hat{g}}\hat{V}\{1-\frac{3}{2}\frac{\kappa^2 H^2}{4\pi^2}\log(a/a_0)\}
\end{aligned}
$$

$$\sqrt{-\hat{g}}\hat{V}=3a^4H^2M_{\rm pl}^2$$

In a similar way, the contribution to the curvature operator is evaluated as

$$
\begin{aligned}
& \frac{M_{\rm pl}^2}{2}\sqrt{-\hat{g}}\hat{R}\{1+4\kappa^2\langle\omega^2\rangle\} \\
\simeq & \frac{M_{\rm pl}^2}{2}\sqrt{-\hat{g}}\hat{R}\{1-\frac{3}{4}\frac{\kappa^2 H^2}{4\pi^2}\log(a/a_0)\}
\end{aligned}
$$

$$h^{00}\simeq 2\omega$$

$$\sqrt{-\hat{g}}\hat{R}=6a\partial_0^2 a$$

The dimensionless ratio $H^2/M_{\rm pl}^2$ does not receive IR correction

Contribution to kinetic term

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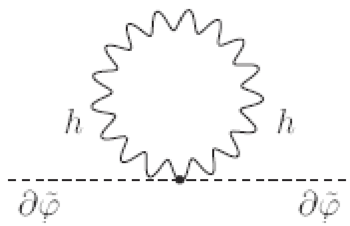
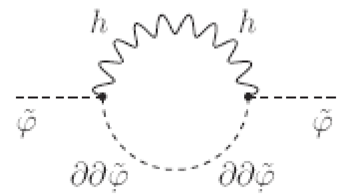
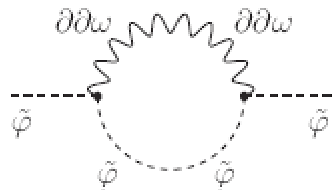
For $\hat{\varphi} + \tilde{\varphi} \equiv e^{\kappa\omega}(\hat{\varphi} + \varphi)$, the total contribution to the kinetic term respects the Lorentz symmetry at the sub-horizon scale





soft gravitons $\rightarrow$ IR logs.

External momenta are set as $p \gg Ha$



soft inflatons $\rightarrow$ IR logs.

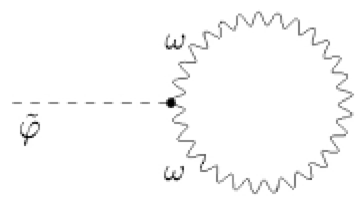
$$\Rightarrow \qquad -\frac{1}{2}\sqrt{-\hat{g}}\hat{g}^{\mu\nu}\partial_\mu\tilde{\varphi}\partial_\nu\tilde{\varphi}\{1 + \frac{1}{2}\frac{\kappa^2 H^2}{4\pi^2}\log(a/a_0)\}$$

This effect can be canceled by the wave function renormalization:

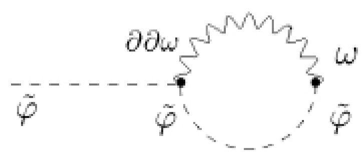
$$\tilde{\varphi} \rightarrow Z\tilde{\varphi}, \quad Z \simeq 1 - \frac{1}{4}\frac{\kappa^2 H^2}{4\pi^2}\log(a/a_0)$$

Contribution to 1-pt operator

After $\tilde{\varphi} \to Z\tilde{\varphi}$, $\quad -\sqrt{-\hat{g}}\hat{V}' e^{3\kappa\omega} Z\tilde{\varphi}$



soft gravitons → IR logs.

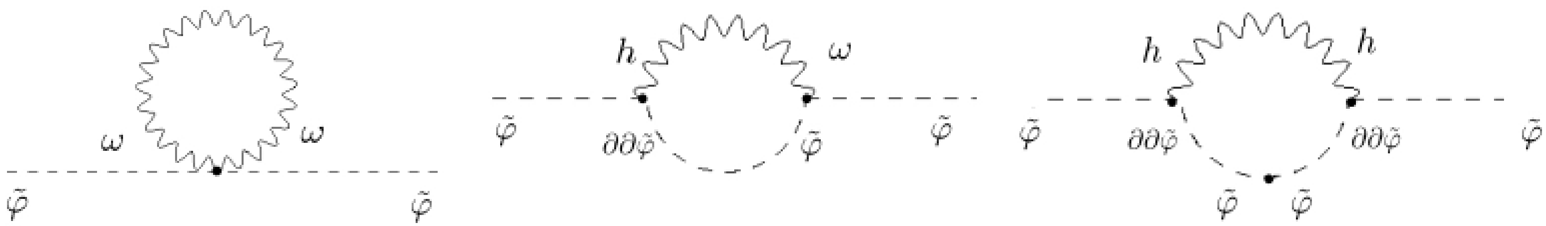


soft inflatons → IR logs.

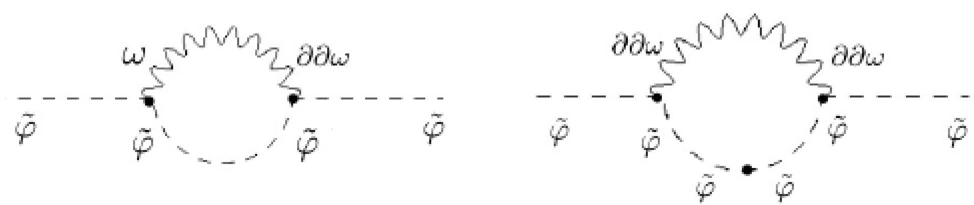
$$\Rightarrow \qquad -\sqrt{-\hat{g}}\hat{V}'\tilde{\varphi}\{1 - \frac{11}{32}\frac{\kappa^2 H^2}{4\pi^2}\log(a/a_0)\}$$

Contribution to 2-pt operator

After $\tilde{\varphi} \to Z\tilde{\varphi}$, $\qquad -\frac{1}{2}\sqrt{-\hat{g}}\hat{V}''e^{2\kappa\omega}Z^2\tilde{\varphi}^2$



soft gravitons $\to$ IR logs.



soft inflatons $\to$ IR logs.

$$\Rightarrow \qquad -\frac{1}{2}\sqrt{-\hat{g}}\hat{V}''\tilde{\varphi}^2\{1 - \frac{3}{4}\frac{\kappa^2 H^2}{4\pi^2}\log(a/a_0)\}$$