Energy dynamics and maximal quantum chaos

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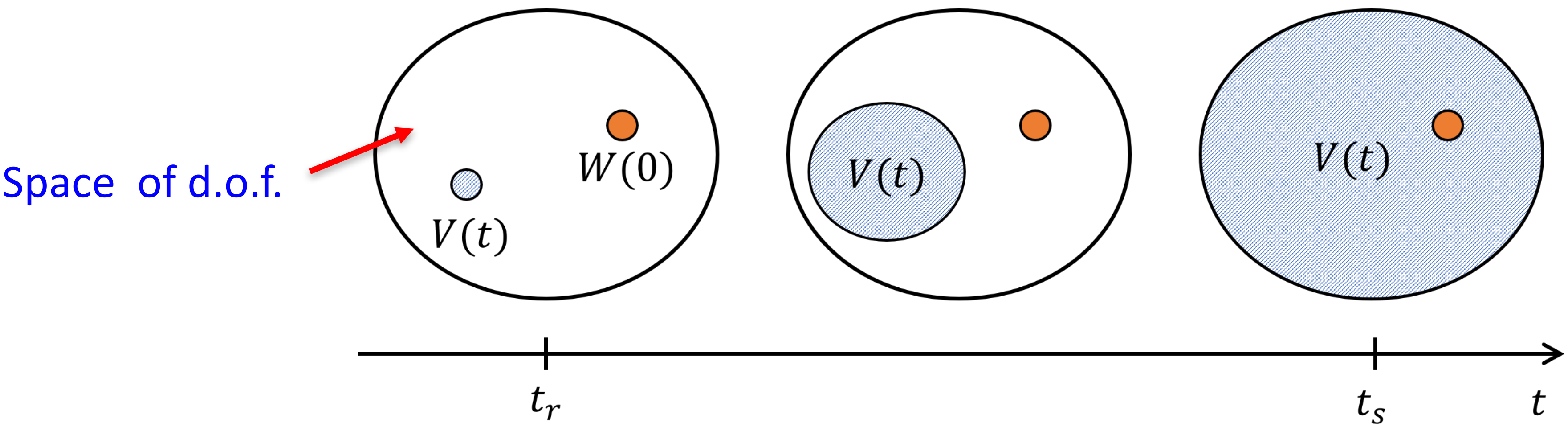
Mike Blake, Hyunseok Lee, HL, arXiv: 1801.00010

Mike Blake, Richard Davison, Saso Grozdanov, HL, 1809.01169

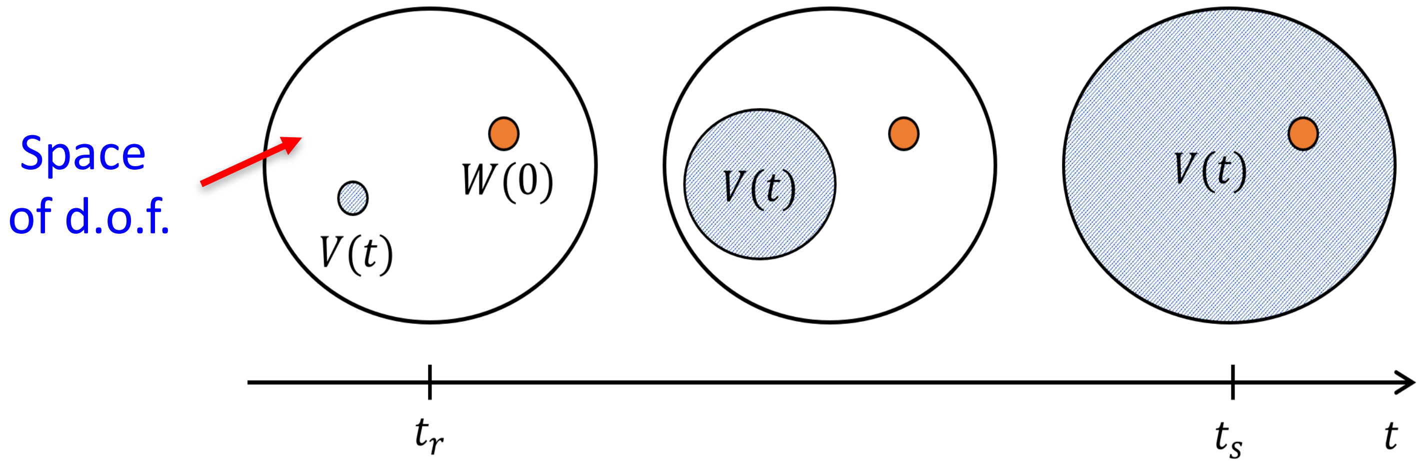
Mike Blake, HL, to appear

Scrambling

$$V(t) = e^{iHt} V(0) e^{-iHt}$$



Scrambling and quantum chaos



V, W generic
few-body operators

$$C(t) = -\langle [V(t), W(0)]^2 \rangle_{\beta_0}$$

$$\sim \frac{1}{\mathcal{N}} e^{\lambda t}$$ (chaotic systems)

Larkin and Ovchinnikov, 1969,
Shenker, Stanford, Kitaev 2013

quantum
Chaos bound: $$\lambda \leq \frac{2\pi}{\beta_0 \hbar}$$

Maldacena, Shenker,
Stanford, 2015

Ballistic spreading

$$C(t,\vec{x}) \equiv -\langle [V(t,\vec{x}), W(0)]^2 \rangle_{\beta_0}$$

$$\sim \frac{1}{\mathcal{N}} e^{\lambda(t - \frac{|\vec{x}|}{v_B})}$$

holographic theories, SYK chain

Roberts, Shenker, Stanford, Susskind, Gu, Qi, …

……..

v_B : Butterfly velocity

Out of time order correlation functions

$$C(t) = -\langle [V(t), W(0)]^2 \rangle_{\beta_0} = C_1(t) - C_2(t)$$

$$C_1(t) = \langle V(t)W(0)W(0)V(t) \rangle_{\beta_0} + \langle W(0)V(t)V(t)W(0) \rangle_{\beta_0}$$

Time ordered correlators (TOCs)

$$C_2(t) = \langle W(0)V(t)W(0)V(t) \rangle_{\beta_0} + \langle V(t)W(0)V(t)W(0) \rangle_{\beta_0}$$

Out-of-Time ordered correlators (OTOCs)

Maximally chaotic systems

$$\lambda_{\max} = \frac{2\pi}{\beta_0 \hbar}$$

1. all holographic systems in the classical gravity limit

2. Sachdev-Ye-Kitaev model and its variants in the low temperature limit

3. CFT correlation functions in the light-cone limit

What is special about maximally chaotic systems?

A common theme:

Only stress tensor is exchanged in the OTOCs

A natural conjecture:

Maximal chaos always appears in the limit involving only stress tensor exchanges

The exponential behavior and butterfly spreading were obtained from detailed calculations of OTOCs in each specific systems.

We developed a universal description of maximally chaotic behavior in terms of quantum hydrodynamics.

with interesting implications and predictions

Hydrodynamics

For a generic finite temperature system at sufficiently long distances and times, only dynamics of conserved quantities are relevant, all other details are washed out by interactions.

Hydrodynamics is the theory of the conserved quantities:

$$\partial_\mu T^{\mu\nu} = 0$$

thus a universal theory for non-equilibrium dynamics of generic many-body systems at sufficiently long distances and times

$$\delta L \gg \ell_{\rm relax}, \quad \delta t \gg \tau_{\rm relax}$$

Traditional formulation: equations of motion, no statistical or quantum fluctuations.

Need for a quantum hydrodynamics

- Quantum chaos operates outside the standard hydro regime.

Standard Hydro: $\delta L \gg \ell_{\rm relax}, \quad \delta t \gg \tau_{\rm relax}$

$$C(t) = -\langle [V(t), W(0)]^2 \rangle_{\beta_0} \sim \frac{1}{\mathcal{N}} e^{\lambda t}$$

$$\delta t \sim \frac{1}{\lambda} \qquad \lambda \sim \frac{1}{\tau_{\rm relax}} \sim \frac{T}{\hbar} \qquad \boxed{\delta t \sim \tau_{\rm relax}}$$

This is quantum chaos, different regime from turbulence.

- Need both statistical and quantum fluctuations
- How can exponential growth be compatible with energy conservation?

We will use a new formulation of hydrodynamics we developed recently which is based on symmetries and action principle:

arXiv: 1511.03646, 1612.07705, 1701.07817 Crossley, Glorioso, HL

a review: 1805.09331 Paolo Glorioso, HL

Such a formulation:

1. Captures both statistical and quantum fluctuations

2. extends regime of validity of hydrodynamics to much shorter time and length scales.

   applicable: $\delta t \sim \tau_{\rm relax}$

EFT formulation of hydrodynamics

How do one formulate energy-momentum conservation in terms of symmetries?

Put the system in a curved spacetime: because of energy-momentum conservation, the system should be diffeomorphism invariant

Dynamical variables for energy-momentum conservation are Stueckelberg variables for diffeomorphisms.

Dynamical variables: $X^\mu(\sigma^0, \sigma^i), \quad X^\mu_a(\sigma^0, \sigma^i)$

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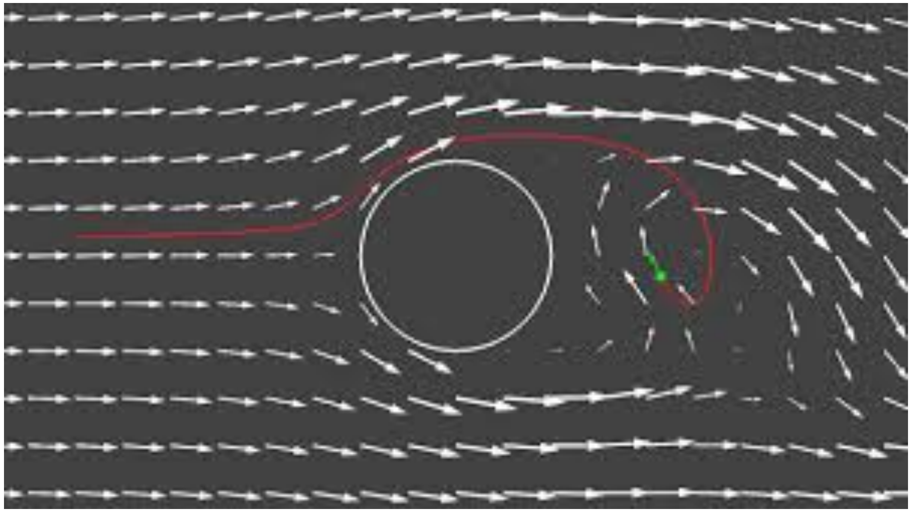
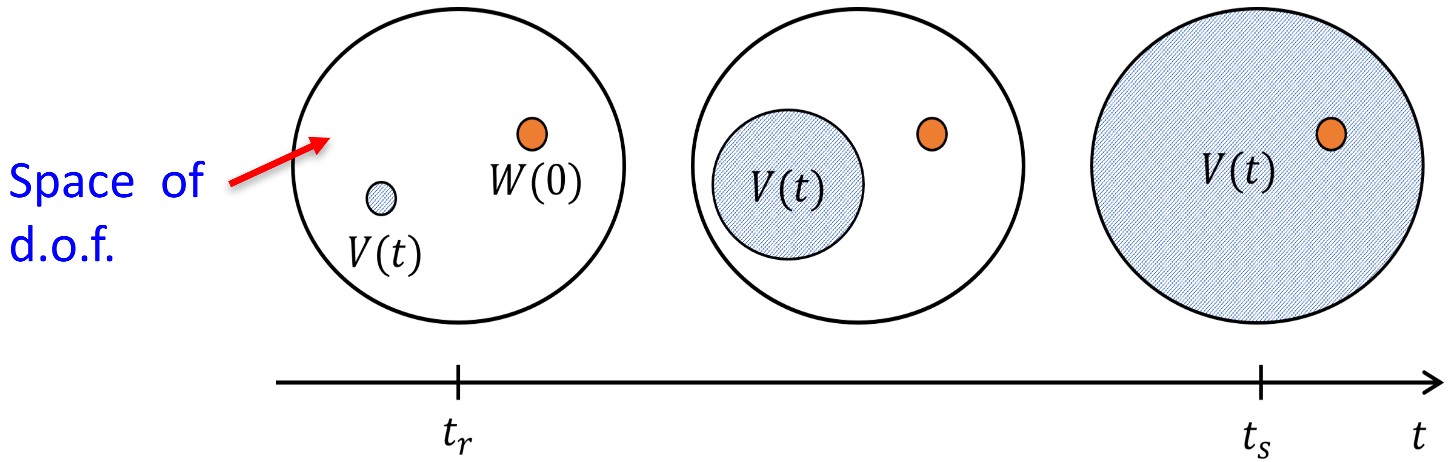
This is just a generalization of the Lagrange description!



σ^i: label fluid elements

$$x^i(t, \sigma^i),$$

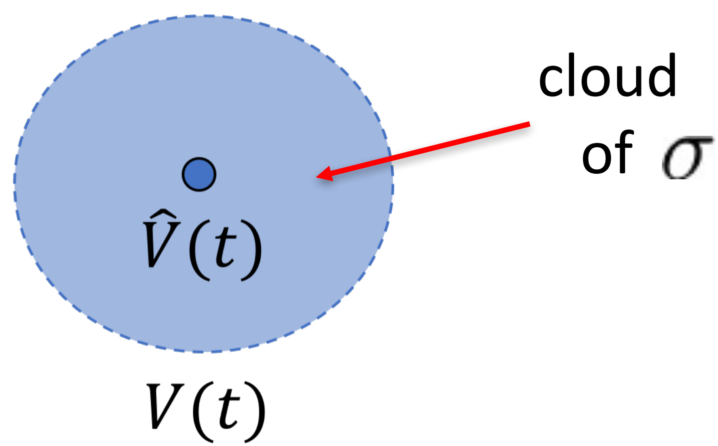
Quantum hydrodynamics for chaos (I)



Scrambling can be described by the field associated with energy conservation (i.e. Stueckelberg field for time diff).

dressing function

$$V(t) = f\left(\hat{V}(t), \sigma\right) =$$



Quantum hydrodynamics for chaos (II)

1. Write down the most general effective theory for energy conservation (momentum may or may not be conserved).

Dynamical variables: $\sigma(t, x^i), X_a(t, x^i), \cdots$

Including all derivatives, nonlocal ....

2. a chaotic system with Lyapunov exponent λ has an emergent shift symmetry:

$$e^{-\lambda\sigma} \to e^{-\lambda\sigma} + a$$

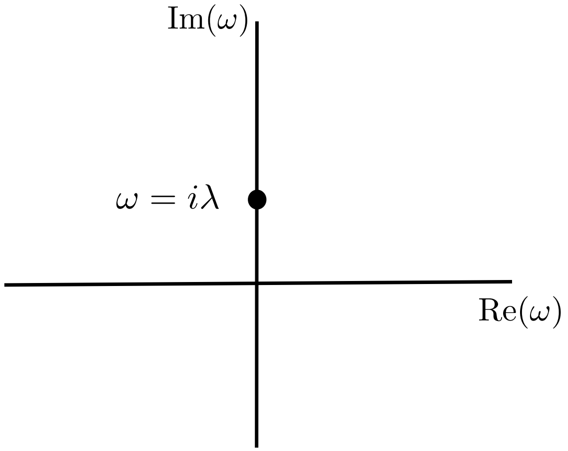
An emergent gauge symmetry. (motivated from the behavior of horizon)

Immediate implication of the shift symmetry

$$\sigma(t,x^i) = t + \epsilon(t,x^i)$$

$$G_R = \langle \epsilon(t,x^i)\epsilon(0)\rangle_{\beta_0}$$

$$\sim \theta(t) e^{\lambda(t - \frac{|\vec{x}|}{v_B})} + \cdots ,$$



$$v_B = \#\sqrt{D\lambda}$$

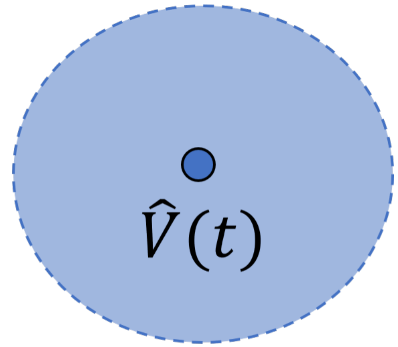
D: energy diffusion constant

Energy density and energy flux are invariant under the shift symmetry:

They are blind to the exponential growth of σ

Four-point functions

$$V(t) = f\big(\hat{V}(t), \sigma\big) =$$

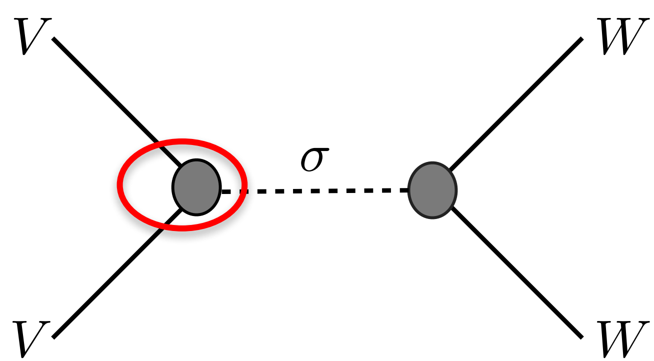


$V(t)$

$$\langle \hat{V} \hat{W} \rangle_{\beta_0} = 0$$

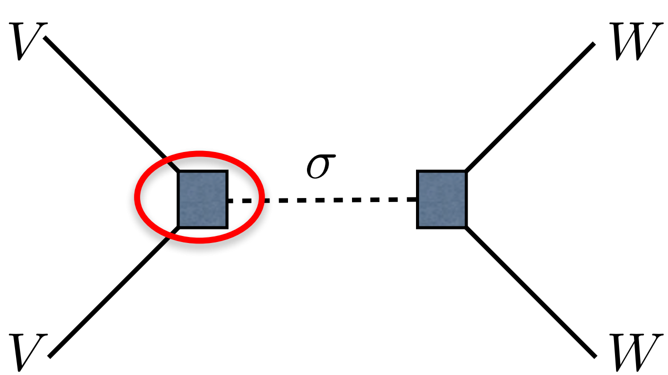
Shift symmetry on f

TOC



Invisible to exponential

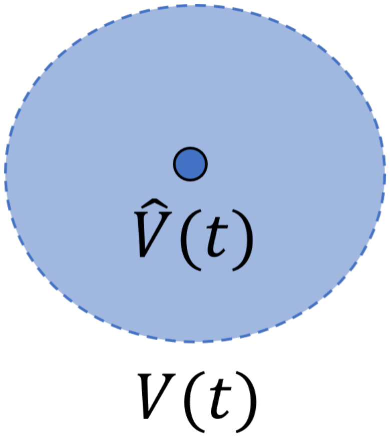
OTOC



Exponential survives

A self-consistency check

$$V(t) = f\big(\hat{V}(t), \sigma\big) =$$



Require the coupling f also respect the shift symmetry

$$e^{-\lambda\sigma} \to e^{-\lambda\sigma} + a$$

Consistency with fluctuation-dissipation relation requires that the Lyapunov exponent be maximal $\lambda_{\max} = \dfrac{2\pi}{\beta_0 \hbar}$

Implications and predictions (I)

In various holographic examples and CM models the butterfly velocity appear to be related to the thermal/energy diffusion constant (Blake, Donos, Gu, Qi,Stanford ….)

Explains connection between energy diffusion constant and chaos

$$D = O(1) v_B^2 \tau = O(1) \frac{v_B^2}{\lambda_{\max}}, \quad \tau = \frac{2\pi}{\beta}$$

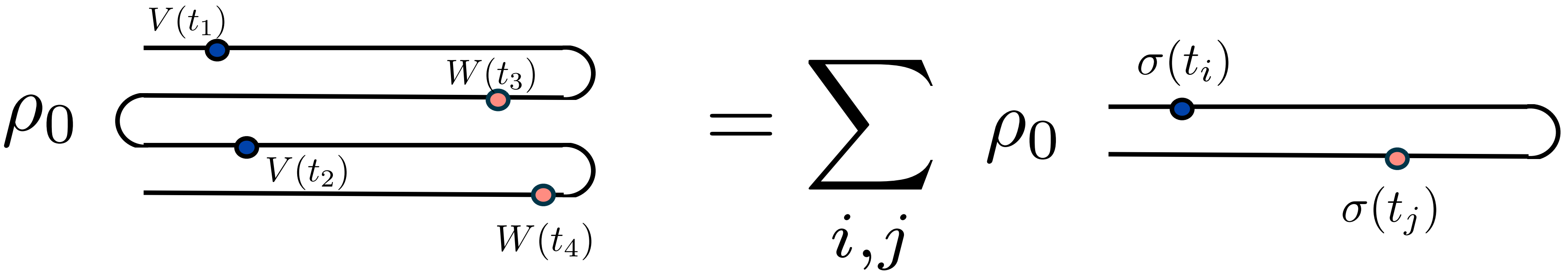
Implications and predictions (II)

$$C(t) = -\langle [V(t), W(0)]^2 \rangle_{\beta_0} = \mathrm{TOCs} - \mathrm{OTOCs}$$

$$\mathrm{OTOCs} = \langle W(0)V(t)W(0)V(t)\rangle_{\beta_0} + \langle V(t)W(0)V(t)W(0)\rangle_{\beta_0}$$

It turns out the chaotic behavior in these two terms precisely cancel in our theory, to leading order in large $\mathcal{N}$

$$C(t) = (ia - ia) \times e^{\lambda t} + \cdots = 0 \times e^{\lambda t}$$



Also found in all maximally chaotic systems

Phenomenon of pole skipping

The same mode is responsible for both energy conservation and chaos → Predicts a new phenomenon

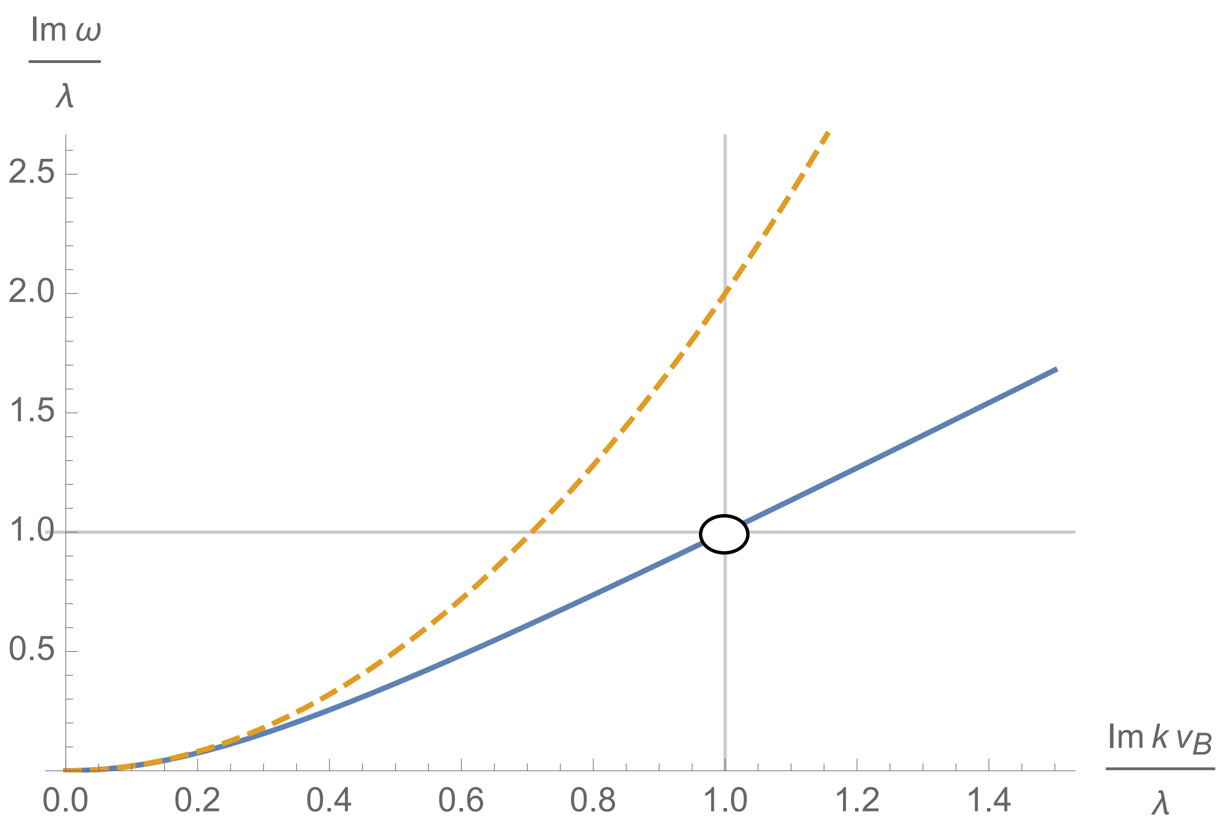
Consider energy density two-point function:

$$\omega = -iD_E k^2 + \cdots \, , \quad \text{small } \omega, k$$

A line of poles in complex frequency plane when changing k

Analytic continue k to imaginary values

Predictions:



1. the pole line passes the point

$$\omega = i\lambda,\; k = \pm i\frac{\lambda}{v_B}$$

2. Precisely at that point the pole is skipped (with zero residue).

Pole-skipping was in fact already seen earlier in holographic systems dual to a AdS_5 Schwarzschild by Grozdanov, Schalm, Scopelliti (arXiv:1710.00921), in a serach for connections between hydro and chaos.

This phenomenon was also implicit in the earlier study of SYK chain,

from analytic continuation of Gu, Qi, Stanford

arXiv:1609.07832

$$\mathcal{G}_R(\omega, k) \sim \frac{i\omega\left(\frac{\omega^2}{\lambda_{\max}^2} + 1\right)}{-i\omega + D_E k^2}$$

Precisely the structure predicted by hydro pole-skipping universal in all holographic systems at a finite temperature.

Due to a very interesting feature of linearized Einstein equation around a black hole geometry at the special value of

$$\omega = i\lambda,\ k = \pm i\frac{\lambda}{v_B}$$

Blake, Davison, Grozdanov, HL

Also holds in higher derivative gravity systems. Grozdanov

For non-maximal chaos, there should be a corresponding effective theory, but the effective field only partially overlaps with the hydro fields.

Need an effective field theory of Pomeron.

Thank You !