Fixing Amplitudes by Single/Double Soft Theorem

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Outline

- Soft theorem constrains effective action

  $$\mathcal{A}_4^{(2)} = c_4^{(2)}(s^2 + t^2 + u^2)$$

- Single soft theorem: holds along the RG flow (symmetry protected)

- Double soft theorem $\begin{cases} \text{Type A: holds along the RG flow} \\ \text{Type B: will be quantum corrected} \end{cases}$

- Soft + Locality $\longrightarrow$ Unitarity

Single Soft Theorems

Single soft theorems up to order λ,

$$M_{n+1}(\tau p, \cdots)|_{\tau\to 0} = \left(S^{(0)} + \cdots + \tau^{\lambda} S^{(\lambda)}\right) M_n(\cdots) + O\left(\tau^{\lambda+1}\right)$$

- Vanishing soft limit in certain degree λ. (0 for NLSM, 1 for DBI, 2 for sGal)

$$\tau^0 : M_{n+1}|_{\mathcal{O}(\tau^0)} = 0$$
$$\vdots$$
$$\tau^{\lambda} : M_{n+1}|_{\mathcal{O}(\tau^{\lambda})} = 0$$

- Non-vanishing soft limit, e.g. Dilaton.

$$\text{Leading} : M_{n+1}|_{\mathcal{O}(\tau^0)} = \mathcal{S}^{(0)} M_n\Big|_{\mathcal{O}(\tau^0)}$$
$$\text{Sub-leading} : M_{n+1}|_{\mathcal{O}(\tau^1)} = \left(\mathcal{S}^{(0)} + \tau\mathcal{S}^{(1)}\right) M_n\Big|_{\mathcal{O}(\tau^1)}$$

Different Expansion Schemes: Type A and Type B

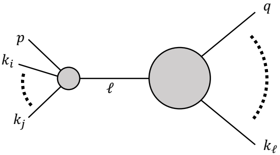
Type A : τ_p and τ_q, perform bivariate Taylor expansion.

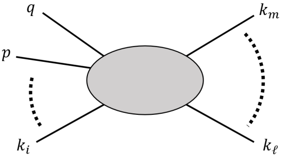
Type B : τ, for both soft legs, perform single-variate Taylor expansion.

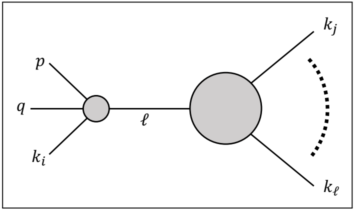
$$
\begin{aligned}
& M_{n+2}\left(\tau_p p, \tau_q q, \cdots\right)|_{\text{type A}} \\
& = M_{n+2}\left(0, 0, \cdots\right) \\
& + \left[\left(\tau_p \frac{\partial}{\partial \tau_p}\right) + \left(\tau_q \frac{\partial}{\partial \tau_q}\right)\right] M_{n+2}\left(\tau_p p, \tau_q q, \cdots\right)\Big|_{\tau_p=\tau_q=0} \\
& + \left[\frac{1}{2}\left(\tau_p^2 \frac{\partial}{\partial \tau_p^2}\right) + \left(\tau_p \frac{\partial}{\partial \tau_p}\right)\left(\tau_q \frac{\partial}{\partial \tau_q}\right) + \frac{1}{2}\left(\tau_q^2 \frac{\partial}{\partial \tau_q^2}\right)\right] M_{n+2}\left(\tau_p p, \tau_q q, \cdots\right)\Big|_{\tau_p=\tau_q=0} \\
& + \cdots
\end{aligned}
$$

$$
\begin{aligned}
& M_{n+2}\left(\tau p, \tau q, \cdots\right)\Big|_{\text{type B}} \\
& = M_{n+2}\left(0, 0, \cdots\right) \\
& + \tau \tfrac{\partial}{\partial \tau} M_{n+2}\left(\tau p, \tau q, \cdots\right)\Big|_{\tau=0} \\
& + \tfrac{1}{2}\tau^2 \tfrac{\partial^2}{\partial \tau^2} M_{n+2}\left(\tau p, \tau q, \cdots\right)\Big|_{\tau=0} + \ldots
\end{aligned}
$$

Different Expansion Schemes: Type A and Type B







$$\frac{1}{(k_i+p+q)^2} \longrightarrow \frac{1}{k_i^2} \longrightarrow \frac{1}{0}$$

Double Soft Theorems: Type A

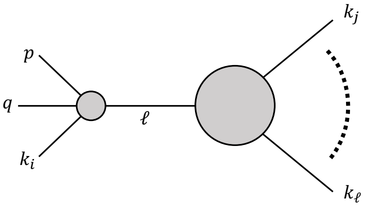
Type A(sequentially soft): Applying single soft theorem twice

$$
\begin{aligned}
M_{n+2}(p,q,\cdots)|_{\text{type A}} &= \left(S^{(0)}_{n+1} + p\cdot S^{(1)}_{n+1} + \cdots + p^{\lambda}\cdot S^{(\lambda)}_{n+1}\right) M_{n+1}(q,\cdots) + O\left(p^{\lambda+1}\right) \\
&= \left(S^{(0)}_{n+1} + p\cdot S^{(1)}_{n+1} + \cdots p^{\lambda}\cdot S^{(\lambda)}_{n+1}\right)\left(S^{(0)}_{n} + q\cdot S^{(1)}_{n} + \cdots q^{\lambda}\cdot S^{(\lambda)}_{n}\right) M_n(\cdots) \\
&\quad + \left(S^{(0)}_{n+1} + p\cdot S^{(1)}_{n+1} + \cdots p^{\lambda}\cdot S^{(\lambda)}\right) O\left(q^{\lambda+1}\right) + O\left(p^{\lambda+1}\right) \\
&= \left[S^{(0)}_{n+1}S^{(0)}_{n} + q^{\mu}S^{(0)}_{n+1}S^{(1)}_{n,\mu} + p^{\mu}S^{(1)}_{n+1,\mu}S^{(0)}_{n} + \cdots + \left(p^{\lambda}\cdot S^{(\lambda)}_{n+1}\right)\left(q^{\lambda}\cdot S^{(\lambda)}_{n}\right)\right] M_n(\cdots) \\
&\quad + \cdots \\
&= \left[S^{(0)}_{\mathrm{d}} + S^{(1)}_{\mathrm{d}} + \cdots + S^{(\lambda_d)}_{\mathrm{d}}\right] M_n(\cdots) + \cdots
\end{aligned}
$$

Double Soft Theorems: Type B

Type B(simultaneously soft):

Can't be obtained by acting consecutive single soft theorem, but can be related to Type A by a four-point vertex.



$$\frac{V_4}{(k_i+p+q)^2} M_n$$

$$M_{n+2}|_{\text{type B}} = M_{n+2}|_{\text{type A}} + \left(M_{\text{pole}}|_{\text{type B}} - M_{\text{pole}}|_{\text{type A}} \right)$$

$$\left(M_{\text{pole}}|_{\text{type B}} - M_{\text{pole}}|_{\text{type A}} \right) = \sum_{i,j,\ldots} \left(\mathcal{S}^{(i)}_{s^2} + \mathcal{S}^{(j)}_{s^3} + \ldots \right) M_n$$

Difference between Type A and Type B : $\mathcal{S}^{(i)}_{s^2} M_n$

$$M_{\text{pole},s^2}\Big|_{\text{type B}} - M_{\text{pole},s^2}\Big|_{\text{type A}}$$

$$=0 \qquad (\mathcal{S}^{(0)}_{s^2} M_n)$$

$$+c_4^{(2)} \sum_i \left[\frac{[k_i \cdot (p-q)]^2}{k_i \cdot (p+q)} + k_i \cdot (p+q) \right] M_n(\cdots, p_i, \cdots) \qquad (\mathcal{S}^{(1)}_{s^2} M_n)$$

$$+2c_4^{(2)} \sum_i \left\{ \frac{2\,(k_i \cdot p)\,(k_i \cdot q)}{[k_i \cdot (p+q)]^2} (p \cdot q) + \frac{k_i \cdot (p-q)}{[k_i \cdot (p+q)]} \left(p_\mu q_\nu J_i^{\mu\nu}\right) + k_i \cdot (p+q) \right\} M_n(\cdots k_i, \cdots) \qquad (\mathcal{S}^{(2)}_{s^2} M_n)$$

$$+ \ldots \qquad (\mathcal{S}^{(3)}_{s^2} M_n)$$

(DBI: 4-pt s^2)

Loop Correction

- Single soft: survive loop correction
- Type A double soft: survive loop correction
- Type B double soft: will be modified

For example, DBI 4-pt vertex is of order s^2

and in 4D, its loop correction will introduce a 4-pt s^4 vertex,

so the τ^3 double soft theorem is modified.

(Guerrieri, Huang, Li, Wen 1705.10087)

M_{s^2}: τ^1 τ^2 τ^3

M_{s^3}: τ^3 τ^4 τ^5

Example: dilaton 6-pt s^3 fixed by single soft

(known) $M_4^{(2)} = c_4^{(2)}(s_{12}^3 + \mathcal{P}_4)$

(known) $M_5^{(3)} = c_5^{(3)}(s_{12}^3 + \mathcal{P}_5)$

$$M^{(3)}_{6\_ansatz} = c_{6p,1}(s_{12}^3 + \mathcal{P}_6) + c_{6p,2}(s_{123}^3 + \mathcal{P}_6) + (c_4^{(2)})^2 \left((s_{12}^2 + s_{13}^2 + s_{23}^2)\frac{1}{s_{123}}(s_{45}^2 + s_{46}^2 + s_{56}^2) + \mathcal{P}_6 \right)$$

$s_{i\ldots k} = (p_i + \ldots + p_k)$,

$\mathcal{P}_n$: sum over full permutations on n legs

Apply leading and sub-leading soft: (need to put numerics into s_{ij})

$$\text{Leading: } M^{(3)}_{6\_ansatz}\Big|_{\mathcal{O}(\tau^0)} = \mathcal{S}^{(0)} M_5^{(3)}\Big|_{\mathcal{O}(\tau^0)}$$

$$\text{Sub-leading: } M^{(3)}_{6\_ansatz}\Big|_{\mathcal{O}(\tau^1)} = \left(\mathcal{S}^{(0)} + \tau \mathcal{S}^{(1)}\right) M_5^{(3)}\Big|_{\mathcal{O}(\tau^1)}$$

Example: dilaton 6-pt s^3 amplitude fixed by single soft

$$M_6^{(3)} = \boxed{-c_5^{(3)}}(s_{12}^3 + \mathcal{P}_6)\boxed{-(\frac{c_5^{(3)}}{2} + (c_4^{(2)})^2)}(s_{123}^3 + \mathcal{P}_6)$$
$$+ (c_4^{(2)})^2 \left((s_{12}^2 + s_{13}^2 + s_{23}^2)\frac{1}{s_{123}}(s_{45}^2 + s_{46}^2 + s_{56}^2) + \mathcal{P}_6 \right)$$

$M_6^{(3)}$ Can be used to determine $M_7^{(3)}$ and so on...

Locality: simple pole

Unitaritiy: Factorization on pole

$$
\begin{aligned}
M^{(3)}_{6\_\text{ansatz}} &= c_{6p,1}(s_{12}^3 + \mathcal{P}_6) + c_{6p,2}(s_{123}^3 + \mathcal{P}_6) \\
&+ c_{6f,1}\left((s_{12}^2 + s_{13}^2 + s_{23}^2)\frac{1}{s_{123}}(s_{45}^2 + s_{46}^2 + s_{56}^2) + \mathcal{P}_6 \right) \\
&+ (\textbf{non-unitary})
\end{aligned}
$$

$(\textbf{non-unitary}) =$
$c_{6f,2}\left(\frac{1}{s_{123}}\left(s_{14}^4 + s_{15}^4 + s_{16}^4 + s_{24}^4 + s_{25}^4 + s_{26}^4 + s_{34}^4 + s_{35}^4 + s_{36}^4\right) + \ldots\right) +$
$c_{6f,3}\left(\ldots\right) + \ldots$

$(1, 2, 3)$ and $(4, 5, 6)$ permutation invariant

Non-unitary terms will be "ruled out" by soft theorem

Soft + Locality ⟶ Unitarity

Soft + Locality → Unitarity

Compatible with soft theorem

Only allow Simple pole

Factorization on pole

Unitarity is an emergent property!

Example: DBI 6-pt amplitude by Type B double soft

(known) $M_4^{(2)} = c_4^{(2)}(s_{12}^2 + \mathcal{P}_4)$

$$
\begin{aligned}
M^{(3)}_{6\_\mathrm{ansatz}} = &\; c_{6p,1}(s_{12}^3 + \mathcal{P}_6) + c_{6p,2}(s_{123}^3 + \mathcal{P}_6) \\
&+ (c_4^{(2)})^2 \left((s_{12}^2 + s_{13}^2 + s_{23}^2)\frac{1}{s_{123}}(s_{45}^2 + s_{46}^2 + s_{56}^2) + \mathcal{P}_6 \right)
\end{aligned}
$$

Apply DBI double soft: (need to put numerics into s_{ij})

$$
\begin{aligned}
\tau^0 &: M^{(3)}_{6\_\mathrm{ansatz}}\Big|_{\mathcal{O}(\tau^0)} = 0 \\
\tau^1 &: M^{(3)}_{6\_\mathrm{ansatz}}\Big|_{\mathcal{O}(\tau^1)} = \tau S_d^{(1)} M_4^{(2)}\Big|_{\mathcal{O}(\tau^1)} \\
\tau^2 &: M^{(3)}_{6\_\mathrm{ansatz}}\Big|_{\mathcal{O}(\tau^2)} = \left(\tau S_d^{(1)} + \tau^2 S_d^{(2)}\right) M_4^{(2)}\Big|_{\mathcal{O}(\tau^2)} \\
\tau^3 &: M^{(3)}_{6\_\mathrm{ansatz}}\Big|_{\mathcal{O}(\tau^3)} = \left(\tau \mathcal{S}_d^{(1)} + \tau^2 \mathcal{S}_d^{(2)} + \tau^3 \mathcal{S}_d^{(3)}\right) M_4^{(2)}\Big|_{\mathcal{O}(\tau^3)}
\end{aligned}
$$

Type B double soft can solve $c_{6p,1}$ and $c_{6p,2}$ in terms of $c_4^{(2)}$

Conclusion

- Amplitudes fixed by single soft or Type A double soft survive quantum correction
- Type B double soft is associated with 4-pt vertex, so they will be quantum corrected, for example, DBI τ^3
- Soft + Locality $\longrightarrow$ Unitarity

  DBI 6-pt, 8-pt (Arkani-Hamed, Rodina, Trnka: 1612.02797)
  Dilaton 8-pt s^3 (Li, Lin, Zhang: 1710.00480)

Thank you for your attention.