NCTS Annual Theory Meeting 2017, Hsinchu 2017.12.7

Gravitational waves in the string axiverse

Jiro Soda
Kobe University

D. Yoshida & J.S., arXiv: 1708.09592

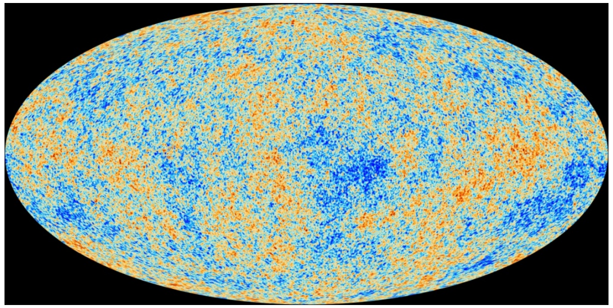
Contents

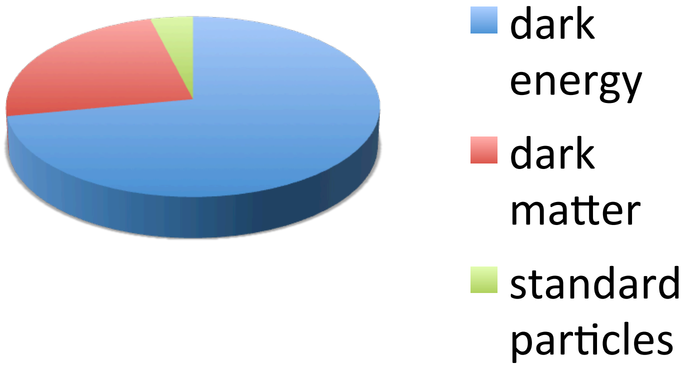
- Axion dark matter
- Gravitational wave detector as a probe
- Chern–Simons gravity
- Exploring the axion dark matter
  with gravitational waves
- Summary

Axion dark matter

Dark matter problem

There are many evidences for the dark matter constituting 25% of the total energy in the universe.

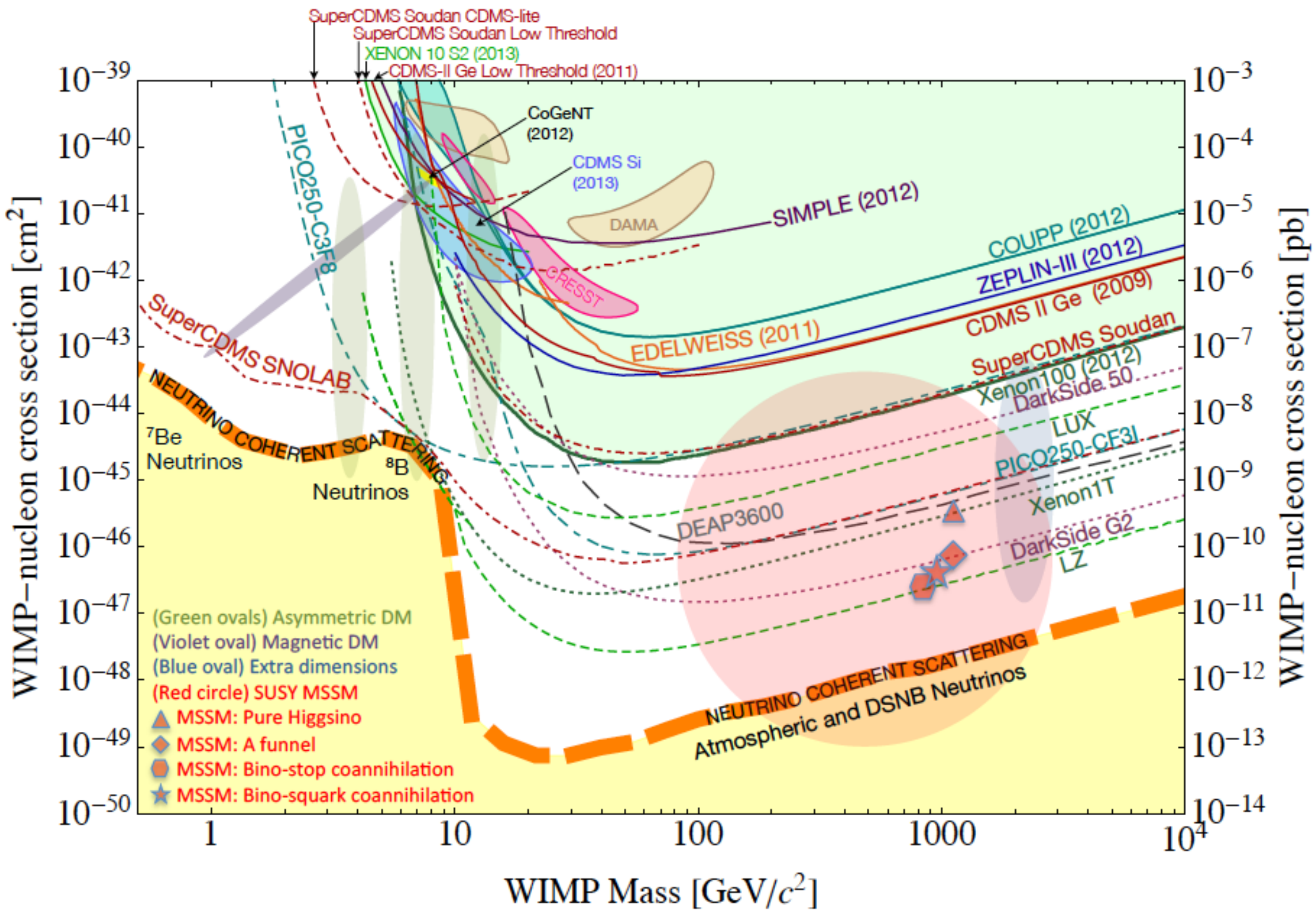




Main question is the following:

What is the dark matter?

Neutralino as CDM?



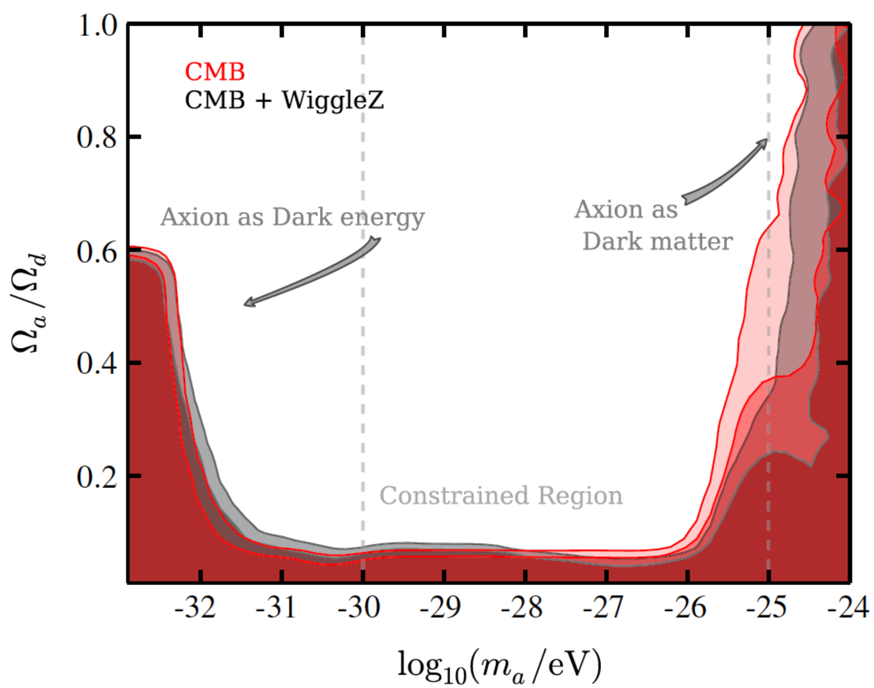
No evidence has been found so far.

Axion dark matter?

Moreover, no evidence of supersymmetry has been found at the LHC.
Hence, there is no reason to stick to neutralinos.

Therefore, it is worth investigating the axion dark matter seriously.
We consider both dominant and subdominant axion dark matter with a wide mass range.

As to the axion mass, there is no serious constraint from CMB observations.



Marsh 2016

The string axiverse

QCD axion

Resolve the Strong CP problem

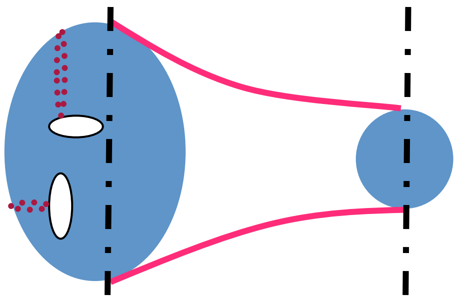
$$m_a \approx 6\times 10^{-6}\,\mathrm{eV}\left(\frac{10^{12}\,\mathrm{GeV}}{f_a}\right)$$

f_a : decay constant

string axions

Model independent axion $H = dB = *d\theta$

Model dependent axion $a_i = \int_{C_{p_i}} F_p$



$$m_a \propto e^{-\#\mathrm{moduli}/2}$$

Mass distribution is logarithmically flat

Thus, theoretically, a wide range of axion mass is allowed.

Axion as a classical field

The model

$$S=\frac{1}{2}\int d^4x\sqrt{-g}\,R-\int d^4x\sqrt{-g}\left[\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi+\frac{1}{2}m^2\phi^2\right]$$

Occupation number

$$\frac{N}{\Delta x^3\Delta p^3}\approx\frac{n}{k^3}=\frac{\rho_{DM}}{m k^3}\approx 10^{38}\left(\frac{\rho_{DM}}{0.3\,\mathrm{GeV/cm^3}}\right)\left(\frac{10^{-10}\,\mathrm{eV}}{m}\right)^4$$

Since the occupation number is so high, classical field description is quite good.

Since the typical velocity of the dark matter $v\approx 10^{-3}$

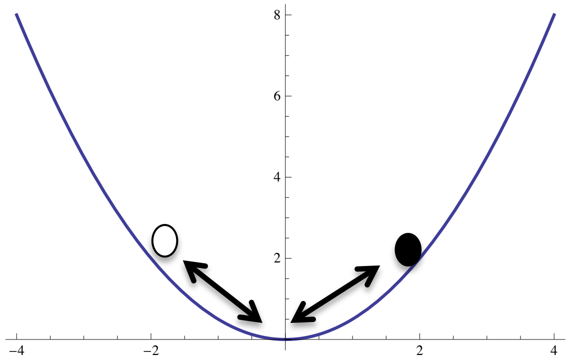
We have a monochromatic frequency

$$E\approx m+\frac{1}{2}mv^2\approx m$$

Axion looks like CDM on large scale

The axion is an coherently oscillating scalar field

$$\phi = \phi_0 \cos mt$$



The energy density becomes

$$\rho_{DM} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 \approx \frac{1}{2}m^2\phi_0^2$$

$$\longrightarrow \quad \phi_0 = \frac{\sqrt{2\rho}}{m} \approx 2.1\times 10^7 \left(\frac{10^{-10}\,\mathrm{eV}}{m}\right)\sqrt{\frac{\rho}{0.3\,\mathrm{GeV/cm^3}}}\ \mathrm{eV}$$

The pressure is given by

$$p_{DM} = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2 \approx -\frac{1}{2}m^2\phi_0^2\cos(2mt)$$

The average value of the pressure over the oscillation period is zero.
Hence, the axion cannot be distinguished from the CDM on cosmological scales.

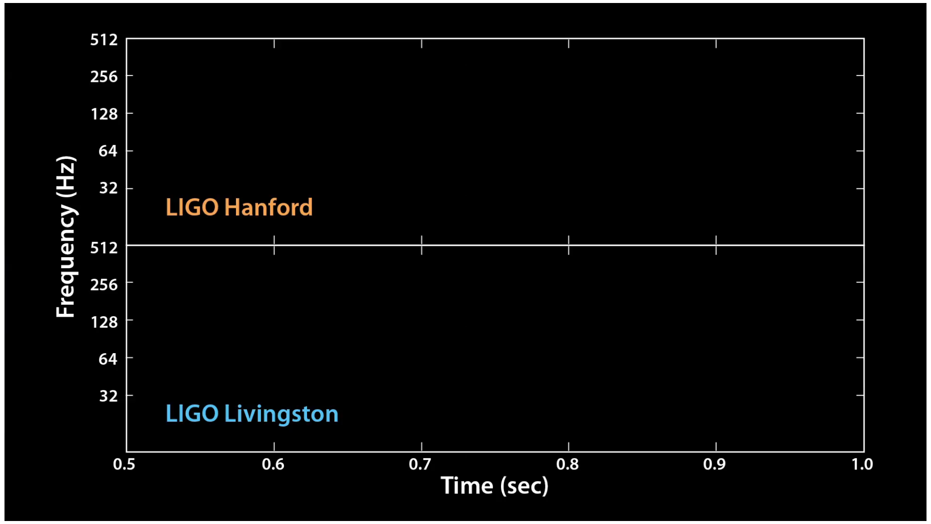
If the axion dark matter cannot be distinguished from CDM on the cosmological scales, how can we prove the existence of axion dark matter?

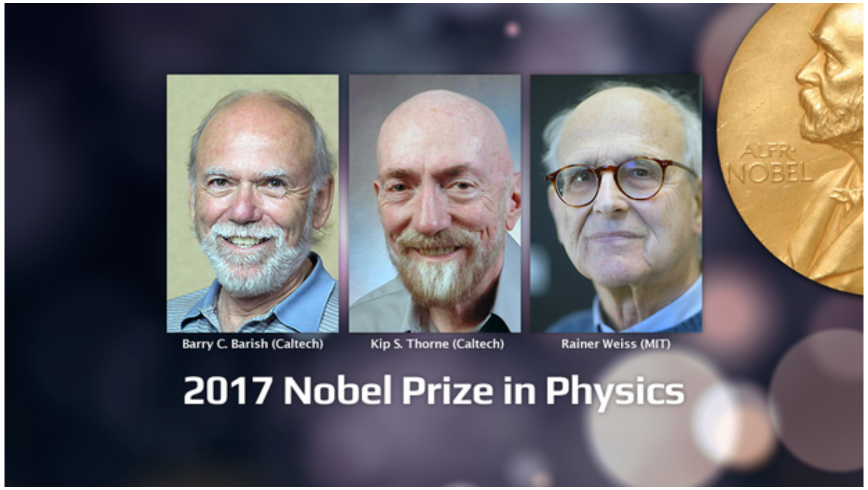
The key is the coherent oscillation of the axion dark matter and gravitational waves.

Gravitational wave detector as a probe

Gravitational waves have been detected !!

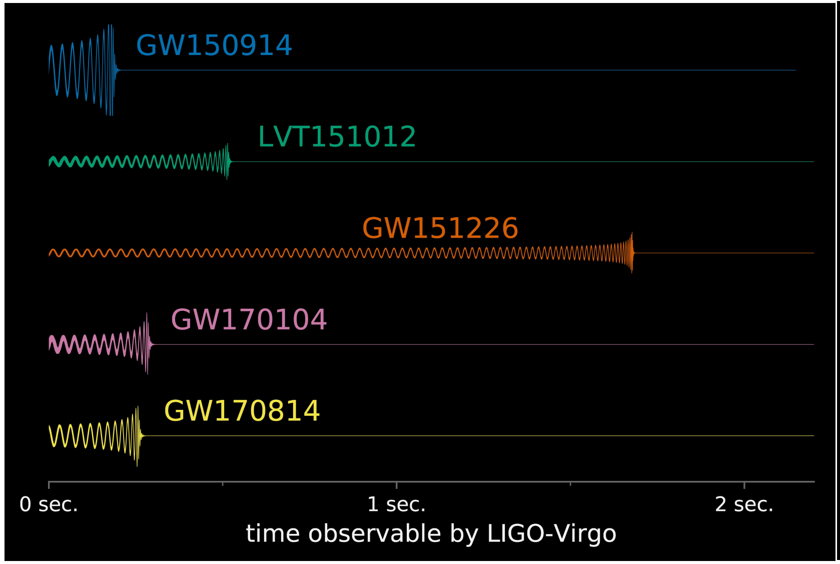
September 14, 2015

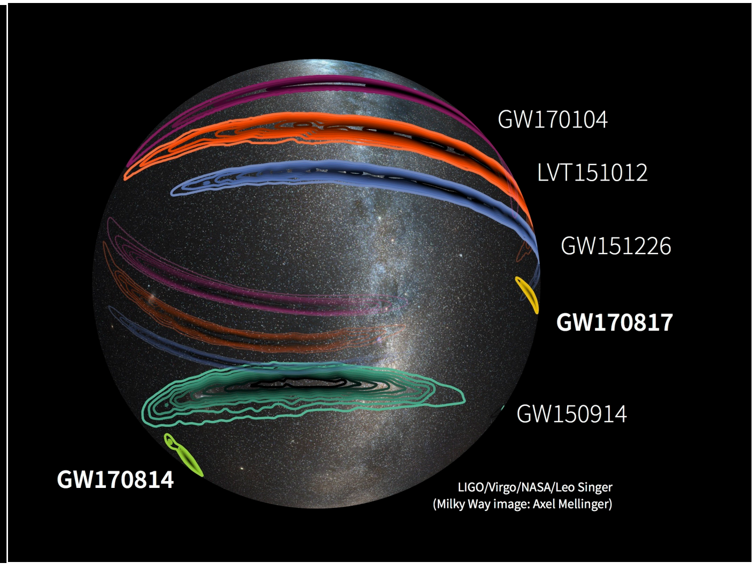


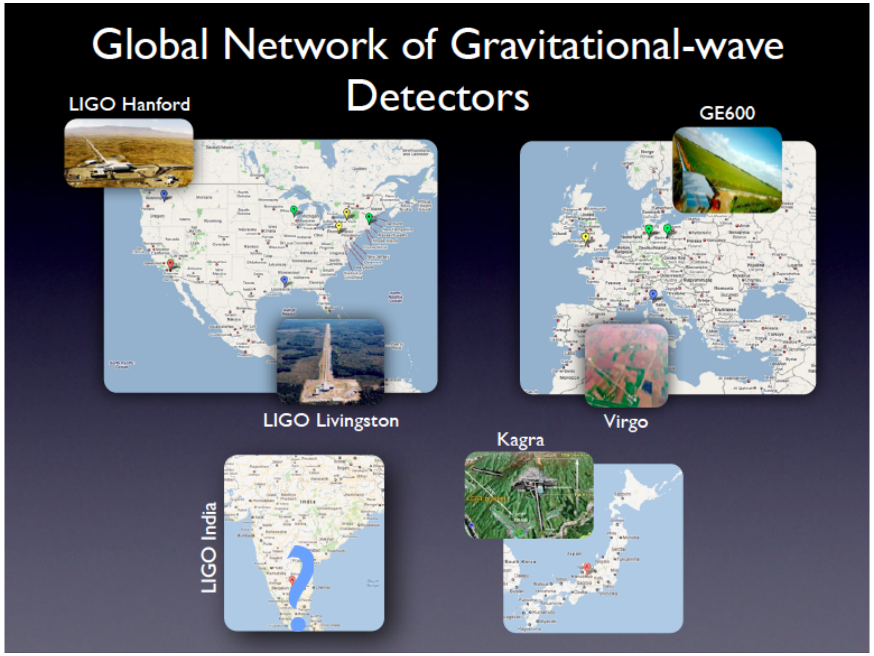


We did it !!

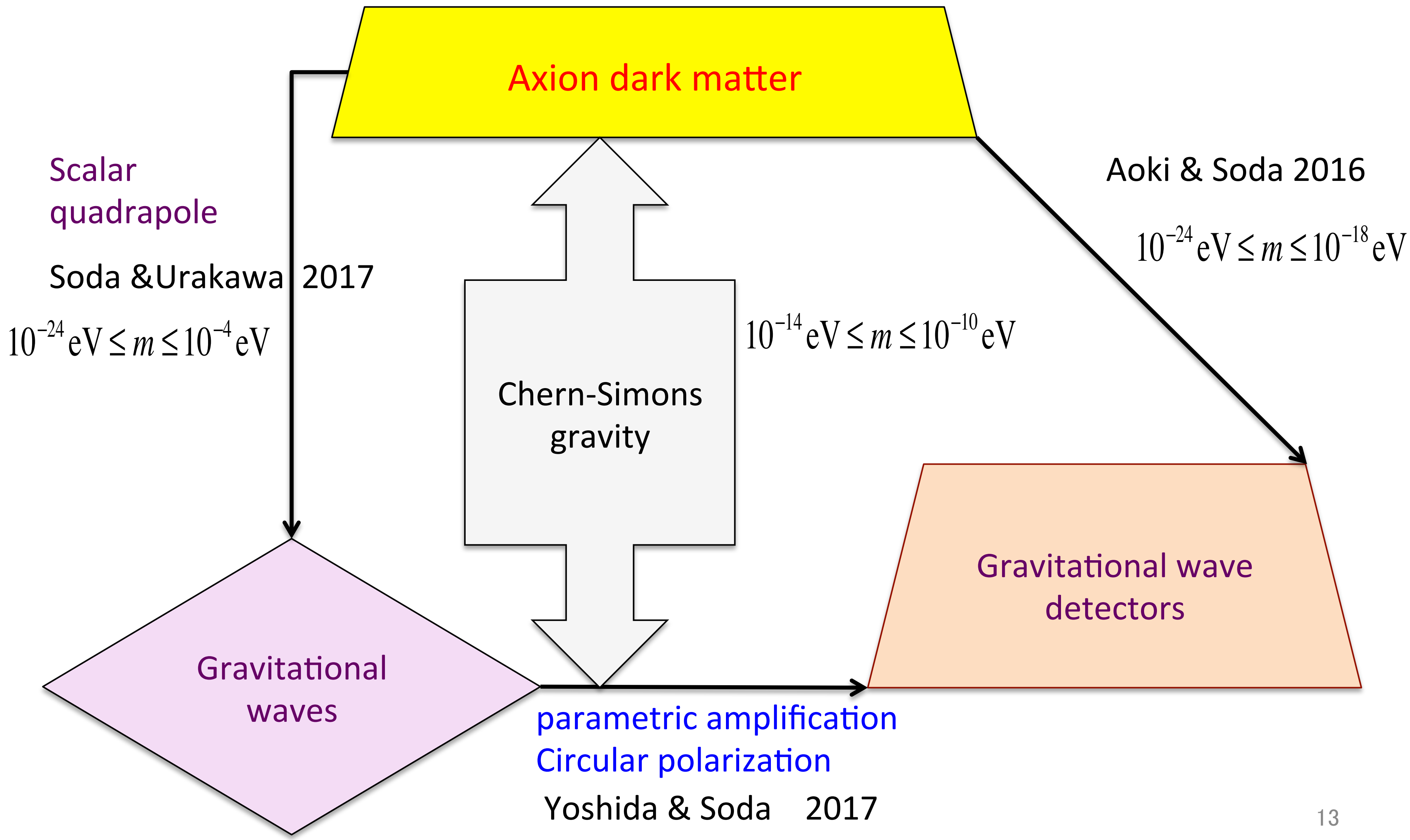
GW astronomy era !!







Gravitational wave detector can probe axion



Chern–Simons gravity

Axion search through Chern-Simons portal

In the presence of the axion, there arises the Chern-Simons term in the gravity sector.

$$S=\frac{M_p^2}{2}\int d^4x\sqrt{-g}\,R+\frac{M_p}{8}\ell^2\int d^4x\sqrt{-g}\,\phi\,\varepsilon^{\mu\nu\lambda\rho}R_{\alpha\beta\mu\nu}R^{\alpha\beta}{}_{\lambda\rho}-\int d^4x\sqrt{-g}\left[\frac{1}{2}(\partial\phi)^2+\frac{1}{2}m^2\phi^2\right]$$

↑ **coupling constant**

The Chern-Simons term appears in string theory.
It is also natural from the effective theory point of view.

Indeed, the Chern-Simons gravity has been investigated as a modified theory of gravity for a long time.

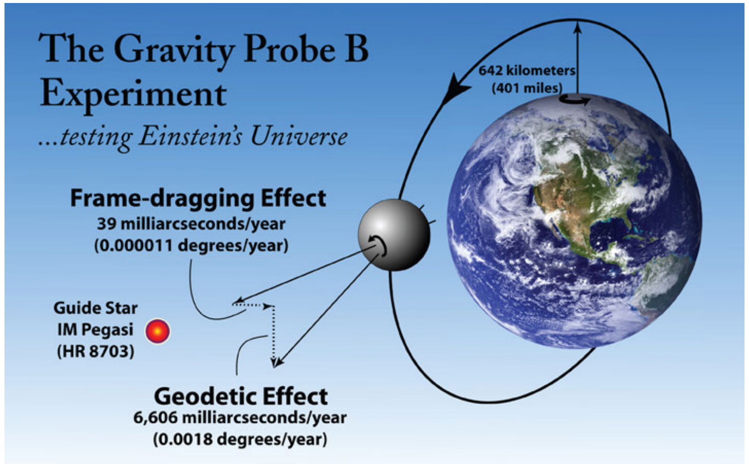
There already exists a Post-Newtonian constraint on the coupling from the gravity probe B experiment.

The future observation of gravitational waves will also give a constraint on the coupling constant.

Current constraint on CS coupling

Gravity probe B has measured the gyroscopic precession due to the frame dragging.
The result was in agreement with GR to an accuracy of 20%.

$$\left| \frac{\omega\left(1.1R_{\oplus}\right)}{\frac{2J}{\left(1.1R_{\oplus}\right)^3}} - 1 \right| \leq 20\%$$



Y.Ali-Haimoud & Y.Chen 2011

$$\ell \leq 10^8\,\mathrm{km}$$

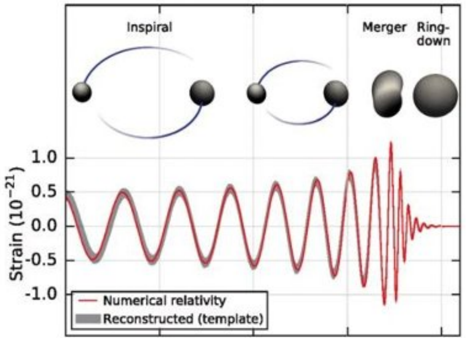
Future Constraint on CS coupling with GW

The gravitational waves emitted from the binary system can be calculated as

$$\tilde{h}(f) = A f^{-7/6} \exp\left[i\Psi(f) \right] \qquad A = 30^{-1/2} \pi^{-2/3} M^{5/6} D_L^{-1} \qquad M = \mu^{3/5} (m_1 + m_2)^{2/5}$$

$$\Psi(f) = \Psi_{\mathrm{GR}}(f) + \delta\Psi(f) \qquad \delta\Psi(f) = \frac{3}{128} (\pi M f)^{-5/3} \left(-10 \delta C (m\omega)^{4/3} \right)$$

Kerr parameter ↓



$$\delta C = \frac{330845}{1107456} \frac{\ell^4}{(m_1+m_2)^2 m_1^2} \chi_1^2 \left[1 - \frac{190107}{66169} \left(\hat{S}_1 \cdot \hat{L} \right)^2 \right]$$
$$- \frac{41525}{158208} \frac{\ell^4}{(m_1+m_2)^4} \frac{\chi_1 \chi_2}{\eta} \left[\left(\hat{S}_1 \cdot \hat{S}_2 \right) - \frac{4743}{1661} \left(\hat{S}_1 \cdot \hat{L} \right) \left(\hat{S}_2 \cdot \hat{L} \right) \right] + (1 \leftrightarrow 2)$$

In future, in principle, the ground-based detectors could constrain the CS coupling to be

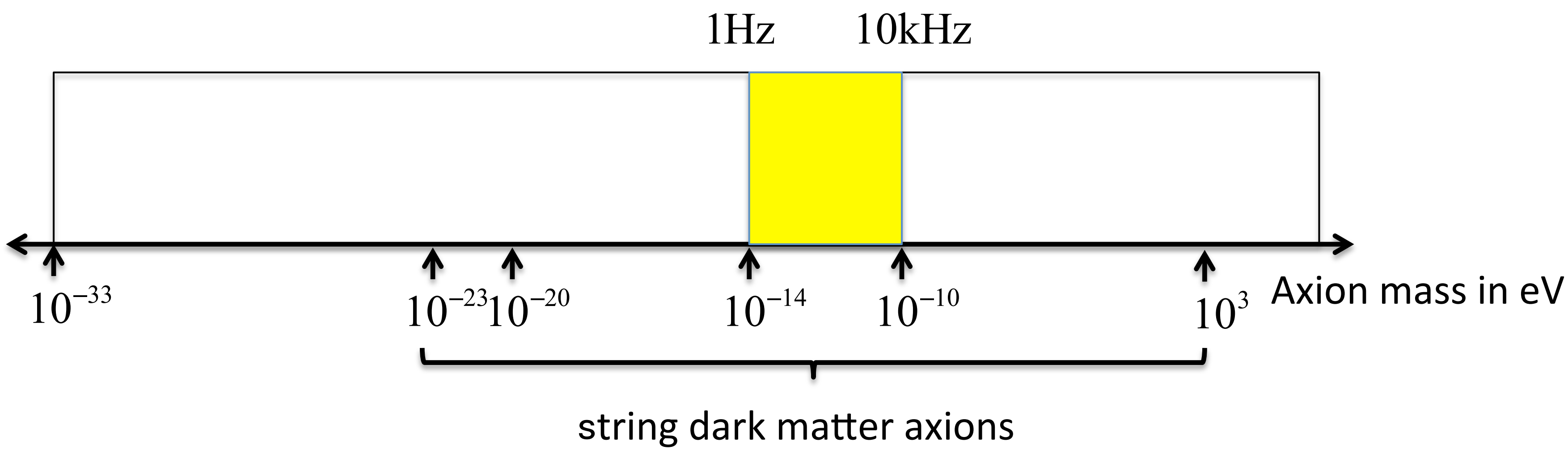
$$\ell \le (10 - 100)\,\mathrm{km}$$

or prove the presence of the CS term. Yagi, Yunes,Tanaka 2012

Ground interferometer can probe axion

In this talk, we show the Chern-Simons coupling gives rise to
a new probe into the axion dark matter.

From a different point of view, it provides an alternative way
to give a constraint on the Chern-Simons coupling.

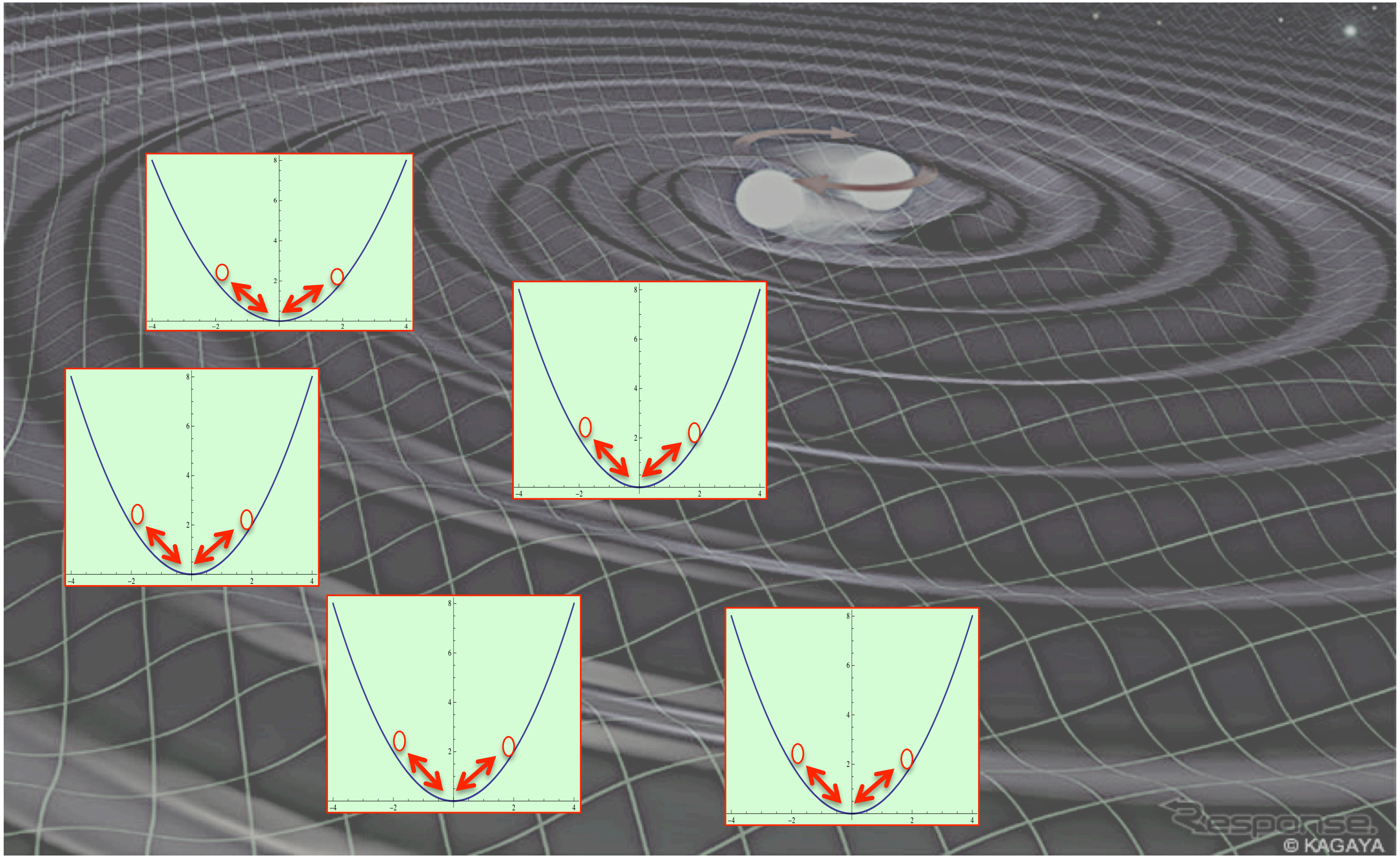


The mass range we are looking into is $10^{-14}\,\mathrm{eV} \le \ell \le 10^{-10}\,\mathrm{eV}$
corresponding to the 1Hz to 10kHz range where Chern-Simons coupling is relevant.

In this frequency range,
we can explore the axion dark matter with gravitational wave interferometers.

Exploring the axion dark matter with gravitational waves

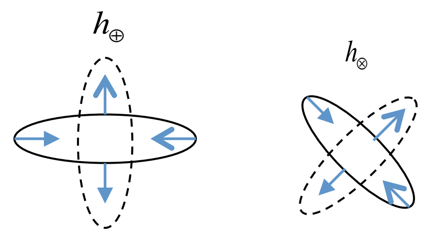
Gravitational waves are propagating in the string axiverse



Polarization of Gravitational Waves

Gravitational waves propagating to z-direction is descrived by

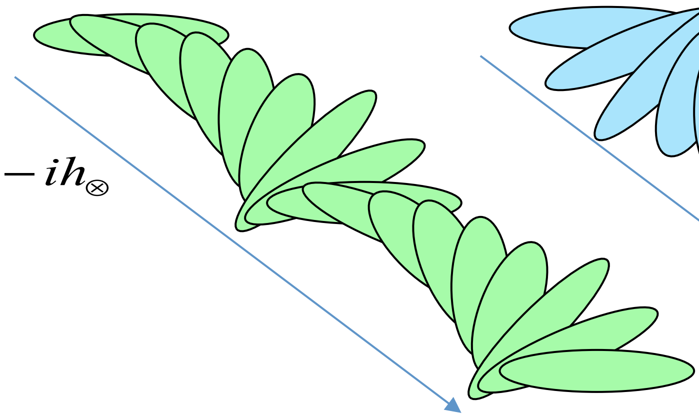
$$ds^2 = -dt^2 + dz^2 + (1+h_\oplus)dx^2 + (1-h_\oplus)dy^2 + 2h_\otimes dxdy$$



For the discussion of parity violation, circular polarization basis are useful.

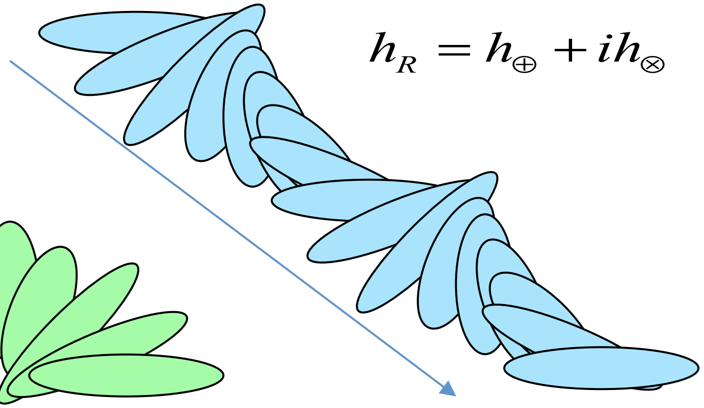
Left-handed circular polarization

$$h_L = h_\oplus - ih_\otimes$$



Right-handed circular polarization

$$h_R = h_\oplus + ih_\otimes$$



$$h_{ij} = h_R e_{ij}^R(\mathbf{n}) + h_L e_{ij}^L(\mathbf{n})$$

$$i\varepsilon_{ilm} n_l e_{mj}^{R/L}(\mathbf{n}) = \pm e_{ij}^{R/L}(\mathbf{n})$$

Parametric amplification of GWs

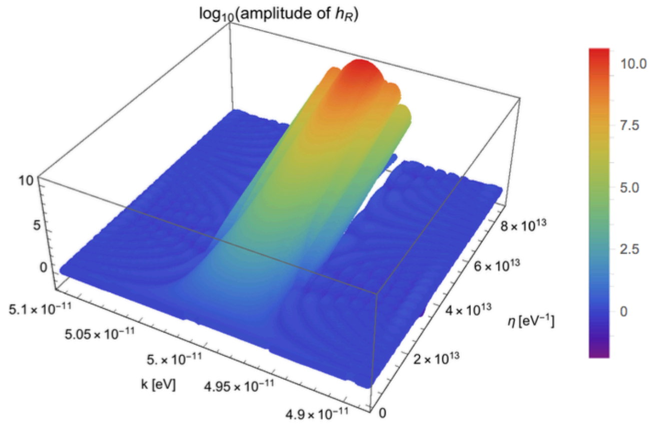
Dynamical Chern-Simons gravity

$$S=\frac{M_p^2}{2}\int d^4x\sqrt{-g}\,R+\frac{M_p}{8}\ell^2\int d^4x\sqrt{-g}\,\phi\,\varepsilon^{\mu\nu\lambda\rho}R_{\alpha\beta\mu\nu}R^{\alpha\beta}{}_{\lambda\rho}-\int d^4x\sqrt{-g}\left[\frac{1}{2}(\partial\phi)^2+\frac{1}{2}m^2\phi^2\right]$$

We can neglect the cosmic expansion

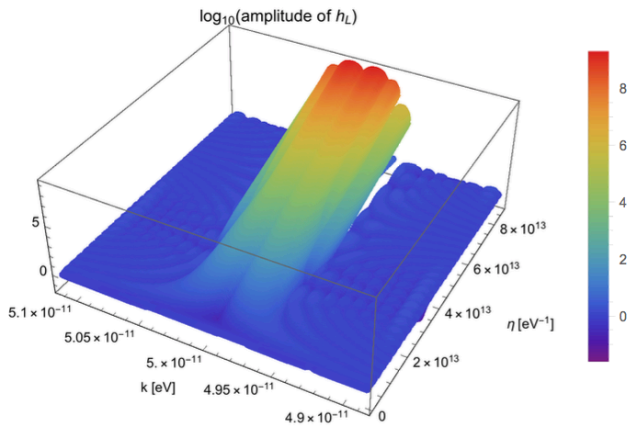
$$S=\frac{M_p^2}{2}\int dt\,d^3k\sqrt{-g}\left(1-\varepsilon_A k\frac{\ell^2}{M_p}\dot{\phi}\right)\left[\dot{h}_A{}^2-k^2h_A{}^2\right]$$

$$\varepsilon_A=\begin{cases}1 & \text{for R}\\ -1 & \text{for L}\end{cases}\qquad \phi=\phi_0\cos mt$$



GWs in the axion background

$$\ddot{h}_A+\frac{m\varepsilon_A\delta\cos mt}{m+\varepsilon_A k\delta\cos mt}k\dot{h}_A+k^2h_A=0\qquad \delta=m^2\ell^2\frac{\phi_0}{M_p}$$



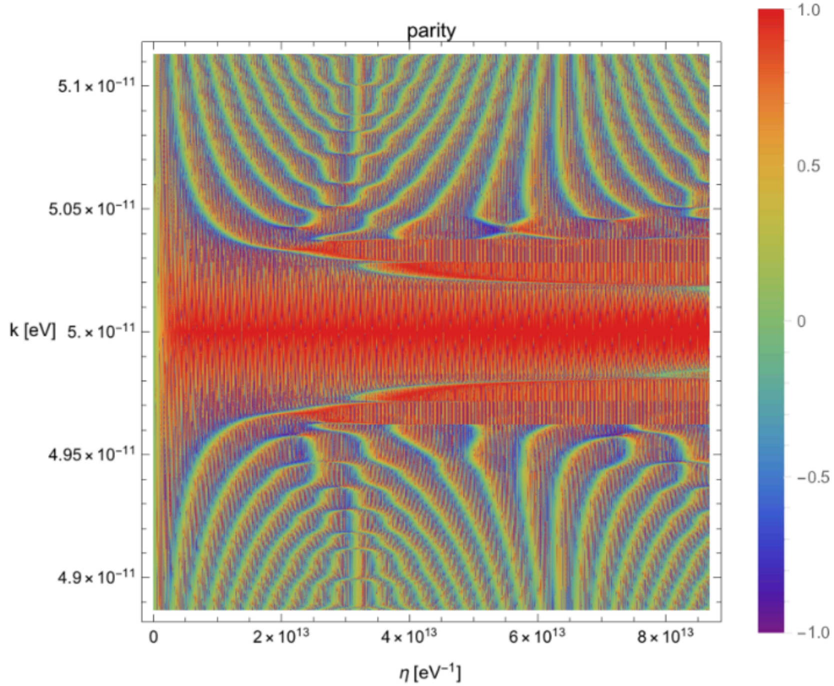
Parametric resonance

$$k_{\rm res}=\frac{m}{2}=1.2\times10^4\left(\frac{m}{10^{-10}\,\text{eV}}\right)\text{Hz}$$

Parity violation in gravity

Since the wave equations for right and left handed polarizations rae different,
the parametric resonance due to the axion oscillation induces parity violation in gravity.

$$\text{parity\_violation} = \frac{|h_R|^2 - |h_L|^2}{|h_R|^2 + |h_L|^2}$$



If this parity violation pattern is observed,
it should be an evidence of the axion dark matter.

A new constraint on Chern-Simons gravity

The current constraint provided by the Gravity Probe B is $\ell \leq 10^{8}\,\mathrm{km}$

Suppose the axion is the main component of the dark matter.

We can calculate the growth rate analytically as

$$\Gamma = \frac{m\delta}{8} = 2.8\times 10^{-16}\,\mathrm{eV}\left(\frac{m}{10^{-10}\,\mathrm{eV}}\right)\left(\frac{\ell}{10^{8}\,\mathrm{km}}\right)^{2}\sqrt{\frac{\rho}{0.3\mathrm{GeV/cm^{3}}}}$$

$$c\,t_{\times 10} = 10^{-8}\left(\frac{10^{-10}\,\mathrm{eV}}{m}\right)\left(\frac{10^{8}\,\mathrm{km}}{\ell}\right)^{2}\sqrt{\frac{0.3\mathrm{GeV/cm^{3}}}{\rho}}\ \mathrm{pc}$$

From this growth rate, we can say that
after 10kpc propagation the amplitude of 10kHz GWs is enhanced by $10^{10^{12}}$

However, we have never observed these phenomena.

Thus, taking $c\,t_{\times 10} = 100\mathrm{Mpc}$, we obtain a new stringent constraint

$$\ell \leq 1\mathrm{km}$$

Exploring the string axion dark matter

Suppose the coupling constant is determined by the GW observations to be

$$\ell = 100\mathrm{km}$$

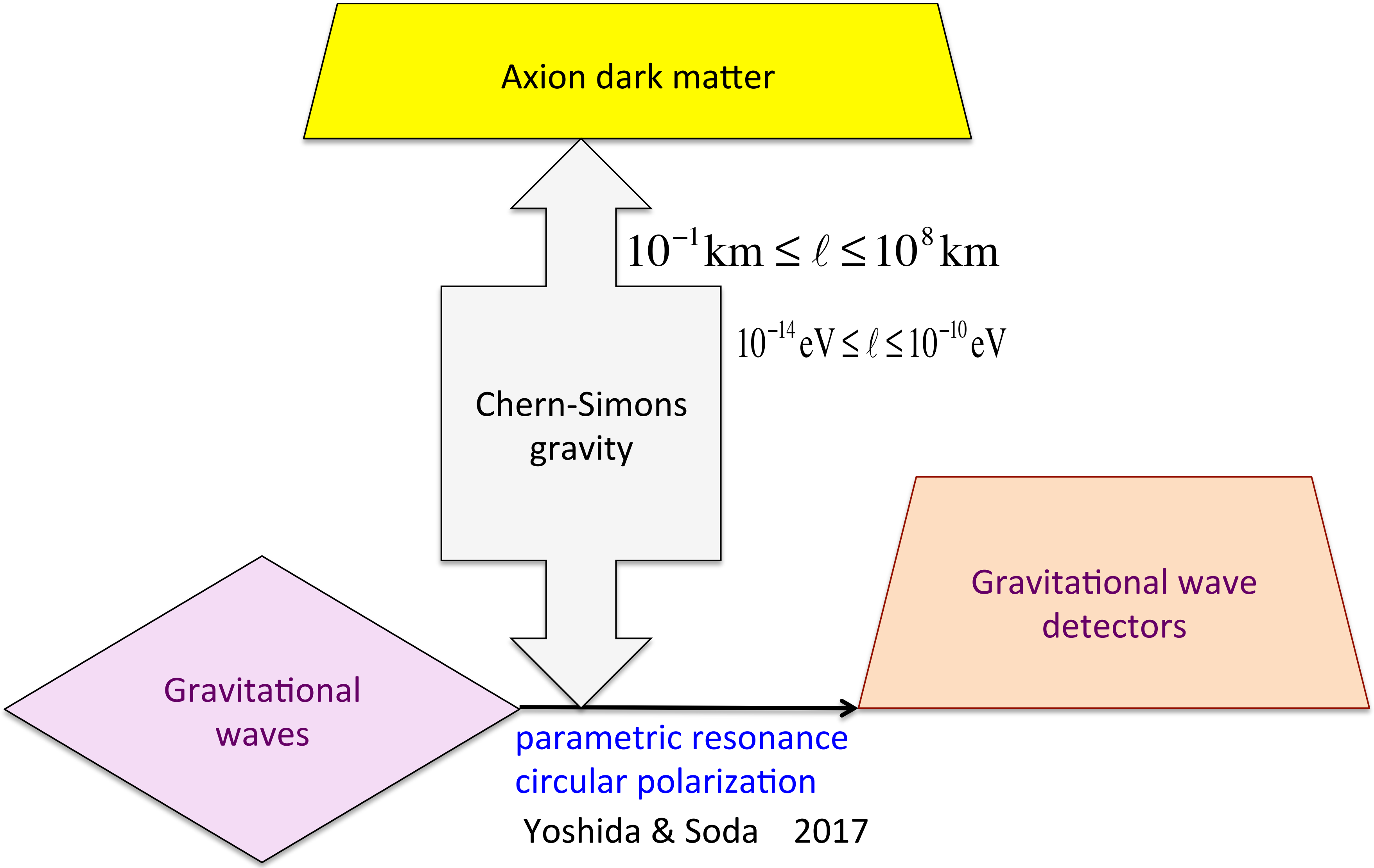
Since the gravitational waves are propagating in the axion background,
we have the growth time

$$c\,t_{\times 10} = 1.0 \times \left(\frac{10^{-10}\mathrm{eV}}{m}\right)\left(\frac{100\mathrm{km}}{\ell}\right)^{2}\sqrt{\frac{10^{-29}\mathrm{g/cm}^{3}}{\rho}}\ \mathrm{Mpc}$$

From this growth rate, we can conclude that
the axion dark matter can be observed by GW interferometers up to $\Omega_a = 0.01$.

At the same time, we can observe the parity violation in the gravity sector.

Summary


Axion dark matter
$10^{-1}\mathrm{km} \leq \ell \leq 10^{8}\mathrm{km}$
$10^{-14}\mathrm{eV} \leq \ell \leq 10^{-10}\mathrm{eV}$
Chern-Simons gravity
Gravitational wave detectors
Gravitational waves
parametric resonance
circular polarization
Yoshida & Soda 2017

感謝!!