Equivalence Principle for Quantum Systems

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ongoing work with

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(last May) lectures on gravitational cat
Three elements: Q I G
Quantum, Information and **Gravity**

- **Quantum** ← Quantum Mechanics ← **Quantum Field Theory**
  Schroedinger Equation |
- **Gravity** ← Newtonian Mechanics ← **General Relativity**
  GR+QFT= **Semiclassical Gravity** (SCG)

- **Laboratory** conditions: | **Strong Field** Conditions:
  Weak field, nonrelativistic limit: | Early Universe, Black Holes

(*work in both regimes ongoing since the 80's*)

e.g., **Newton-Schroedinger Eq** | **Semiclassical Einstein Eq.**
(NSE) ← *beware of problems*! | **Einstein-Langevin Equation**

Quantum Information Issues in gravitational quantum physics

- **Quantum Decoherence:** *Environment-induced*
  - Gravity as ubiquitous environment
  - "Universal / Fundamental / Intrinsic" Decoherence

- **Quantum Entanglement:** *Entangled states*
  - Bell states
  - Gravitational Cat State

This talk, **Equivalence Principle for Quantum Systems,** focuses on **Gravity and Quantum**, but necessarily involve entanglement.

Here, more about *GQP than RQI*

Gravitational Quantum Physics than

Relativistic Quantum Information

GR + QFT In fact, only

weak field, nonrelativistic QM

Part I: Why worry about EP for Quantum Systems?

I. Introduction
Weak Equivalence Principle

- Newton

  F= ma (m inertial mass)

  F= GMm/r^2 (m gravitational mass)

- m (i) = m (g) weak equivalence principle

- E”otv”os expts, Torsion pendulum expts

- Laboratory results correct to very high accuracy.

Classical description

Galileo: [G]

All masses fall (in vacuum) at the same rate:

$x = \frac{1}{2} g t^2$ $\qquad\qquad$ $g = GM/r^2$

Einstein: [E]

Gravity can be "replaced" by acceleration:

Physics in a *freely falling frame* (Einstein elevator)
FFF is the same as in an *inertial frame*

EP assumed in QFT / CST

- **Wave equation for a quantum field ϕ in curved spacetime with metric g, curvature scalar R** $(\Box - m^2 - \xi R)\hat{\phi} = 0.$

Box: **Laplace-Beltrami Operator** in CST with metric g: Kinetic term: m_i **inertial** mass

Potential terms: m_g **grav.** mass

(g_00: expansion: 1 -2GM/r)

- **Einstein Equation**-- as field equation: grav. Mass; as equation of motion: inertial mass

What is different in a quantum world?

Quantum description in terms of:

- state preparation
- measurements
- Probabilities

Quantum states and processes:

- pure / mixed / entangled states
- Dephasing vs Decoherence

How does EP manifest in Quantum physics?

- **Quantum systems** with **internal** dof: Atoms

External dof (center of mass): Trajectory

(Caution: *Quantum histories need be sufficiently decohered before classical trajectories can be defined*.)

- Consider two separate cases of:

1) an elementary (non-composite) **particle**

2) an atom (composite) **in free fall**

- Describe its motion in QM language

Our Findings: Equiv. Principle for Quantum Systems: 2 versions

A. [**Einstein**] The probability distribution of the position for a **free-falling** particle is the same as the probability distribution of a **free** particle, modulo a ***mass-independent*** shift of its mean. (the ½ gt^2 term)

B. [**Galileo**]: Any two particles with the same **velocity** wave-function **behave identically in free fall, *irrespective of their masses*.**

II. Elementary Particle in Free Fall:

$$\hat{H}_g = m\hat{1} + \frac{\hat{p}^2}{2m} + mg\hat{x},$$

The Hamiltonian (1) has continuous spectrum over the whole real generalized eigenstates $|E\rangle$,

$$\hat{H}_g|E\rangle = (m+E)|E\rangle.$$

We readily evaluate $|E\rangle$ in the momentum representation,

$$\langle p|E\rangle = \frac{1}{\sqrt{2\pi mg}} e^{-\frac{i}{mg}(Ep - \frac{p^3}{6m})}.$$

The generalized eigenstates (3) are normalized so that $\langle E|E'\rangle = \delta(E-E')$.

We define the propagator

$$G_t^{(g)}(x,x') := \langle x|e^{-i\hat{H}_g t}|x'\rangle = \int dE e^{-i(m+E)t}\langle x|E\rangle\langle E|x'\rangle.$$

Using (3) we find

$$G_t^{(g)}(x,x') = e^{-imgtx - \frac{img^2t^3}{6}} G_t^{(0)}(x + \frac{1}{2}gt^2, x'),$$

where

$$G_t^{(0)}(x,x') = \sqrt{\frac{m}{2\pi it}} e^{i\frac{m(x-x')^2}{2t}} e^{-imt}$$

is the propagator of a free particle.

Let $\psi_0(x)$ be the initial state of the system, and $\psi_t^{(g)}(x)$ the state at time t evolved with the Hamiltonian $\hat{H}_g$. Eq. (5) implies that

$$\psi_t^{(g)}(x) = e^{-imgtx - \frac{img^2t^3}{6}} \psi_t^{(0)}(x + \frac{1}{2}gt^2). \tag{7}$$

Eq. (7) can be written equivalently as

$$|\psi_t^{(g)}\rangle = e^{i\frac{mg^2t^3}{3}} \hat{V}(-mgt, -\frac{1}{2}gt^2)|\psi_t^{(0)}\rangle, \tag{8}$$

where $\hat{V}(a,b) = e^{ia\hat{x} - ib\hat{p}}$ is the Weyl translation operator.

2.2. Position measurements

EPQS Version A [Einstein]

Eq. (7) implies that

$$|\psi_t^{(g)}(x)|^2 = |\psi_t^{(0)}(x + \frac{1}{2}gt^2)|^2, \tag{9}$$

i.e., that the probability distribution of position is the same as that of a free particle with the same initial state, but with a time-dependent shift of the center. The shift *does not depend on the particle's mass.*

- Applies to particles prepared in any initial state, not just those prepared in a state with a direct classical analogue.
- In particular, Eq. (9) applies also to cat states, i.e., superpositions of macroscopically distinct configurations.
- Valid for composite particles. It remains unaffected by the coupling between internal and translational degrees of freedom that is induced by free-fall.
- Thus, quantum tests of the EP could be used to constrain / discern different models of gravitational decoherence.

EPQS Version B [Galileo] **in terms of Velocity Wigner function**

2.5. Alternative formulation of the equivalence principle

We can provide a better formulation of the equivalence principle by examining free fall in the Wigner picture. For any density matrix $\hat{\rho}$, we define the Wigner function

$$W(x,p) = \int \frac{d\xi}{2\pi} e^{-ip\xi} \langle x + \frac{1}{2}\xi|\hat{\rho}|x - \frac{1}{2}\xi\rangle \tag{14}$$

as a quasi-probability density on the classical phase space Γ spanned by position x and momentum p .

By Eq. (8), we readily evaluate the Wigner function $W_t^{(g)}(x,p)$ at time t for a particle of mass m freely falling in a gravitational field g,

$$W_t^{(g)}(x,p) = W_0(x + \frac{p}{m}t + \frac{1}{2}gt^2, p + mgt), \tag{15}$$

where W_0 is the Wigner function at time $t = 0$. In this case, the time evolution of the Wigner function is identical to the time evolution of a classical probability distribution according to the Liouville equation.

It is evident that the mass-dependence in the Wigner function originates from its momentum dependence. If we change variables to (x,v) where $v = p/m$ is the velocity, the mass-dependence disappears. To this end we define the velocity Wigner function

EPQS Version B [Galileo]

$$\bar{W}(x, v) = \frac{1}{m} W(x, mv), \qquad (16) \Rightarrow$$

$$\bar{W}_t^{(g)}(x, v) = \bar{W}_0(x + vt + \frac{1}{2}gt^2, v + gt). \tag{17}$$

Two particles of different masses m_1 and m_2, but with the same velocity Wigner function *behave exactly the same* in free fall. Equality of the velocity Wigner function implies that the state vectors $|\psi_1\rangle$ and $|\psi_2\rangle$ of the particles satisfy

$$\langle p/m_1|\psi_1\rangle = \langle p/m_2|\psi_2\rangle, \tag{18}$$

i.e., their wave functions in the 'velocity basis' coincide. Thus, we are led to an equivalent but stronger statement of the equivalence principle for quantum systems:

Equivalence principle for quantum system, Version B:
Any two particles with the same velocity wave-function behave identically in free fall, irrespective of their masses.

Why Velocity Wigner Function?

In general, there is no correspondence between Hilbert spaces that correspond to particles of different mass. The reason is that particles of different mass correspond to unitarily inequivalent representations of the Galileo group (or the Poincaré group), and thus, there is no natural identification of observables defined on the different Hilbert spaces.

Given the restriction above, the choice of the velocity basis provides the most natural way of identifying states for particles of different mass, as long as relativity is fully taken into account. The reason is that the defining representation for a relativistic particle of mass m consists of square integrable functions on the positive-energy cone $V_m^+ = \{p_\mu, p_\mu p^\mu = m^2, p_0 > 0\}$. Since $V_m^+ \cap V_{m'}^+ = \emptyset$ for $m \neq m'$, the only natural way of comparing the quantum states of particles with different mass is using wave functions defined with respect to the four-velocity $v^\mu = p^\mu/m$, since the four-velocity is a unit four-vector for all masses.

III. Composite particle: An atom in Free Fall

- **Effect of internal dof on** the translational motion: **Dephasing** in the position basis

- **Effect of free fall on its internal dof**:
  Gravitational **phase shift**

At the fundamental level, composite particles are described by interacting QFTs. Composite particles correspond to poles of appropriate correlation functions of the quantum fields. Consider a two-point correlation function for a quantum field with a sequence of poles labeled by the integers $n = 0, 1, 2, \ldots$. Each pole characterized by a different rest mass m_n, so that $m_0 < m_1 \leq m_2 \leq \ldots$, and by different values of spin s_n.

III. One free composite particle: Effect of internal dof on translational dof

A single composite particle for this QFT is described by the Hilbert space

$$\mathcal{H} = \oplus_n \mathcal{H}_{m_n, s_n}, \tag{19}$$

where $\mathcal{H}_{m,s}$ is the Hilbert space associated with an irreducible representation of the Poincaré group with mass m and spin s. The Hilbert space $\mathcal{H}_{m,s}$ can be written as $\mathcal{H}_0 \otimes \boldsymbol{C}^{2s+1}$ where $\mathcal{H}_0 = L^2(V_1^+)$ contains square integrable wave functions over four-velocities u^μ with $u_0 > 0$. Then,

$$\mathcal{H} = \mathcal{H}_0 \otimes \mathcal{H}_{int}, \tag{20}$$

where $\mathcal{H}_{int} = \oplus_n C^{2s_n+1}$ describes all internal (i.e., non translational) degrees of freedom of the composite particles. The Hilbert space $\mathcal{H}_{int}$ is spanned by a basis $|n\rangle$, that defines the Hamiltonian for the internal degrees of freedom, $\hat{H}_{int} = \sum_n m_n |n\rangle\langle n|$.

The Hamiltonian for a composite particle in a weak homogeneous gravitational field is given by a matrix with respect to the basis |n> of H_internal

$$\hat{H}_g = \begin{pmatrix} m_0 + \frac{\hat{p}^2}{2m_0} + m_0 g\hat{x} & 0 & 0 & \dots \\ 0 & m_1 + \frac{\hat{p}^2}{2m_1} + m_1 g\hat{x} & 0 & \dots \\ 0 & 0 & m_2 + \frac{\hat{p}^2}{2m_2} + m_2 g\hat{x} & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}. \quad (21)$$

Given Eq. (20), a general initial state $|\Psi_0\rangle$ is decomposed with respect to the basis $|n\rangle$ of $\mathcal{H}_{int}$ as

$$|\Psi_0\rangle = \sum_n c_n |\psi_{n,0}\rangle \otimes |n\rangle \quad (22)$$

where the vectors $|\psi_{n,0}\rangle \in \mathcal{H}_0$ correspond to the translational degrees of freedom.

The state $|\Psi_0\rangle$ evolves under the Hamiltonian (21) to

$$|\Psi_t^{(g)}\rangle = \sum_n c_n e^{i\frac{m_n g^2 t^3}{3}} \hat{V}(-m_n g t, -\frac{1}{2} g t^2) |\psi_{n,t}^{(0)}\rangle \otimes |n\rangle, \quad (23)$$

where $|\psi_{n,t}^{(0)}\rangle$ is the evolution of the initial state $|\psi_{n,0}\rangle$ with the free-particle Hamiltonian $\hat{H}_n = m_n + \frac{\hat{p}^2}{2m_n}$.

Consider measurements only of the translational degrees of freedom. All information about such a measurement is encoded in the reduced density matrix on H_0 that is obtained by a partial trace of the internal degrees of freedom

$$\langle x|\hat{\rho}^{(g)}_{red}(t)|x'\rangle := \sum_n \langle x,n|\Psi^{(g)}_t\rangle\langle\Psi^{(g)}_t|x',n\rangle$$
$$= \sum_n |c_n|^2 e^{-im_n gt(x-x')}\psi^{(0)}_{n,t}(x+\frac{1}{2}gt^2)\psi^{*(0)}_{n,t}(x'+\frac{1}{2}gt^2). \quad (24)$$

The probability density for position is obtained from the diagonal elements of the reduced density matrix

$$\langle x|\hat{\rho}^{(g)}_{red}(t)|x\rangle = \sum_n |c_n|^2|\psi^{(0)}_{n,t}(x+\frac{1}{2}gt^2)|^2 = \langle x+\frac{1}{2}gt^2|\hat{\rho}^{(0)}_{red}(t)|x+\frac{1}{2}gt^2\rangle \quad (25)$$

Eq. (25) manifestly verifies Version A of the EP.

QEP Version A [Einstein]

Version B [Galileo] of QEP

Regarding Version B of the EP, we have to employ the velocity density matrix for the translational degrees of freedom. This is naturally defined, because of the splitting (20) of the Hilbert space $\mathcal{H}$. $\mathcal{H}_0$ is naturally defined in the velocity basis.

It is simpler to work with the velocity Wigner function of composite particles. This is defined as follows. Let $|\Psi\rangle = \sum_n c_n |\psi_n\rangle \otimes |n\rangle$ be a general state on $\mathcal{H}$ and let $\bar{W}_n(x, v)$ be the velocity Wigner function associated with the vectors $|\psi_n\rangle$ according to Eq. (16). Then, the reduced velocity Wigner function for the translational degrees of freedom is defined as

$$\bar{W}_{red}(x, v) = \sum_n |c_n|^2 \bar{W}_n(x, v) \qquad (26)$$

We readily verify that the time evolution of $\bar{W}_{red}$ is given by Eq. (17). Thus, Version B of the equivalence principle is also satisfied when expressed in terms of $\bar{W}_{red}$.

Dephasing

3.3. Dephasing of the translational dof by the internal dof

3.3.1. The evolution of a factorized initial state Consider now the special case of a factorized initial state

$$|\Psi_0\rangle = |\psi_0\rangle \otimes \sum_n c_n |n\rangle, \tag{27}$$

i.e., a state where all vectors $|\psi_{n,0}\rangle$ in Eq. (22) coincide with $|\psi_0\rangle$.

Time evolution entangles the translational and internal degrees of freedom

$$|\Psi_t^{(g)}\rangle = \sum_n e^{i\frac{m_n g^2 t^3}{3}} \hat{V}(-m_n g t, -\frac{1}{2} g t^2) |\psi_{n,t}^{(0)}\rangle \otimes |n\rangle. \tag{28}$$

In this case, the dependence of $|\psi_{n,t}^{(0)}\rangle$ on n in Eq. (28) is not due to the initial condition, but due to the fact that the time-evolution of any state is mass-dependent, and the mass depends on n. A measure of the dependence of $|\psi_{n,t}^{(0)}\rangle$ on n is the difference δ_n in the position dispersion $\Delta x^2(t)$ between $|\psi_{n,t}^{(0)}\rangle$ and $|\psi_{0,t}^{(0)}\rangle$. By Eq. (13),

$$\delta_n^2 = \frac{t^2}{4(\Delta x)_0^2} (m_0^{-2} - m_n^{-2}). \tag{29}$$

We assume that the excitation energies

$$\omega_n = m_n - m_0, \tag{30}$$

are much smaller than the energy m_0 of the ground state. Then, Eq. (29) becomes

$$\delta_n^2 = \frac{t^2 \omega_n}{2m_0^3 (\Delta x)_0^2}. \tag{31}$$

Observe that the states $|\psi_{n,t}^{(0)}\rangle$ are almost identical if $\delta_n << (\Delta x)_0$, i.e., for times

$$t << \sqrt{m_0/\omega_n} m_0 (\Delta x)_0^2. \tag{32}$$

In this regime, all $\psi_{n,t}^{(0)}(x)$ in Eq. (28) coincide with

$$\psi_t^{(0)}(x) = \int dx' G_t^{(0)}(x, x') \psi_0(x'), \tag{33}$$

where the propagator $G^{(0)}$ is defined with respect to the mass m_0.

The state (28) still remains entangled. Therefore, the reduced density matrix for the translational degrees of freedom is a mixed state,

$$\langle x|\hat{\rho}_{red}^{(g)}(t)|x'\rangle := \Gamma_t(x - x') e^{-im_0 gt(x-x')} \psi_t^{(0)}(x + \frac{1}{2}gt^2) \psi_t^{*(0)}(x' + \frac{1}{2}gt^2), \tag{34}$$

where

$$\Gamma_t(\Delta x) = \sum_n |c_n|^2 e^{-i\omega_n tg\Delta x}. \tag{35}$$

3.3.2. Dephasing due to internal degrees of freedom Assume that the internal degrees of freedom are in a thermal state at temperature β^{-1}, whence $|c_n|^2 \sim e^{-\beta\omega_n}$. Then,

$$\Gamma_t(\Delta x) = \frac{Z(\beta + igt\Delta x)}{Z(\beta)}, \tag{36}$$

where $Z(\beta) = \sum_n e^{-\beta\omega_n}$ is the partition function for the internal degrees of freedom.

For $g\beta^{-1}t\Delta x << 1$, we expand the partition function $\log Z(\beta + \delta) = \log Z(\beta) - \langle E\rangle\delta + \frac{1}{2}\beta^{-2}C_v\delta^2$, in terms of the mean energy $\langle E\rangle$ and the heat capacity C_v, to obtain

$$|\Gamma_t(\Delta x)| \simeq e^{-\frac{1}{2}C_v(g\beta^{-1}(\Delta x)t)^2}. \tag{37}$$

Hence, time evolution typically suppresses the off-diagonal elements of the density matrix (34), i.e., superpositions of states with position localizations that differ by Δx. The relevant time scale τ is

$$\tau = \frac{\beta}{g\Delta x\sqrt{C_v}}. \tag{38}$$

There is no suppression at low temperatures, since $Z(\beta) \to 1$ as $\beta \to \infty$.

We also note that the time-scale τ depends on the internal structure of the particle, as the latter is encoded in C_v, but it does not depend on the particle mass. This is a consequence of the EP, i.e., the equality between inertial and gravitational mass.

Universal Decoherence?

Eq. (34) coincides with an analogous equation of *Pikovsky et al 2015*, where it was claimed that the suppression of interferences due to Γ_t(Δx) corres-ponds to a process of **universal decoherence**.

We disapprove this claim.

1) Not universal:

Result depends on choice of initial state

2) Not decoherence, but dephasing:

No loss of information

Non-Markovian Evolution of entangled state

- Since the Hamiltonian involves coupling between translational and internal degrees of freedom, the generic state for a composite particle is entangled.

- The evolution law Eq. (24) is non-Markovian

- Memory of the initial state can persist in time.
- Depends on specific choices of the initial condition.

IV. Effects of translational dof on the internal dof (qubit here):

Gravitational Phase Shifts

Phase Shift from Free Fall

Consider a factorized initial state (27), and assume that Eq. (32) applies. We can evaluate the reduced density matrix for the internal degrees of freedom

$$\hat{\rho}_{qub}(t) = \begin{pmatrix} |c_0|^2 & c_0 c_1^* e^{i\omega t - i\omega g^2 t^3/3}\zeta_t \\ c_0^* c_1 e^{-i\omega t + i\omega g^2 t^3/3}\zeta_t^* & |c_1|^2 \end{pmatrix}, \tag{39}$$

where

$$\zeta = \int dx |\psi_t^{(0)}(x)|^2 e^{-i\omega g t x}. \tag{40}$$

Consider a localized initial state with position spread σ_x and with vanishing mean momentum. Assume that the qubit is recorded at distance L from each source. Hence, the detection time is strongly peaked around $t_d = \sqrt{2L/g}$. If

$$b := \omega g \sqrt{2L/g}\sigma_x << 1, \tag{41}$$

then $\zeta_{t_d} \simeq 1$, and it can be ignored in Eq. (39).

Hence, the qubit density matrix has developed a phase due to the free fall,

$$\phi_g = \frac{1}{3}\omega g^2 t_d^3 = \frac{2\sqrt{2}}{3}\omega g^{1/2} L^{3/2}, \tag{42}$$

in addition to the phase ωt due to free evolution.

Gravitational Phase-Shift

Albeit of quantum origin

ϕ_g has a classical interpretation:

- half originates from gravitational red-shift
- half from special-relativistic time dilation.

To see this, consider the radial free-fall of a particle in Schwarzschild spacetime which models the gravitational field of the Earth. The proper time τ of the particle is related to the coordinate time t and the radial coordinate r by

$$d\tau^2 = (1 - \frac{2GM}{r})dt^2 - \frac{dr^2}{1 - \frac{2GM}{r}}. \tag{44}$$

We rewrite Eq. (44) as

$$d\tau = dt\sqrt{1 - \frac{2GM}{r} - \frac{v^2}{1 - \frac{2GM}{r}}}, \tag{45}$$

where $v = dr/dt$. For a weak gravitational field ($GM/r << 1$) and non-relativistic velocity ($v << 1$), we expand the square root in Eq. (45) to obtain

$$d\tau \simeq dt(1 - \frac{GM}{r} - \frac{1}{2}v^2). \tag{46}$$

Suppose we drop a body from $r = R$. For $r = R - x$ with $x << R$,

$$d\tau \simeq dt(1 - \frac{GM}{R} - gx - \frac{1}{2}v^2), \tag{47}$$

where $g = GM/R^2$ is the gravitational acceleration, approximately constant as long as $x << R$.

Let the trajectory of the falling particle be given by the function $x(t)$. Then,

$$\tau = (1 - \frac{GM}{R})t - g\int_0^t ds x(s) - \frac{1}{2}\int_0^t ds \dot{x}^2(s) \tag{48}$$

The term $g\int_0^t ds x(s)$ corresponds to gravitational redshift and the term $\frac{1}{2}\int_0^t ds\dot{x}^2(s)$ to special relativistic time dilation. For a general path $x(s)$ those two terms are different. However, for a free-falling particle, $x(t) = \frac{1}{2}gt^2$, both terms turn out to be equal to $\frac{1}{6}g^2t^3$, so that

$$\tau = (1 - \frac{GM}{R})t - \frac{1}{3}g^2t^3. \tag{49}$$

Thus, the phase shift for a qubit of frequency ω in the rest frame is

$$\omega\tau = \omega(1 - \frac{GM}{R})t - \phi_g = \omega\tau_0 - \phi_g, \tag{50}$$

where the phase shift $\phi_g = \frac{1}{3}\omega g^2 t^3$ coincides with that of Eq. (42) and τ_0 is the proper time for a static observer at $r = R$.

This phase ϕ_g is measurable, at least in principle. For ω in the microwave range, and L of the order of $100m$, ϕ_g varies between 10^{-4} and 10^{-2} radians. If ϕ_g is of order unity or smaller, the condition (41) is always satisfied since $b/\phi_g \sim \sigma_x/L << 1$. Thus, we predict a rotation of a qubit's Bloch vector by a phase ϕ_g due to free fall.

We emphasize that the mass independence of the phase shift ϕ_g is a direct consequence of the mass independence of free-falling trajectories, i.e., of the classical EP.

Summary I. Equivalence Principle for quantum systems: 2 statements

A: The probability distribution of position for a free-falling particle is the same as the probability distribution of a free particle, modulo a *mass-independent* shift of its mean.

B: Any two particles with the same velocity wave-function behave identically in free fall, irrespective of their masses.

Summary 2: Coupling between internal dof & translational dof

Free fall induces a coupling between the internal and translational degrees of freedom.

- **It depends on the initial state of the system** and on the observable that is being measured.
- For a particular class of initial states, we show that the internal degrees of freedom can lead to a suppression of the off-diagonal terms of the density matrix in the position basis: **Dephasing of position**
- This phenomenon is **not universal** and that **it is not decoherence**, because it does not involve irreversible loss of information.

Summary 3: Effect of free fall on the internal dof

- We found a **gravitational phase shift** in the reduced density matrix of the internal degrees of freedom.

- While this phase shift is a fully quantum effect, it has a natural classical interpretation in terms of **gravitational red-shift and special relativistic time-dilation**.

Thank You.

Happy Holidays !