Atom-Field-Medium Interaction

via Graded Influence Actions

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ongoing work with

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-- Based on two papers in preparation and two earlier papers:

Ryan Behunin and B. L. Hu, *Nonequilibrium Forces between two Neutral Atoms Mediated by a Quantum Field* Phys. Rev. A 82, 022507 (2010) *Nonequilibrium Atom-Dielectric Forces Mediated by a Quantum Field* Phys. Rev. A 84, 012902 (2011)

(All the pretty slides, esp those with long equations, are the artwork of Dr. Hsiang)

Atom-Field-Medium Interaction:

A unified theoretical framework for fluctuation forces, quantum friction, and quantum optomechanics

We present a unified theoretical framework for studying the interaction between an atom or a mirror with a dielectric medium via a quantum field [1]. The range of problems covered includes a) atom-quantum field interaction as in cavity QED, b) the mirror–quantum field interactions as in the Casimir and dynamical Casimir effects, c) the atom-medium forces, such as the Casimir-Polder force, and for moving atoms in the vacuum, the Unruh-Davies effect [2] and near a dielectric plane, the quantum friction forces. We construct a model of the medium with a harmonic lattice, and allow the atom/mirror to have both external and internal dynamical degrees of freedom (dof) [3]. The atom/mirror and dielectric slabs in a cavity field are the essential components in a typical setup in optomechanics. A problem studied recently with this model is that of atom-field entanglement in quantum optomechanics [4]. Here we use the influence functional formalism of quantum open systems to handle the interplay of these five dynamic variables. We perform graded coarse-grainings [5,6] to first obtain the effects of the dielectric on the quantum field and then the effects of the modified quantum field on the atom or mirror's internal degree of freedom. We show the advantages of a) using the Green function obtained from coarse-graining the dielectric dof *over* the existent macroscopic and stochastic electrodynamics approaches and b) deriving the equations of motion for the atom/mirror from the influence actions, for fully nonequilibrium conditions, *over* the linear response theory approaches valid only for near-equilibrium conditions. In this talk we describe the key procedures in our framework and report on the results from the first two papers where we treat a static atom in three aspects: spontaneous emission, spatial decoherence and atom-field entanglement. Our next stage of work will treat a moving atom first in a prescribed trajectory for motional decoherence [7] and entanglement, and then include back-action effects [8] for the treatment of quantum friction. Memory (nonMarkovian) effects are naturally included in all of these processes because this approach guarantees self-consistency in the dynamics of all relevant physical variables.

[1] J. T. Hsiang and B. L Hu, Atom-Field-Medium Interaction I. Graded Influence Action, Spontaneous Emission and Spatial Decoherence. II. Atom-Field and Field-Medium Entanglement (in preparation)

[2] B. L. Hu, A. Roura and S. Shresta, Vacuum Fluctuations and Moving Atoms/Detectors: From Casimir-Polder to Unruh -Davies-DeWitt-Fulling Effect, Journal of Optics B – Quantum Semiclass. Opt. **6** (2004) S698-S705

[3] Chad R. Galley, Ryan Behunin and B. L. Hu, Oscillator-Field Models of Moving Mirrors in Quantum Optomechanics, Phys. Rev. A 87, 043832 (2013)

[4] Kanupriya Sinha, S. Y. Lin and B. L. Hu, Mirror-Field Entanglement in a Microscopic model for Quantum Optomechanics, Phys. Rev. A 92, 023852 (2015)

[5] Ryan Behunin and B. L. Hu, Nonequilibrium Forces between two Neutral Atoms Mediated by a Quantum Field Phys. Rev. A 82, 022507 (2010)

[6] Ryan Orson Behunin, Bei-Lok Hu, Nonequilibrium Atom-Dielectric Forces Mediated by a Quantum Field Phys. Rev. A 84, 012902 (2011)

[7] S. Shresta and B. L. Hu, Moving Atom-Field Interaction: Quantum Motional Decoherence and Relaxation, Phys. Rev. A **68** 012110 (2003)

[8] E.g., S. Shresta, B. L. Hu and N. G. Phillips, Moving Atom-Field Interaction: Correction to Casimir-Polder Effect from Coherent Backreaction, Phys. Rev. A **68**, 062101 (2003)

Coherence and Fluctuations in the Interaction between Moving Atoms and a Quantum Field *

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[quant-ph/9710061] in *Macroscopic Quantum Coherence*,
edited by J. Swain and A. Widom (World Scientific, Singapore, 1998)

Based on work with J. Anglin, PRD53, 7003 (1996), and with D. Koks, PRD55, 4795 (1997)

ABSTRACT

Mesoscopic physics deals with three fundamental issues: quantum coherence, fluctuations and correlations. Here we analyze these issues for atom optics ,using a simplified model of an assembly of atoms (or detectors, which are particles with some internal degree of freedom) moving in arbitrary trajectories in a quantum field. Employing the influence functional formalism, we study the self-consistent effect of the field on the atoms, and their mutual interactions via coupling to the field. We derive the coupled Langevin equations for the atom assemblage and analyze the relation of dissipative dynamics of the atoms (detectors) with the correlation and fluctuations of the quantum field. This provides a useful theoretical framework for analyzing the coherent properties of atom-field systems.

Goals & Scope

- A unified theoretical framework to treat moving atoms (or mirror) interacting with a medium (dielectric, conductor, mirror) via a quantum field *self-consistently*.

This includes:

- **Atom-Field Interaction**: Quantum Optics, Atom Optics (50- 90s) *Old topics* like spontaneous emission, cooling, *New issues* (95-15): Q decoherence, Entanglement ← Q Information

- **Atom-Mirror / Dielectric**: Casimir-Polder effects

Atom / Mirror – Field interaction

- Atom-Field interaction: Casimir Polder Effect

For a stationary atom with polarizability α at a distance *R* from a conductor / dielectric

The potential energy is:

$$U_{sa}(R) \to -\frac{\alpha_o \hbar \omega_0}{8} \frac{1}{R^3} \quad \text{for} \quad R \ll \frac{c}{\omega_0},$$

Attractive force:

$$U_{sa}(R) \to -\frac{3\alpha_o \hbar c}{8\pi} \frac{1}{R^4} \quad \text{for} \quad R \gg \frac{c}{\omega_0},$$

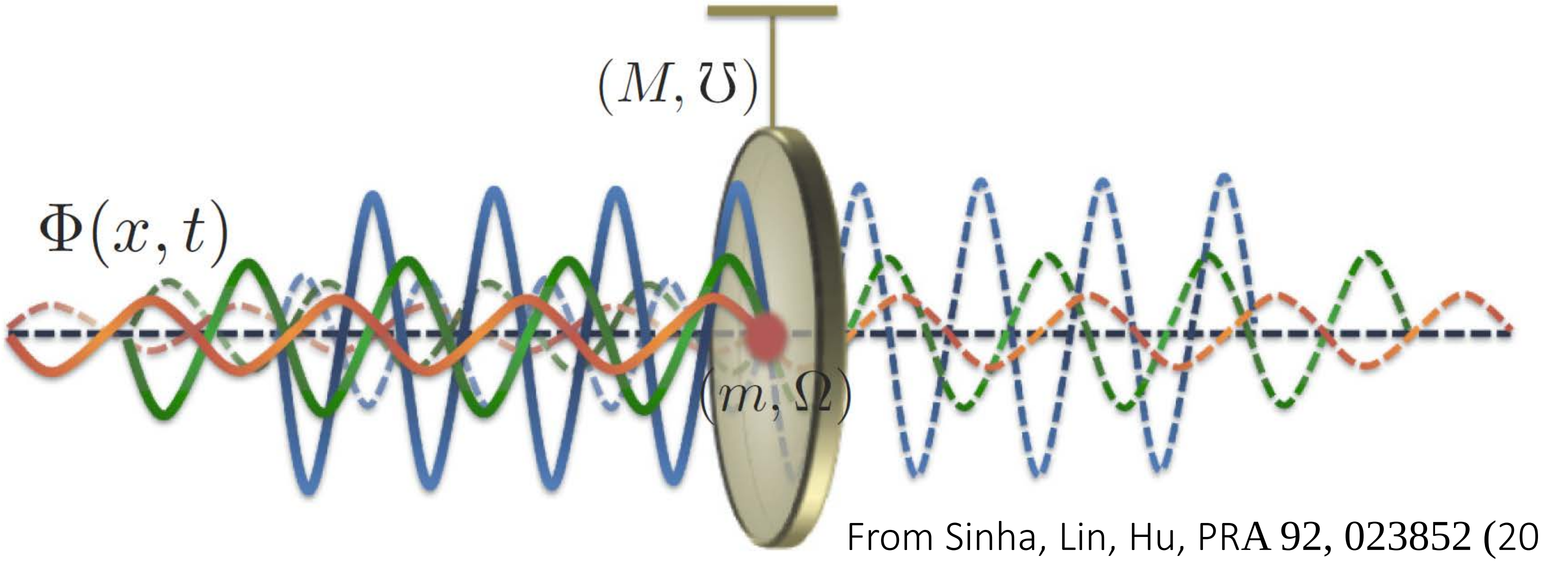
[From Shresta, Hu, Phillips, PRA **68**, 062101 (2003)]

- Mirror-Field Interaction: vacuum fluctuations

Casimir Effect: Boundary changes the field configurations

Dynamical Casimir: Parametric amplification of v fluctuations

- **Quantum Optomechanics**: Moving atoms or moving mirrors of all sorts: atom mirror etc.



From Sinha, Lin, Hu, PRA 92, 023852 (2015)

FIG. 1. (Color online) Schematic representation of the interaction of a mirror with a field via its internal degree of freedom.

- **Q. Friction:** Atom moving close to a dielectric feels at one point a reactive-dissipative force which depends on its entire past trajectory: back-action, strongly non-Markovian

Traditional Approaches

in atomic physics and quantum optics:

- Time-dependent Perturbation Theory

  e.g., in spontaneous emission: weak coupling to field, only short time behavior

- Born-Markov approximation

  e.g, use of Lindblad master equation. No memory

- Rotating Wave approximation

  Consistency conditions and domain of Applicability

  Chris Fleming, Nicholas Cummings, C. Anastopoulos and B.L. Hu, J. Phys. A: Math. Theor. 43 (2010) 405304. [arXiv:1003.1749]

- Neglecting A^2 terms (will not see Casimir-Polder effect)

What’s new?

Our approach can offer what traditional methods (e.g., assuming weak coupling, Markovian) cannot:

- Strong field, strong coupling
- Fully nonequilibrium dynamics
- Low temperature
- Non-Markovian processes
- Back-action included, self-consistent treatment

• A clear pathway. Just have to work a bit harder.

Better than other nonperturbative approaches, e.g., **CP Effect:** Buhmann Knöll Welsch Dung, PRA 70, 052117 (2004). **Casimir Physics:** Dalvit, Milonni, Roberts, da Rosa, (ed) Springer (2011)

1. Atom-Field Interaction
via Influence Functional method

Quantum Brownian (QBM) Model- Open Q Systems

- Atom (system): modeled by a harmonic oscillator
- Field (Environment): NHOs
- Bilinear coupling

Quantum Open System

Closed System: **Density Matrix** $$\hat{\rho}(t) = \mathcal{J}(t, t_i)\hat{\rho}(t_i).$$

$\mathcal{J}(x, \mathbf{q}, x', \mathbf{q}', t \mid x_i, \mathbf{q}_i, x'_i, \mathbf{q}'_i, t_i)$ is the (**unitary**) **evolutionary operator** of the system from initial time t_i to time t.

OPEN SYSTEM: System (s) interacting with an Environment (e) or Bath (b): Integrate out (coarse-graining) the bath dof renders the system open. Its evolution is described by the Reduced Density Matrix

$$\rho_r(x, x') = \int_{-\infty}^{+\infty} dq \int_{-\infty}^{+\infty} dq' \rho(x, \mathbf{q}; x', \mathbf{q}')\delta(\mathbf{q} - \mathbf{q}')$$

$$\rho_r(x, x', t) = \int_{-\infty}^{+\infty} dx_i \int_{-\infty}^{+\infty} dx'_i \; \mathcal{J}_r(x, x', t \mid x_i, x'_i, t_i) \; \rho_r(x_i, x'_i, t_i \;).$$

Influence Functional

Assume **factorizable condition** between the system (s) and the bath (b) initially

$$\hat{\rho}(t = t_i) = \hat{\rho}_s(t_i) \times \hat{\rho}_b(t_i),$$

:

Evolutionary operator for the reduced density matrix is

$$\mathcal{J}_r(x_f, x'_f, t \mid x_i, x'_i, t_i) = \int_{x_i}^{x_f} Dx \int_{x'_i}^{x'_f} Dx' \; \exp\left(\frac{i}{\hbar}\left\{S[x] - S[x']\right\}\right) \; \mathcal{F}[x, x']$$

Influence Functional

$$\mathcal{F}[x, x'] = \int_{-\infty}^{+\infty} d\mathbf{q}_f \int_{-\infty}^{+\infty} d\mathbf{q}_i \int_{-\infty}^{+\infty} d\mathbf{q}'_i \int_{\mathbf{q}_i}^{\mathbf{q}_f} D\mathbf{q} \int_{\mathbf{q}'_i}^{\mathbf{q}_f} D\mathbf{q}'$$

$$\exp\left(\frac{i}{\hbar}\left\{S_b[\mathbf{q}] + S_{\text{int}}[x, \mathbf{q}] - S_b[\mathbf{q}'] - S_{\text{int}}[x', \mathbf{q}']\right\}\right) \times \rho_b(\mathbf{q}_i, \mathbf{q}'_i, t_i)$$

Influence Action

$$= \exp\left(\frac{i}{\hbar}\delta\mathcal{A}[x, x']\right)$$

Quantum Brownian Motion via the Influence Functional / closed-time path integral Methods

Feynman Vernon 1963,
Caldeira & Leggett 1983,
Hu, Paz & Zhang 1992,
….

10 slides courtesy Prof. Hing-Tong Cho

System (1HO) interacting *bilinearly* with an Environment (NHO): All Gaussian

$$
\begin{aligned}
S[x] &= \int_0^t ds \left[\frac{1}{2}M\dot{x}^2 - V(x)\right] \\
S_e[q_n] &= \int_0^t ds \sum_n \left[\frac{1}{2}m_n\dot{q}_n^2 - \frac{1}{2}m_n\omega_n^2 q_n^2\right] \\
S_{int}[x,\{q_n\}] &= \int_0^t ds \sum_n (-C_n x\, q_n)
\end{aligned}
$$

Closed-Time-Path /Schwinger-Keldysh/ in-in Effective Action

[Schwinger 61, Keldysh 63, Chou, Hao, Su, Yu 1981, Calzetta Hu 1987, …]

$$
\begin{aligned}
e^{i\Gamma[x_+,x_-]} &= e^{iS[x_+]-iS[x_-]} \times \\
&\quad \int_{CTP} \prod_n Dq_{n+} Dq_{n-} \left(e^{iS_e[\{q_{n+}\}]-iS_e[\{q_{n-}\}]} \right. \\
&\qquad \left. e^{iS_{int}[x_+,\{q_{n+}\}]-iS_{int}[x_-,\{q_{n-}\}]} \right) \\
&= e^{iS[x_+]-iS[x_-]+iS_{IF}[x_+,x_-]}
\end{aligned}
$$

$$
\begin{aligned}
S_{IF}[x_+, x_-] = & \sum_n \frac{1}{2} \int ds\, ds' \\
& \left[x_+(s) G_{n++}(s,s') x_+(s') - x_+(s) G_{n+-}(s,s') x_-(s') \right. \\
& \left. -x_-(s) G_{n-+}(s,s') x_+(s') + x_-(s) G_{n--}(s,s') x_-(s') \right]
\end{aligned}
$$

where Gn are the Schwinger-Keldysh or closed time path (+, -) propagators:

$$
\begin{aligned}
G_{n++}(s,s') &= -\eta_n(s-s')\,\mathrm{sgn}(s-s') + i\nu_n(s-s') \\
G_{n+-}(s,s') &= \eta_n(s-s') + i\nu_n(s-s') \\
G_{n-+}(s,s') &= -\eta_n(s-s') + i\nu_n(s-s') \\
G_{n--}(s,s') &= \eta_n(s-s')\,\mathrm{sgn}(s-s') + i\nu_n(s-s')
\end{aligned}
$$

The influence action S_{IF} can be written as

$$
\begin{aligned}
e^{iS_{IF}} &= e^{-i\int_0^t ds \int_0^s ds' [\Delta x(s) \eta(s-s') \Sigma x(s')]} \\
& \quad e^{-\frac{1}{2}\int_0^t ds \int_0^t ds' \, [\Delta x(s) \nu(s-s') \Delta x(s')]}
\end{aligned}
$$

where $\Delta x(s) = x_+(s) - x_-(s)$ and $\Sigma x(s) = x_+(s) + x_-(s)$, and

$$
\begin{aligned}
\eta(s-s') &= \sum_n \eta_n(s-s') = -\sum_n \frac{C_n^2}{2m_n\omega_n} \sin\omega_n(s-s') \\
\nu(s-s') &= \sum_n \nu_n(s-s') = \sum_n \frac{C_n^2}{2m_n\omega_n} \cos\omega_n(s-s')
\end{aligned}
$$

Spectral density $\mathcal{J}(\omega)$ (later)

Feynman-Vernon Gaussian Integral Identity, Noise Kernel

Rewriting the imaginary part of S_{IF} as

$$
\begin{aligned}
& e^{-\frac{1}{2}\int \Delta x \nu \Delta x} \\
= \quad & N \int D\xi e^{-\frac{1}{2}\int \xi \nu^{-1} \xi} e^{-\frac{1}{2}\int \Delta x \nu \Delta x} \\
= \quad & N \int D\xi e^{-\frac{1}{2}\int (\xi - i\nu\Delta x)\nu^{-1}(\xi - i\nu \Delta x)} e^{-\frac{1}{2}\int \Delta x \nu \Delta x} \\
= \quad & N \int D\xi P[\xi] e^{i\int \xi \Delta x}
\end{aligned}
$$

where $P[\xi] = e^{-\frac{1}{2}\int \xi \nu^{-1} \xi}$ is the Gaussian probability density of the stochastic force ξ.

Due to this probability density one has the stochastic average $\langle \xi(s)\xi(s')\rangle_s = \nu(s-s')$ which is called the noise kernel.

Equation of Motion from the influence action

After this procedure the effective action

$$\begin{aligned}\Gamma[x_+, x_-] &= S[x_+] - S[x_-] \\ &\quad - \int_0^t ds \int_0^s ds' \Delta x(s) \eta(s-s') \Sigma x(s') \\ &\quad + \int_0^t ds \Delta x(s) \xi(s)\end{aligned}$$

The equation of motion for the particle is then given by

$$\left.\frac{\delta \Gamma[x_+, x_-]}{\delta x_+}\right|_{x_+ = x_- = x} = 0$$

Langevin Equation
Dissipation kernel

The equation of motion is a Langevin equation with the stochastic force $\xi(t)$,

$$M\ddot{x} + V'(x) + \int_0^t ds\, \eta(t-s)x(s) = \xi(t)$$

The integral term is related to dissipation as one can write

$$\eta(t) = \frac{d}{dt}\gamma(t) \Rightarrow \gamma(t) = \sum_n \frac{C_n^2}{2m_n\omega_n^2}\; cos\omega_n t$$

and we have

$$M\ddot{x} + V'(x) + \int_0^t ds\, \gamma(t-s)\dot{x}(s) = \xi(t)$$

$\eta(s-s')$ is called the dissipation kernel.

Fluctuation-Dissipation Relation

$$\nu(s) = \int_{-\infty}^{\infty} ds' \, K(s-s')\gamma(s')$$

in this simple case

$$K(s) = \int_0^{\infty} \frac{d\omega}{\pi} \omega \, \cos \omega s$$

Note the existence of FDR is a condition of self-consistency between the system dynamics with backaction from the environment.
This relation originates from the unitarity In the original closed system.

The coarse-grained environmental variables are now represented by noise and fluctuations. Their backreaction on the system imparts to it dissipative dynamics in the now opened system.

η for dissipation kernel in HPZ92

$$\mathcal{F}_n^{(1)}[x,x'] = \exp\Big\{ -\frac{2i}{\hbar}\int_0^t ds_1 \int_0^{s_1} ds_2\, [x(s_1)-x'(s_1)]\, \mu_n(s_1-s_2)\, [x(s_2)+x'(s_2)]$$
$$- \frac{1}{\hbar}\int_0^t ds_1 \int_0^{s_1} ds_2\, [x(s_1)-x'(s_1)]\, \nu_n(s_1-s_2)\, [x(s_2)-x'(s_2)]\Big\}$$

Summing up the modes, get:

(Noise kernel)

$$\nu(s) = \sum_n \nu_n(s) = \int \frac{d\omega}{\pi} I(\omega) \coth\tfrac{1}{2}\beta\hbar\omega \cos\omega s \qquad s = s_1 - s_2$$

(Dissipation kernel)

$$\mu(s) = \sum_n \mu_n(s) = \frac{d}{ds}\gamma(s), \quad \gamma(s) = \int \frac{d\omega}{\pi} I(\omega)\frac{1}{\omega}\cos\omega s$$

(damping kernel)

Spectral density function $I(\omega) = \rho_D(\omega)\cdot \dfrac{\pi c^2(\omega)}{2m\omega}$

density of state $\rho_D(\omega)$

simplest case: $= \sum \delta(\omega-\omega_n)$

▹ Different environments with the same spectral density function will have the same influence on the Brownian particle

A common form for $I(\omega) \sim \omega^\alpha e^{-\omega/\Gamma}$ — Γ cutoff frequency

$\alpha = 1$ ohmic — QBM at high temp. → $\delta(s)$ white noise

$\alpha > 1$ superohmic

$0 < \alpha < 1$ subohmic

$$\boxed{\begin{aligned}\nu(s) &= \frac{2kT}{\hbar}\gamma(s)\\ \gamma(s) &= 2m\gamma_0\,\delta(s)\end{aligned}}$$ (Markov)

Ohmic with high T ⇒ local dissipation

2. Field-Medium Interaction

Classical electrodynamics: Treats medium as a dielectric <- polarization tensor / susceptibility fcn in response theory.

QED applied to dissipative medium: How to: 1) **preserve the canonical relation** in field quantization 2) explain **how dissipation enters** and 3) identify the **source of noise** from microscopic physics.

2 Popular approaches : A. M**acroscopic Electrodynamics:** **D**iagonalization of the full field + medium. Huttner & Barnett 1992. Review by Scheel and Buhmann (2008); Eberline & Zietal (2012)
B. **Stochastic Electrodynamics** (Lifshitz): Pitaevsky, Azetta – see comments in Behunin & Hu, PRA (2011, 2012) for Casimir-Polder etc

We treat this problem by an **open system approach**.

Review: Models for Mirror

- For a **perfectly reflecting mirror** one can impose certain boundary conditions on the field. E.g., Dirichlet / Neumann or combination (Robin) boundary condition.
- But it misses **physical effects** like evanescent waves, which could be important.
- Also specifying an exact value of the field on the boundary tends to introduce divergent behavior on the boundary due to the Heisenberg uncertainty principle.

Renormalized energy density still diverges on the boundary of a perfect conductor because the conjugated variable of the field diverges*.

- The divergent behavior can be softened by
    - mode-dependent boundary conditions (divergence doesn't grow as fast),

* Sopova & Ford, PRD 66, 045026 (2002); PRD 72, 033001 (2005)

    - blurring the location of the boundary, e.g.
        - the boundary does not ideally rest at a fixed position* (finite).

Ford & Svaiter, PRD 58, 065007 (1998)

The boundary condition method is an idealization of (and thus contains approximations to) the overall effects of the material that make up the mirrors.

MICRO SCALE

- The micro-mirror in quantum optomechanics used for the study of macro quantum phenomena cries out for a more sophistical description than imposition of boundary conditions.
- the shorter-wavelength modes of the field can pick up possible irregularity on the surface or in the medium, which makes a simple boundary condition insufficient.

How to account for the interactions of the field with the material constituents of mirror: Let the mirror possess some internal degrees of freedom (IDF).

This is the motivation for the construction of the so called "Mirror Oscillator Field (MOF)" model: Galley, Behunin & Hu, PRA 87, 043832 (2013)

MATTER-FIELD INTERACTION :

- We mention a few popular approaches: e.g., Macro QED / Stochastic ED (Lifshitz):

  E.g., Review: Scheel & Buhmann, acta phys. slov. 58, 675 (2008)

  - Macroscopic Maxwell's eqs. with (micro?) bosonic sources
    - the sources are fixed by the requirement that the equal-time commutation relations of the quantized EM fields should be preserved,
    - limited to strictly local sources, physical meaning of the sources and the corresponding quantum states not clear.
  - Phenomenological, polarization field + EM field → polariton
  - Susceptibility function posited is required to satisfy several general properties: Kramer-Kronig relation, reality condition and reciprocal theorem. Input from experiments,
  - Applicable mainly to near-equilibrium or steady state conditions

An earlier approach we can accept: Huttner & Bartlett, PRA 46, 4306 (1992)

*

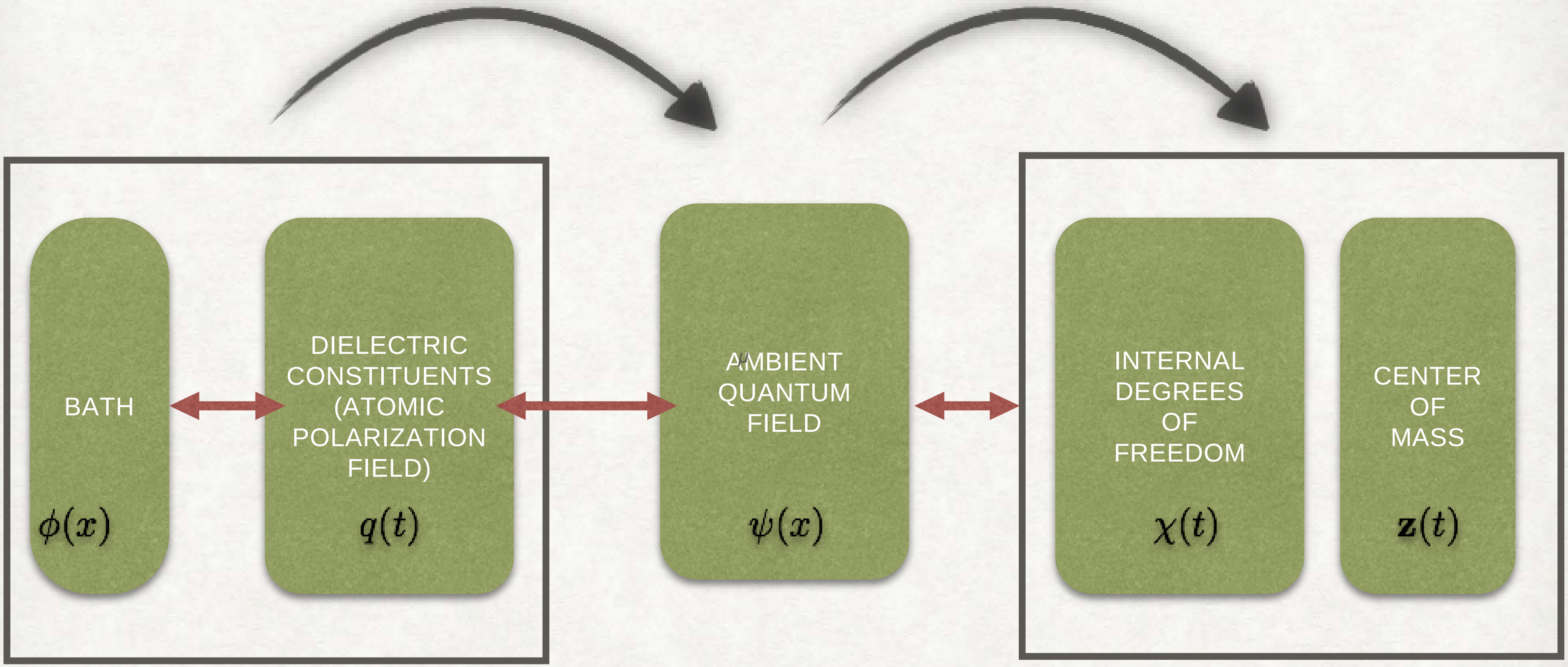
Diagonalization of the Hamiltonian of the atom+ field + medium closed system. (This is like the projection operator formalism: Formally complete and pretty. But you have to buy it wholesale. Unless all parts couple with bilinear coupling, difficult to get specific relevant subsystem's dynamics.)

But the **quantum state of the diagonalized closed system** is not that of the atom or field or medium,

- in the **nonequilibrium cases**, the dynamics of the atom or field or medium has the added difficulty of a well-defined vacuum in a time-dependent background –
- similar to the difficulty for QFT in time-dependent curved space, e.g., particle creation in the early universe

In treating the most general cases, some approximation need be introduced (e.g., weak coupling, coarse-graining) to select out the sub-system of interest from the other variables and also include their effects.

3. HIERARCHY OF INFLUENCES



X medium *P*   *Quantum Field* *A*   *Q* atom *z*

$$e\int d^4x\ \rho(\mathbf{x})\dot{q}(t)\psi(x) \qquad g\int d^4x\ \dot{\chi}(t)\delta^{(3)}[\mathbf{x}-\mathbf{z}(t)]\psi(x)$$

: coupling

: influence

(Symbols in red are used in Behunin Hu, PRA2001)

THE MODEL

CONFIGURATION

* Behunin & Hu, PRA 84, 012902 (2011)

One or more 1) neutral polarizable atom in 2) an ambient quantum field outside 3) a neutral dielectric medium*: (symbols in red are used in B&H01)

1) **Atom**'s **Internal degree of freedom (IDF) χ of the atom** is model by a quantum oscillator: Dipole moment. ***Q***

Atom's **CoM coordinate z**. The movement of the atom as a whole can be either **prescribed** or **dynamically determined**.

[The role of the atom in our model can also describe an imperfect mirror (the MOF model of Galley Behunin Hu 2013) or **any system with IDF**.]

2) The scalar **field** ψ mocks the EM field – 2 polarizations (vector pot **A_μ**)

3) The **dielectric material** is a collection of harmonic atoms, fixed at the lattice sites, with their private baths (Hopfield) or a shared bath, yields **Polarization.** **P** (variable frequencies depicts dynamical Casimir effect.)

5 players, 2 coarse-grained parties

- Dielectric-induced modified quantum field,
  - provides a more satisfactory description of the interplay between the quantum field and the (quantum) dielectric material,
- Atom/mirror interacting with this modified field
  - Influence of the quantum fluctuations of the modified field on the IDF of the atom/mirror : spontaneous emission, spatial decoherence
  - Atom-field entanglement: Enhanced when idf included (Sinha Lin Hu PRA 2015)
  - atom-atom (mirror-mirror) entanglement
- Moving Mirror:
  - Mirror cooling (heating), / Dynamical Casimir Effect.
- Moving atoms:
  * Inertial motion: Motional decoherence, Dynamical CP effect, Q Friction: dielectric
  - non-inertial motional: Acceleration radiation from moving-atom or mirror; **Unruh effect**, Analogue models of **Hawking radiation** from black holes.

TABLE I. Summary of Coarse-graining and detailed physics remaining at each tier.

Tier	Coarse-graining	Influence functional	Detailed physics	Remaining variables
0			All parties	$\vec{z}, \vec{Q}, A^{\mu}, \vec{P}, \vec{X}_{\nu}$
1	Tr_X	$\mathcal{F}_X$	Atom + field + matter	$\vec{z}, \vec{Q}, A^{\mu}, \vec{P}$
2	Tr_P	$\mathcal{F}_M$	Atom + field	$\vec{z}, \vec{Q}, A^{\mu}$
3	Tr_A	$\mathcal{F}_{\xi}$	Atom	$\vec{z}, \vec{Q}$
4	Tr_Q	$\mathcal{F}_Z$	Atom's motion	$\vec{z}$

ACTION: DIELECTRIC-MODIFIED FIELD

- For dielectric-modified field

$$S = \frac{1}{2}\int d^4x\, \partial_\mu\psi\partial^\mu\psi + e\int d^4x\, \rho(\mathbf{x})\dot{q}(t)\,\psi(x) + S_q + \lambda\int d^4x\, \rho(\mathbf{x})q(t)\,\phi(x) + \frac{1}{2}\int d^4x\, \partial_\mu\phi\partial^\mu\phi$$

- ψ is the ambient scalar field, e is the coupling constant

  between the field and the IDF ***q* of the dielectric constituents**,

- ρ is the **density distribution** of the constituents,

- ϕ is the bath attached to the constituents.

- **the inclusion of the bath** naturally provides the damping mechanism for the constituents of the dielectric, so this model can account for the linear dispersive and dissipative dielectric and simple conductors,
- the **private bath** setup is easier to implement, as Hopfield did, but it is not as intuitive as the shared bath,
- the **shared bath** will induce **non-Markovian effects** between the constituents of the dielectric, so discretion is advised when assuming an infinite extension of the dielectric. Relaxation takes longer time or stays incomplete.
- the coupling of the **ambient scalar field** mimics the coupling of the EM field, albeit polarization effects are ignored.

  (There is a difference between minimal qϕ vs \dot q ϕ derivative coupling as in EM: P\cdot E or dP/dt ϕ; radiation reaction)

Reduced Density Matrix

- Use the **influence functional formalism** to find the evolution of each subsystem, given by the reduced density matrix.

  integrating over the (private) bath -> the EoM of the constituents,

  $$m\,\ddot{q}(t) + 2m\gamma\,\dot{q}(t) + m\Omega_R^2\,q(t) = \xi^{(\phi)}(\mathbf{a}_i, t)\,,$$

  - damping + additional noise from the bath: they are not arbitrarily assigned but are derived self-consistently.

    $\mathbf{a}_i$ specifies the location of the lattice sites,

  - they behave like a bunch of **driven damped oscillators**,

  - consistent with the results from **the Lorentz model** in classical electrodynamics.

Damping

Integrating over the IDF of the dielectric, get EoM of the modified scalar field,

$$\partial_\mu \partial^\mu \psi(x) + e \frac{d}{dt} \int d^4x' \; G_R^{(P)}(x,x') \Big[e\,\dot{\psi}(x') - \xi^{(\phi)}(x') \Big] = e\,\dot{\xi}^{(P)}(x)\,,$$

yields a wave equation as in the vacuum but with damping (*GR* Retarded Green function) and noise:

- **Damping**: on wave propagation; associated with the **dissipation kernel** of the medium polarization (*P*),

$$G_R^{(P)}(x,x') = \sum_{i,j} \delta_{ij} \delta^{(3)}(\mathbf{x} - \mathbf{a}_i)\, g_R^{(P)}(t-t')\, \delta^{(3)}(\mathbf{x}' - \mathbf{a}_j) \qquad \overline{g}_R^{(P)}(\kappa) = m^{-1} \Big[-\kappa^2 + \Omega_R^2 - i\,2\gamma\,\kappa \Big]^{-1}$$

- Writing the wave equation in the frequency domain:

$$\nabla^2 \overline{\psi}(\mathbf{x};\omega) + \omega^2 \Big[1 + \chi_e(\mathbf{x};\omega) \Big] \overline{\psi}(\mathbf{x};\omega) = i\,\omega e^{-1}\, \chi_e(\mathbf{x};\omega)\, \overline{\xi}^{(\phi)}(\mathbf{x};\omega) + i\,\omega e\, \overline{\xi}^{(P)}(\mathbf{x};\omega)\,,$$

we obtain the **susceptibility function**:

$$\chi_e(\mathbf{x};\omega) = e^2 \varrho(\mathbf{x}) \overline{g}_R^{(P)}(\omega) = \frac{e^2 \varrho(\mathbf{x})}{m(-\omega^2 + \Omega_R^2 - i\,2\gamma\,\omega)}$$

Noise

- The right-hand side consists of the combined (intrinsic + induced) **quantum polarization noise of the dielectric**,
$$\epsilon(\mathbf{x};\omega) = 1 + \chi_e(\mathbf{x};\omega)$$
- The dielectric function of the medium is in principle spatially-inhomogeneous; if the observer is sufficiently far away from the dielectric or can only see longer-wavelength modes, we usually **average the dielectric function over the lattice unit volume** to have*
$$\chi_e(\omega) = \frac{e^2 n_0}{m(-\omega^2 + \Omega_R^2 - i\,2\gamma\,\omega)}$$
- We thus recover
$$\nabla^2 \overline{\psi}(\mathbf{x};\omega) + \omega^2 \epsilon(\omega)\,\overline{\psi}(\mathbf{x};\omega) = i\,\omega e\,\overline{\xi}_{tot}^{(P)}(\mathbf{x};\omega)$$
in the QFT + quantum dielectric context.

* consistent with:
   1) Jackson, Classical Electrodynamics 2nd Ed.
   2) Eberlein et al, PRA 86, 022111 (2012)

- This approach provides a mathematically sound and physically intuitive description of the **medium-modified field**. Using the open quantum system approach we can construct more sophisticated models for the medium including multi-frequency & nonlinearity.

ATOM IN MEDIUM-MODIFIED FIELD

- We see that **the medium affects the ambient quantum field in two ways**: 1) modifying its amplitude, and 2) introducing the **driving noise**. Consider 1) first.
- Consider an atom/mirror outside the dielectric interacting with the medium-modified scalar field, the IDF χ of the atom/mirror couples with the field by

$$g \int d^4x \; j(x) \, \psi(x)$$

  - the **current** associated with the atom: $j(x) = \varrho_a(\mathbf{x})\dot{\chi}(t)$
  - the **density** of the atom: $\varrho_a(\mathbf{x}) = \delta^{(3)}[\mathbf{x} - \mathbf{z}(t)]$
  - ***z(t)*** is the external/CoM coordinate of the atom. It can be
    - 1) a constant: the atom **stays at rest**,
    - 2) a given function of time: the atom follows a **prescribed trajectory**,
    - 3) a dynamical variable: it is **self-consistently and dynamically determined**,
  - In the latter case its Langevin equation can become nonlinear from the coupling.

- The full action takes the form

$$S[\mathbf{z},\chi,\psi] = \int dt \left\{ \frac{M}{2}\,\dot{\mathbf{z}}^2(t) - V(\mathbf{z}) + \frac{\mu}{2}\,\dot{\chi}^2(t) - \frac{\mu\omega^2}{2}\,\chi^2(t) \right\} + g\int d^4x\;\dot{\chi}(t)\,\delta^{(3)}[\mathbf{x}-\mathbf{z}(t)]\,\psi(x) + S_\psi[\psi]$$

- Even if we keep only the first order in g^2, the Eq of Motion in the corresponding order is still highly nonlinear

$$M\,\ddot{\mathbf{z}}(t) + \frac{\partial V(\mathbf{z})}{\partial \mathbf{z}(t)} = g^2 \int dt' \left\{ \overline{G}_H^{(\chi)}(t,t')\,\frac{\partial}{\partial \mathbf{z}(t)}\,G_R^{(\psi)}[\mathbf{z}(t),\mathbf{z}(t')] + \overline{G}_R^{(\chi)}(t,t')\,\frac{\partial}{\partial \mathbf{z}(t)}\,G_H^{(\psi)}[\mathbf{z}(t),\mathbf{z}(t')] \right\}$$

  - the righthand side is a vacuum fluctuation force, the origin of **quantum friction**, it is nonzero even if ***z*** is in uniform motion.
  - It is a consequence of the interaction between dipole fluctuations and field fluctuations. [see work of Behunin, Dalvit, Intravaia]
  - It can be perceived as the frictional force between the neutral atom and its delayed image. Thus highly non-Markovian

ONE ATOM IN A FIXED POSITION

HSIANG & HU, PAPERS I & II (2018)

- To illustrate how our formalism works, we‘ll work out 3 examples:
    - Spontaneous transition probability,
    - Spatial (distance-dependent) decoherence
    - Atom-field entanglement

A STATIC ATOM IN A MEDIUM-MODIFIED FIELD

- For a static atom, z (external DoF) is fixed. Action becomes

$$S[\chi,\psi] = \frac{m}{2}\int dt'\,\left[\dot{\chi}^2(t') - \omega^2\chi^2(t')\right] + \int d^4x'\; j(x')\psi(x') + S[\psi]$$

- The reduced density matrix of the internal DoF of the atom χ, upon integrating over ψ is

$$\rho_\chi(\chi_f,\chi'_f,t_f) = \int d\chi_i d\chi'_i \int d\psi_i d\psi'_i \int_{(\chi_i,\chi'_i)}^{(\chi_f,\chi'_f)} \mathcal{D}\chi_\pm \oint_{\psi_i}^{\psi'_i} \mathcal{D}\psi\; \exp\left\{ i\,S[\chi_+,\psi_+] - i\,S[\chi_-,\psi_-]\right\} \rho(\chi_i,\chi'_i;\psi_i,\psi'_i;t_i)$$

$$= \int d\chi_i d\chi'_i \int_{(\chi_i,\chi'_i)}^{(\chi_f,\chi'_f)} \mathcal{D}\chi_\pm\; \exp\left\{ i\,S_{\mathrm{CG}}[\chi_+,\chi_-]\right\} \rho_\chi(\chi_i,\chi'_i,t_{i)}$$

Knowledge of the reduced density matrix allows us to readily find:

- 1) transition probability of the atom, (to the ground state for example)

$$P_0(t_f) = \mathrm{Tr}\Big\{ |\psi_0\rangle\langle\psi_0|\,\rho_\chi(t_f) \Big\}$$

- 2) spatial decoherence: from the off-diagonal elements,
- 3) covariance matrix elements: $\langle\chi_f^2\rangle$ from which we can construct the linear entropy to determine the atom-field entanglement.

- The presence of the dielectric modifies the amplitudes of mode functions of the ambient field, thus the net effects of the ambient field on an atom/mirror outside the dielectric should be different.

- If the dielectric occupies the half-space, due to symmetry we expect that the effects of the modified field may **depend on the distance** from the atom to the vacuum-dielectric interface.

- More interestingly, the radiation field out of the atom/mirror may **bounce off the material interface** and back-reacts on the atom/mirror with a time delay => **a non-Markovian effect**.

- We will illustrate these aspects with a few simple examples.

EXAMPLE 1: STATIC ATOM: TRANSITION PROBABILITY

- A harmonic atom can be viewed as an approximation to the real atom; in particular when $\beta\omega_{tr} \geq 1$ (low T) when only the ground state (0) and first excited state (1) are populated, it is close to a two-level atom.

- Suppose the atom is initially in the first excited state, then **the reduced density matrix of the internal DoF** is

$$\rho_\chi^{(1)}(\Sigma_f^{(x)}, \Delta_f^{(x)}, t_f) = \left(\frac{m^3\Omega^3}{4\pi}\right)^{\frac{1}{2}} \left(\alpha\, \Delta_f^{(x)\,2} + 2\beta\, \Delta_f^{(x)}\Sigma_f^{(x)} + \gamma\, \Sigma_f^{(x)\,2} + \delta\right)$$
$$\times \exp\left[-a\, \Delta_f^{(x)\,2} - i\, 2b\, \Delta_f^{(x)}\Sigma_f^{(x)} - c\, \Sigma_f^{(x)\,2}\right]$$

where

$$a = \frac{1}{2\mathcal{V}_{11}^{(0)}}\left\{\mathcal{V}_{11}^{(0)}\mathcal{V}_{22}^{(0)} - \left[\mathcal{V}_{12}^{(0)}\right]^2\right\}, \qquad b = -\frac{\mathcal{V}_{12}^{(0)}}{2\mathcal{V}_{11}^{(0)}}, \qquad c = \frac{1}{2\mathcal{V}_{11}^{(0)}}$$

$$\alpha = -\frac{4m\Omega\,\dot{\mu}_i^5}{(\Omega^2 + \dot{\nu}_i^2)^{\frac{5}{2}}}\left\{(\mathcal{V}_{22}^{(1)} - \mathcal{V}_{22}^{(0)})\mathcal{V}_{11}^{(0)} + \mathcal{V}_{12}^{(0)}\left[(\mathcal{V}_{11}^{(1)} + \mathcal{V}_{11}^{(0)})\mathcal{V}_{12}^{(0)} - 2\mathcal{V}_{12}^{(1)}\mathcal{V}_{11}^{(0)}\right]\right\},$$

$$\beta = -i\,\frac{4m\Omega\,\dot{\mu}_i^5}{(\Omega^2 + \dot{\nu}_i^2)^{\frac{5}{2}}}\left\{\mathcal{V}_{11}^{(1)}\mathcal{V}_{12}^{(0)} - \mathcal{V}_{11}^{(0)}\mathcal{V}_{12}^{(1)}\right\}, \qquad \gamma = \frac{4m\Omega\,\dot{\mu}_i^5}{(\Omega^2 + \dot{\nu}_i^2)^{\frac{5}{2}}}\left\{\mathcal{V}_{11}^{(1)} - \mathcal{V}_{11}^{(0)}\right\},$$

$$\delta = \frac{4m\Omega\,\dot{\mu}_i^5}{(\Omega^2 + \dot{\nu}_i^2)^{\frac{5}{2}}}\left\{\mathcal{V}_{11}^{(0)}\left[3\mathcal{V}_{11}^{(0)} - \mathcal{V}_{11}^{(1)}\right]\right\}.$$

Everything can be obtained from the **covariance matrix elements**.

At late time $t_f \to \infty$ the transition probability to the ground state is

$$P_{1\to 0} = \left[\left(\frac{m\Omega}{2} + \mathcal{V}_{22}^{(0)}\right)\left(\frac{1}{2m\Omega} + \mathcal{V}_{11}^{(0)}\right) - \left(\mathcal{V}_{12}^{(0)}\right)^2\right]^{-\frac{3}{2}} \left\{\frac{1}{2}\left[-\left(\frac{1}{2m\Omega} + \mathcal{V}_{11}^{(0)}\right)\mathcal{V}_{22}^{(1)} + \left(\frac{3}{2m\Omega} + 4\mathcal{V}_{11}^{(0)} - \mathcal{V}_{11}^{(1)}\right)\mathcal{V}_{22}^{(0)}\right]\right.$$
$$\left. + \frac{1}{4}\left[1 + m\Omega\left(3\mathcal{V}_{11}^{(0)} - \mathcal{V}_{11}^{(1)}\right) + 4\mathcal{V}_{12}^{(0)}\left(\mathcal{V}_{12}^{(1)} - 2\mathcal{V}_{12}^{(0)}\right)\right]\right\}$$

- This very general result applies to all Gaussian configurations of the fields: 1) **vacuum** field or **therma**l field in free space, 2) in confined space* with a (i) perfect conductor or (ii) dielectric boundary.

  * Compatible in the perturbative sense with Eberlein & Zietal,. PRA 86, 022111 (2012) and Scheel & Buhmann, acta phys. slov. 58, 675 (2008)

- E.g. in unbounded space, when there is no coupling between the internal state and the field, the probability is zero, but it becomes nonzero when the coupling is nonzero,

- The field has an infinite number of DoF's, so if they are initially in the vacuum state, the atom sooner or later will lose energy to the surrounding field, and the lost energy has no chance to come back,

- It is straightforward to check that at low temperature $\beta\Omega \gg 1$, the probability is very close to 1.

- As a check, we show the *late-time probability that the atom remains at the first excited state*,

$$P_{1\to 1} = \left[\mathcal{V}_{11}^{(0)}\mathcal{V}_{22}^{(0)} - \frac{1}{4}\right]\left[\left(\frac{m\Omega}{2} + \mathcal{V}_{22}^{(0)}\right)\left(\frac{1}{2m\Omega} + \mathcal{V}_{11}^{(0)}\right)\right]^{-\frac{3}{2}}$$

- and the *probability of transiting to the second excited state,*

$$P_{1\to 2} \simeq \left[\frac{1}{16} + \frac{1}{8}\left(\frac{\mathcal{V}_{22}^{(0)}}{m\Omega} - m\Omega\,\mathcal{V}_{11}^{(0)}\right)^2 + \mathcal{V}_{11}^{(0)}\mathcal{V}_{22}^{(0)}\left(\mathcal{V}_{11}^{(0)}\mathcal{V}_{22}^{(0)} - \frac{1}{2}\right)\right]\left[\left(\frac{m\Omega}{2} + \mathcal{V}_{22}^{(0)}\right)\left(\frac{1}{2m\Omega} + \mathcal{V}_{11}^{(0)}\right)\right]^{-\frac{5}{2}}$$

- We see when the bath is at zero temperature, these two results are almost zero; in particular it says that **leakage to the second excited state is negligible**.

- At medium temperature $\beta\Omega \sim \mathcal{O}(1)$, both have substantial values but at high temperature $\beta\Omega \ll 1$, they fall off like $\mathcal{O}(\beta\Omega)$, because the atom is excited to higher states.

- How the medium affects the transition probability can be easily calculated in terms of the covariance matrix elements.

- A harmonic atom initially in either the first excited state or the ground state sitting in the scalar field vacuum. (no stimulated phenomena)

  Comparison with Time Dependent Perturbation Theory: suitable only for

  - 1) weak coupling,
  - 2) early evolution time,

  TDPT + Einstein's AB coefficient – enables one to find the asymptotic value of transition probability (only for weak coupling)

  For the spontaneous emission case of the atom in field vacuum,

  the atom always ends up in the ground state.

  - modification of atomic state,
  - renormalization of the atomic parameters,
  - cutoff scale in the configuration

- Our approach based on Graded Influence Functional Formalism shows some distinct features lacking in TDPT (+AB coefficients):
  - Weak coupling condition not required. Finite coupling implies :
    - 1) the asymptotic late time transition probability of spontaneous emission is lower than 1, ← new phenomenon
    - 2) the difference is proportional to the coupling constant,
    - 3) cutoff dependence is physical (no room for hand-waving),
    - 4) qualitatively distinct behavior at strong coupling, even at early time

Interesting Consequences:

1) There will be non-zero transitions to other levels, even spontaneous excitation!

2) At finite coupling, the product states of the atom and the field are not the energy eigenstates of the Hamiltonian of the combined system (Jordan & Buttiker 09); the ``difference” between the product state and the genuine eigenstate is determined by the strength of coupling.

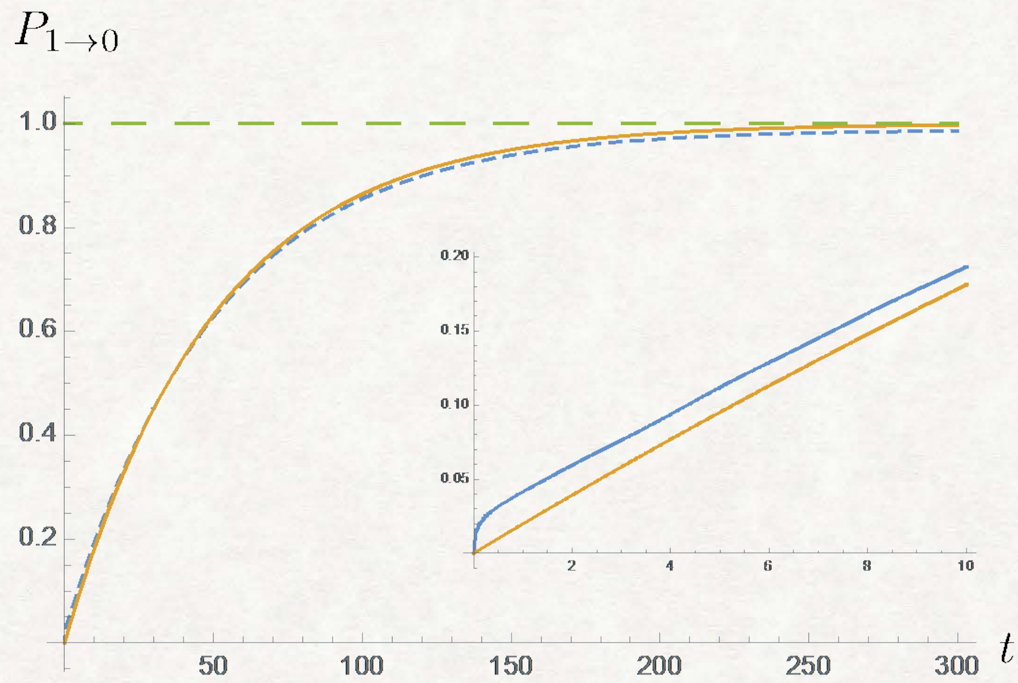
3) thus one can be written as a linear combination of the other
  => non-zero transition probability.

4) at stronger atom-field coupling the concept of ``energy levels” is a poor
  approximation (Hanke & Zwerger 95).
    wider level / resonance width => no discrete levels any more.

5) effects of the cutoff can be seen at very early time.

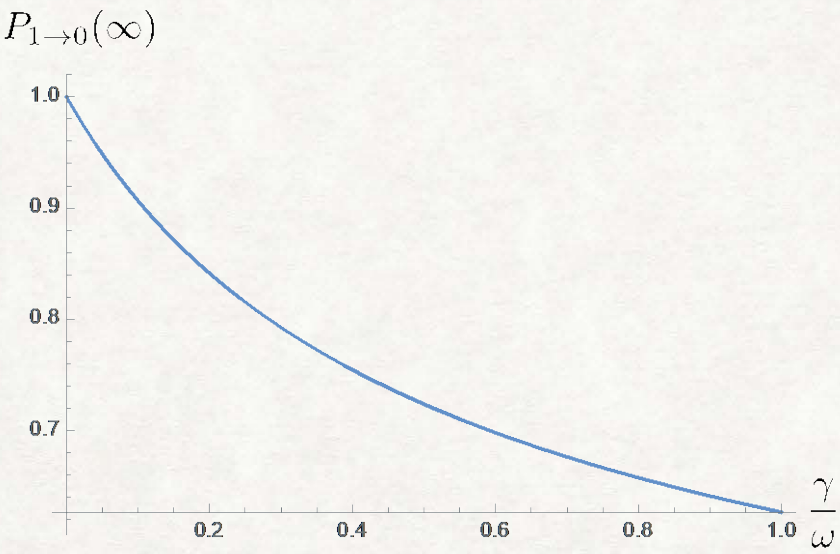
Spontaneous Emission at Weak Coupling

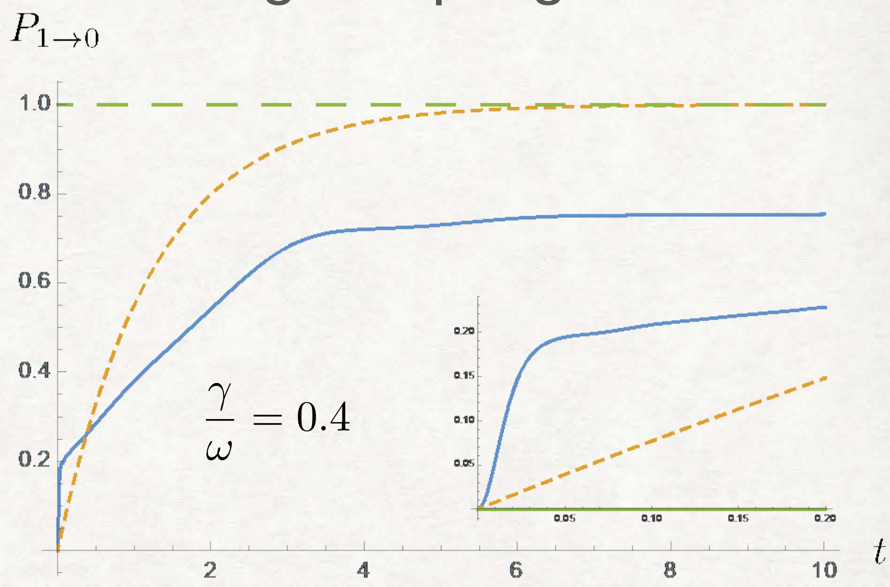
$$\frac{\gamma}{\omega} = 0.01$$



- At weak coupling both theories are compatible.
- Probability predicted by GIA (blue curve) is slightly lower than 1.
- At early time, the probability is slightly larger than TDPT (orange curve) due to the finite cutoff of the ambient field.
- The differences diminishes in the limit of vanishing coupling strength.

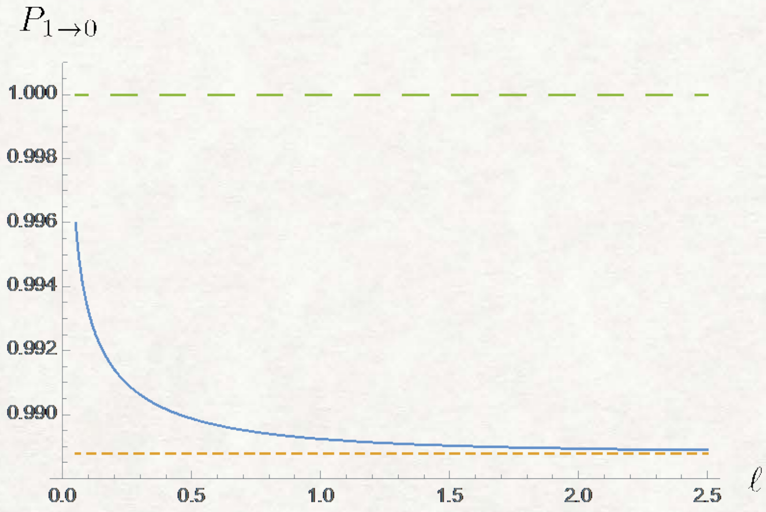
Spontaneous Emission at Strong Coupling

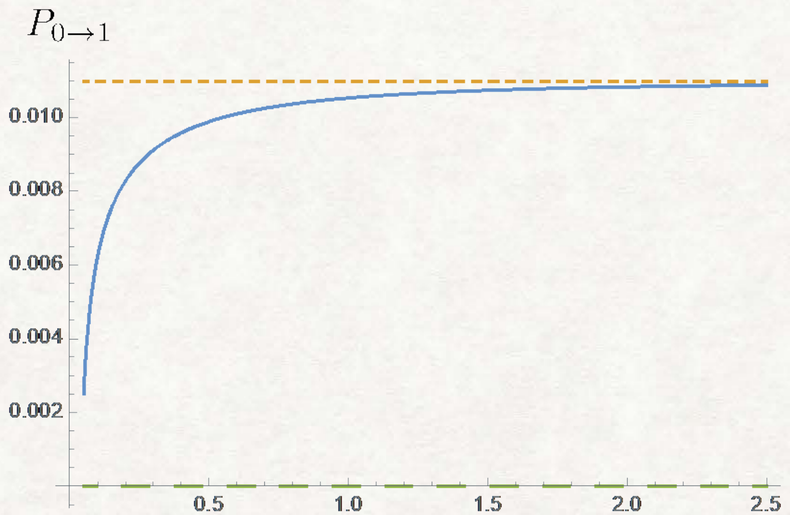




- Very different predictions from TDPT vs. GIA
- Cutoff effects are not negligible at earlier time.
- The linear regime grows at a rate way different from 2γ by TDPT.
- The asymptotic probability is significantly smaller than 1.

Boundary Effects at Weak Coupling





- When the atom is near a perfectly conducting plate, the boundary has different effects on atoms initially at different levels
- It tends to enhance spontaneous emission, but to suppress spontaneous excitation.
- The boundary effects vary exponentially at shorter separation, but algebraically at larger separation.
- They all vanish in the large separation limit.

Example 2: For a Static Atom:
(DISTANCE-DEPENDENT) SPATIAL DECOHERENCE

- **Quantum fluctuations of the field** can disrupt the coherence between the IDF of the atom, leading to loss of coherence,
- **Modification in the field configuration** thus can alter the degree of decoherence,
- Decoherence is most easily seen from the decay of the off-diagonal elements of the density matrix, [(s) here is (χ)]

$$\rho^{(S)}(t_f) = \int \mathcal{D}\chi_{\pm}^{(S)} \; \exp\Big\{ i\, S_{CG}^{(S)}[\chi_{\pm}] \Big\} \circ \rho^{(S)}(t_i)$$

with

$$S_{CG}^{(S)}[\chi_{\pm}] = S^{(S)}[\chi_+] - S^{(S)}[\chi_-] + g^2 \int_{t_i}^{t_f} ds ds' \; \dot{\Delta}^{(\chi)}(s)\, G_R^{(\psi)}(s,s')\, \dot{\Sigma}^{(\chi)}(s')$$
$$+ i\, \frac{g^2}{2} \int_{t_i}^{t_f} ds ds' \; \dot{\Delta}^{(\chi)}(s)\, G_H^{(\psi)}(s,s')\, \dot{\Delta}^{(\chi)}(s')$$

$$\Sigma^{(\chi)} = (\chi_+ + \chi_-)/2$$

$$\Delta^{(\chi)} = \chi_+ - \chi_-$$

- The imaginary part of $S_{CG}^{(S)}[\chi_{\pm}]$: $\mathrm{Im}\, S_{CG}^{(S)}[\chi_{\pm}] = \dfrac{g^2}{2} \displaystyle\int_{t_i}^{t_f} ds ds' \; \dot{\Delta}^{(\chi)}(s)\, G_H^{(\psi)}(s,s')\, \dot{\Delta}^{(\chi)}(s')$

introduces a decaying behavior of the density matrix w/ larger $\Delta^{(\chi)}$,

- We know, and can see here, that the **noise kernel** representing the effect of the **quantum fluctuations of the field is responsible for decoherence.**

- In an interference experiment, this results in the contrast of the fringes getting dimmer.

- Different field configurations will show different changes in the fringe contrast.

The reduced density matrix : $\rho^{(S)}(t_f) = \mathcal{N} \exp\left\{ i\, \overline{S}^{(S)}_{CG}[\bar{\chi}_\pm] \right\}$

where $\overline{S}^{(S)}_{CG}[\bar{\chi}_\pm]$ is the coarse-grained effective action evaluated along the **extremal path** $\bar{\chi}_\pm$, obtained from:

$$\frac{\delta S^{(S)}_{CG}[\chi_\pm]}{\delta \Delta^{(x)}} = 0$$

In a generic form, (superscript (0) indicates atom initially is in the ground state)

$$\rho_\chi^{(S)}(\Sigma_f^{(\chi)}, \Delta_f^{(\chi)}, t_f) = \left(\frac{m\Omega}{2\pi}\right)^{\frac{1}{2}} \exp\left[-a\,\Delta_f^{(\chi)\,2} - i\,2b\,\Delta_f^{(\chi)}\Sigma_f^{(\chi)} - c\,\Sigma_f^{(\chi)\,2}\right]$$

where $a = \frac{1}{2\mathcal{V}_{11}^{(0)}}\left\{\mathcal{V}_{11}^{(0)}\mathcal{V}_{22}^{(0)} - \left[\mathcal{V}_{12}^{(0)}\right]^2\right\}$, $b = -\frac{\mathcal{V}_{12}^{(0)}}{2\mathcal{V}_{11}^{(0)}}$, $c = \frac{1}{2\mathcal{V}_{11}^{(0)}}$

- A useful **decoherence measure** for a single atom is the **linear entropy** $S_\ell = 1 - \mathrm{Tr}\{\varrho^2\} = 1 - \frac{\sqrt{c}}{2\sqrt{a}}$ $\quad 0 \le S_\ell \le 1$

  where $\frac{a}{c} = \mathcal{V}_{11}^{(0)}\mathcal{V}_{22}^{(0)} - \left[\mathcal{V}_{12}^{(0)}\right]^2$ is the **Robertson-Schrodinger uncertainty function. I**t essentially measures the mixedness, entanglement, and decoherence of the pure state as a consequence of interaction with the environ.

- The **covariance matrix elements** $\mathcal{V}_{ij} = \frac{1}{2}\langle\{R_i, R_j\}\rangle$ with $R^T = (\chi, p)$ register the influence from the medium-modified field, e.g.

$$\mathcal{V}_{11}^{(0)}(t) = \langle\chi^2(t)\rangle$$

$$= d_1^2(t)\langle\chi^2(0)\rangle + d_2^2(t)\,\frac{\langle p^2(0)\rangle}{m^2} + \frac{g^2}{m^2}\int_0^t ds ds'\;\dot{d}_2(t-s)\,G_H^{(\psi)}(s,s')\,\dot{d}_2(t-s')$$

- Where d_i are the homogeneous solutions of
$$m\,\ddot{d}_i(t) + m\omega^2\, d_i(t) + \partial_t \int_0^t dt'\; G_R^{(\psi)}(t-t')\,\dot{d}_i(t') = 0$$
which depend on $G_R^{(\psi)}(s,s')$ of the field.

- The medium-modified field affects the covariance matrix elements through two routes:
  - amplitude change in d_i via $G_R^{(\psi)}(s,s')$
  - driving noise force via $G_H^{(\psi)}(s,s')$

  Both influences are connected by the

  Fluctuation-Dissipation Relation (FDR) of the field,

- In general $G^{(\psi)}(s,s')$ contains two distinct contributions
  - $G_0^{(\psi)}(s,s')$: in the absence of the medium, $G^{(\psi)} = G_0^{(\psi)} + G_M^{(\psi)}$
  - $G_M^{(\psi)}(s,s')$: correction due to the **medium**,

- This shows how the dielectric affects the field, which in turn modifies the coherence of the IDF's of an atom.
- An alternative measure of decoherence is to compare the position dispersion $\mathcal{V}_{11}^{(0)}$ with or without coupling with the field,
- It describes the change in the spreading of the wavefunction of IDF's due to the quantum fluctuations of the field, modified by the presence of the dielectric.
- That is how the distance dependence enters.

EXAMPLE 3: STATIC ATOM: ATOM-FIELD ENTANGLEMENT

Allow the atom to sit outside the dielectric for a while, the atom's internal states tend to get entangled with the field states,

$$|\varphi_i\rangle \otimes |0_i\rangle \longrightarrow \sum_k |\varphi_k\rangle \otimes |\gamma_k\rangle \stackrel{?}{=} |\Psi\rangle \otimes |\eta_\gamma\rangle$$

- The degree of atom-field entanglement can be quantified by **purity** μ_s or **linear entropy** $S_L = 1 - \mu_s \qquad \mu_s = \mathrm{Tr}\{\rho^{(S)\,2}\} = \frac{1}{2\sqrt{\mathcal{V}}}$
- $\mathcal{V}$ is the determinant of the covariance matrix, whose elements at late time are

$$\mathcal{V}_{11} = \langle\chi_f^2\rangle = \frac{1}{M}\,\mathrm{Im}\int_0^\infty \frac{d\omega}{\pi}\,\widetilde{G}_R^{(\chi)}(\omega)$$

$$\mathcal{V}_{22} = \langle p_f^2\rangle = M\,\mathrm{Im}\int_0^\infty \frac{d\omega}{\pi}\,\omega^2\,\widetilde{G}_R^{(\chi)}(\omega)$$

and $\widetilde{G}_R^{(\chi)}(\omega) = \left[-\omega^2 + \Omega^2 - i\,2\tau\omega^3 - \frac{g^2}{m}\,\omega^2\widetilde{G}^{(\psi)}_{R,\,M}(\omega)\right]^{-1}$ is the retarded Green function which enters into the "noise kernel" of the atom system.

- The medium modifies the field which in turn affects the entanglement.

- For example, outside a **perfect conductor**, we have

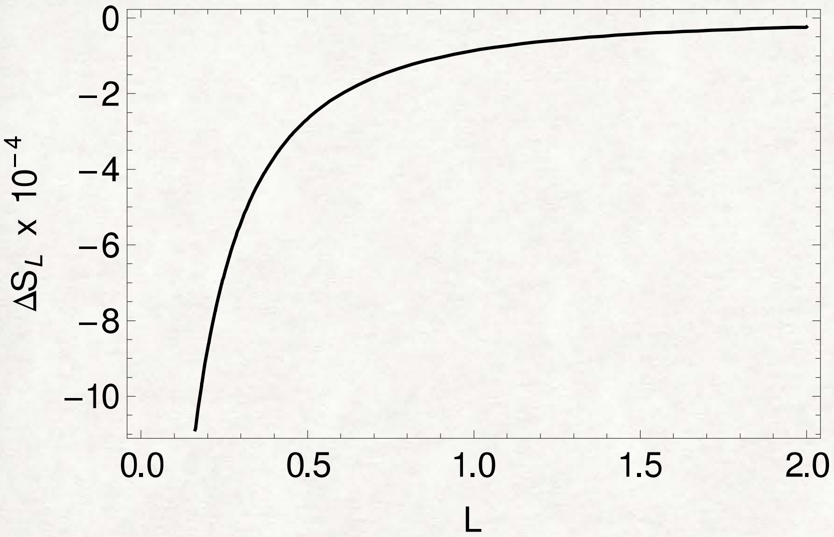
$$\Delta S_L = -\Delta\mu < 0$$

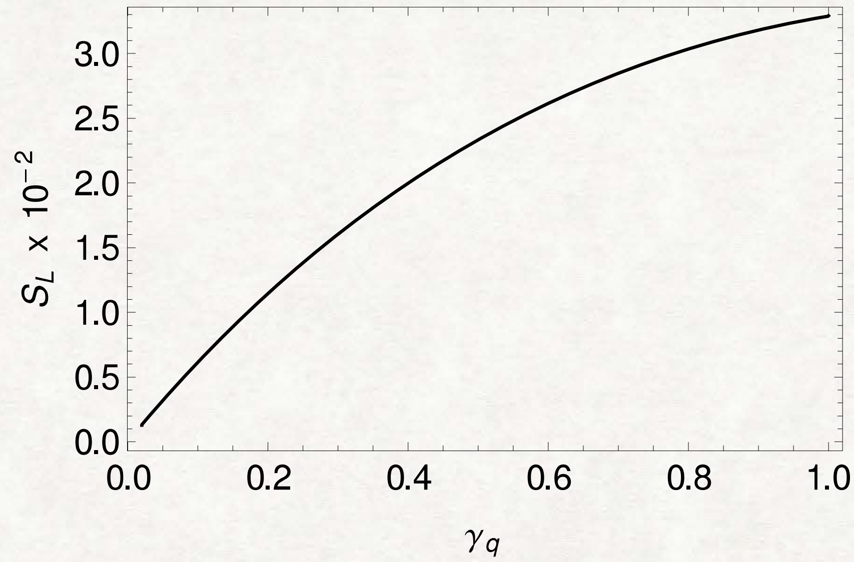
- The presence of the mirror increases the purity over the case without, thereby reducing the entanglement between the atom and the modified field.

- As the atom approaches the conductor, entanglement actually decreases. [Rong Zhou, R. Behunin, S. Y. Lin and B. L. Hu JHEP 08 (2013) 040]

- We can understand the result in terms of the image atom, which carries the opposite induced polarization, thus counteracting the original field's contribution. The counter action is the stronger the closer the atom is to the dielectric, thus reducing the entanglement.

- For the dielectric, the change in the entanglement measure will be similar but not so strong because the image atom is "blurred".





- Linear entropy as a measure of the decoherence of the atom, and atom-field entanglement.

- Right plot indicates a stronger atom-field entanglement at stronger coupling γ_q ← expected

- Left plot shows that the presence of a perfectly conducting plate tends to suppress entanglement between atom and field. The suppression is stronger at shorter distance L. ← counter- intuitive!

SUMMARY

- We have established from first principles **a unified framework** to treat atom/mirror + quantum field + (quantum) medium interactions.

- We gave several examples to show how this framework **reproduces well-known results**. More importantly, it gives **unexpected new results in well established issues,** like spontaneous emission.

- This framework can be applied to treat **new problems** in quantum information such as **decoherence and entanglement**, and **quantum fluctuation phenomena** (Casimir, Casimir-Polder, quantum friction).

- It serves as a sound basis for investigations of **quantum optomechanics**. M**oving atom / mirror near a dielectric** (motional decoherence, cooling, quantum friction) will be treated in Papers III, IV, V.

- The strength of this framework is in its mathematical rigor, **systematic thoroughness, self-consistency**, and clarity in physical meanings. The modality established here can be extended to treat more general configurations.

This is a long and strenuous journey,
but hopefully an invigorating and rewarding one. So,

Thank you!

for your good company.