Hawking radiation as tunneling from supertranslated horizon

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Introduction

- BMS supertranslations - Asymptotic symmetries of asymptotically Minkowski spacetime

- Supertranslation hair on black hole - new kind of hair which may be used to distinguish black holes with the same M, Q, J [Hawking, Perry and Strominger (2016)]

- Question: Does Hawking radiation carry information of supertranslation hair ?

- Hawking raidation as tunneling [Parikh and Wilczek (2000)]

  In this work, we adopt Hamilton-Jacobi method

❑ BMS supertranslation

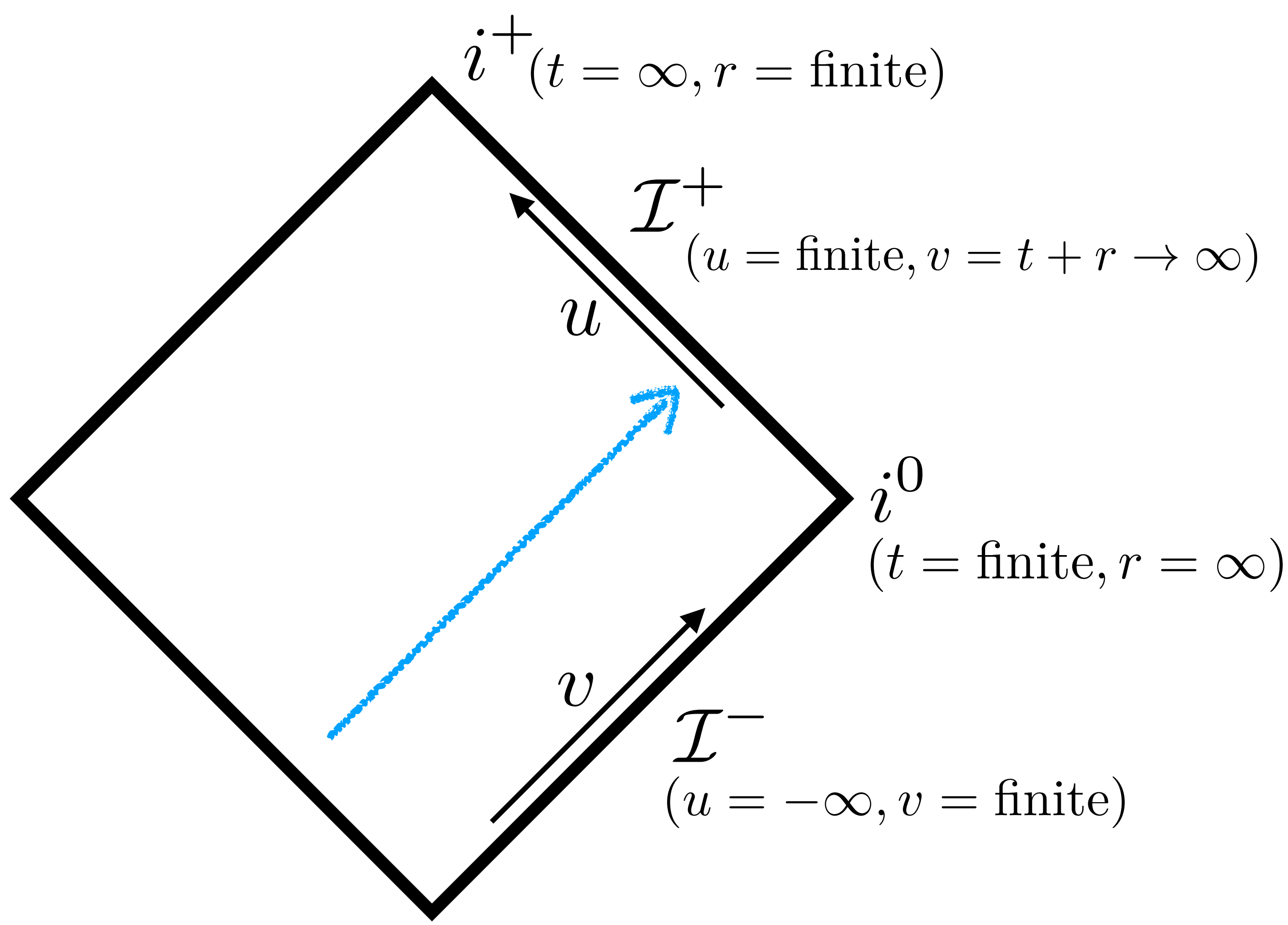
- Minkowski space in advanced Bondi coordinates $(v, r, z, \bar{z})$

$$ds^2 = -\,dv^2 + 2dvdr + r^2\gamma_{AB}d\Theta^A d\Theta^B$$

$$v = t + r \qquad (A, B) = (z, \bar{z})$$

$$z = e^{i\phi}\tan\theta/2$$

$$\gamma_{z\bar{z}} = \frac{2}{(1+|z|^2)^2} \qquad \gamma_{zz} = 0$$



Asymptotically Minkowski metric

- Bondi gauge: $g_{rr} = 0, \quad g_{rA} = 0, \quad \partial_r \det\left(\frac{g_{AB}}{r^2}\right) = 0.$

- Boundary conditions at large r :

$$g_{vv} = -1 + \mathcal{O}\left(\frac{1}{r}\right), \qquad g_{vA} = \mathcal{O}(1), \qquad g_{vr} = 1 + \mathcal{O}\left(\frac{1}{r^2}\right),$$
$$g_{z\bar{z}} = r^2\gamma_{z\bar{z}} + \mathcal{O}(1), \qquad g_{zz} = \mathcal{O}(r)$$

- Asymptotic expansion of metric $r \to \infty$: past null infinity $\mathcal{I}^-$

$$\begin{aligned} ds^2 =& - dv^2 + 2dvdr + r^2\gamma_{AB}d\Theta^A d\Theta^B \\ &+ \frac{2m}{r}dv^2 + rC_{AB}d\Theta^A d\Theta^B - (D^B C_{AB} + \frac{4}{3r}N_A)dvd\Theta^A + \cdots \end{aligned}$$

m, C_{AB}, N_A : functions of $(v, z, \bar{z})$, $\quad \gamma^{AB}C_{AB} = 0,$

BMS supertranslation : [Bondi,Metzner,van der Burg,Sachs]

diffeomorphism which preserves Bondi gauge conditions and asymptotic falloff conditions at null infinity of asymptotically Minkowski spacetime

The vector field ζ which generates BMS supertranslation is

$$\zeta_f = f\partial_v - \frac{1}{2}D^2 f\partial_r + \frac{1}{r}D^A f\partial_A, \qquad f = f(\Theta) \text{: arbitrary function}$$

Supertranslation is parametrized by a function ⇒ infinitely many

✳ Conserved charge associated with supertranslation: $Q_f = \int_{i^0} f(\Theta) m d^2\Omega$

$$f(\Theta) = \sum_{l,m} a_{lm} Y_{lm}(\Theta)$$

☐ Supertraslation hair

Schwarzschild spacetimes with different supertranslations

$(M_{\rm ADM}, f(\Theta))$ $(M_{\rm ADM}, \tilde{f}(\Theta))$

Classically, black holes are characterized by $M_{\rm ADM}$ and $Q_f = Q_{\tilde{f}}$ which are independent of f, $\tilde{f}$

Black holes do not carry the classical supertranslation hair

(Superrotation charge can be interpreted as the classical supertranslation hair)
[Hawking, Perry and Strominger (2016)]

Quantum mechanically, black holes evaporate
If Hawking radiation depends on f , the black holes with different supertranslations can be distinguished

➡ Black holes have supertranslation hair carried by Hawking radiation

Setup

- During evaporation, black hole will be dynamical due to the backreaction of Hawking radiation
- Vaidya spacetime gives a simple model of dynamical black holes
- We study Hawking radiation as tunneling from Vaidya black hole with supertranslation hair (supertranslated Vaidya black hole)

1. Supertraslation of Vaidya spacetime

- Vaidya metric in advanced Bondi coordinates

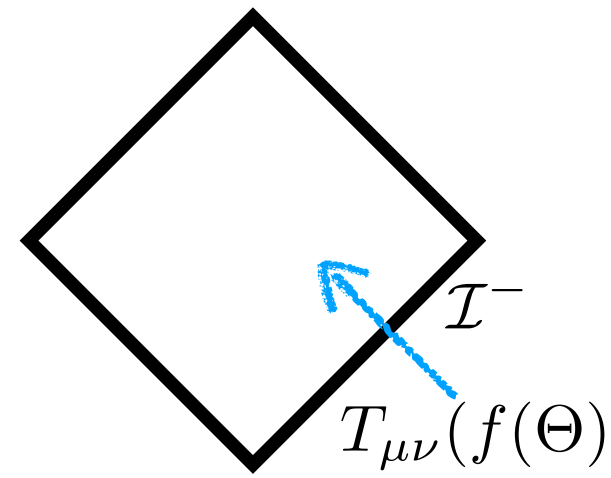
$$\bar{g}_{\mu\nu}dx^\mu dx^\nu = -V dv^2 + 2dvdr + r^2\gamma_{AB}d\Theta^A d\Theta^B, \qquad V \equiv 1 - \frac{2M(v)}{r}$$

Asymptotically Minkowski space containing Vaidya black hole

- Supertranslated Vaidya metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + \mathcal{L}_\zeta \bar{g}_{\mu\nu}$

$$g_{\mu\nu}dx^\mu dx^\nu = -\Big(V - \frac{2fM'}{r} - \frac{MD^2 f}{r^2}\Big)dv^2 + 2dvdr - D_A(2Vf + D^2 f)dvd\Theta^A$$
$$+ (r^2\gamma_{AB} + 2rD_A D_B f - r\gamma_{AB}D^2 f)d\Theta^A d\Theta^B \qquad (M' = \partial_v M(v))$$

✳We can reproduce the supertranslated Vaidya spacetime by injecting matter energy momentum flux which has angular dependence



X ✳ADM mass does not depend on the supertranslation function $f(\Theta)$

$$M_{\rm ADM} = M(v = \infty)$$

Vaidya spacetime with different supertranslations has the same ADM mass

✳The supertranslated metric can be extended from null infinity to the interior of spacetime since the metric solves the Einstein eq. for all r.

- matter crossing horizon induces supertranslation of horizon

2. Properties of supertranslated Vaidya black hole

2.1. Trapping horizon

Let θ_- and θ_+ be the expansions of the bundles of ingoing and outgoing radial null geodesics, respectively. In our case,

$$\theta_- = -\frac{2}{r}, \quad \theta_+ = \frac{1}{r}\Big(V - \frac{2fM'}{r} - \frac{MD^2 f}{r^2}\Big),$$

$\theta > 0$: bundle is expanding

$\theta < 0$: bundle is contracting

❑ Future outer marginally trapped surface

$$\theta_+ = 0 \quad \text{(marginally trapped)}, \quad \theta_- < 0 \quad \text{(future type)}, \quad l^\mu \partial_\mu \theta_+ < 0 \quad \text{(outer type)}$$

- $\theta_+ = 0 \;\Rightarrow\; r = r_h = 2M + 2fM' + \frac{1}{2}D^2 f$ ✳Location of horizon

❑ Future outer trapping horizon (FOTH) [Hayward]

- A foliation of future outer marginally trapped surface

✳ FOTH gives the local definition of supertranslated Vaidya black hole

2.2. Surface gravity

❑ Kodama vector and surface gravity in spherical symmetry

- Kodama vector satisfies $\nabla^\mu (G_{\mu\nu} K^\mu) = 0, \qquad \nabla_\mu K^\mu = 0$ [Kodama]

✳Kodama vector gives a preferred time direction in dynamical situation

- Surface gravity: $K^\mu \nabla_{[\nu} K_{\mu]} = -\kappa K_\nu$ at FOTH [Hayward]
- In spherical Vaidya: $K^\mu = \delta^\mu_v \qquad \kappa = (4M(v))^{-1}$

❑ Extension to supertranslated Vaidya spacetime

- We could find a Kodama-like vector which satisfies the above conditions and the corresponding surface gravity at FOTH

$$K^\mu = \delta^\mu_v$$

$$\kappa = \frac{1}{4M}\left(1 - f\frac{M'}{M}\right)$$

✳The surface gravity has angular dependence from the supertraslations

3. Hawking radiation from supertranslated Vaidya BH

❑ Hamilton-Jacobi method [Angheben etal., Srinivasan etal.]

Massless scalar field minimally coupled to gravity $\Box\phi = 0$

- WKB ansatz: $\phi = A(x)e^{iS/\hbar} + \mathcal{O}(\hbar)$

At the lowest order,

$$g^{\mu\nu}\partial_\mu S \partial_\nu S = 0 \qquad \text{Hamilton-Jacobi equation}$$

Near a horizon, WKB (particle) approx. is reliable: $\nu_{\rm em} \propto (1 - 2\frac{M}{r})^{-1/2}\nu_{\rm ob}$

- Reconstruction of the phase (particle action): $S = \int_P dx^\mu \partial_\mu S$

  ✳We are interested in imaginary part of S = signal of tunneling

- Semiclassical emission rate: $\Gamma_{\rm em} \propto \exp\Big(-2\,{\rm Im}S\Big)$

❑ Supertranslated Vaidya black hole

$$2\partial_r S\partial_v S + \Big(V - \frac{2fM'}{r} - \frac{MD^2 f}{r^2}\Big)(\partial_r S)^2$$
$$+ \frac{1}{r^2} D^A(2Vf + D^2 f)\partial_r S\partial_A S + \frac{1}{r^4}(r^2\gamma^{AB} - 2rD^A D^B f + r\gamma^{AB} D^2 f)\partial_A S\partial_B S = 0$$

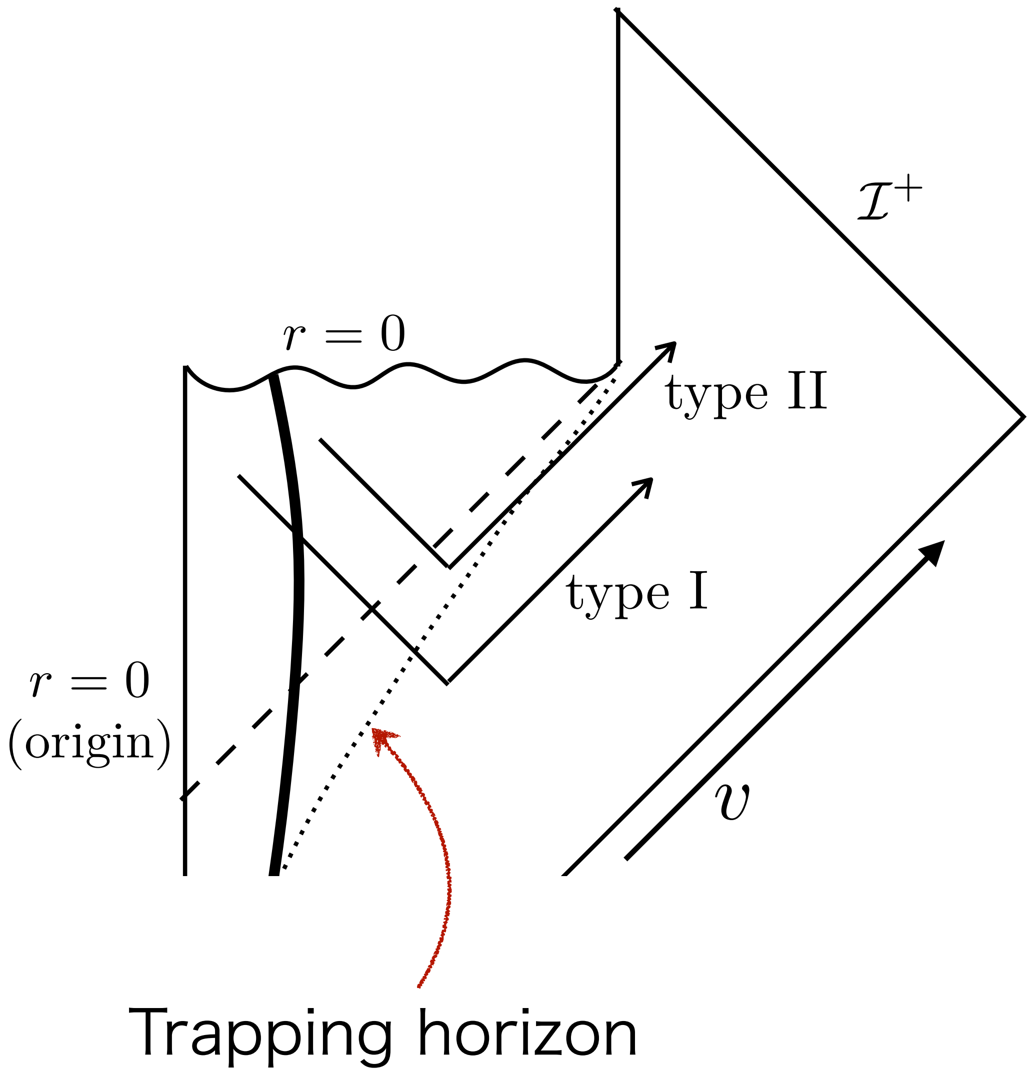
- particle energy: $\omega \equiv -K^\mu \partial_\mu S = -\partial_v S$ $\quad K^\mu$: Kodama-like vector
- We concentrate on radial null geodesics: $\Theta^A = \text{const.}$ $\Rightarrow$ $\partial_A S = 0$

Then HJ equation yields

$$\partial_r S_{out} = 2\omega\left(V - \frac{2fM'}{r} - \frac{MD^2 f}{r^2}\right)^{-1}, \quad \partial_r S_{in} = 0$$

$$\Rightarrow \quad S_{out} = -\int \omega dv + \int \frac{2\omega r^2 dr}{Vr^2 - 2fM'r - MD^2 f}, \qquad S_{in} = -\int \omega dv.$$

Two types of tunneling path for S_{out}



- Type-I: classically forbidden trajectory backward in time
- Type-II: classically allowed trajectory forward in time. [Vanzo etal.]

Along these paths,

$$v = \text{const.} \quad (\text{type-I}),$$

$$\frac{dr}{dv} = \frac{1}{2}\left(V - \frac{2fM'}{r} - \frac{MD^2 f}{r^2}\right) \quad (\text{type-II}).$$

X For type-II path, we have

$$S_{out} = -\int_{\mathrm{II}} dv\omega + \int_{\mathrm{II}} \partial_r S_{out} dr = 0$$

No tunneling at all.

For type-I path, we have

$$\mathrm{Im}S_{out} = \mathrm{Im}\int_{\mathrm{I}} \frac{2\omega r^2 dr}{(r-r_h)(r-r_f)} = \mathrm{Im}\int_{\mathrm{I}} \frac{2\omega r^2 dr}{(r-r_h-i\epsilon)(r-r_f)}$$

$$= \frac{2\pi\omega r_h^2}{r_h - r_f} = \pi\omega\cdot 4M\left(1+f\frac{M'}{M}\right)$$

where $r = r_h = 2M + 2fM' + \frac{1}{2}D^2 f$ and $r_f = -\frac{1}{2}D^2 f$

↑ outer trapping horizon

✳Tunneling occurs and radiation forms at the trapping horizon

❑ Semiclassical emission rate

$$\Gamma_{\mathrm{em}} \propto e^{-2\mathrm{Im}S_{out}} = \exp\left(-8\pi\omega M\left(1+f\frac{M'}{M}\right)\right) = \exp\left(-\frac{2\pi\omega}{\kappa}\right)$$

✳The emission rate of Hawking radiation has a dependence on supertranslation $f(\Theta)$!

X ❑ Entropy and semiclassical emission rate

$$S_{\rm dyn} = \frac{A_h}{4} = 4\pi M^2 + \sqrt{16\pi} a_{00} M M'$$

$$f(\Theta) = \sum_{l,m} a_{lm} Y_{lm}(\Theta)$$

After an emission of Hawking radiation, $M \to M - \omega, \quad S_{\rm dyn} \to S_{\rm dyn} + \Delta S$

$$\Delta S = -\omega (8\pi M + \sqrt{16\pi} a_{00} M') + \mathcal{O}(\omega^2)$$

Thus, $\Delta S = \int \frac{d\Omega}{4\pi} \ln \Gamma_{\rm em}$

$$\Rightarrow \quad \frac{d\Delta S}{d\Omega} = \ln \Gamma_{\rm em}$$

Hawking radiation carries away from the black hole different amount of entropy at different angles

4. Summary

- We found that Hawking radiation will actually carry information of supertranslation. In the static limit, the dependence disappears.

- We can distinguish black hole with different supertranslation hair by Hawking radiation

- Tunneling of massless particles occurs at the trapping horizon.

- The spectrum of Hawking radiation can be expressed in terms of the horizon surface gravity κ which can serve as local measure of Hawking temperature

❑ Some questions

- Energy flux of Hawking radiation will produce the gravitational memory effects near future null infinity?

- Like energy conservation, Hawking radiation carry part of charges associated with supertranslation/ superrotation? If so, how much amount of the charges?

- What's the implication to information problem?

[HPS, Bousso etal, Gomez etal, Mirababayi etal, Carney etal, …]

Infinite degeneracy of BMS vacua

Under the supertranslation, asymptotic data of metric change

$$\delta_f m = f\partial_u m + \frac{1}{4}N^{AB}D_A D_B f + \frac{1}{2}D_A f D_B N^{AB} \qquad N_{AB} = \partial_u C_{AB}$$

$$\delta_f C_{AB} = f\partial_u C_{AB} - 2D_A D_B f + \gamma_{AB} D^2 f$$

- BMS vacuum: $\partial_u m = 0, \quad N_{AB} = 0,$

$$D_{\bar{z}}^2 C_{zz} - D_z^2 C_{\bar{z}\bar{z}} = 0 \;\Rightarrow\; C_{zz} = -2D_z^2 C(z,\bar{z})$$

But $\delta_f C_{zz} = -2D_z^2 f \;\Rightarrow\; C(z,\bar{z}) \to C(z,\bar{z}) + f(z,\bar{z})$

BMS vacua are related by supertranslation.

- Once a vacuum is chosen, supertranslation invariance of BMS vacua is spontaneously broken.
- Different BMS vacua have different angular momentum

Gravitational memory effect

[Zeldovich etal.,Christodoulou,Braginsky etal.]

Before radiation: vacuum with $C_{AB}=0$

$$\delta s(z_1,z_2)=L=\sqrt{2}r_0\gamma_{z\bar{z}}\sqrt{|\delta z|^2}$$

$$\delta z=z_1-z_2$$

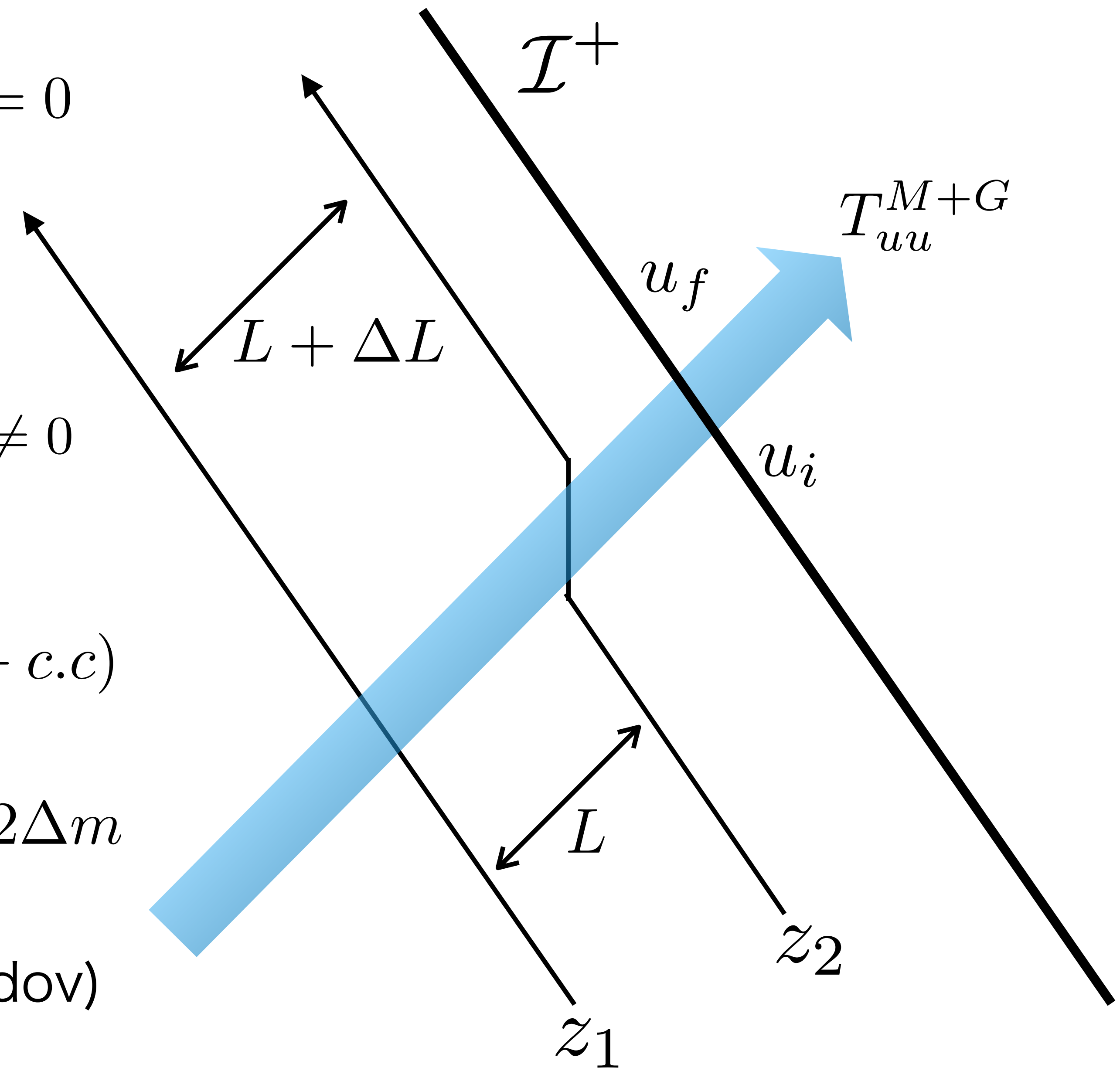
After radiation: vacuum with $\Delta C_{AB}\neq 0$

Especially $\Delta C\neq 0$

$$L\to L+\Delta L \qquad \Delta L=\frac{r_0}{2L}(\Delta C_{zz}\delta z^2+c.c)$$

$$D_z^2\Delta C_{zz}=2\int_{u_i}^{u_f}du\lim_{r\to\infty}[r^2T_{uu}^{M+G}]+2\Delta m$$

(Formular for ΔC : Strominger, Zhiboedov)



- ΔC contains the information of massless/massive matters
- Memory effect: permanent displacement of detectors due to a passage of gravitational wave/null matter T_{uu}^{M+G} (null memory) and/or a change in state of massive matter Δm (ordinary memory)

Tunneling in Schwarzschild black hole

HJ eq.: $\left(1-\dfrac{2M}{r}\right)(\partial_r S)^2 - 2\omega\partial_r S = 0$

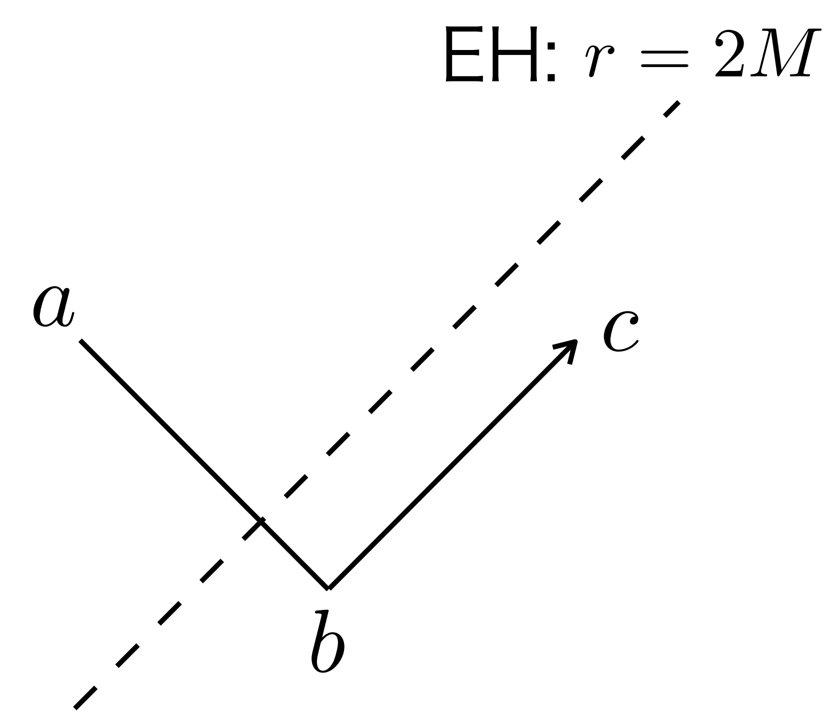
Trajectory on $(v-r)$ plane

$$\Rightarrow \quad \partial_r S_{out} = \frac{2\omega}{1-\frac{2M}{r}}, \qquad \partial_r S_{in} = 0$$

- particle energy: $\omega = -\xi^\mu p_\mu = -\xi^\mu \partial_\mu S = -\partial_v S$

Particle action is given by a line integral along a path

$$S_{out} = -\int \omega dv + \int \frac{2\omega dr}{1-\frac{2M}{r}}$$



Imaginary part of the action is

$$\mathrm{Im} S_{out} = \mathrm{Im}\int_{a\to b\to c} \frac{2\omega dr}{1-\frac{2M}{r}} = \mathrm{Im}\int_{a\to b} \frac{2\omega dr}{1-\frac{2M}{r}}$$

$$= \mathrm{Im}\int_{a\to b} \frac{2\omega r dr}{r-2M-i\epsilon} = 4\pi\omega M.$$

tunneling path $\overrightarrow{ab}$
(classically forbidden)

Semiclassical emission rate

$$\Gamma_{\rm em} \propto \exp(-2{\rm Im}S_{out}) = \exp(-8\pi\omega M) = \exp(-\omega/T_{\rm H})$$

We can read off the Hawking temperature: $T_{\rm H} = \dfrac{\kappa_s}{2\pi}, \quad \kappa_s = \dfrac{1}{4M}$