Stability of Electroweak Vacuum in the Standard Model and Beyond

Takeo Moroi (Tokyo)

Endo, TM, Nojiri, Shoji, PLB 771 ('17) 281 [1703.09304]

Endo, TM, Nojiri, Shoji, JHEP 1711 ('17) 074 [1704.03492]

Chigusa, TM, Shoji, PRL 119 ('17) 211801 [1707.09301]

Chigusa, TM, Shoji, work in progress

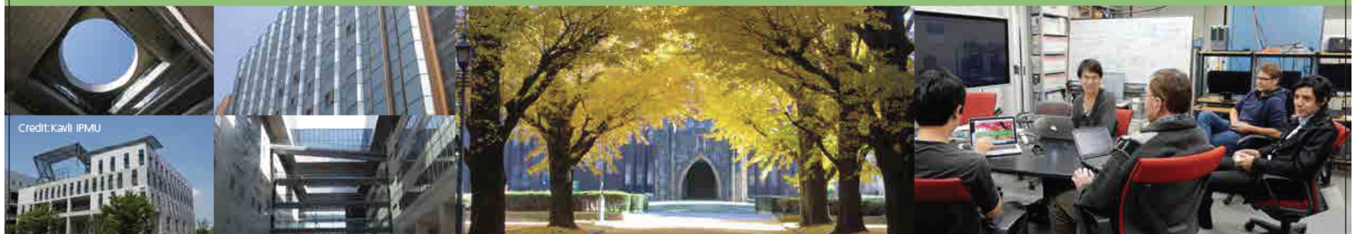
NCTS Workshop, Taiwan, '17.12.07

THE UNIVERSITY OF TOKYO

Department of Physics

SCHOOL OF SCIENCE
THE UNIVERSITY OF TOKYO

GLOBAL SCIENCE GRADUATE COURSE



Astrophysics

Theoretical/Observational

Cosmology, Gravity Theory, Astrophysics, Extra-solar Planet, High-Energy Astrophysics, Cosmic-Ray Physics, Gravitational-Wave Astrophysics, Optical and Infrared Astronomy, Radio Astronomy, Neutrino Physics and Astrophysics, Dark Matter Search

Condensed Matter Physics

Theoretical/Experimental

Superconductivity, Magnetism, Strongly Correlated Electronic Systems, Semiconductors, Surfaces and Interfaces, Quantum Many-Body Theory, Statistical Physics, Mathematical Physics, Quantum Information, Computational Physics, Superfluidity, Non-Equilibrium Systems, High Pressure Physics, Soft Matter, High Magnetic Fields, Laser and Synchrotron Spectroscopy

Nuclear Physics

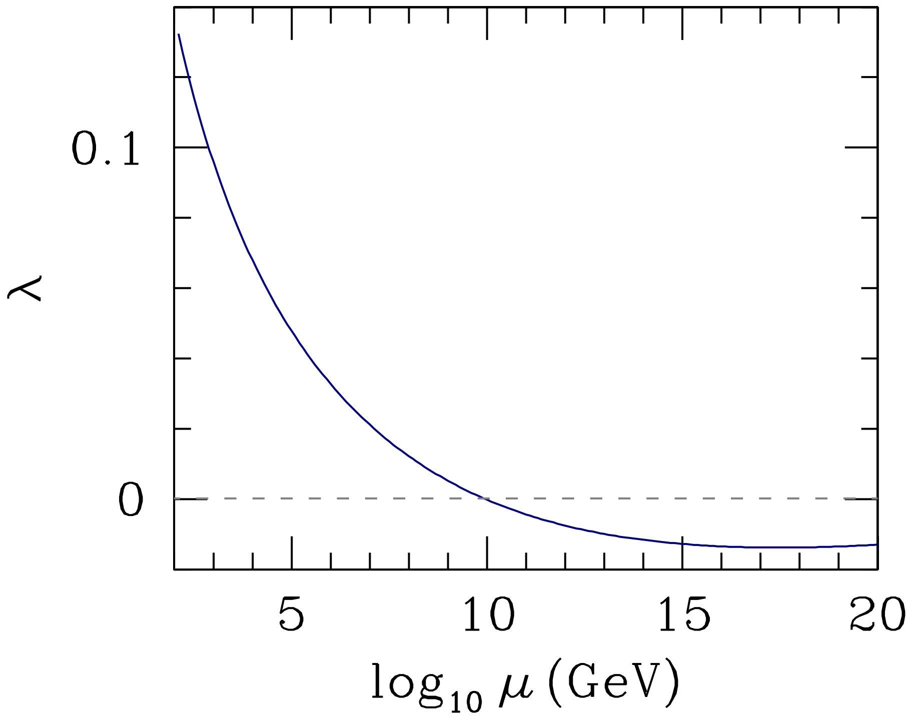
Theoretical/Experimental

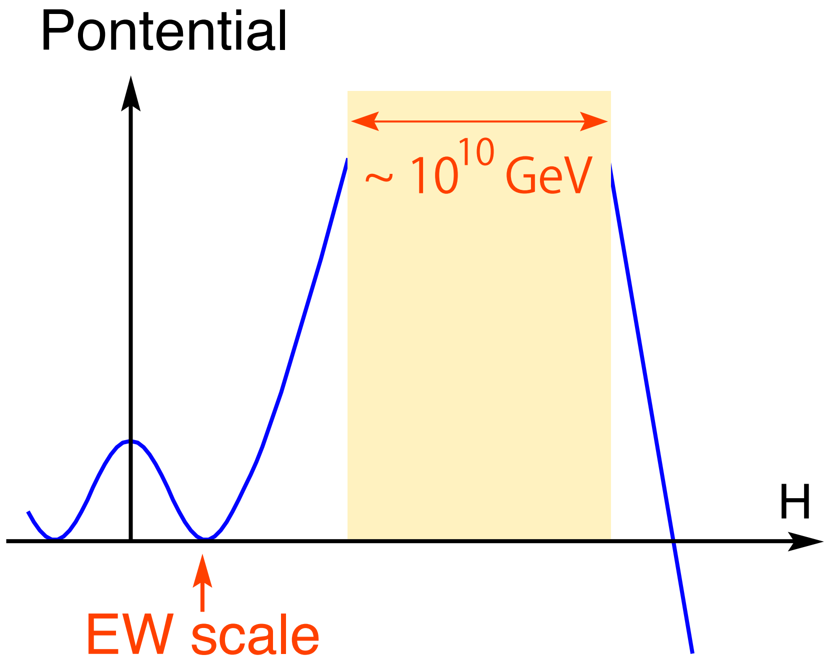
Nuclear Many-Body Problems, Structure of Exotic Nuclei, Quark-Hadron Many-Body Problem, Fundamental Study on the Strong Interaction

1. Introduction

EW vacuum is (probably) not stable in the standard model

$m_h \simeq 125$ GeV $\Rightarrow$ $V = \lambda(|H|^2 - v^2)^2$ with $\lambda(m_h) \simeq 0.13$





- $\lambda(\mu \gtrsim 10^{10}$ GeV$) < 0$
- λ is minimized at $\mu \sim 10^{17}$ GeV

Is the decay rate small enough so that $t_{\text{now}} \simeq 13.6$ Gyr?

$\Rightarrow$ (Probably) yes

[Isidori, Ridolfi & Strumia; Degrassi et al.; Alekhin, Djouadi & Moch; Espinosa et al.; Plascencia & Tamarit; Lalak, Lewicki & Olszewski; Espinosa, Garny, Konstandin & Riotto; $\cdots$]

In the previous studies:

Effects of zero-modes were not properly taken into account

How precisely can we estimate the decay rate?

$\Leftrightarrow$ There has been progresses in the calculation of the decay rate of false vacuum

[Endo, TM, Nojiri & Shoji; Chigusa, TM & Shoji; see also Andreassen, Frost & Schwartz]

Today, I discuss

- A calculation of the decay rate of EW vacuum
- Effects of extra matters

Outline

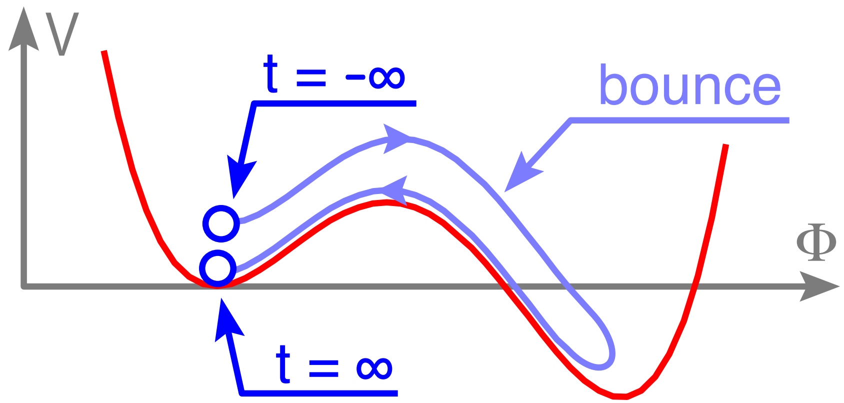
2. Bounce in the SM

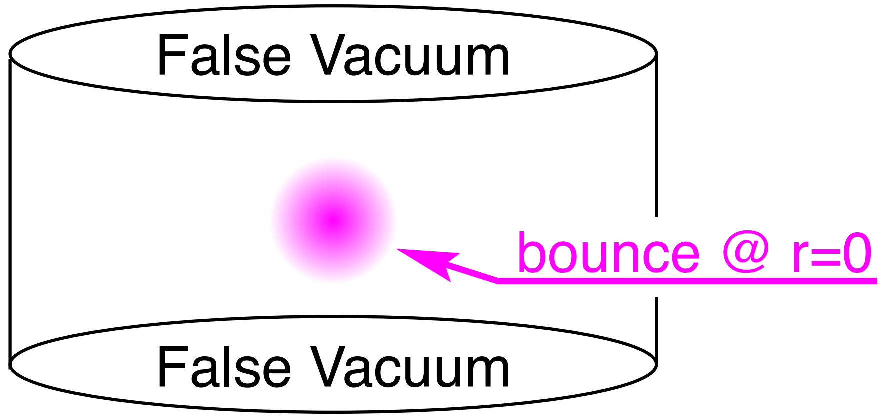
The decay rate is related to 4D Euclidean partition function [Coleman]

$$Z = \langle \mathsf{FV} | e^{-HT} | \mathsf{FV} \rangle \simeq \int \mathcal{D}\Psi\, e^{-S_{\mathsf{E}}} \propto \exp(i\gamma V T)$$

The path integral is dominated by the "bounce"

Bounce: $O(4)$ symmetric saddle-point solution of EoM





$$\left[\partial_r^2 \Phi + \frac{3}{r}\partial_r \Phi - \frac{\partial V}{\partial \Phi}\right]_{\Phi \to \bar{\phi}} = 0 \quad \text{with} \quad \bar{\phi}(\infty) = (\text{false vacuum})$$

The decay rate per unit volume w.r.t. one-bounce action

[Callan & Coleman]

$$\gamma \simeq \frac{1}{VT} \,\mathrm{Im}\left[\frac{\int_{\text{1-bounce}} \mathcal{D}\Psi\, e^{-S_{\mathsf{E}}}}{\int_{\text{0-bounce}} \mathcal{D}\Psi\, e^{-S_{\mathsf{E}}}}\right] \equiv \mathcal{A} e^{-\mathcal{B}} \quad \text{with } \mathcal{B} = S_{\mathsf{E}}[\bar{\phi}] - S_{\mathsf{E}}[v]$$

We expand the action around the "classical path"

$$S_{\mathsf{E}}[\bar{\phi} + \Psi] = S_{\mathsf{E}}[\bar{\phi}] + \frac{1}{2}\int d^4x \Psi \mathcal{M} \Psi + O(\Psi^3)$$

$$S_{\mathsf{E}}[v + \Psi] = S_{\mathsf{E}}[v] + \frac{1}{2}\int d^4x \Psi \widehat{\mathcal{M}} \Psi + O(\Psi^3)$$

Prefactor $\mathcal{A}$ (for bosonic contribution)

$$\mathcal{A} \simeq \frac{1}{VT}\left|\frac{\mathrm{Det}\mathcal{M}}{\mathrm{Det}\widehat{\mathcal{M}}}\right|^{-1/2}$$

Higgs potential in the SM: $V = m^2 H^\dagger H + \lambda (H^\dagger H)^2$

- We consider very large Higgs amplitude for which $\lambda < 0$
- It happens when $|H| \gg m$, so we neglect m^2-term

We use the following potential: $V = -|\lambda|(H^\dagger H)^2$

$\Rightarrow$ The "bounce solution" for this potential

$$H_{\text{bounce}} = \frac{1}{\sqrt{2}} e^{i\sigma^a \theta^a} \begin{pmatrix} 0 \\ \bar{\phi} \end{pmatrix} \quad \text{with} \quad \partial_r^2 \bar{\phi} + \frac{3}{r}\partial_r \bar{\phi} + 3|\lambda|\bar{\phi}^2 = 0$$

$\Rightarrow$ Explicit form of the bounce:

$$\bar{\phi}(r) = \frac{8\bar{\phi}_C}{8 + |\lambda|\bar{\phi}_C^2 r^2} \quad \Leftarrow \quad \begin{cases} \bar{\phi}_C \equiv \bar{\phi}(r=0) : \text{free parameter} \\ 1/\sqrt{|\lambda|}\bar{\phi}_C : \text{size of the bounce} \end{cases}$$

Bounce action for the SM

$$\mathcal{B} = \frac{8\pi^2}{3|\lambda|}$$

Expansion around the bounce:

$$H = \frac{1}{\sqrt{2}} e^{i\sigma^a\theta^a} \begin{pmatrix} \varphi^1 + i\varphi^2 \\ \bar{\phi} + h - i\varphi^3 \end{pmatrix}, \quad W_\mu^a = w_\mu^a, \quad B_\mu = b_\mu$$

Possible deformations of the bounce

- Scale transformation: parameterized by $\bar{\phi}_C$
- $SU(2)$ transformation: parameterized by θ^a

  Effects of zero-modes in association with these transformations were not properly taken into account before

- Translation

3. Effects of the Higgs Mode

We need to calculate the functional determinant of $\mathcal{M}^{(h)}$

$$\mathcal{L} \ni \frac{1}{2}h\,(-\partial^2 - 3|\lambda|\bar{\phi}^2)\,h = \frac{1}{2}h\,\mathcal{M}^{(h)}\,h$$

Expansion of h w.r.t. 4D spherical harmonics $\mathcal{Y}_{J,m_A,m_B}$

$$h(x) = \sum_{J,m_A,m_B,n} \alpha_{n,J,m_A,m_B}\,\rho_{n,J}(r)\,\mathcal{Y}_{J,m_A,m_B}(\hat{\mathbf{r}})$$

$J = 0,\ 1/2,\ 1,\ 3/2,\ \cdots$

$\rho_{n,J}$: radial mode function

α_{n,J,m_A,m_B}: expansion coefficient (integration variable)

Fluctuation operator for angular-momentum eigenstates:

$$\mathcal{M}_J^{(h)} \equiv -\left(\Delta_J + 3|\lambda|\bar{\phi}^2\right) \equiv -\left[\partial_r^2 + \frac{3}{r}\partial_r - \frac{4J(J+1)}{r^2} + 3|\lambda|\bar{\phi}^2\right]$$

Higgs-mode contribution to the prefactor $\mathcal{A}$

$$\mathcal{A}^{(h)} = \prod_J \left[\frac{\text{Det}\mathcal{M}_J^{(h)}}{\text{Det}\widehat{\mathcal{M}}_J^{(h)}} \right]^{-(2J+1)^2/2}$$

The ratio of the functional determinants can be evaluated with Gelfand-Yaglom theorem

Zero-modes exist for $\mathcal{M}^{(h)}$

- Conformal zero-mode (for $J = 0$)

$$\rho_{\text{conf}}(r) \propto \frac{\partial \bar{\phi}}{\partial \bar{\phi}_C} \quad \Leftrightarrow \quad \mathcal{M}_0^{(h)} \rho_{\text{conf}}(r) = 0$$

- Translational zero-modes (for $J = 1/2$)

[Callan & Coleman]

Path integral over conformal zero-mode = integral over $\bar{\phi}_C$

$$H \ni \frac{1}{\sqrt{2}}(\bar{\phi} + h) = \frac{1}{\sqrt{2}}\left[\bar{\phi} + \alpha_{\text{conf}}\mathcal{N}_{\text{conf}}\frac{\partial\bar{\phi}}{\partial\bar{\phi}_C} + \cdots\right]$$

$$\Rightarrow \int \mathcal{D}h^{(\text{conf})} \equiv \int d\alpha_{\text{conf}} \to \int \frac{d\bar{\phi}_C}{\mathcal{N}_{\text{conf}}}$$

$$\Rightarrow \left[\frac{\text{Det}\mathcal{M}_0^{(h)}}{\text{Det}\widehat{\mathcal{M}}_0^{(h)}}\right]^{-1/2} \to \int \frac{d\bar{\phi}_C}{\mathcal{N}_{\text{conf}}}\left[\frac{\text{Det}'\mathcal{M}_0^{(h)}}{\text{Det}\widehat{\mathcal{M}}_0^{(h)}}\right]^{-1/2}$$

Det': zero-eigenvalue is omitted from the Det

Higgs-mode contribution:

[Chigusa, TM & Shoji; Andreassen, Frost & Schwartz]

$$\frac{\mathcal{A}^{(h)}}{VT} \to \frac{1}{VT}\int \frac{d\bar{\phi}_C}{\bar{\phi}_C}\left(\frac{16\pi}{|\lambda|}\right)^{1/2} \prod_{J\geq 1/2}\left[\frac{\text{Det}\mathcal{M}_J^{(h)}}{\text{Det}\widehat{\mathcal{M}}_J^{(h)}}\right]^{-(2J+1)^2/2}$$

4. Total Decay Rate

Decay rate:

$$\gamma = \int d\ln\bar{\phi}_C \left[I^{(h)} I^{(W,Z,\text{NG})} I^{(t)} e^{-\mathcal{S}_{\text{C.T.}}} e^{-\mathcal{B}} \right]_{\mu\sim\bar{\phi}_C}$$

We derived complete and gauge-invariant expressions of $I^{(X)}$

$I^{(h)}$: Higgs contribution

$I^{(W,Z,\text{NG})}$: gauge and NG contribution

$I^{(t)}$: top contribution

The renormalization scale is taken to be $\mu \sim \bar{\phi}_C$

⇒ The effects of μ-dependent terms from higher loops, i.e., $\sim \ln^p(\bar{\phi}_C/\mu)$, are expected to be minimized

⇒ This is important for the convergence of the integral

We use:

- $m_h = 125.09 \pm 0.24$ GeV
- $m_t = 173.1 \pm 1.1$ GeV
- $\alpha_s(m_Z) = 0.1181 \pm 0.0011$
- 2- or 3-loop RGEs (with relevant threshold corrections)

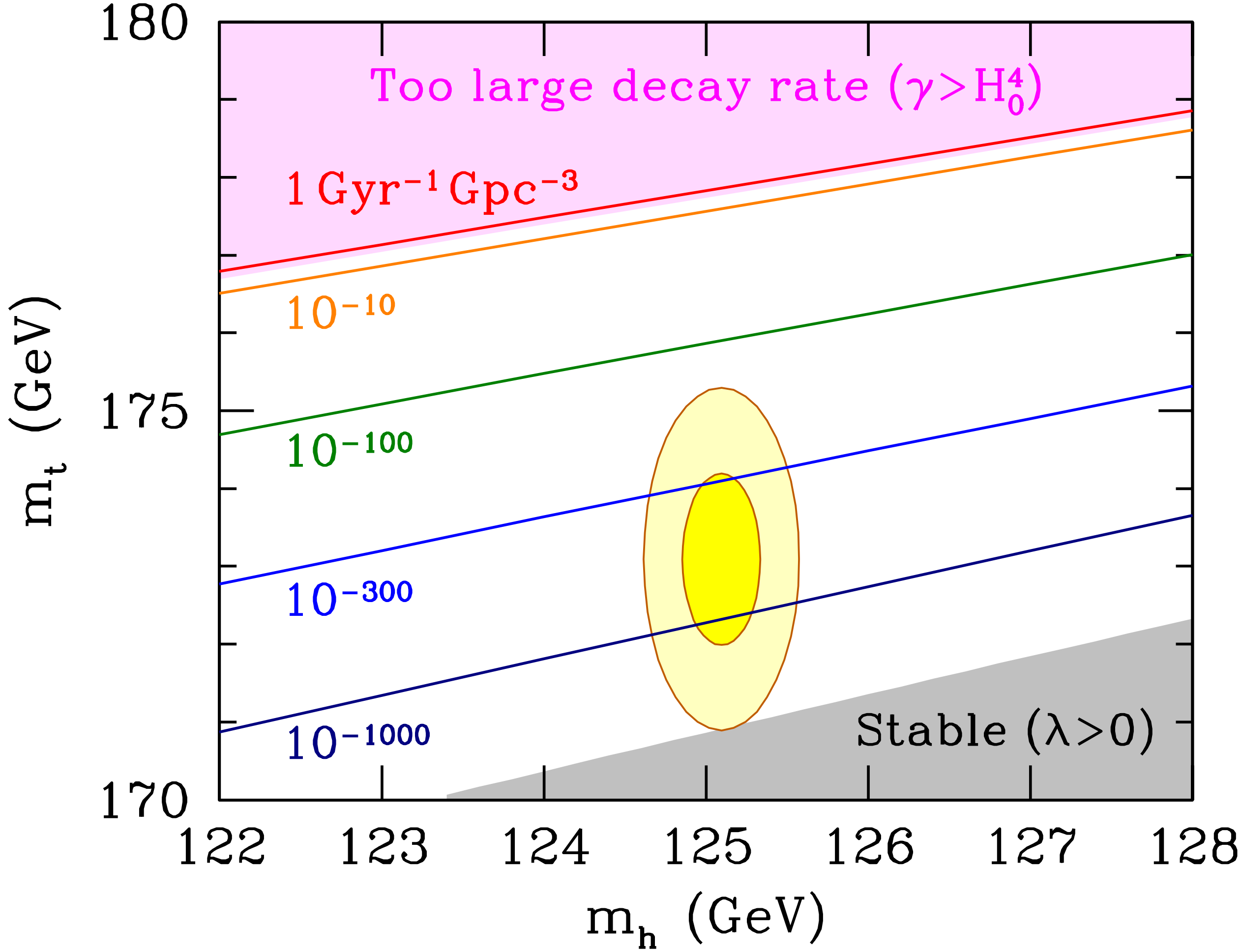
Decay rate of the EW vacuum (taking $\mu = \bar{\phi}_C$)

- $\log_{10}[\,\gamma\ (\text{Gyr}^{-1}\text{Gpc}^{-3})\,] \simeq -554^{+38+270+137}_{-41-817-204}$

For the present universe:

- Cosmic age: $t_0 \simeq 13.6$ Gyr
- Horizon scale: $H_0^{-1} \simeq 4.5$ Gpc

Decay rate per unit volume as a function of m_h and m_t



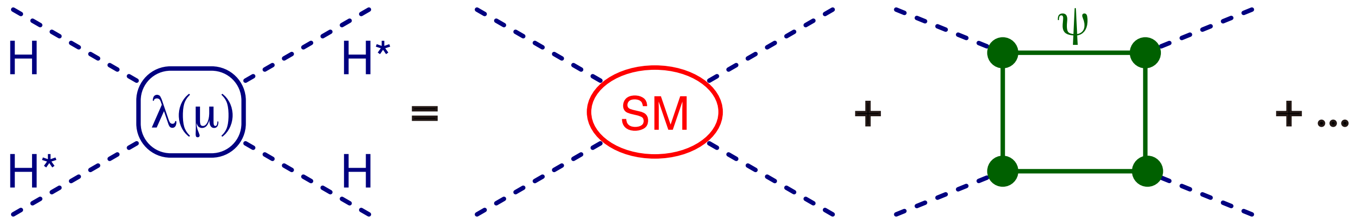
- $\alpha_s(m_Z) = 0.1181$
- $\mu = \bar{\phi}_C$

5. Case with Extra Matters

Let us consider vector-like fermions coupled to H

$$\mathcal{L} = \mathcal{L}_{\mathsf{SM}} + y_\psi H \psi_L \psi_R + y_{\bar\psi} H \bar\psi_L \bar\psi_R + M_\psi \bar\psi_L \psi_L + M_\psi \bar\psi_R \psi_R + \cdots$$

RGE for λ



$$\frac{d\lambda}{d\ln\mu} = \left[\frac{d\lambda}{d\ln\mu}\right]_{\mathsf{SM}} - \frac{1}{4\pi^2}\sum_\psi N_C^{(\psi)} y_\psi^4 + \cdots$$

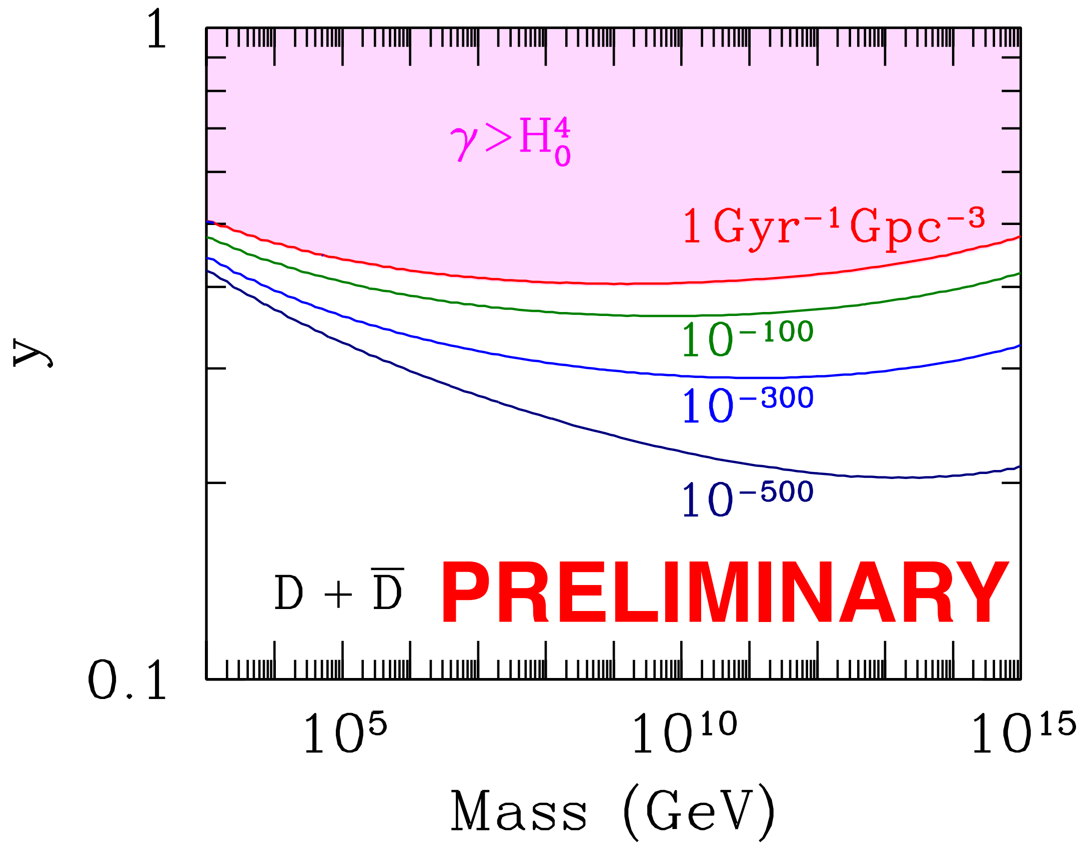
With extra fermions, λ may become smaller (at high scale)

$\Rightarrow$ Enhancement of the decay rate

C.f., $\gamma = \mathcal{A} e^{-\mathcal{B}}$ with $\mathcal{B} = \dfrac{8\pi^2}{3|\lambda|}$

Case 1: Down-quark-like colored fermions

$\Rightarrow \psi_L(\mathbf{3}, \mathbf{2}, 1/6)$ and $\psi_R(\bar{\mathbf{3}}, \mathbf{1}, -1/3)$

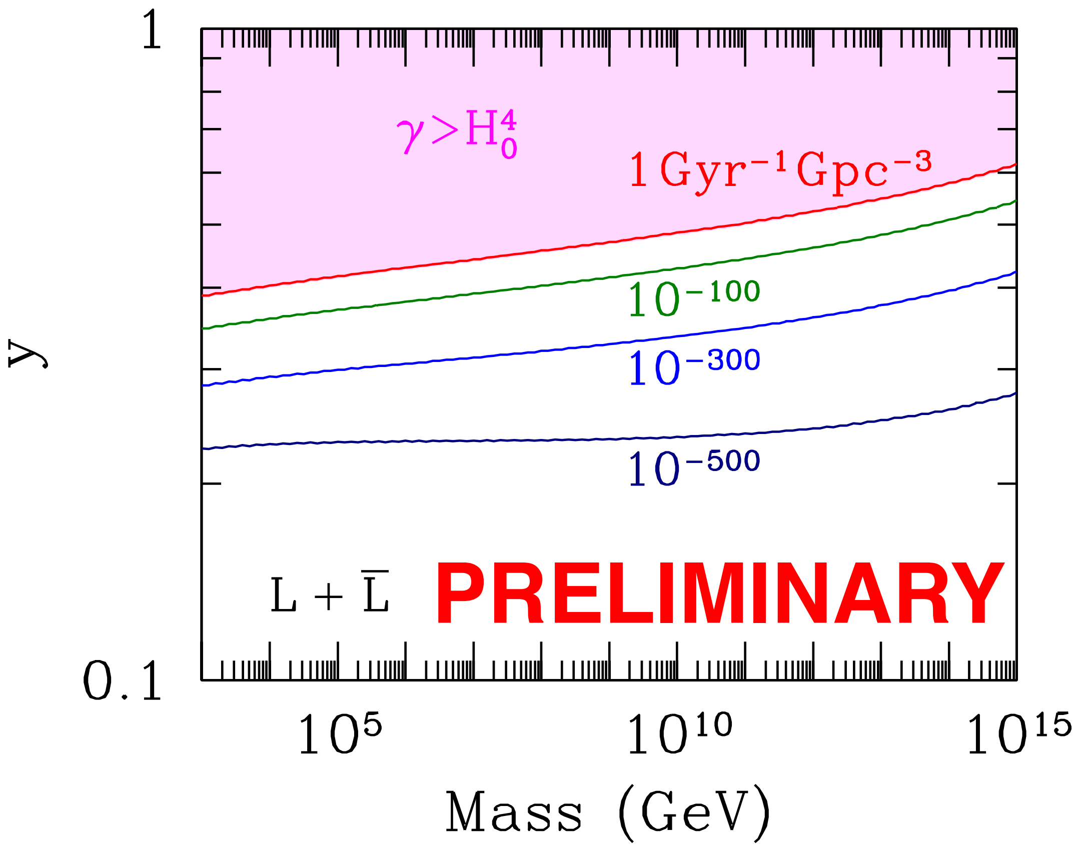


- $\bar{\phi}_C^{(\text{max})} = M_{\text{Pl}}$

$\Rightarrow$ Yukawa coupling larger than $\sim 0.4 - 0.5$ is dangerous

Case 2: Charged-lepton-like fermions

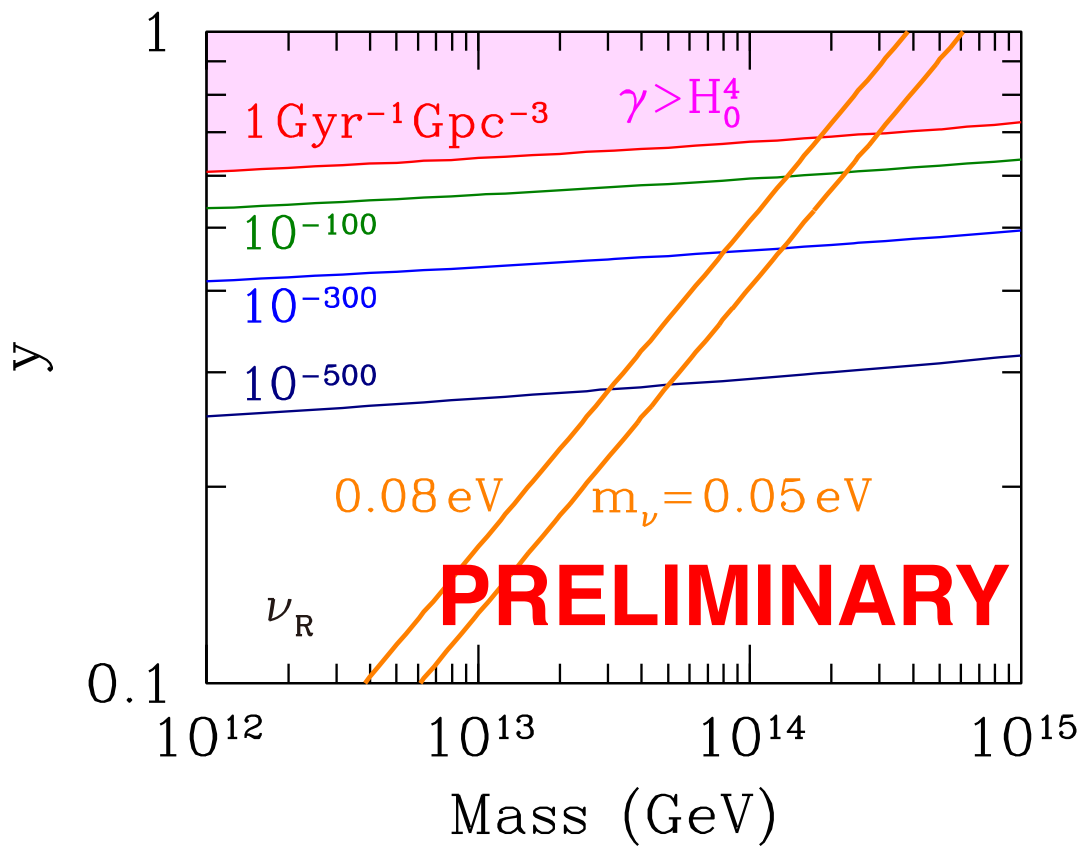
$\Rightarrow \psi_L(\mathbf{1}, \mathbf{2}, 1/2)$ and $\psi_R(\mathbf{1}, \mathbf{1}, -1)$



- $\bar{\phi}_C^{(\mathrm{max})} = M_{\mathrm{Pl}}$

Case 3: Right-handed neutrino

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + y_\nu H \ell_L \nu_R^c + \frac{1}{2} M_\nu \nu_R^c \nu_R^c + \cdots$$



- $\bar{\phi}_C^{(\text{max})} = M_{\text{Pl}}$

6. Summary

I have discussed the decay rate of the EW vacuum

Path integrals over the conformal and gauge zero-modes are properly performed

Numerical result

$$\log_{10}[\,\gamma\ (\mathsf{Gyr}^{-1}\mathsf{Gpc}^{-3})\,] \simeq -554^{+38+270+137}_{-41-817-204}$$

The decay rate is extremely small: $\gamma \ll H_0^4$

$\Rightarrow$ We will fall into another vacuum if we wait $\sim 10^{552}$ Gyr (assuming that the dark energy is cosmological constant)

Extra fermions may change the above conclusion

$\Rightarrow$ $y \gtrsim 0.4 - 0.6$ is dangerous

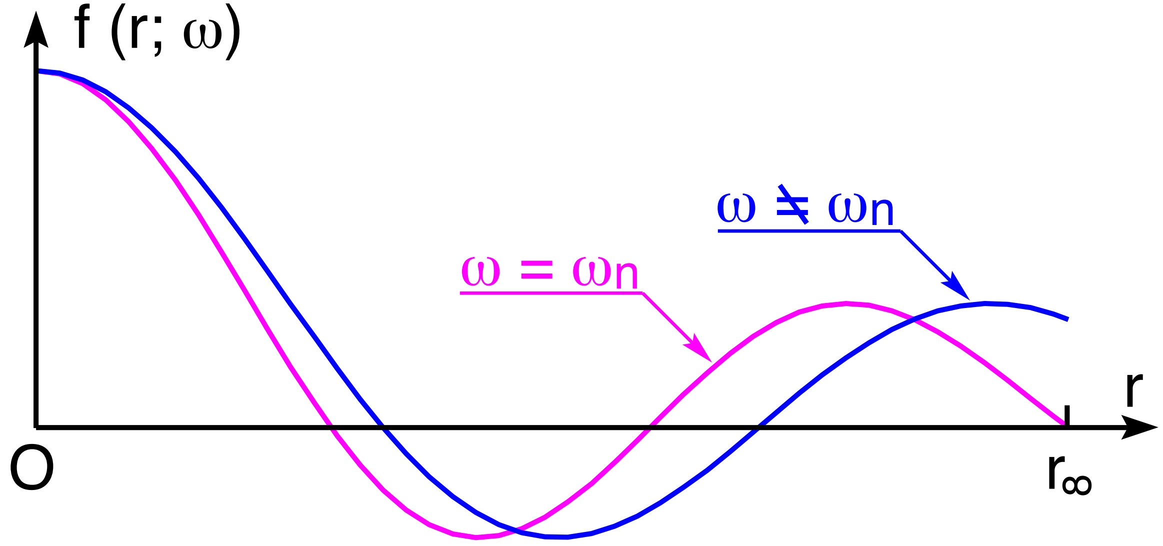
Back Up

Functional determinant for operators defined in $0 \le r \le r_\infty$

$$\text{Det}\mathcal{M} \simeq \prod_n \omega_n \text{ with } \begin{cases} \mathcal{M}\rho_n = \omega_n \rho_n \text{ with } \mathcal{M} = -\Delta_J + \delta W(r) \\ \rho_n(0) < \infty \\ \rho_n(r_\infty) = 0 \end{cases}$$

We introduce a function f which obeys: $\mathcal{M} f(r;\omega) = \omega f(r;\omega)$

- $f(r = r_\infty;\omega)|_{\omega=\omega_n} = 0$
- $\text{Det}(\mathcal{M} - \omega)|_{\omega=\omega_n} = 0$



Gelfand-Yaglom theorem

[Gelfand & Yaglom; Coleman; Dashen, Hasslacher & Neveu; Kirsten & McKane; $\cdots$]

$$\frac{\mathrm{Det}(\mathcal{M}-\omega)}{\mathrm{Det}(\widehat{\mathcal{M}}-\omega)} = \frac{f(r=r_\infty;\omega)}{\hat{f}(r=r_\infty;\omega)} \quad \text{with} \quad \begin{cases} \mathcal{M} f(r;\omega) = \omega f(r;\omega) \\ \widehat{\mathcal{M}} \hat{f}(r;\omega) = \omega \hat{f}(r;\omega) \\ f(r=0) = \hat{f}(r=0) < \infty \end{cases}$$

$\Rightarrow$ Notice: LHS and RHS have the same analytic behavior

- LHS and RHS have same zeros and infinities
- LHS and RHS becomes equal to 1 when $\omega \to \infty$

We take $r_\infty \to \infty$ at the end of calculation

$\Rightarrow$ The results converge (in the case of our interest)