Non perturbative calculations of Parton Distribution Functions

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Introduction

- Quantum Chromodynamics: Theory of strong interactions
  - Describes the forces that bind together quarks to form hadrons such as the proton
- Non-linear and strongly coupled quantum field theory
- Proton is a relativistic many body system (partons)
  - It's structure is described in terms of parton densities
- Proton structure can be in principle accessed with theoretical computations
  - It requires numerical methods: Lattice QCD
- Proton structure is "universal"
  - Once determined it can be used to predict experimental results
  - It is currently determined experimentally and used as input to understand other experiments
    - Example: search for new physics at LHC

PDFs: Definition

Light-cone PDFs:

$$f^{(0)}(\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega^-}{4\pi} e^{-i\xi P^+ \omega^-} \left\langle P \left| T\, \overline{\psi}(0,\omega^-,\mathbf{0}_\mathrm{T}) W(\omega^-,0)\gamma^+ \frac{\lambda^a}{2}\psi(0) \right| P \right\rangle_\mathrm{C} .$$

$$W(\omega^-,0) = \mathcal{P}\exp\left[-ig_0 \int_0^{\omega^-} \mathrm{d}y^- A_\alpha^+(0,y^-,\mathbf{0}_\mathrm{T}) T_\alpha\right]$$

$$\langle P'|P\rangle = (2\pi)^3 2P^+ \delta\left(P^+ - P'^+\right)\delta^{(2)}\left(\mathbf{P}_\mathrm{T} - \mathbf{P}'_\mathrm{T}\right)$$

Moments:

$$a_0^{(n)} = \int_0^1 \mathrm{d}\xi\, \xi^{n-1} \left[f^{(0)}(\xi) + (-1)^n \overline{f}^{(0)}(\xi)\right] = \int_{-1}^1 \mathrm{d}\xi\, \xi^{n-1} f(\xi)$$

Local matrix elements:

$$\left\langle P|\mathcal{O}_0^{\{\mu_1\ldots\mu_n\}}|P\right\rangle = 2a_0^{(n)}\left(P^{\mu_1}\cdots P^{\mu_n} - \mathrm{traces}\right)$$

$$\mathcal{O}_0^{\{\mu_1\cdots\mu_n\}} = i^{n-1}\overline{\psi}(0)\gamma^{\{\mu_1} D^{\mu_2}\cdots D^{\mu_n\}}\frac{\lambda^a}{2}\psi(0) - \mathrm{traces}$$

Introduction (cont.)

- Goal: Compute hadron structure properties from QCD
    - Parton distribution functions (PDFs)
- Operator product: Mellin moments are local matrix elements that can be computed in Lattice QCD
    - Power divergent mixing limits us to few moments
- Few years ago X. Ji suggested an approach for obtaining PDFs from Lattice QCD
- First calculations already available

X. Ji, Phys.Rev.Lett. 110, (2013)

Y.-Q. Ma J.-W. Qiu (2014) 1404.6860

H.-W. Lin, J.-W. Chen, S. D. Cohen, and X. Ji, Phys.Rev. D91, 054510 (2015)

C. Alexandrou, et al, Phys. Rev. D92, 014502 (2015)

- A new approach for obtaining PDFs from LQCD introduced by A. Radyushkin

A. Radyushkin Phys.Lett. B767 (2017)

- Hadronic tensor methods

K-F Liu et al Phys. Rev. Lett. 72 (1994) , Phys. Rev. D62 (2000) 074501
Detmold and Lin 2005
M. T. Hansen et al arXiv:1704.08993.
UKQCD-QCDSF-CSSM Phys. Lett. B714 (2012), arXiv:1703.01153

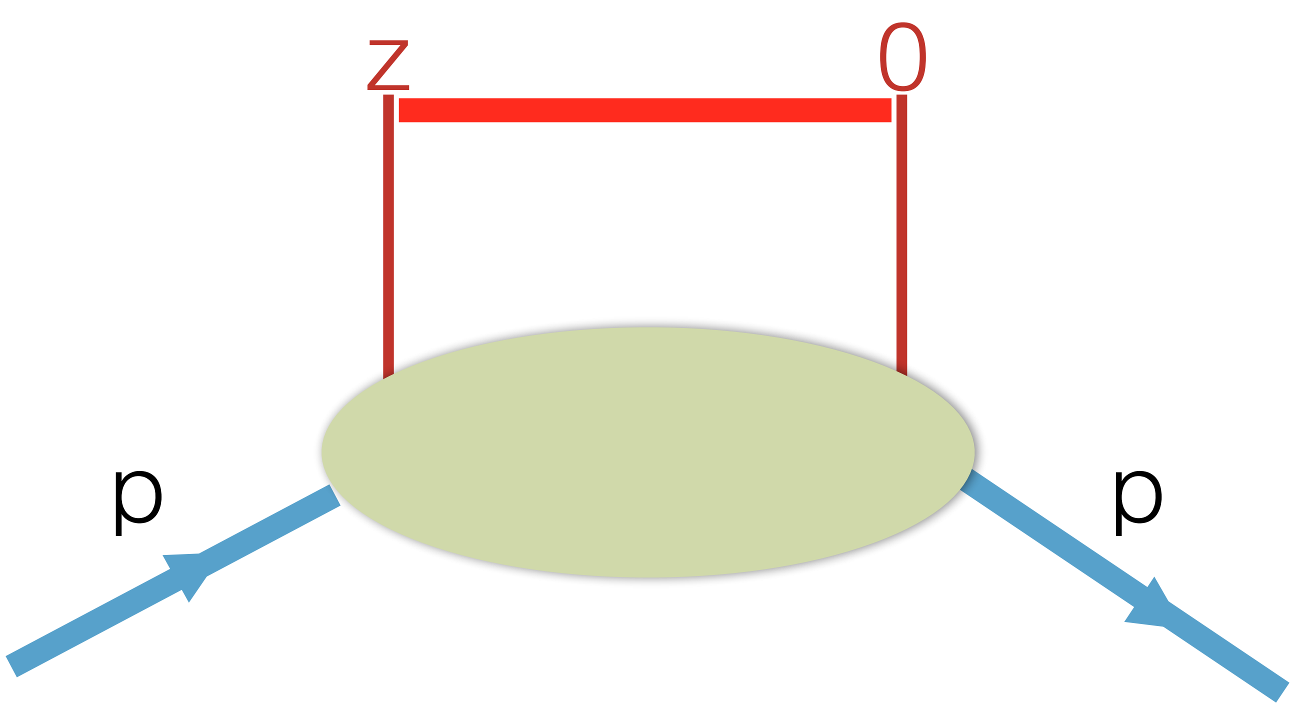
Ma and Qiu : arXiv:1709.03018

Pseudo-PDFs

Unpolarized PDFs proton:

$$\mathcal{M}^{\alpha}(z,p) \equiv \langle p|\bar{\psi}(0)\,\gamma^{\alpha}\,\hat{E}(0,z;A)\psi(z)|p\rangle$$

$$\hat{E}(0,z;A) = \mathcal{P}\exp\left[-ig\int_0^z \mathrm{d}z'_{\mu}\,A^{\mu}_{\alpha}(z')T_{\alpha}\right]$$



A. Radyushkin Phys.Lett. B767 (2017)

Lorentz decomposition:

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar{\psi}(0)\,\gamma^\alpha\,\hat{E}(0,z;A)\psi(z)|p\rangle$$

$$\mathcal{M}^\alpha(z,p) = 2p^\alpha \mathcal{M}_p(-(zp),-z^2) + z^\alpha \mathcal{M}_z(-(zp),-z^2)$$

Collinear PDFs: Choose

$$z = (0, z_-, 0)$$

$$p = (p_+, 0, 0)$$

$$\gamma^+$$

$$\mathcal{M}^+(z,p) = 2p^+ \mathcal{M}_p(-p_+ z_-, 0)$$

Definition of PDF:

$$\mathcal{M}_p(-p_+ z_-, 0) = \int_{-1}^{1} dx\, f(x)\, e^{-ixp_+ z_-}$$

$\mathcal{M}_p(-pz, -z^2)$ is a Lorentz invariant therefore computable in any frame

$\nu = -zp$ $\quad$ ν is called Ioffe time

B. L. Ioffe, Phys. Lett. 30B, 123 (1969)

$$\mathcal{M}_p(\nu, -z^2) \equiv \int_{-1}^{1} dx\, \mathcal{P}(x, -z^2) e^{ix\nu} \qquad \mathcal{P}(x, 0) = f(x)$$

It can be shown that the domain of x is [-1, 1]

A. Radyushkin Phys.Lett. B767 (2017)

One can obtain PDFs in the limit of $z^2 \to 0$

This limit is singular but using OPE, PDFs are defined

$$\mathcal{M}_p(\nu, z^2) = \int_0^1 d\alpha\, \mathcal{C}(\alpha, z^2\mu^2, \alpha_s(\mu))\, \mathcal{Q}(\alpha\nu, \mu) \;\; + \mathcal{O}(z^2\Lambda_{qcd}^2)$$

$\mathcal{Q}(\nu, \mu)$ is called the Ioffe time PDF

V. Braun, et. al Phys. Rev. D 51, 6036 (1995)

$$\mathcal{Q}(\nu, \mu) = \int_{-1}^1 dx\, e^{-ix\nu} f(x, \mu)$$

Matching to $\overline{MS}$

Radyushkin Phys.Rev. D98 (2018) no.1, 014019
Izubuchi et al. Phys.Rev. D98 (2018) no.5, 056004
Zhang et al. Phys.Rev. D97 (2018) no.7, 074508

A. Radyushkin Phys.Lett. B767 (2017)

Lattice QCD calculation:

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar{\psi}(0)\,\gamma^\alpha\,\hat{E}(0,z;A)\psi(z)|p\rangle$$

Choose
$$p = (p_0, 0, 0, p_3)$$
$$z = (0, 0, 0, z_3)$$
γ^0

On shell equal time matrix element computable in Euclidean space

Briceno *et al* arXiv:1703.06072

Obtaining only the relevant

$$\mathcal{M}_p(\nu, z_3^2) = \frac{1}{2p_0}\mathcal{M}^0(z_3, p_3)$$

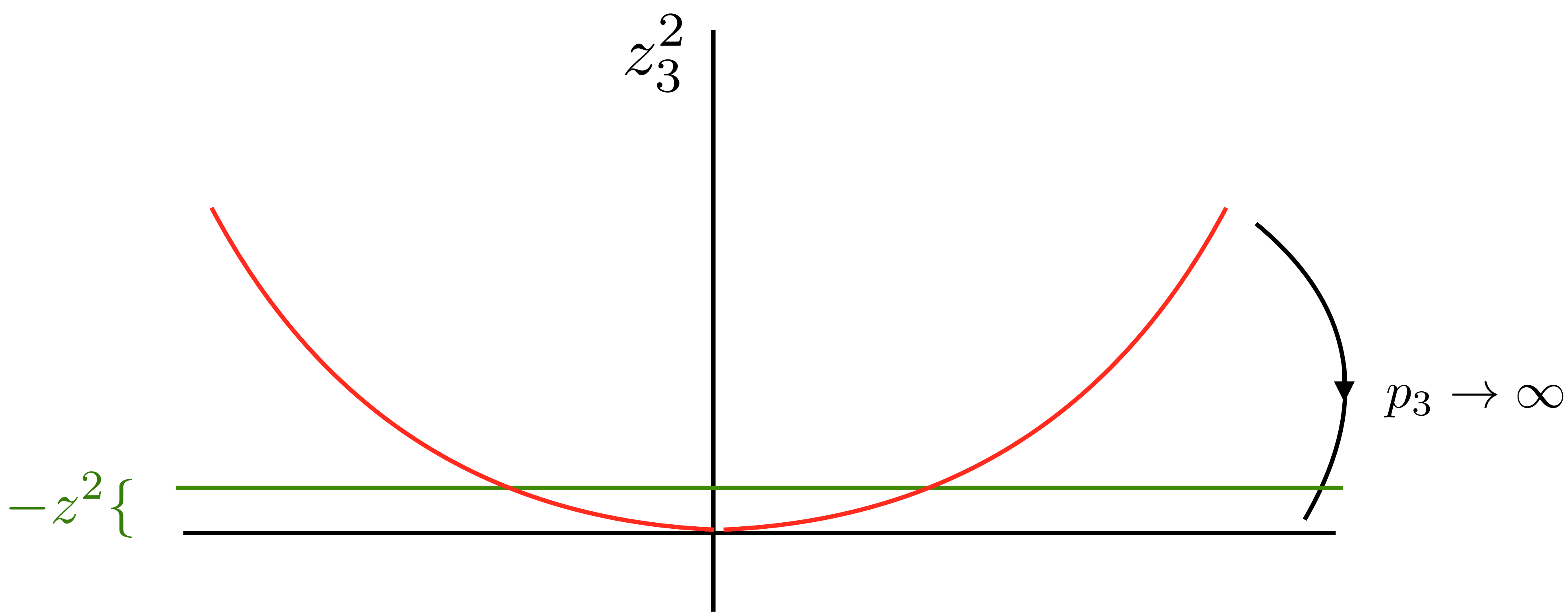
Chosing γ^0 was also suggested also by M. Constantinou at GHP2017 based on an operator mixing argument for the renormalized matrix element

Alexandrou *et al* arXiv:1706.00265

$$Q(y, p_3) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu \mathcal{M}_p(\nu, \nu^2/p_3^2) e^{-iy\nu}$$ Ji’s quasi-PDF

Large values of $z_3 = \nu/p_3$ are problematic

Alternative approach to the light-cone:



$$\mathcal{P}(x, -z^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\nu\, \mathcal{M}_p(\nu, -z^2) e^{-ix\nu}$$

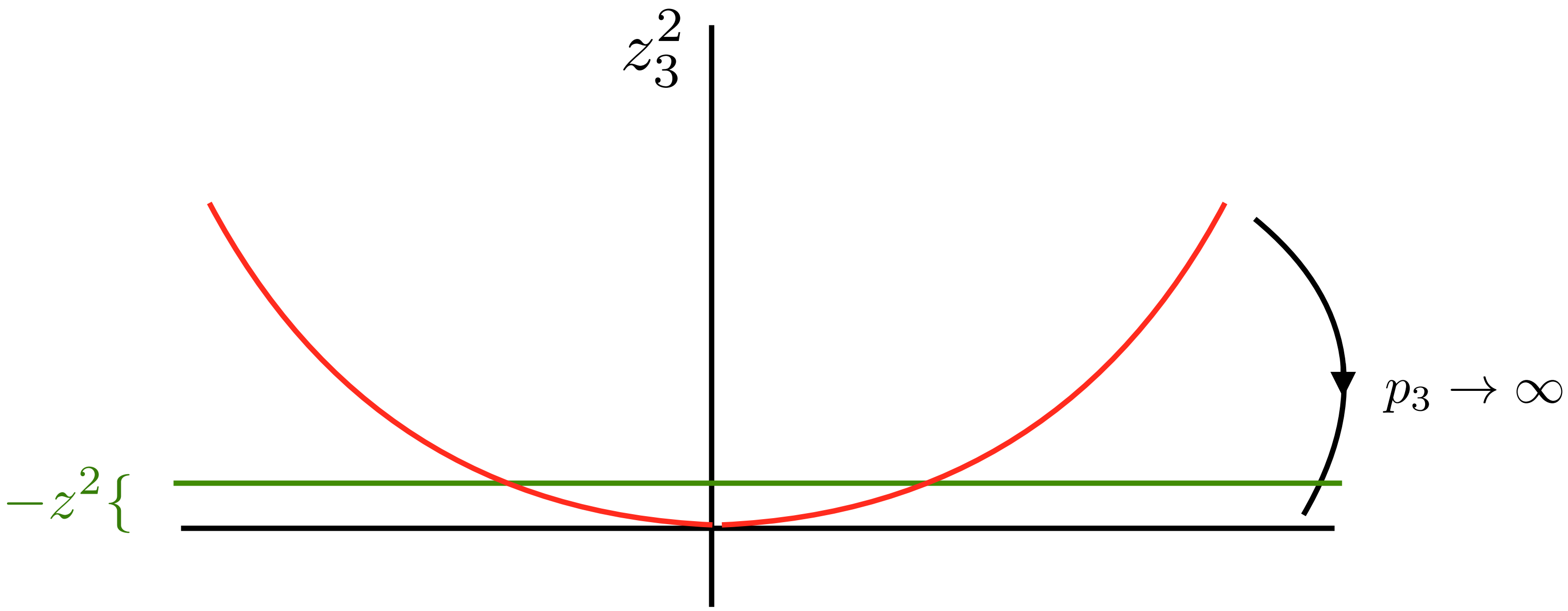
PDFs can be recovered $-z^2 \to 0$

Note that $x \in [-1, 1]$

Radyusking Phys.Lett. B767 (2017) 314-320

$$Q(y, p_3) = \frac{1}{2\pi p_3}\int_{-\infty}^{\infty} dz_3 \mathcal{M}_p(z_3 p_3, z_3^2) e^{-iyz_3p_3}$$

Ji's quasi-PDF



Note that

$$y \in (-\infty, \infty)$$

Rossi & Testa argue that the moments of the $Q(y,p_3)$ are not well defined due to contributions from the region of $|y|>1$, which is coming from contamination from trace terms.

Rossi & Testa: PhysRev D 96, 014507 (2017), PhysRev D 98, 054028

Radyushkin argued that such contributions may be safely ignored as they vanish at the large p_3 limit

Radyushkin arXiv:1807.07509

From Radyushkin arXiv:1807.07509

$$
\begin{aligned}
f_{\mp}(y,\mu^2) &= Q_{\mp}(y,P)\theta(0 \le y \le 1) \\
&- \frac{\alpha_s}{2\pi} C_F \int_0^1 \frac{dx}{x} \left[Q_{\mp}(x,P) - Q_{\mp}(y,P)\right] \\
&\times \left[\theta(x \ge y) \left\{ \frac{1 + y^2/x^2}{1 - y/x} \left(\ln\left[4y(x-y)\frac{P^2}{\mu^2}\right] - 1 \right) + \frac{3/2}{1 - y/x} + 1 \right\} \right. \\
&\left. + \theta(x \le y) R_{>}(y/x) \pm R_{<}(-y/x) \right| + \mathcal{O}(1/P^2) \,. \qquad (5.16)
\end{aligned}
$$

Shows that the PDF $f(y,\mu^2)$ can be computed with out references to the $|y|>1$ values of the quasi-PDF. Those only contribute to the $1/P^2$ terms

Note:

The Rossi-Testa issue appears only if one performs the Fourier transform over z to construct the quasi-PDF

Quasi-PDF:

$$Q(y, p_3) = \int_{-1}^{1} \frac{dx}{|x|} Z(\frac{y}{x}, \frac{\mu}{p_3}) f(x, \mu) + \mathcal{O}(\frac{\Lambda_{qcd}^2}{p_3^2})$$

Chen et al. arXiv:1711.07858

At fixed large momentum p_3

Ioffe time PDF:

$$\mathcal{M}_p(\nu, z^2) = \int_0^1 d\alpha\, \mathcal{C}(\alpha, z^2\mu^2, \alpha_s(\mu))\, \mathcal{Q}(\alpha\nu, \mu) \;\; + \mathcal{O}(z^2\Lambda_{qcd}^2)$$

$$\mathcal{Q}(\nu, \mu) = \int_{-1}^{1} dx\, e^{-ix\nu} f(x, \mu)$$

Matching to $\overline{MS}$

At fixed small z^2

Radyushkin Phys.Rev. D98 (2018) no.1, 014019
Izubuchi et al. Phys.Rev. D98 (2018) no.5, 056004
Zhang et al. Phys.Rev. D97 (2018) no.7, 074508

Lattice QCD requirements

$$aP_{max} = \frac{2\pi}{4} \sim \mathcal{O}(1)$$

$$a \sim 0.1fm \rightarrow P_{max} = 10\Lambda$$
$$a \sim 0.05fm \rightarrow P_{max} = 20\Lambda$$

$$\Lambda \sim 300MeV$$

For practical calculations large momentum is needed

*Higher twist effect suppression (qpdfs)

*Wide coverage of Ioffe time ν

P= 3 GeV is already demanding due to statistical noise

achievable with easily accessible lattice spacings

P= 6 GeV exponentially harder

requires current state of the art lattice spacing

Statistical noise

Nucleon with momentum P two-point function:

$$C_{2p}(P,t) = \langle O_N(P,t) O_N^\dagger(P,0)\rangle \sim \mathcal{Z} e^{-E(P)t}$$

Variance of nucleon two-point function:

$$\mathrm{var}\left[C_{2p}(P,t)\right] = \langle O_N(P,t) O_N(P,t)^\dagger O_N(P,0) O_N^\dagger(P,0)\rangle \sim \mathcal{Z}_{3\pi} e^{-3m_\pi t}$$

Variance is independent of the momentum

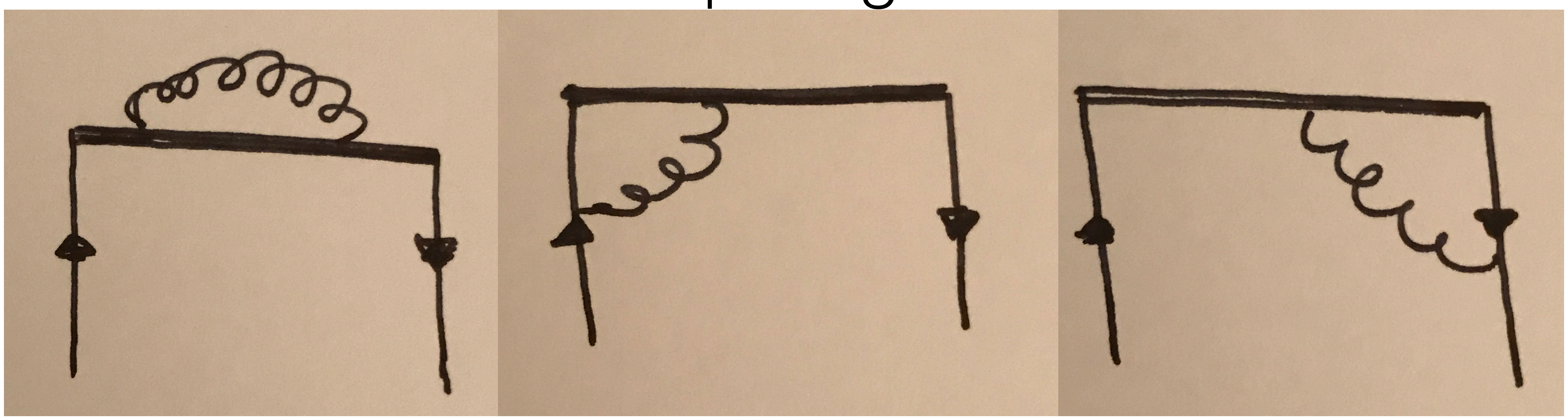
$$\frac{\mathrm{var}\left[C_{2p}(P,t)\right]^{1/2}}{C_{ap}(P,t)} \sim \frac{\mathcal{Z}}{\mathcal{Z}_{3\pi}} e^{-[E(P)-3/2 m_\pi]t}$$

Statistical accuracy drops exponentially with the increasing momentum limiting the maximum achievable momentum.

Renormalization

$$\mathcal{M}^0_{ren}(z,p,\mu) = \lim_{a\to 0} Z_{\mathcal{O}}(z,\mu,a)\mathcal{M}^0(z,P,a)$$

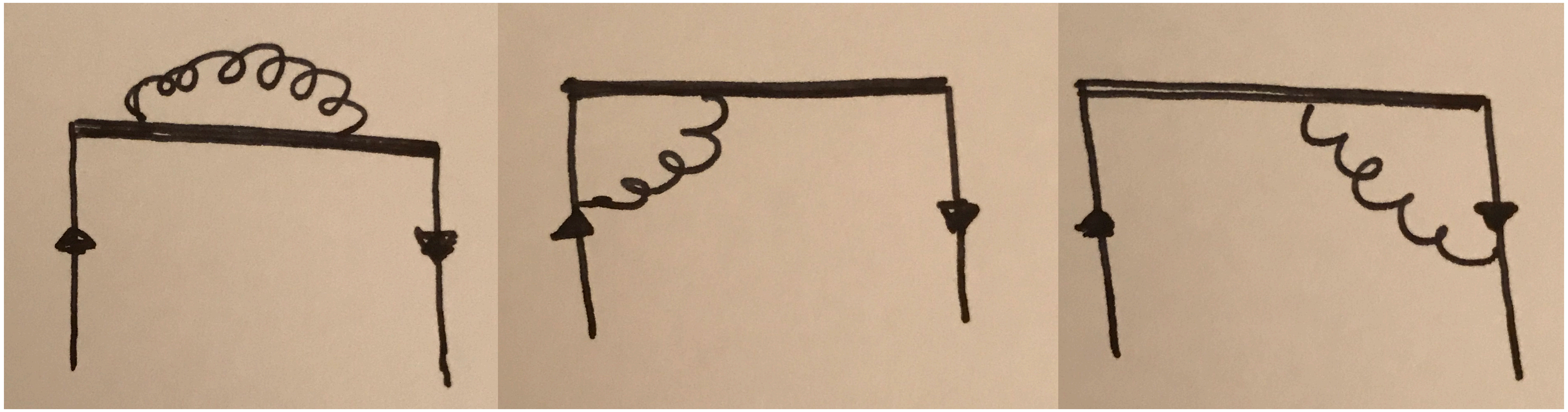
One loop diagrams



Linear divergence

Logarithmic divergence

Dotsenko Nucl.Phys. B169 (1980) 527
Ishikawa et al. arXiv:1707.03107, arXiv:1609.02018
Chen et al. Nucl.Phys. B915 (2017)
Radyushkin arXiv:1710.08813



One loop calculation of the UV divergences results in

$$\mathcal{M}^0(z, P, a) \sim e^{-m|z|/a} \left(\frac{a^2}{z^2}\right)^{2\gamma_{end}}$$

after re-summation of one loop result resulting exponentiation

- J.G.M.Gatheral,Phys.Lett.133B,90(1983)
- J.Frenkel, J.C.Taylor,Nucl.Phys.B246,231(1984),
- G.P.Korchemsky, A.V.Radyushkin,Nucl.Phys.B283,342(1987).

Multiplicatively renormalizable

Consider the ratio $\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}_p(\nu, z_3^2)}{\mathcal{M}_p(0, z_3^2)}$

UV divergences will cancel in this ratio resulting a renormalization group invariant (RGI) function

The lattice regulator can now be removed

$\mathfrak{M}^{cont}(\nu, z_3^2)$ Universal independent of the lattice

$\mathcal{M}_p(0,0) = 1$ Isovector matrix element

$$\mathfrak{M}(\nu, z^2) = \int_0^1 d\alpha\, \mathfrak{C}(\alpha, z^2\mu^2, \alpha_s(\mu))\mathcal{Q}(\alpha\nu, \mu) \; + \sum_{k=1}^{\infty} \mathcal{B}_k(\nu)(z^2)^k$$

$$\mathcal{B}_k(\nu)(z^2)^k \sim \mathcal{O}(\Lambda_{qcd}^{2k})$$

Polynomial corrections to the Ioffe time PDF may be suppressed

B. U. Musch, *et al* Phys. Rev. D 83, 094507 (2011)
M. Anselmino et al. 10.1007/JHEP04(2014)005

A. Radyushkin Phys.Lett. B767 (2017)

Polynomial corrections will vanish in the $z^2 = 0$ limit

Possible mechanism for polynomial correction suppression

Approximate TMD factorization

A. Radyushkin Phys.Lett. B767 (2017)

M. Anselmino et al. 10.1007/JHEP04(2014)005

B. U. Musch, *et al* Phys. Rev. D 83, 094507 (2011)

$$\mathcal{M}_p(\nu, -z^2) \equiv \int_{-1}^{1} dx\, \mathcal{P}(x, -z^2) e^{ix\nu}$$

Taking $z = (0, z_-, z_\perp)$ we can identify $\mathcal{P}(x, z_\perp^2) = \int d^2k_\perp\, \mathcal{F}(x, k_\perp^2) e^{ik_\perp z_\perp}$

$\mathcal{F}(x, k_\perp^2)$ the primordial TMD

Assuming $\mathcal{F}(x, k_\perp^2) = f(x) g(k_\perp^2)$ we obtain $\mathcal{P}(x, z_\perp^2) = f(x)\tilde{g}(z_\perp^2)$

Implying that $\mathcal{M}_p(\nu, -z^2) = \mathcal{Q}(\nu, -z^2)\mathcal{M}_p(0, -z^2)$

where $\mathcal{M}_p(0, -z^2) = \tilde{g}(-z^2)$

$$\mu^2 \frac{d}{d\mu^2} \mathcal{Q}(\nu, \mu^2) = -\frac{2}{3}\frac{\alpha_s}{2\pi} \int_0^1 du\, B(u)\, \mathcal{Q}(u\nu, \mu^2)$$

$$B(u) = \left[\frac{1+u^2}{1-u}\right]_+$$

DGLAP kernel in position space

V. Braun, et. al Phys. Rev. D 51, 6036 (1995)

At 1-loop

$$\mathcal{Q}(\nu, \mu'^2) = \mathcal{Q}(\nu, \mu^2) \; - \frac{2}{3}\frac{\alpha_s}{2\pi} \ln(\mu'^2/\mu^2) \int_0^1 du\, B(u)\, \mathcal{Q}(u\nu, \mu^2)$$

Matching to $\overline{MS}$ computed at 1-loop

$$\mathfrak{M}(\nu, z^2) = \int_0^1 d\alpha\, \mathfrak{C}(\alpha, z^2\mu^2, \alpha_s(\mu))\mathcal{Q}(\alpha\nu, \mu) \; + \sum_{k=1}^{\infty} \mathcal{B}_k(\nu)(z^2)^k$$

Radyushkin Phys.Rev. D98 (2018) no.1, 014019
Zhang et al. Phys.Rev. D97 (2018) no.7, 074508

The Moments

Karpie et al. arXiv:1807.10933

Using OPE:

$$\mathfrak{M}(\nu, z^2) = 1 + \frac{1}{2p^0} \sum_{k=1}^{\infty} i^k \frac{1}{k!} z_{\alpha_1} \cdots z_{\alpha_k} c_k(z^2\mu^2) \langle p | \mathcal{O}_{(k)}^{0\alpha_1 \cdots \alpha_k} | p \rangle_\mu + \mathcal{O}(z^2)$$

$$\langle p | \mathcal{O}_{(k)}^{0\alpha_1 \cdots \alpha_k} | p \rangle_\mu = 2[p^0 p^{\alpha_1} \cdots p^{\alpha_k} - \text{traces}]_{\text{sym}}\, a_{k+1}(\mu)\,,$$

Where $$a_n(\mu) = \int_{-1}^{1} dx\, x^{n-1}\, q(x,\mu)\,,$$

are the moments of the PDFs

The Moments

Karpie et al. arXiv:1807.10933

Using that

$$\mathcal{Q}(\nu, \mu) = \int_{-1}^{1} dx\, q(x, \mu) e^{ix\nu} \,,$$

We can show

$$(-i)^n \left. \frac{\partial^n \mathcal{Q}(\nu, \mu)}{\partial \nu^n} \right|_{\nu=0} = \int_{-1}^{1} dx\, x^n\, q(x, \mu) = a_{n+1}(\mu)$$

The derivatives of Ioffe time distributions are related to the moments of the PDFs

The Moments

Karpie et al. arXiv:1807.10933

As a consequence:

$$(-i)^n \frac{\partial^n \mathfrak{M}(\nu, z^2)}{\partial \nu^n}\Big|_{\nu=0} = c_n(z^2\mu^2) a_{n+1}(\mu) + \mathcal{O}(z^2)\,.$$

Where the Wilson coefficients are

$$c_n(z^2\mu^2) = \int_0^1 d\alpha\, \mathcal{C}(\alpha, z^2\mu^2, \alpha_s(\mu))\alpha^n\,.$$

The Moments

Karpie et al. arXiv:1807.10933

$$\mathcal{C}(\alpha, z^2\mu^2, \alpha_s(\mu)) = \delta(1-\alpha) - \frac{\alpha_s}{2\pi} C_F \left[B(\alpha) \ln\left(z^2\mu^2 \frac{e^{2\gamma_E+1}}{4} \right) + D(\alpha) \right]$$

$$c_n(z^2\mu^2) = 1 - \frac{\alpha_s}{2\pi} C_F \left[\gamma_n \ln\left(z^2\mu^2 \frac{e^{2\gamma_E+1}}{4} \right) + d_n \right] ,$$

$$\gamma_n = \int_0^1 d\alpha \, B(\alpha)\alpha^n = \frac{3}{2} - \frac{1}{1+n} - \frac{1}{2+n} - 2\sum_{k=1}^{n} \frac{1}{k} ,$$

$$d_n = \int_0^1 d\alpha \, D(\alpha)\alpha^n = 2\left[\left(\sum_{k=1}^{n} \frac{1}{k} \right)^2 + \frac{2\pi^2 + n(n+3)(3+\pi^2)}{6(n+1)(n+2)} - \psi^{(1)}(n+1) \right]$$

Numerical Tests

with

J. Karpie, A. Radyushkin, S. Zafeiropoulos

Phys.Rev. D96 (2017) no.9, 094503

Numerical Tests

- Quenched approximation β=6.0

$32^3 \times 64 \quad m_\pi \sim 600 MeV$

- Need series of small z_3
- Need a range of momenta to scan ν
- Goals:
    - Check polynomial corrections
    - Understand the systematics of the approach

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Matrix element calculation

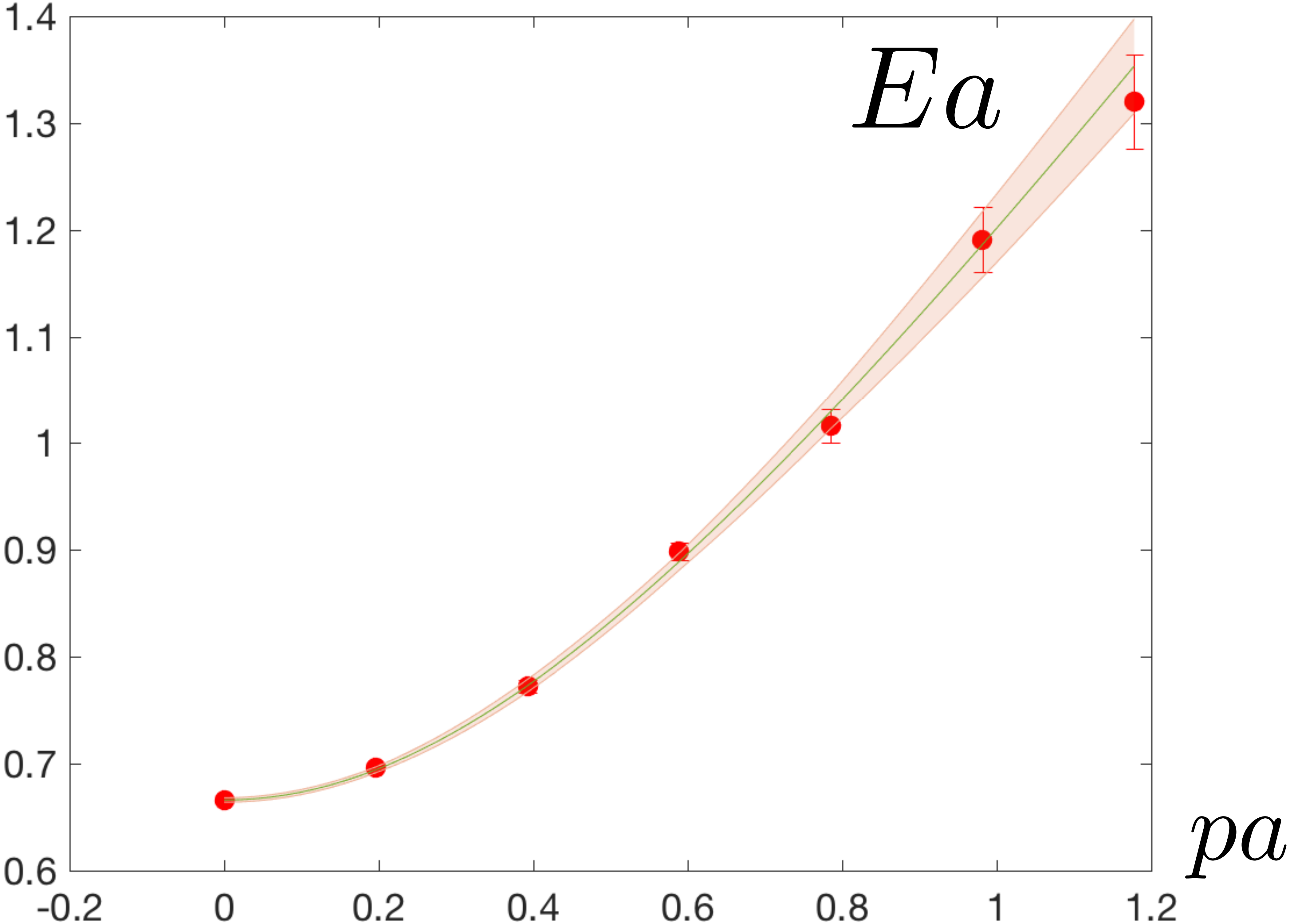
$$C_P(t) = \langle \mathcal{N}_P(t) \overline{\mathcal{N}}_P(0) \rangle \qquad C_P^{\mathcal{O}^0(z)}(t) = \langle \mathcal{N}_P(t) \mathcal{O}^0(z) \overline{\mathcal{N}}_P(0) \rangle$$

$$\mathcal{M}_{\text{eff}}(z_3 P, z_3^2; t) = \frac{C_P^{\mathcal{O}^0(z)}(t+1)}{C_P(t+1)} - \frac{C_P^{\mathcal{O}^0(z)}(t)}{C_P(t)}$$

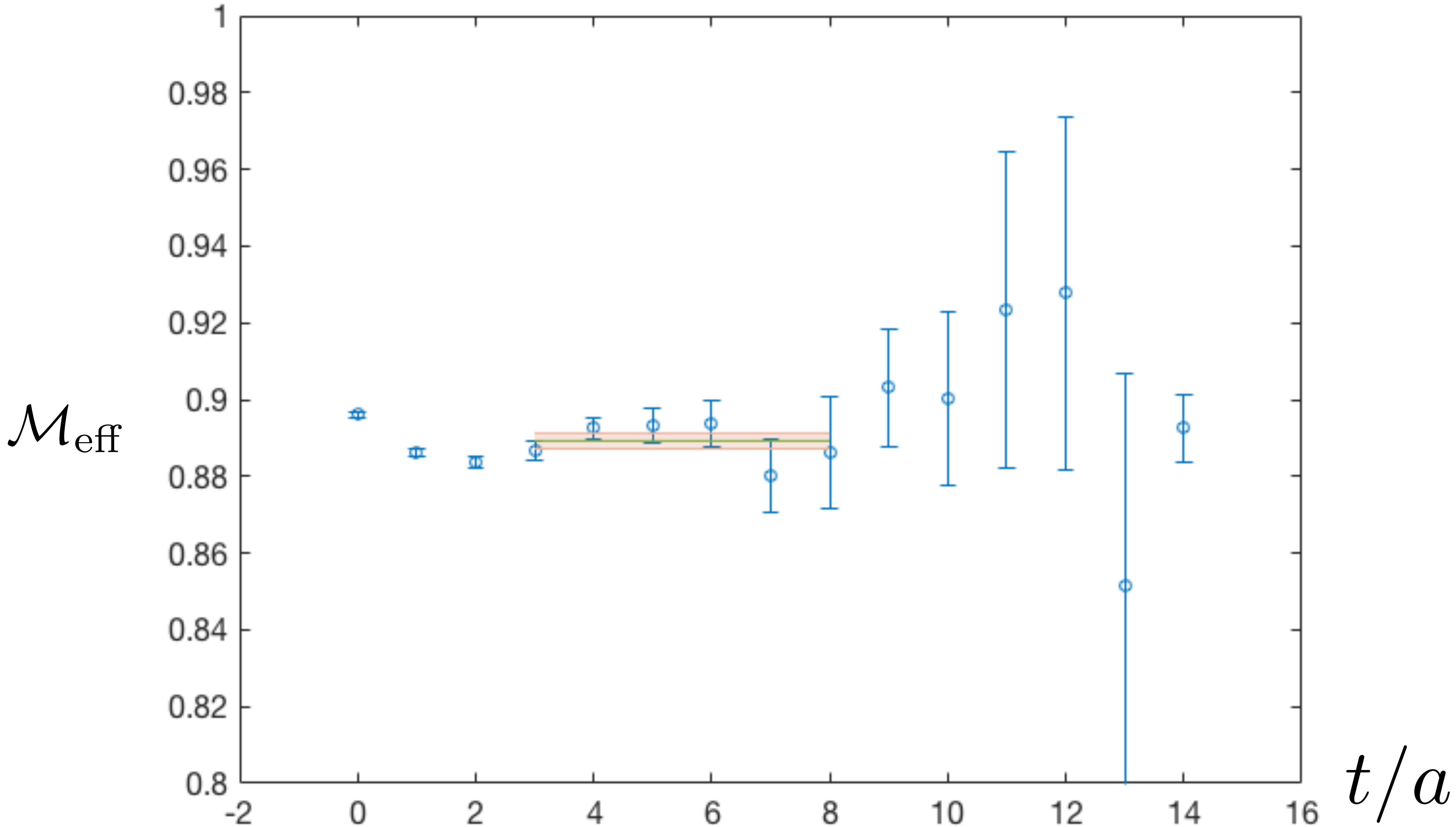
C. Bouchard, et al arXiv:1612.06963 [hep-lat]

$$\mathfrak{M}(\nu, z_3^2) = \lim_{t\to\infty} \frac{\mathcal{M}_{\text{eff}}(z_3 P, z_3^2; t)}{\mathcal{M}_{\text{eff}}(z_3 P, z_3^2; t)\big|_{z_3=0}} \times \frac{\mathcal{M}_{\text{eff}}(z_3 P, z_3^2; t)\big|_{z_3=0}}{\mathcal{M}_{\text{eff}}(z_3 P, z_3^2; t)\big|_{P=0}}$$

Constructed to remove lattice spacing errors



Gaussian smeared sources



1
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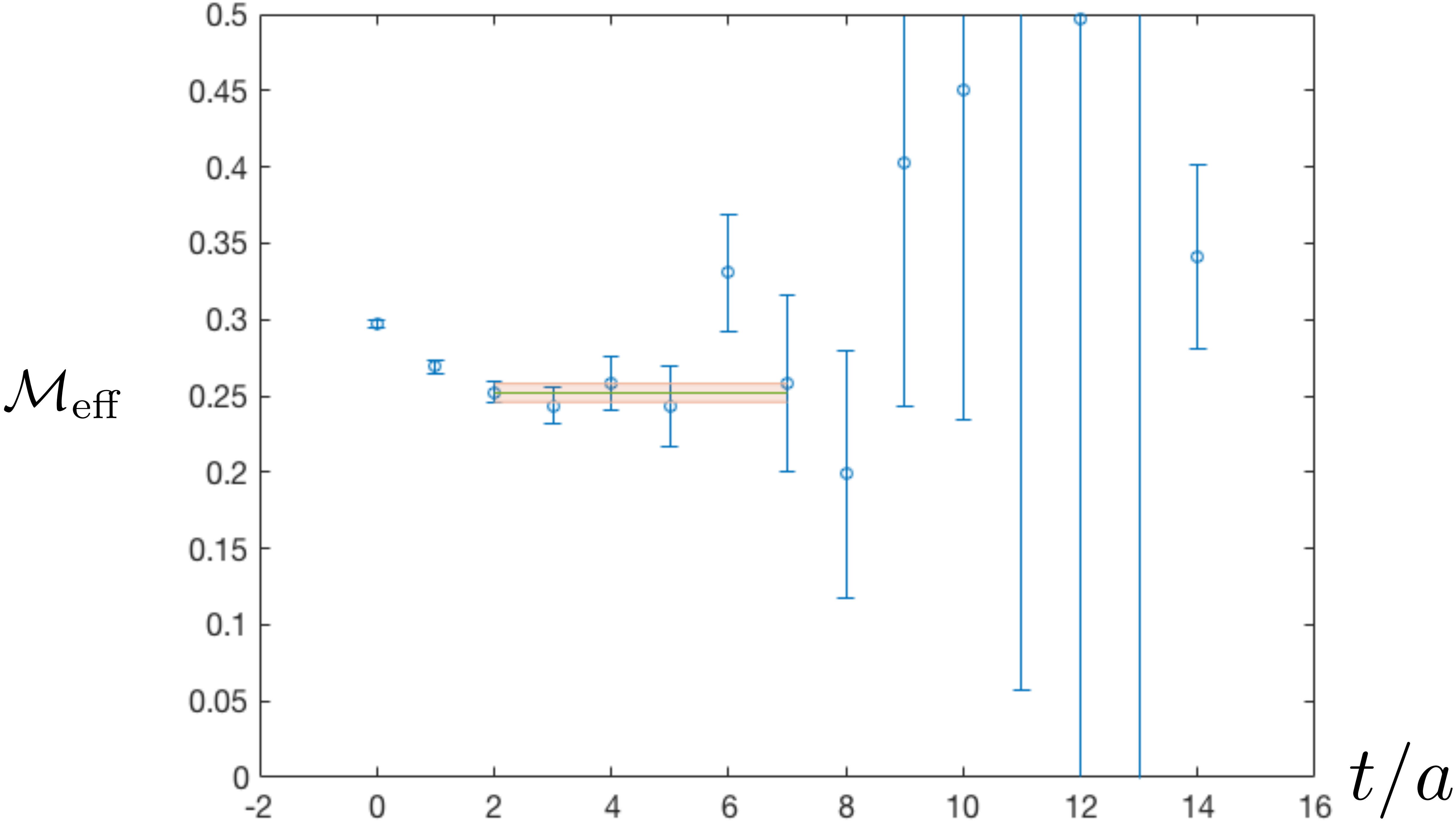
$\mathcal{M}_{\rm eff}$
t/a



$\mathcal{M}_{\text{eff}}$
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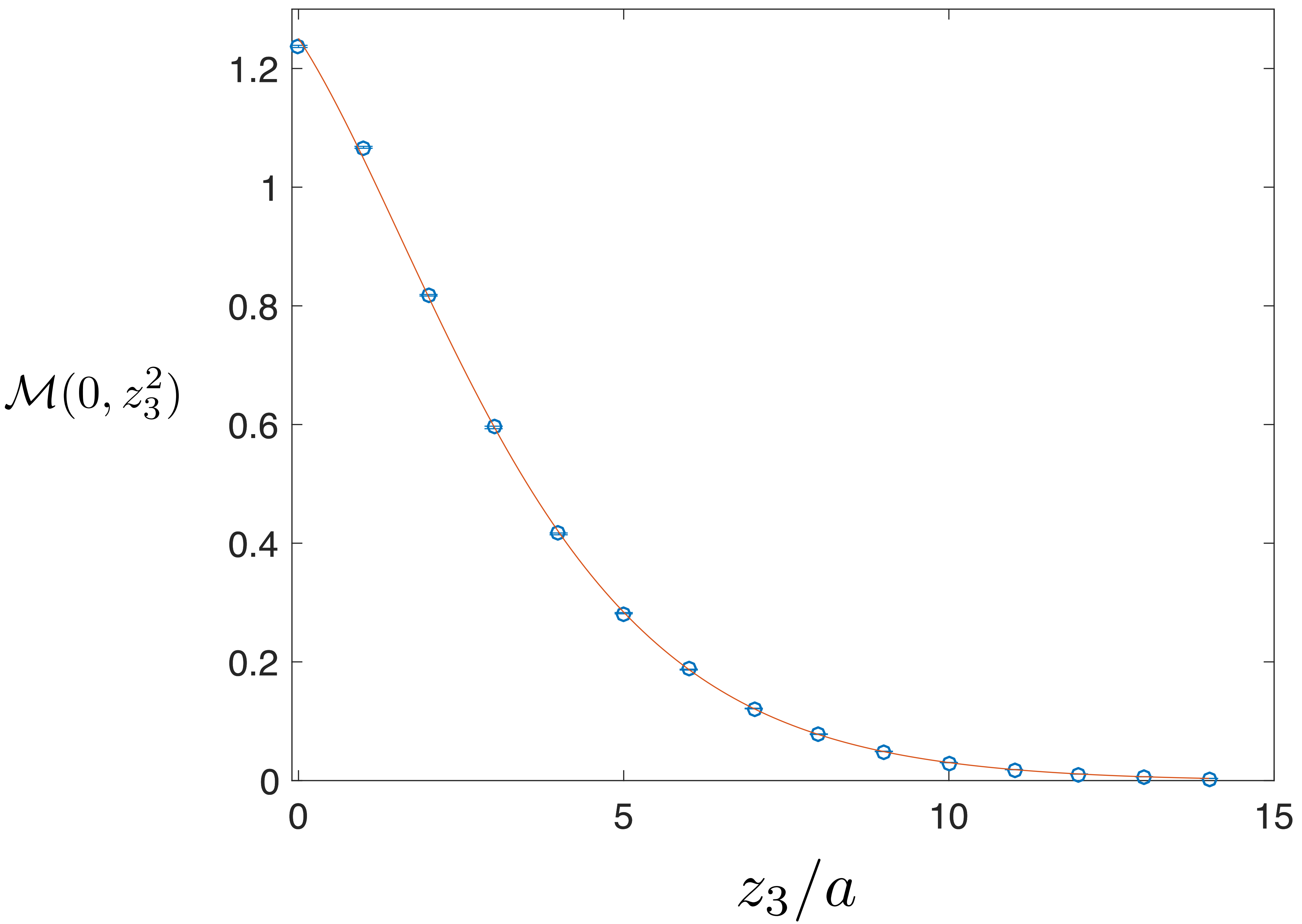
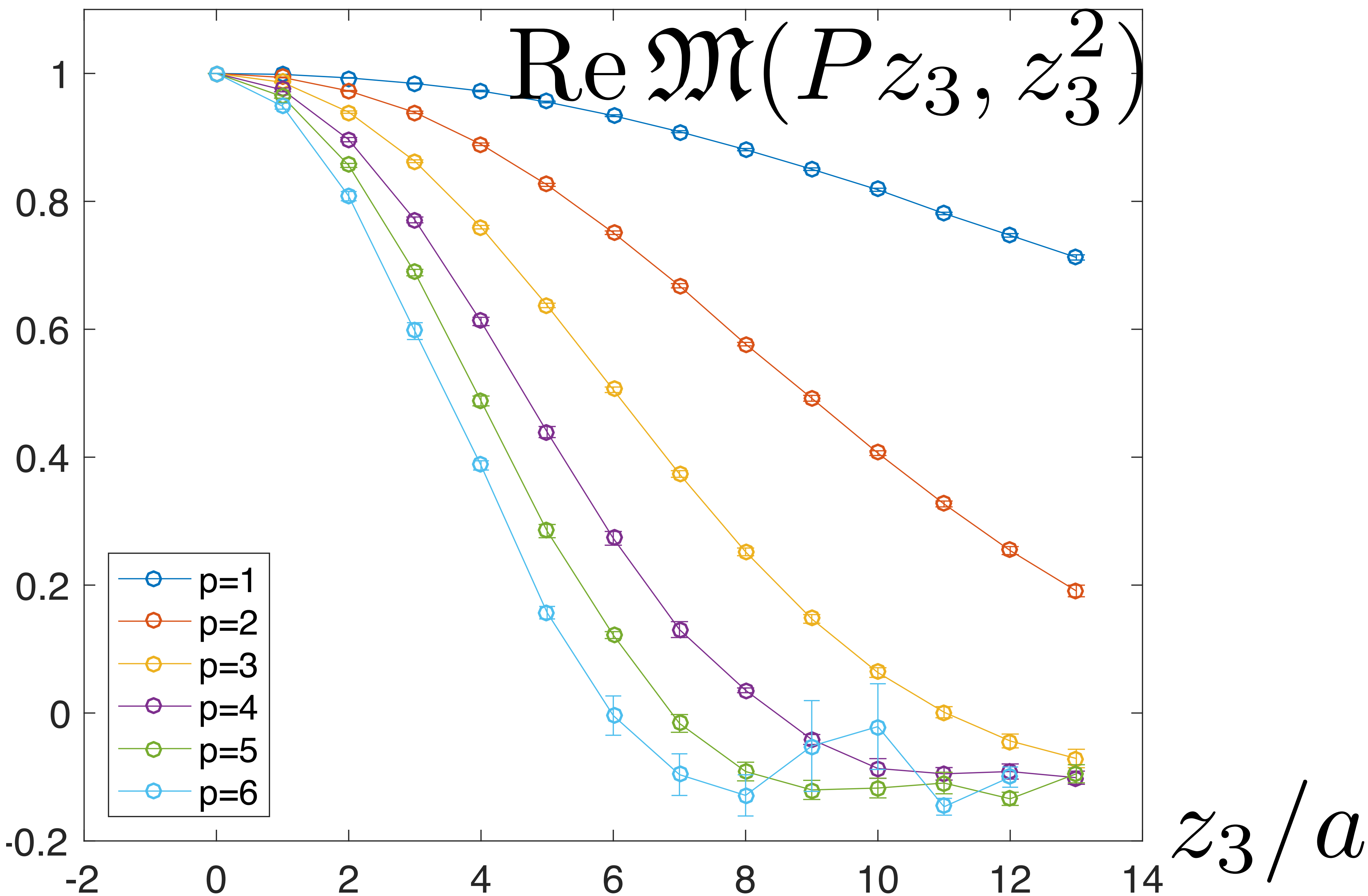
16
t/a



Cusp indicates “linear” divergence of Wilson line



Ratio removes the linear” divergence of Wilson line

Real Part

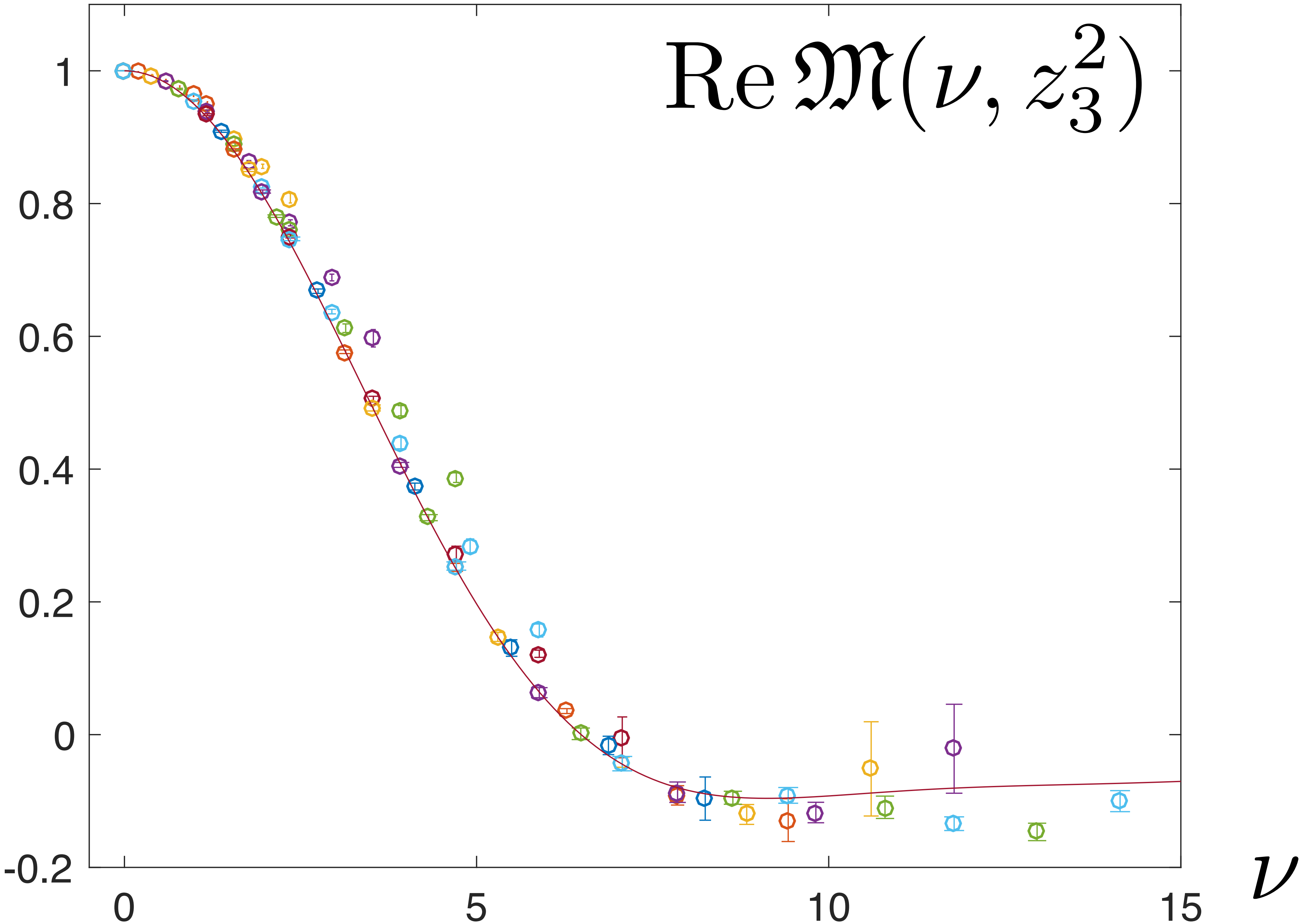
Isovector distribution

$$\mathfrak{M}_R(\nu, z^2 = 1/\mu^2) \equiv \int_0^1 dx\ \cos(\nu x)\, q_v(x, \mu^2)$$

$$q_v(x) = q(x) - \bar{q}(x) \qquad\qquad q(x) = u(x) - d(x)$$

$\overline{MS}$ $\qquad \mu^2 = (2e^{-\gamma_E}/z_3)^2$

Radyushkin arXiv:1710.08813



Points almost collapse on a universal curve

$$q_v(x) = \frac{315}{32}\sqrt{x}(1-x)^3$$

Imaginary Part

Isovector distribution

$$\mathfrak{M}_I(\nu, z^2 = 1/\mu^2) \equiv \int_0^1 dx \, \sin(\nu x) \, q_+(x, \mu^2) \, .$$

$$q_+(x) = q(x) + \bar{q}(x)$$

$$q(x) = u(x) - d(x)$$

$$q_+(x) = q_v(x) + 2\bar{q}(x)$$

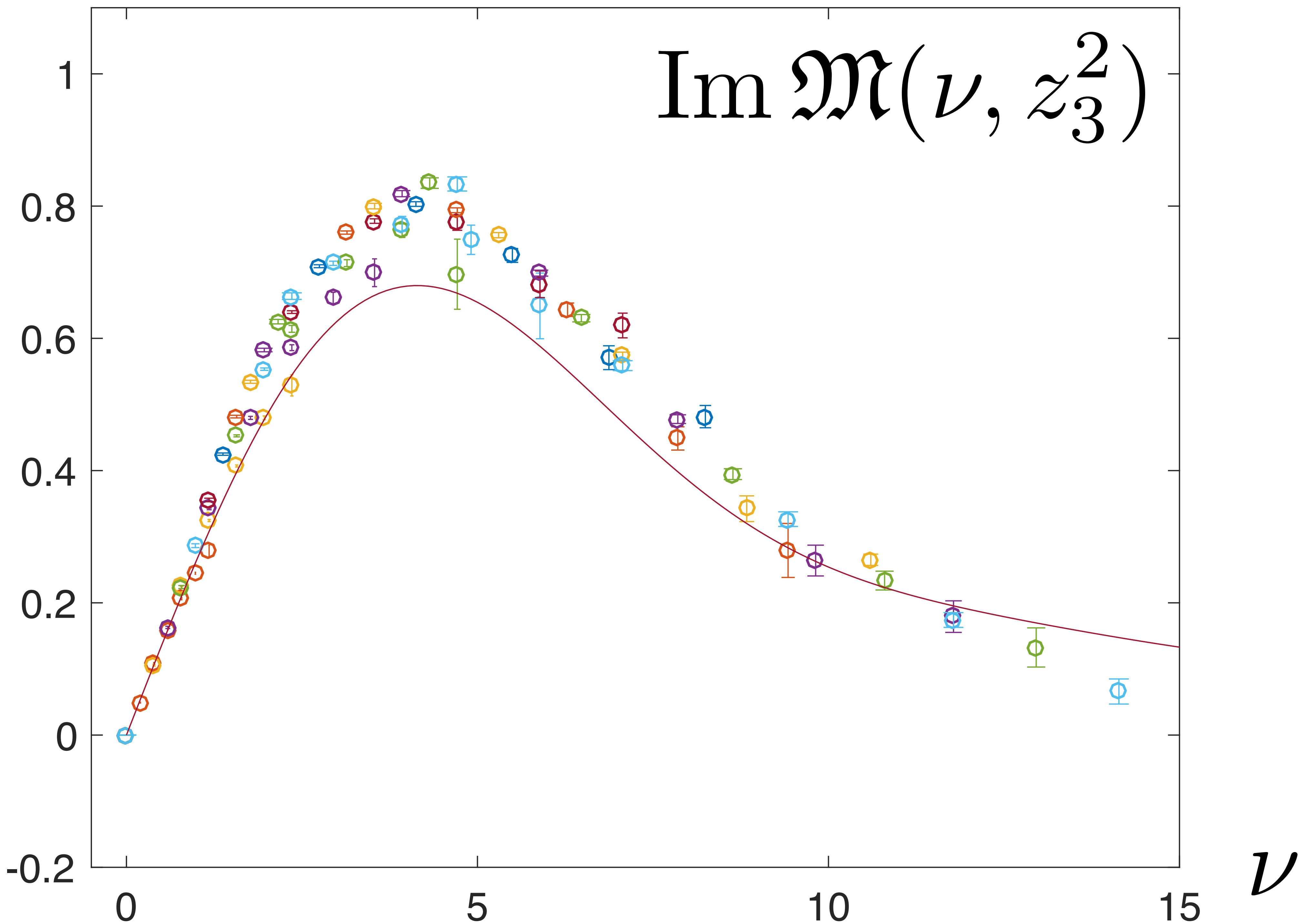
$$q_v(x) = q(x) - \bar{q}(x)$$

$\overline{MS}$ $\quad \mu^2 = (2e^{-\gamma_E}/z_3)^2$

Radyushkin arXiv:1710.08813

anti-quarks contribute to the imaginary part

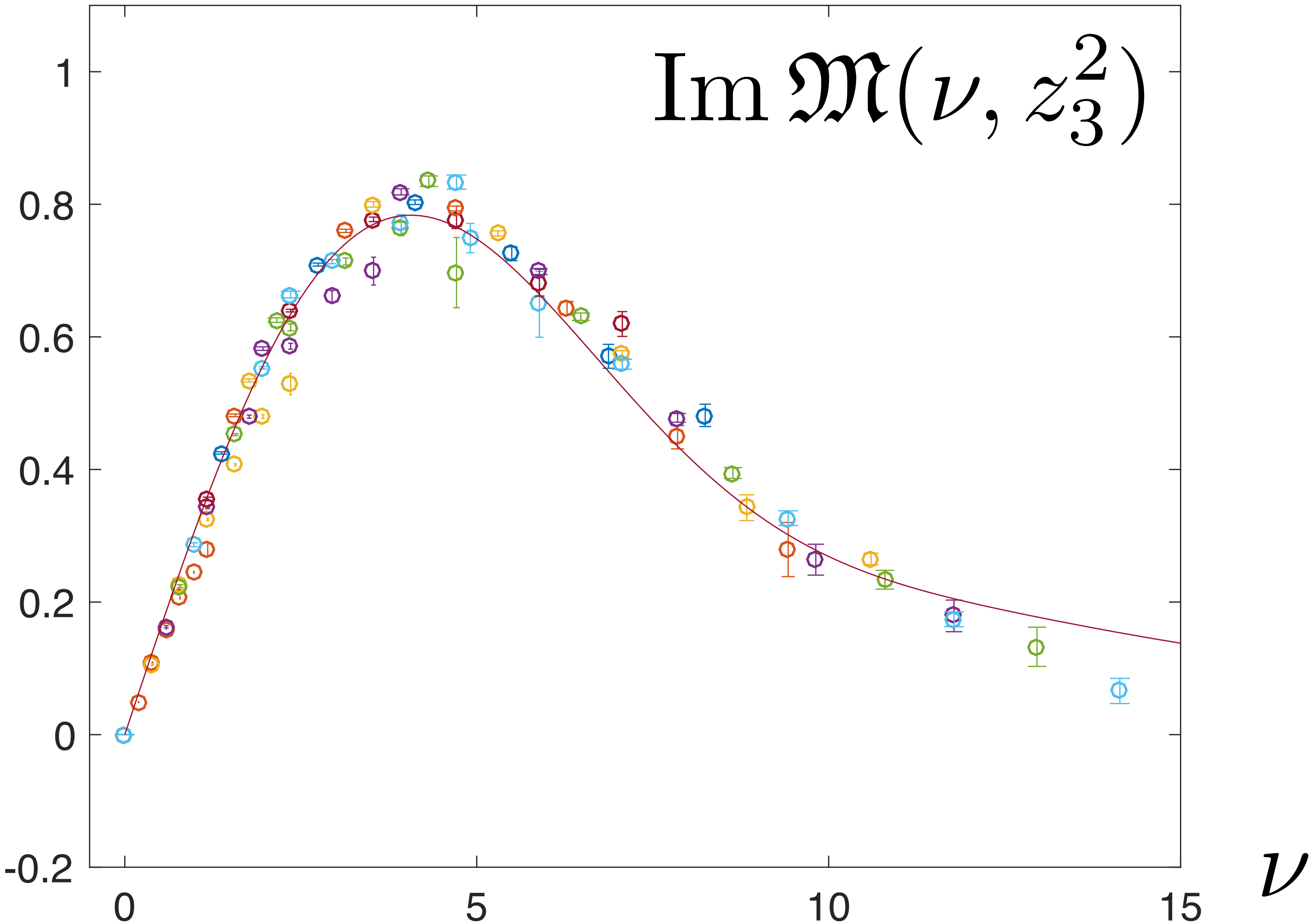
$$q_+(x) = q_v(x) + 2\bar{q}(x)$$



$$q_v(x) = \frac{315}{32}\sqrt{x}(1-x)^3 \qquad \bar{q}(x) = 0$$

anti-quarks contribute to the imaginary part

$$q_+(x) = q_v(x) + 2\bar{q}(x)$$



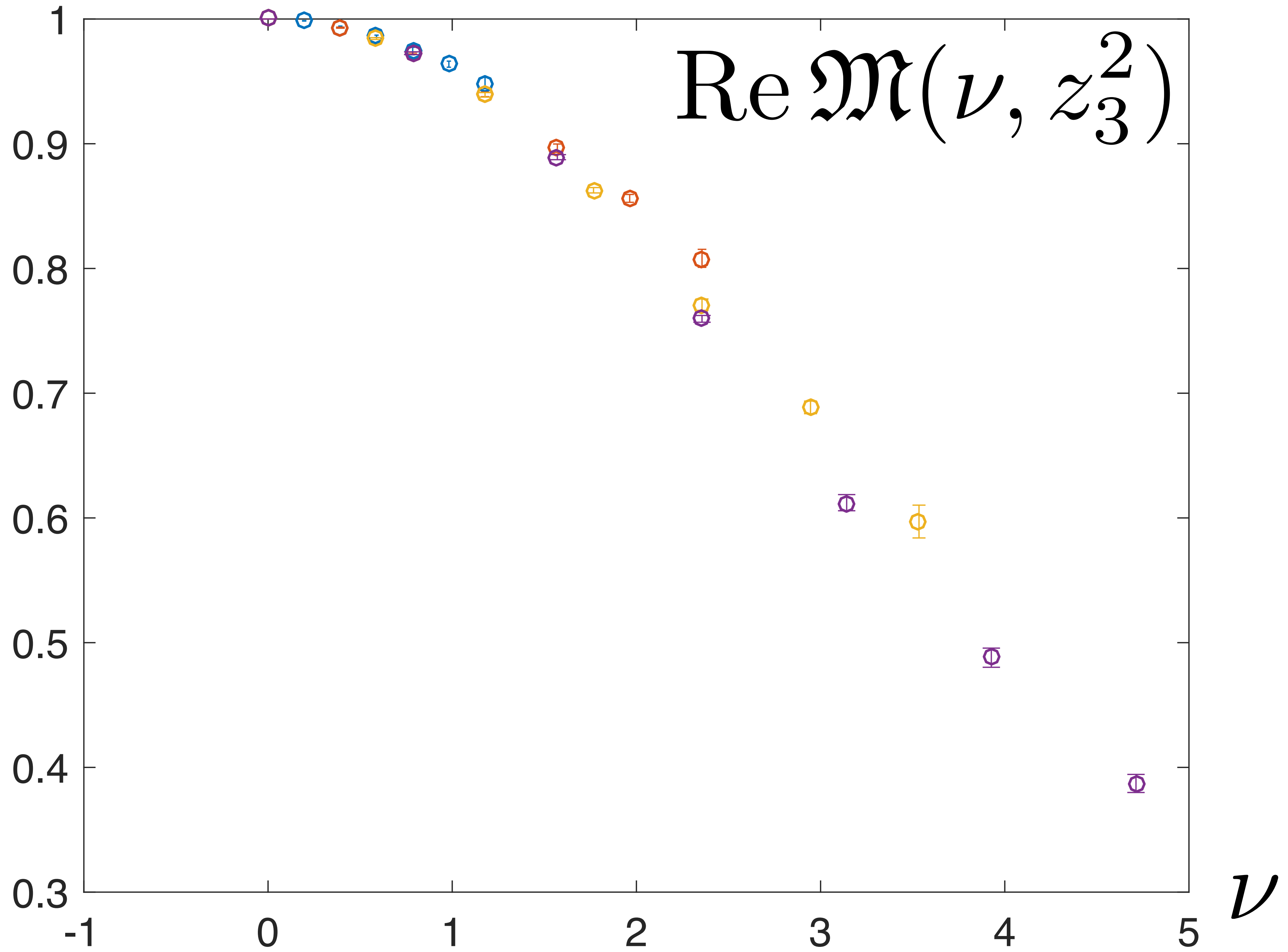
$$q_v(x) = \frac{315}{32}\sqrt{x}(1-x)^3 \qquad \bar{q}(x) = 1.4\,x\,(1-x)^3$$

Points in previous plots obtained in with different z/a
i.e. correspond to the Ioffe time PDF at different scales!

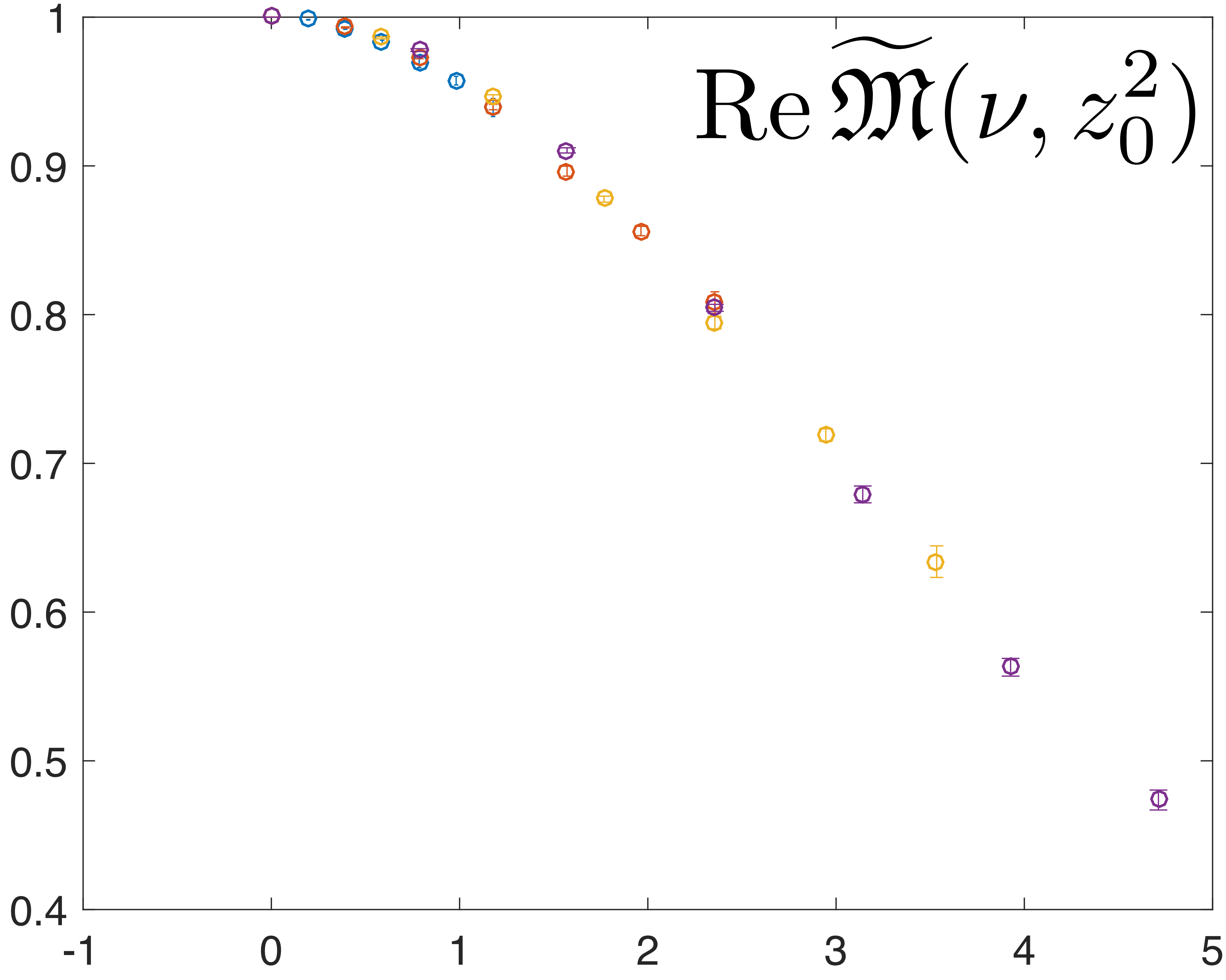
DGLAP evolution:

$$\mathfrak{M}(\nu, {z'}_3^2)=\mathfrak{M}(\nu, z_3^2) \ - \frac{2}{3}\frac{\alpha_s}{\pi}\ln({z'_3}^2/z_3^2) B \otimes \mathfrak{M}\,(\nu, z_3^2)$$

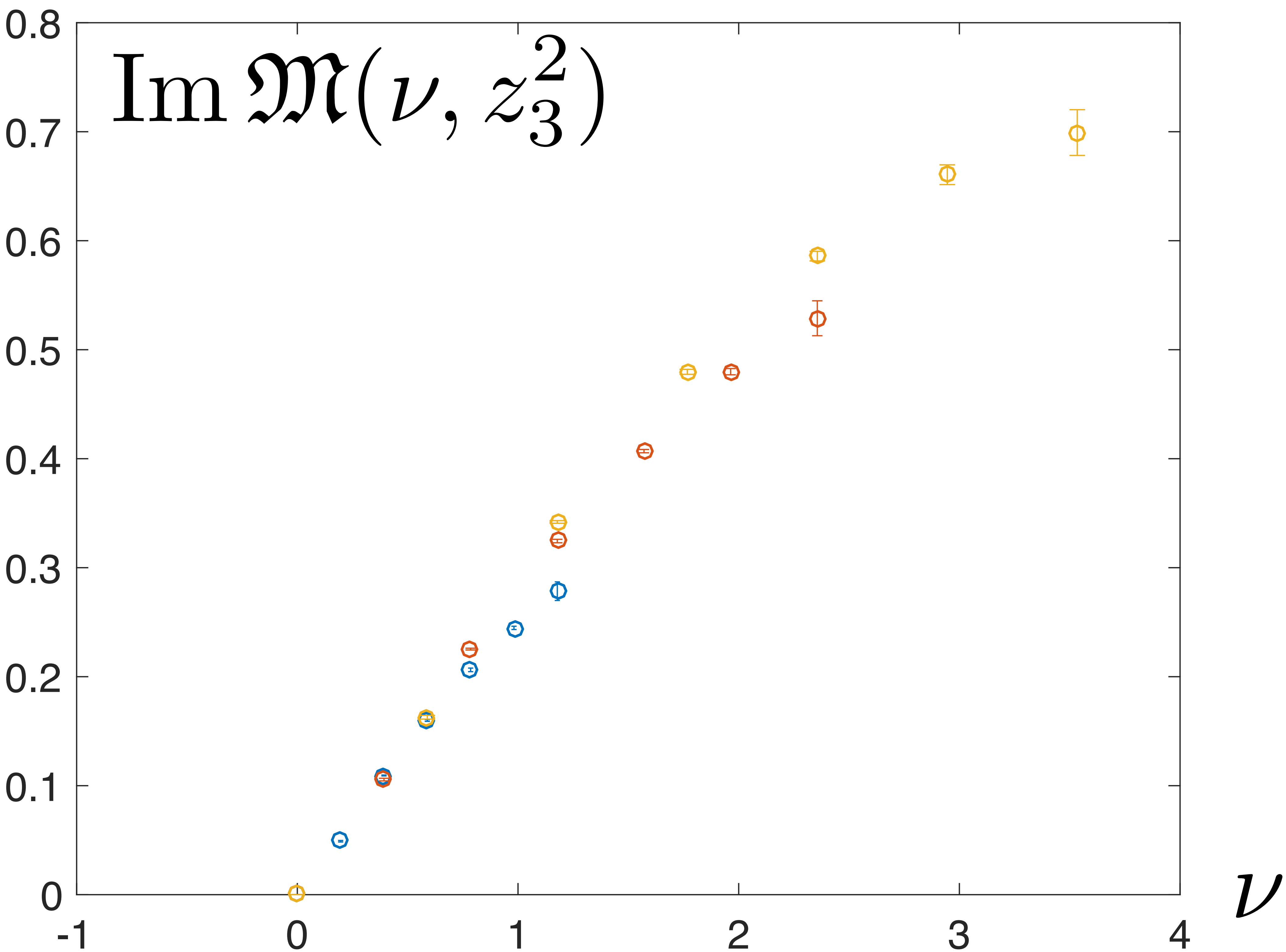
Apply evolution only at short distance points [~1GeV]



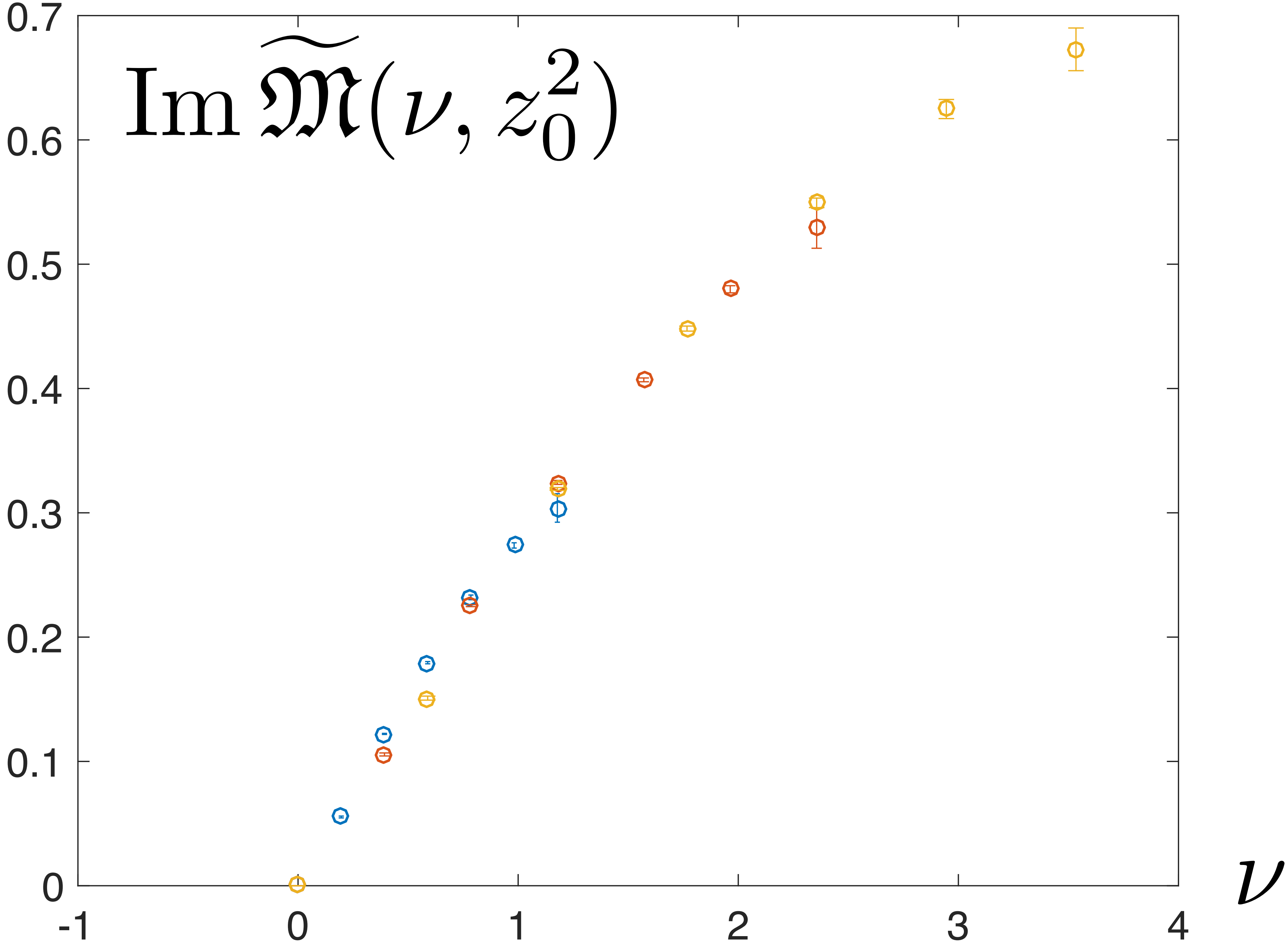
Data corresponding to z/a= 1, 2, 3, 4



Evolved to 1GeV



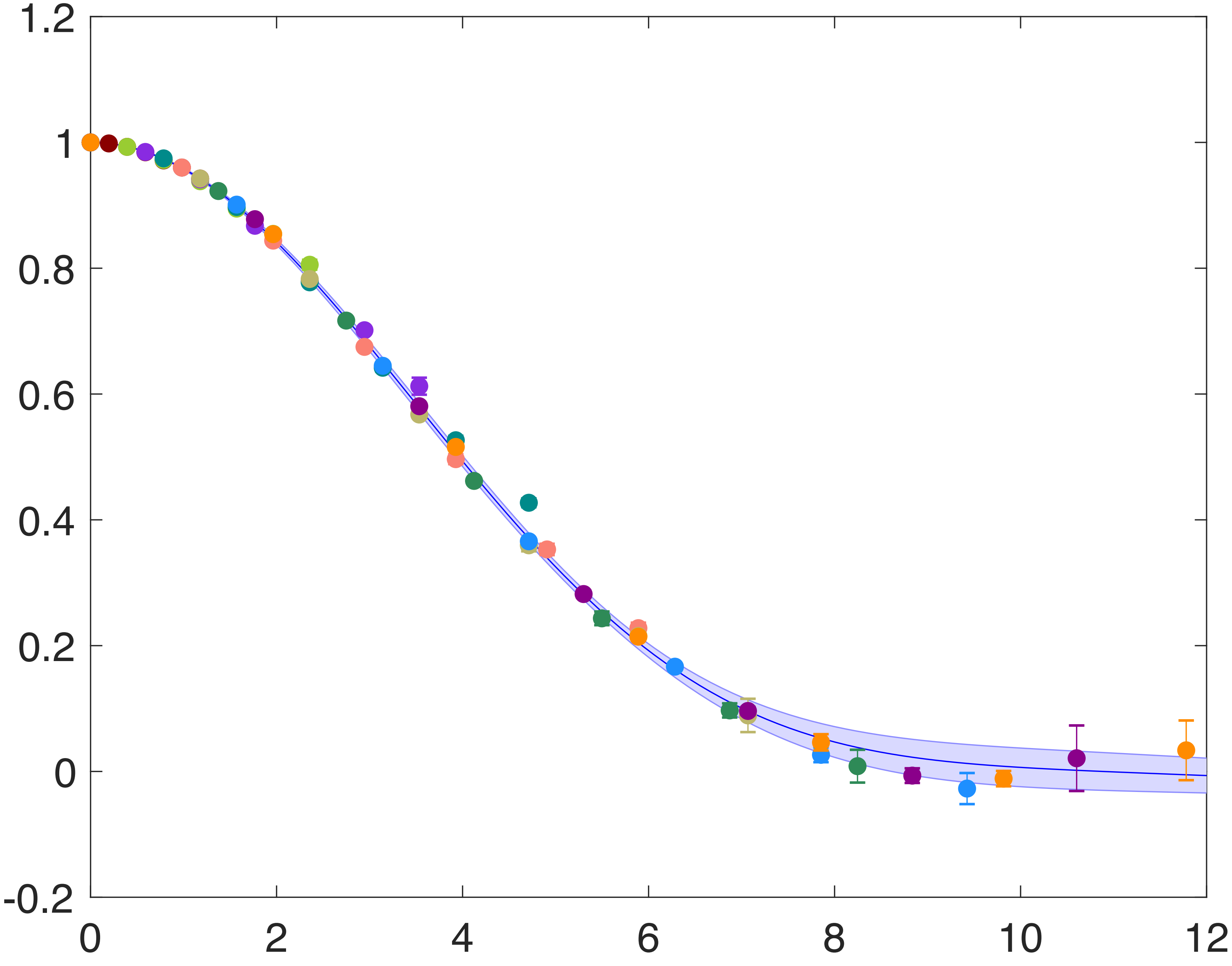
Data corresponding to z/a= 1, 2, 3, 4



Evolved to 1GeV

$$\mathfrak{M}(\nu, z^2) = \int_0^1 d\alpha\, \mathfrak{C}(\alpha, z^2\mu^2, \alpha_s(\mu))\mathcal{Q}(\alpha\nu, \mu)$$

μ = 1 GeV



Fit to:

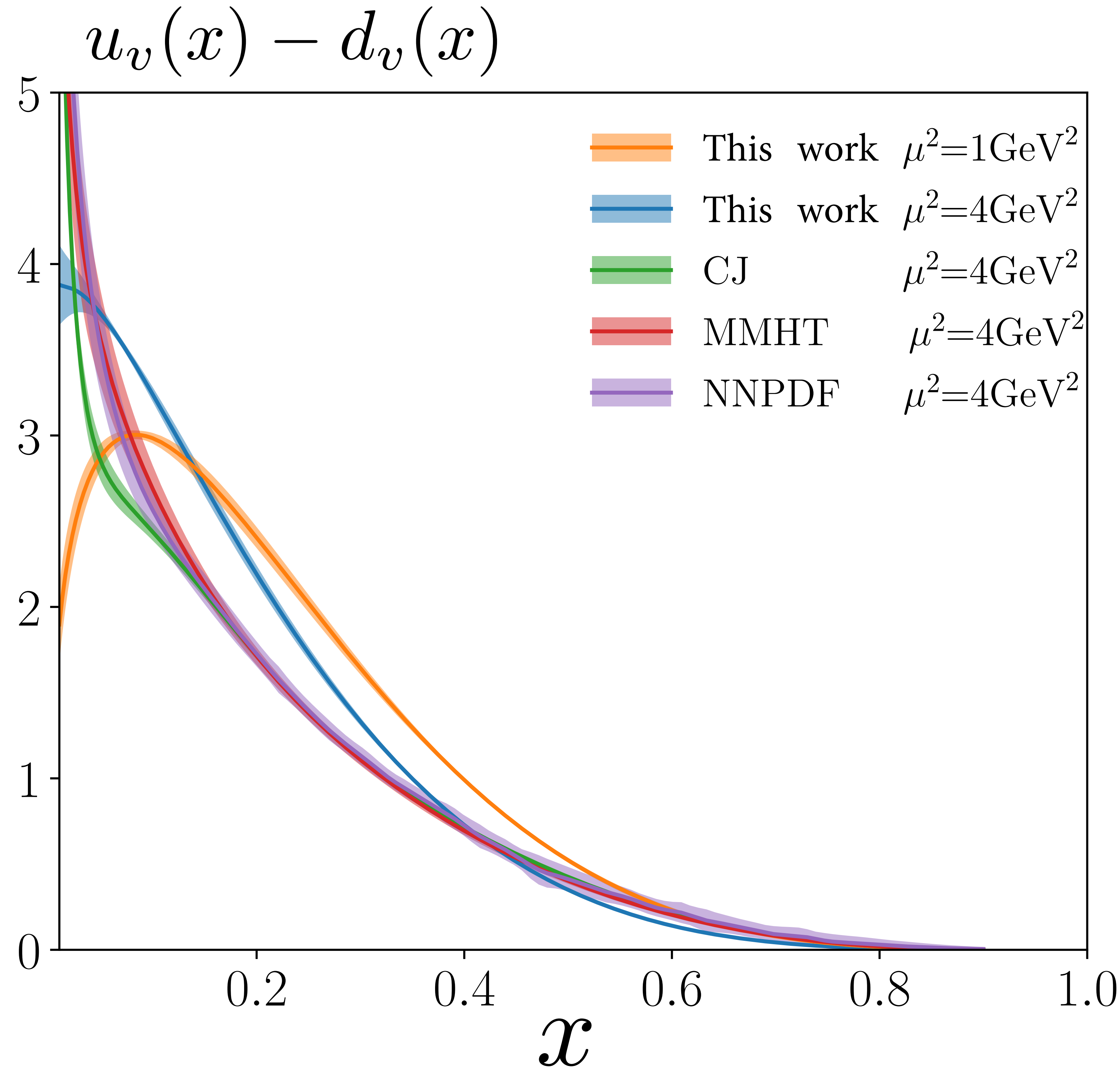
$$N(a,b)x^a(1-x)^b$$

$$a = 0.36(6)$$

$$b = 3.95(22)$$

Ignoring polynomial corrections

$$\mathcal{Q}(\nu, \mu^2) = \int_0^1 dx\, cos(\nu x) q_v(x, \mu^2)$$



Thanks to N. Sato for making this figure

The Moments

Karpie et al. arXiv:1807.10933

As a consequence:

$$(-i)^n \frac{\partial^n \mathfrak{M}(\nu, z^2)}{\partial \nu^n}\bigg|_{\nu=0} = c_n(z^2\mu^2) a_{n+1}(\mu) + \mathcal{O}(z^2)\,.$$

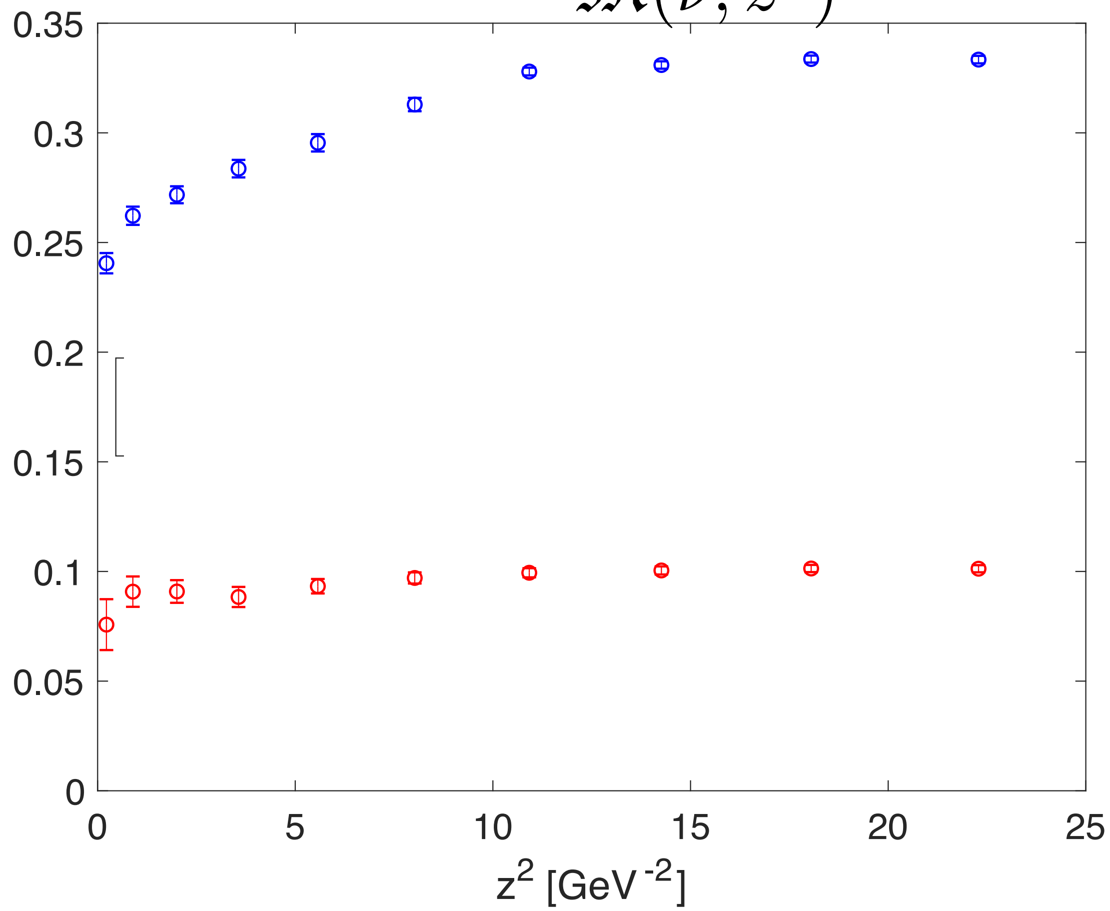
Where the Wilson coefficients are

$$c_n(z^2\mu^2) = \int_0^1 d\alpha\, \mathcal{C}(\alpha, z^2\mu^2, \alpha_s(\mu))\alpha^n\,.$$

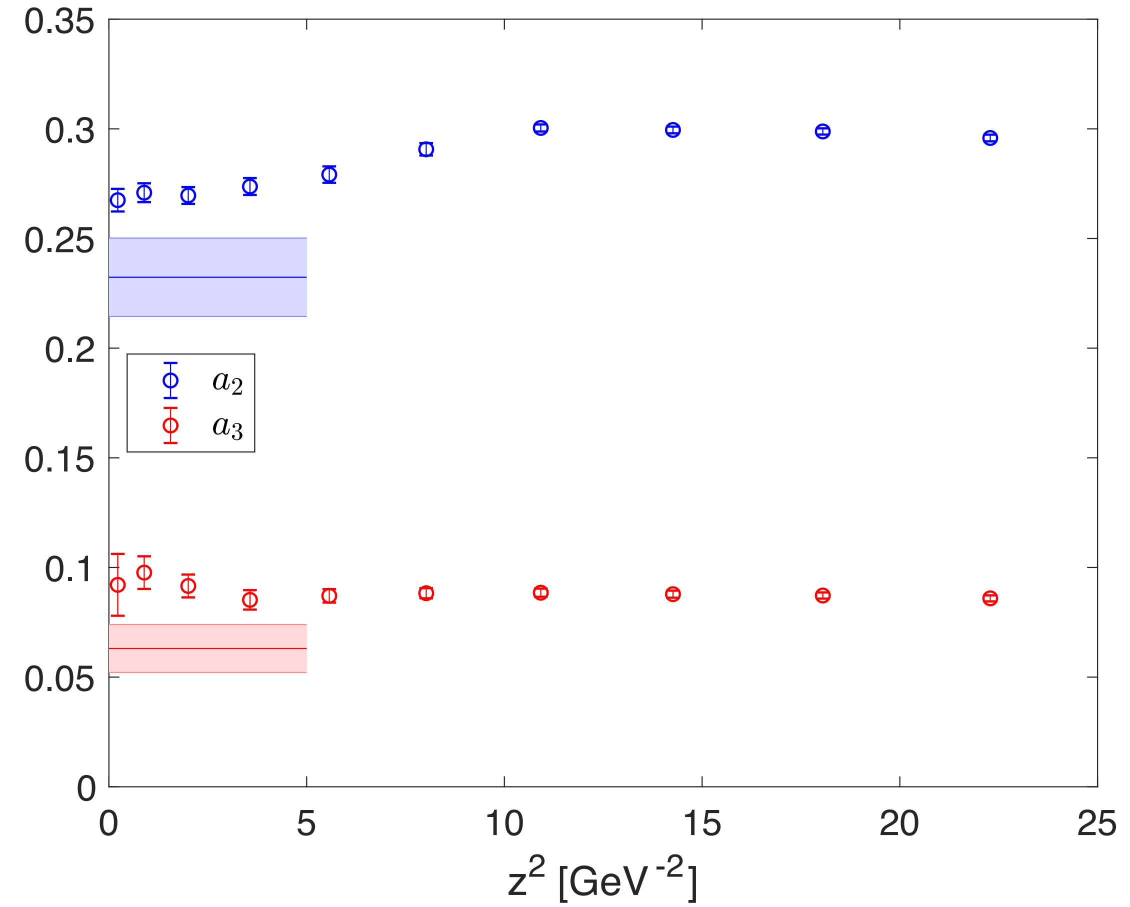
The Moments

Quenched QCD

derivatives of $\mathfrak{M}(\nu, z^2)$



extracted moments



QCDSF: Phys.Rev. D53 (1996) 2317-2325

μ=3 GeV

PDF reconstruction

PDFs cannot be directly computed

PDFs can only be reconstructed from matrix elements

Just they are obtained from cross-sections

Reason: Fourier transform cannot be done with limited data

work done with: *Joe Karpie, Alexander Rothgopf, Savvas Zafeiropoulos*

Ioffe-time Pseudo PDF

$$\mathfrak{M}(\nu, z^2) = \int_0^1 d\alpha\, \mathfrak{C}(\alpha, z^2\mu^2, \alpha_s(\mu))\mathcal{Q}(\alpha\nu, \mu) \; + \sum_{k=1}^{\infty} \mathcal{B}_k(\nu)(z^2)^k$$

$$\mathcal{Q}(\nu, \mu) = \int_{-1}^{1} dx\, e^{-ix\nu} f(x, \mu)$$

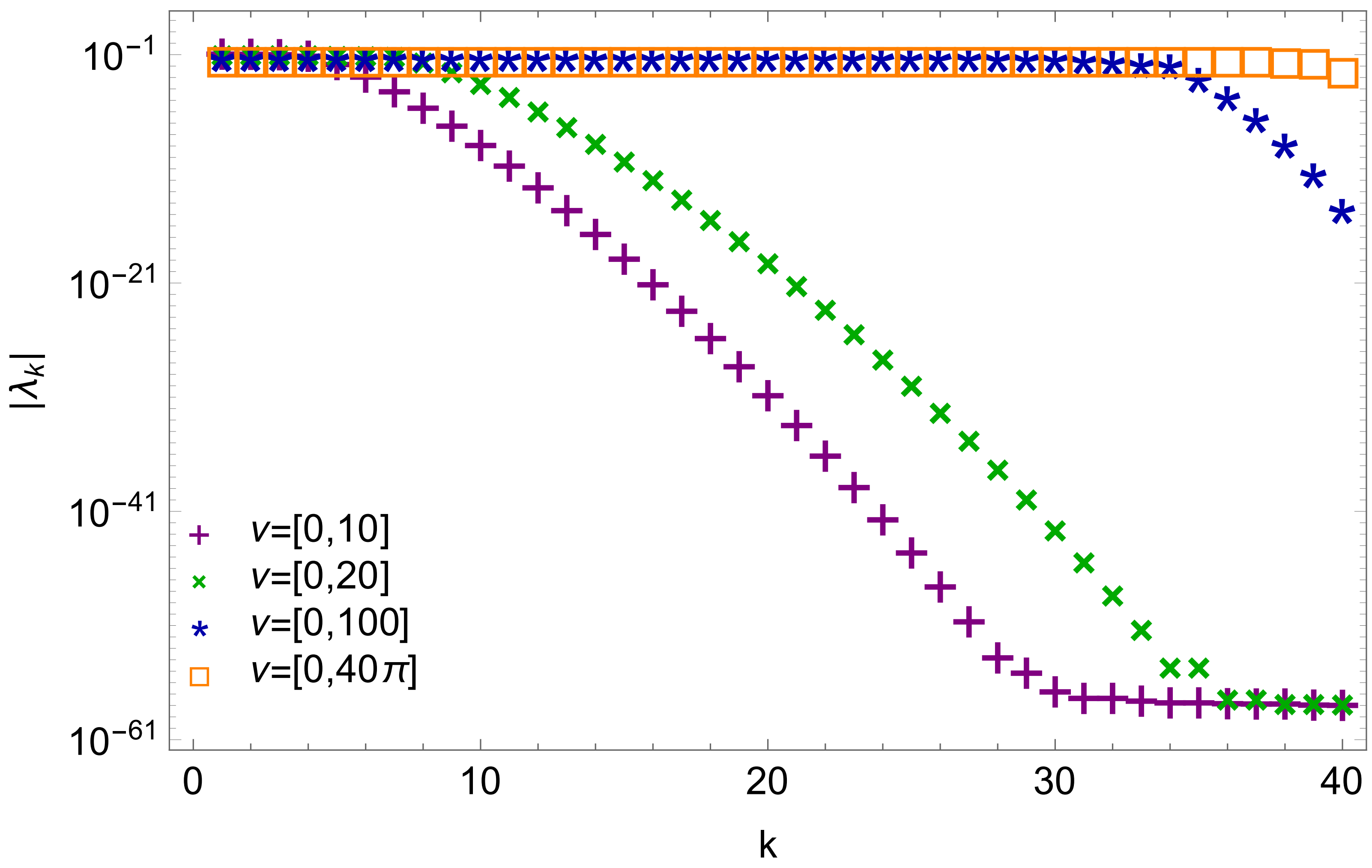
Computed only for limited values ν at fixed z^2

Quasi-PDF

$$Q(y, p_3) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \mathcal{M}_p(\nu, \nu^2/p_3^2) e^{-iy\nu}$$

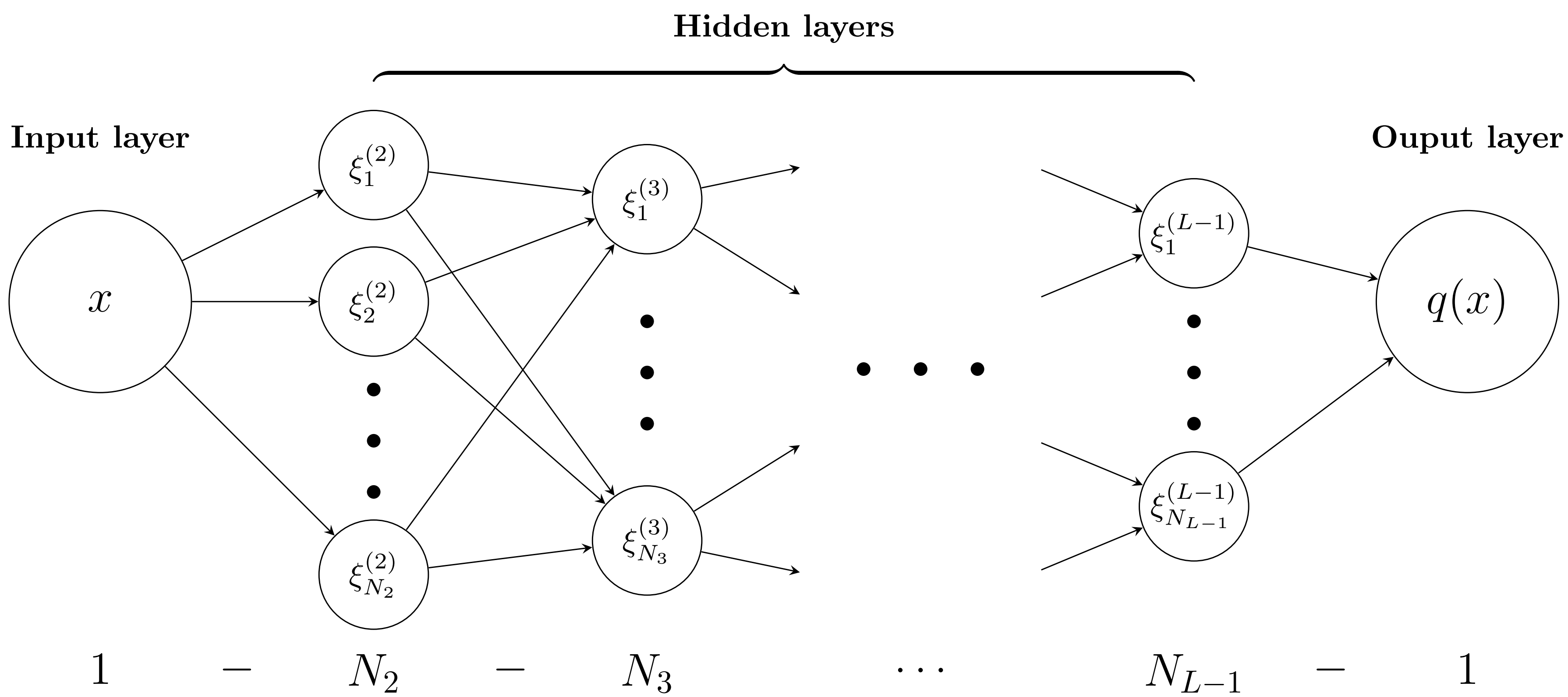
Computed only for limited values ν at fixed p_3

Eigenvalues of the discrete inversion kernel



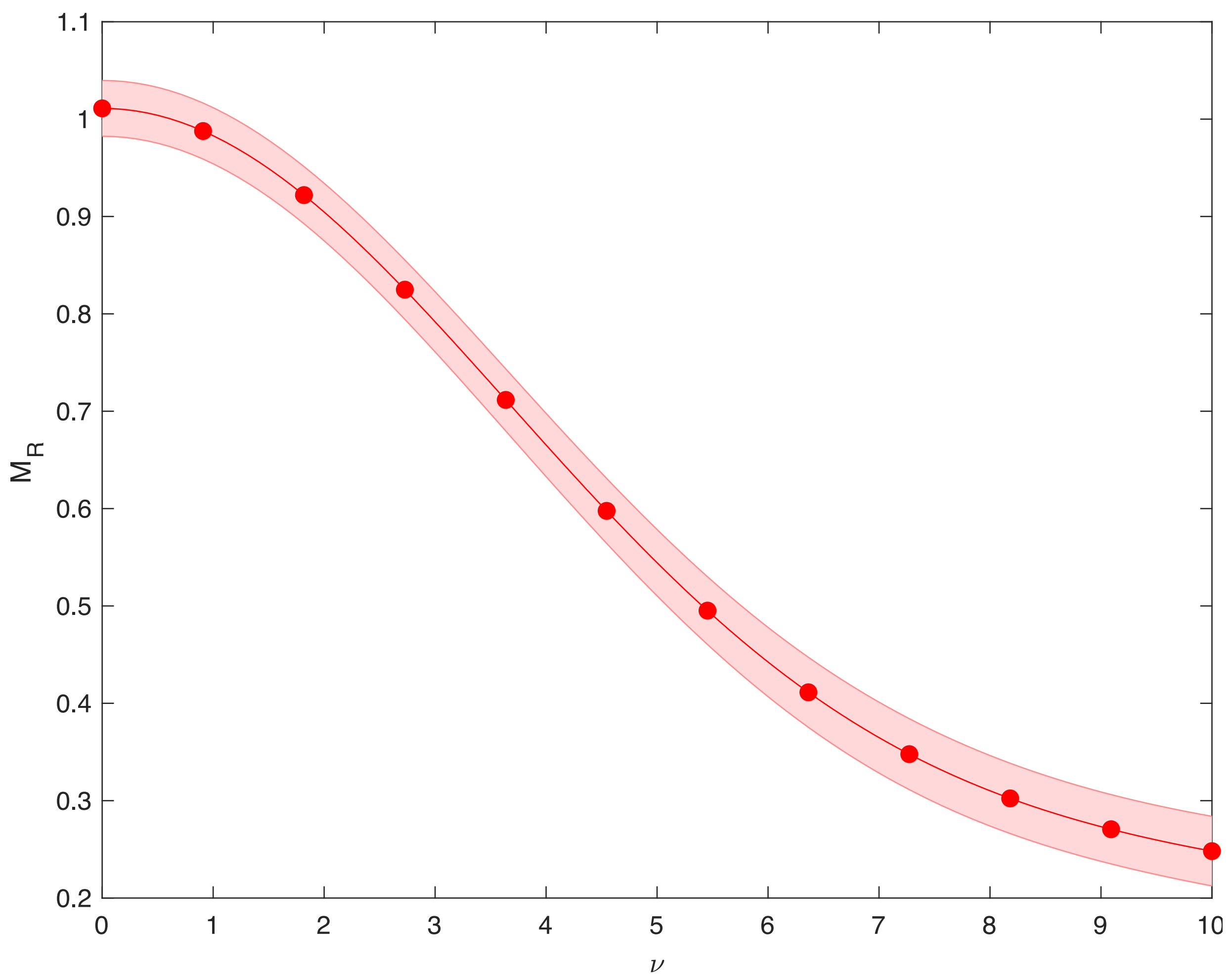
40 data points

Neural networks as fitting form

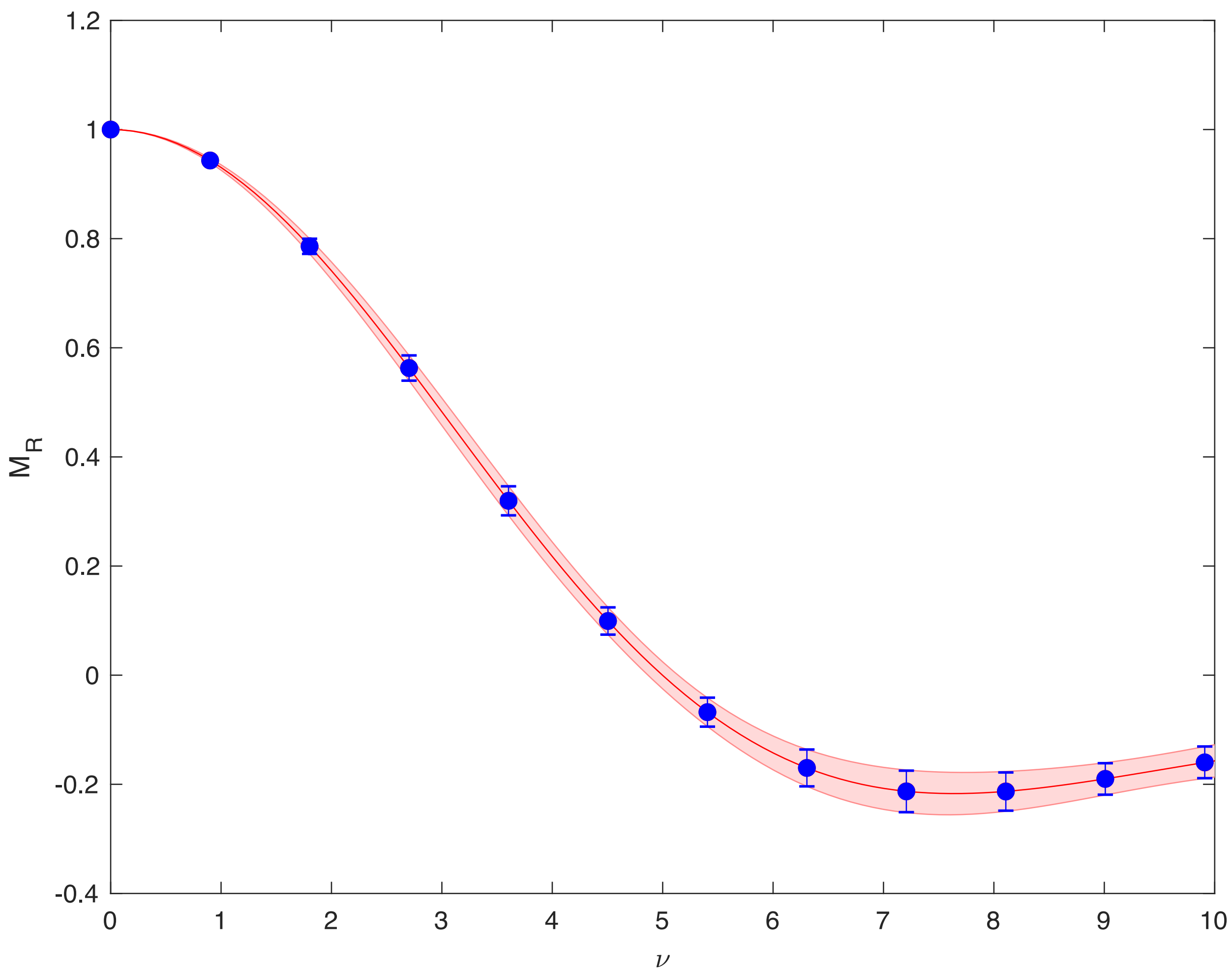


Methodology used by NNPDF

Numerical experiments of reconstruction

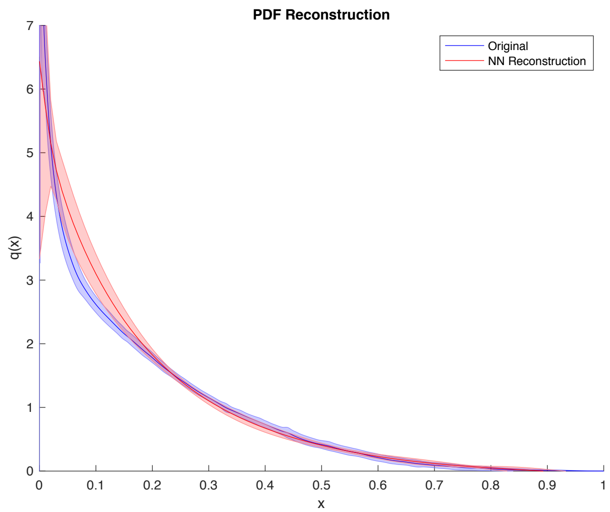


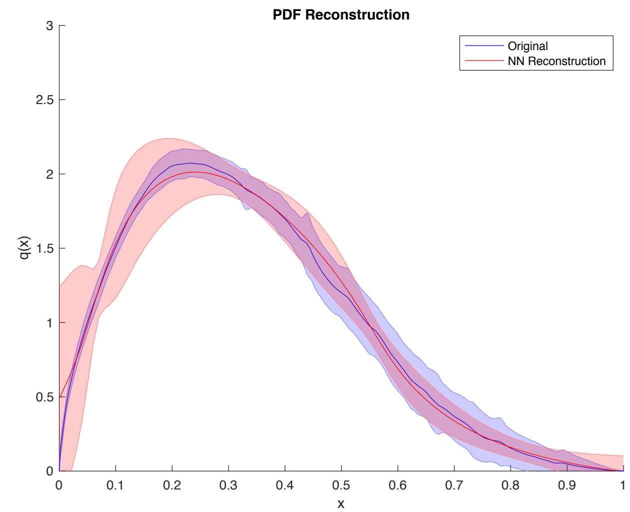
NNPDF 3.1



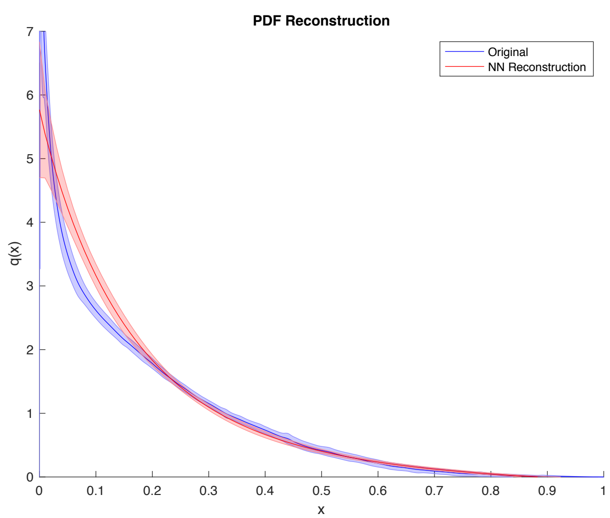
Unphysical model

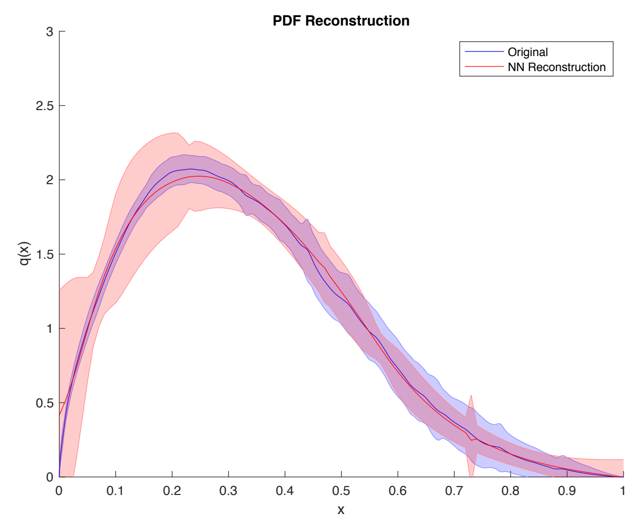
Network topology



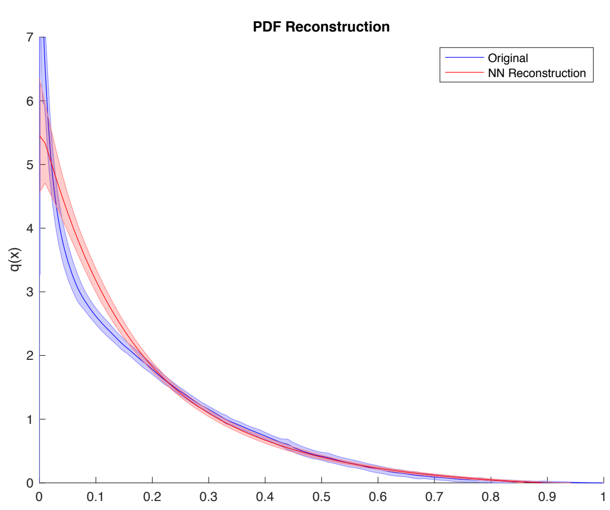


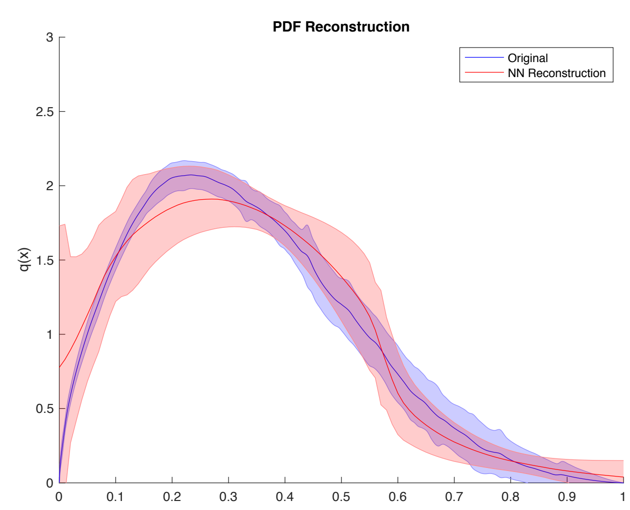
1-3-1





1-4-1





1-2-2–1

Summary

- Methods for obtaining parton distribution from Lattice QCD have now emerged
- An approach based on pseudo-PDFs has been proposed
  - Renormalization is handled in a simple way
  - Light cone limit is obtained by computing real space matrix elements at short Euclidean distances
  - All hadron momenta are useful in obtaining PDFs (including the low momenta)
- WM/JLab: first numerical tests are available in quenched approximation indicating the feasibility of the method
  - Results consistent with DGLAP evolution
- Dynamical fermion simulations are on the way
- Lattice spacing effects under study
- Probing the small x region (or large Ioffe time) remains a challenge
  - Large Ioffe time may be probed with high momentum which requires a small lattice spacing
- Correctly applying evolution, matching and controling polynomial corrections is essential for obtaining reliable results