Revisiting the Simplest Little Higgs

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Based on:

1709.08929, in collaboration with S-P. He, Y-n. Mao & S-h. Zhu, PRD 97, 075005 (2018).

1801.10066, in collaboration with K. Cheung, S-P. He, Y-n. Mao & Y. Zhou, PRD 97, 115001 (2018).

1809.03809, in collaboration with K. Cheung, S-P. He, Y-n. Mao & P-Y. Tseng, PRD 98, 075023 (2018).

Little Hierarchy Problem and the Little Higgs

- Radiative instability of Higgs mass

  $\Rightarrow$ New physics around TeV scale

  $\Rightarrow$ Little Hierarchy Problem

  $\Rightarrow$ Little Higgs Theories

- Little Higgs theories:
  - EFT below some cutoff ~ 10 TeV
  - Higgs is a pNGB of some spontaneous global symmetry breaking
  - Collective symmetry breaking (CSB) N. Arkani-Hamed, JHEP 07 (2002) 034

  The global symmetry is completely broken by a collection of spurions, but not by any single spurion.

EWSB Prediction and NDA

- Making EWSB predictions in Little Higgs theories

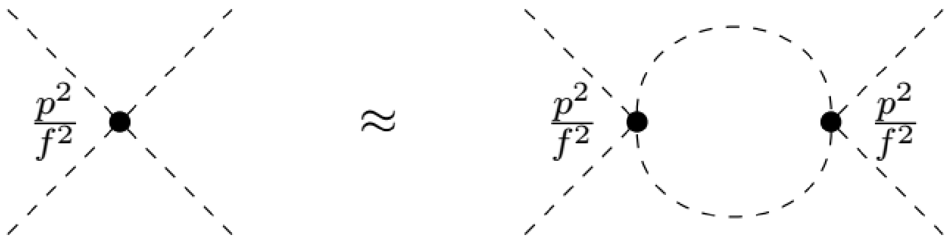
  ⇒Compute the scalar effective potential via Coleman-Weinberg

  ⇒UV divergences appear

- Two approaches (widely) adopted in Little Higgs literature

  1. NDA cutoff approach: $\Lambda = 4\pi f$ qualitative rather than quantitative

     M. Schmaltz, JHEP 08 (2004) 056 J. Reuter et al., JHEP 02 (2013) 077

     C. Csaki et al., 1811.04279 A. Manohar and H. Georgi, NPB 234 (1984) 189

  2. Free parameter approach: predictivity is lost X-F. Han et al., PRD 87 (2013) 055004

- EFT understanding: IR & UV contributions

Introduction to the SLH

- The Simplest Little Higgs is based on global symmetry breaking pattern

  M. Schmaltz, JHEP 08 (2004) 056

$$[SU(3)_1 \times U(1)_1] \times [SU(3)_2 \times U(1)_2] \to [SU(2)_1 \times U(1)_1] \times [SU(2)_2 \times U(1)_2]$$

and enlargement of electroweak gauge group to $SU(3)_L \times U(1)_X$

- We adopt a nonlinear realization, with the Goldstones parametrized as

$$\Phi_1 = \exp\left(\frac{i\Theta'}{f}\right)\exp\left(\frac{it_\beta\Theta}{f}\right)\begin{pmatrix}0\\0\\fc_\beta\end{pmatrix} \qquad \Phi_2 = \exp\left(\frac{i\Theta'}{f}\right)\exp\left(-\frac{i\Theta}{ft_\beta}\right)\begin{pmatrix}0\\0\\fs_\beta\end{pmatrix}$$

η: pseudo-axion

$$\Theta = \frac{\eta}{\sqrt{2}} + \begin{pmatrix}\mathbf{0}_{2\times2} & h\\ h^\dagger & 0\end{pmatrix}, \quad \Theta' = \frac{\zeta}{\sqrt{2}} + \begin{pmatrix}\mathbf{0}_{2\times2} & k\\ k^\dagger & 0\end{pmatrix}$$

$$h = \begin{pmatrix}h^0\\h^-\end{pmatrix}, \quad h^0 = \frac{1}{\sqrt{2}}(v + H - i\chi)$$

$$k = \begin{pmatrix}k^0\\k^-\end{pmatrix}, \quad k^0 = \frac{1}{\sqrt{2}}(\sigma - i\omega)$$

Gauge bosons:

A^3, A^8, B_x=>Z', Z, A; $A^1, A^2, A^4, A^5, A^6, A^7$=>W, X, Y.

CP-even scalars: H, σ. CP-odd scalars: η, ζ, χ, ω

- Counting: 10-8=2 degrees of freedom (H, η) will ultimately be physical.

Introduction to the SLH

- Fermion field content and Yukawa Lagrangian (anomaly-free embedding like 331 models)

F. del Aguila et al, JHEP 03 (2011) 080

$$L_m = (\nu_L, \ell_L, iN_L)^T_m \text{ with } Q_x = -\tfrac{1}{3}$$

$$\ell_{Rm} \text{ with } Q_x = -1 \text{ and } N_{Rm} \text{ with } Q_x = 0.$$

$$Q_1 = (d_L, -u_L, iD_L)^T, \quad d_R, \quad u_R, \quad D_R$$

$$Q_2 = (s_L, -c_L, iS_L)^T, \quad s_R, \quad c_R, \quad S_R$$

$$Q_3 = (t_L, b_L, iT_L)^T, \quad t_R, \quad b_R, \quad T_R$$

Fermion	$Q_{1,2}$	Q_3	u_{Rm}, T_{Rm}	d_{Rm}, D_{Rm}, S_{Rm}	L_m	N_{Rm}	e_{Rm}
Q_x charge	0	1/3	2/3	−1/3	−1/3	0	−1
SU(3) rep.	$\bar{\mathbf{3}}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$

- Yukawa Lagrangian (CSB for top sector)

$$\mathcal{L}_{LY} = i\lambda_N^m \bar{N}_{Rm} \Phi_2^\dagger L_m + \frac{i\lambda_\ell^{mn}}{\Lambda} \bar{\ell}_{Rm} \epsilon_{ijk} \Phi_1^i \Phi_2^j L_n^k + \text{h.c.}$$

$$\mathcal{L}_{QY} = i\lambda_1^t \bar{u}_{R3}^1 \Phi_1^\dagger Q_3 + i\lambda_2^t \bar{u}_{R3}^2 \Phi_2^\dagger Q_3 + i\frac{\lambda_b^m}{\Lambda} \bar{d}_{Rm} \epsilon_{ijk} \Phi_1^i \Phi_2^j Q_3^k$$
$$+ i\lambda_1^{dn} \bar{d}_{Rn}^1 Q_n^T \Phi_1 + i\lambda_2^{dn} \bar{d}_{Rn}^2 Q_n^T \Phi_2 + i\frac{\lambda_u^{mn}}{\Lambda} \bar{u}_{Rm} \epsilon_{ijk} \Phi_1^{*i} \Phi_2^{*j} Q_n^k + \text{h.c.}$$

The Hidden Mass Relation

- We write down the bare potential with a quartic term required by renormalization

$$V_B = -\mu_B^2(\Phi_{1B}^\dagger\Phi_{2B} + \Phi_{2B}^\dagger\Phi_{1B}) + \lambda_B|\Phi_{1B}^\dagger\Phi_{2B}|^2$$

- The soft-U(1)-breaking mu-term is introduced to make the pseudo-axion massive.

- Scalar effective potential is calculated via renormalized perturbation theory (see Peskin & Schroeder, Section 11.4)

$$V_{\text{tree}} = -\mu^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) + \lambda_R|\Phi_1^\dagger\Phi_2|^2$$

- One-loop potential at small field value is expressed as

$$V_{\text{1-loop}} = V_{\text{1-loop}}^{\text{s}} + V_{\text{1-loop}}^{\text{ns}} \qquad V_{\text{1-loop}}^{\text{s}} = \bar{\lambda}|\Phi_1^\dagger\Phi_2|^2 \qquad V_{\text{1-loop}}^{\text{ns}} = \Delta(\hat{h})\hat{h}^4 \qquad \hat{h} \equiv (h^\dagger h)^{1/2}$$

- Combining tree and one-loop, the scalar effective potential at small field value is given by

$$V = -\mu^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) + \lambda|\Phi_1^\dagger\Phi_2|^2 + \Delta(\hat{h})\hat{h}^4 \qquad \lambda \equiv \lambda_R + \bar{\lambda}$$

The Hidden Mass Relation

- In the scalar effective potential

$$V = -\mu^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) + \lambda|\Phi_1^\dagger\Phi_2|^2 + \Delta(\hat{h})\hat{h}^4 \quad \lambda \equiv \lambda_R + \bar{\lambda}$$

The radiative correction contribution from gauge and Yukawa sector is computed to be (in Landau gauge and $\overline{\mathrm{MS}}$ renormalization scheme)

$$\bar{\lambda} = -\frac{3}{8\pi^2}\left[\lambda_t^2\frac{M_T^2}{f^2}\left(\ln\frac{M_T^2}{\mu_R^2} - 1\right) - \frac{1}{4}g^2\frac{M_X^2}{f^2}\left(\ln\frac{M_X^2}{\mu_R^2} - \frac{1}{3}\right) - \frac{1}{8}g^2(1+t_W^2)\frac{M_{Z'}^2}{f^2}\left(\ln\frac{M_{Z'}^2}{\mu_R^2} - \frac{1}{3}\right)\right]$$

$$\Delta(\hat{h}) = \frac{3}{16\pi^2}\left\{\lambda_t^4\left[\ln\frac{M_T^2}{m_t^2(\hat{h})} - \frac{1}{2}\right] - \frac{1}{8}g^4\left[\ln\frac{M_X^2}{m_W^2(\hat{h})} - \frac{1}{2}\right] - \frac{1}{16}g^4(1+t_W^2)^2\left[\ln\frac{M_{Z'}^2}{m_Z^2(\hat{h})} - \frac{1}{2}\right]\right\}$$

Parameter Definition

$$\lambda_t \equiv \frac{\lambda_1^t\lambda_2^t}{\sqrt{\lambda_1^{t2}c_\beta^2 + \lambda_2^{t2}s_\beta^2}}$$

$$m_t^2(\hat{h}) = \lambda_t^2\hat{h}^2$$

$$m_W^2(\hat{h}) = \frac{1}{2}g^2\hat{h}^2$$

$$m_Z^2(\hat{h}) = \frac{1}{2}g^2(1+t_W^2)\hat{h}^2$$

$$M_T^2 \equiv (\lambda_1^{t2}c_\beta^2 + \lambda_2^{t2}s_\beta^2)f^2 \qquad M_T^2 = m_T^2 + m_t^2$$

$$M_X^2 \equiv \frac{1}{2}g^2f^2 \qquad M_X^2 = m_X^2 + m_W^2$$

$$M_{Z'}^2 \equiv \frac{2}{3-t_W^2}g^2f^2 \qquad M_{Z'}^2 = m_{Z'}^2 + m_Z^2$$

The Hidden Mass Relation

- Counting the parameters: 5→3

$f, t_\beta, M_T, \mu, \lambda$ —vev=246, Higgs mass=125→ f, t_β, M_T

Hidden relation between pseudo-axion mass and top partner mass

- The derivation of hidden mass relation is based on stationary point condition and the associated Hessian matrix. The result is

$$m_\eta^2 = [m_h^2 - v^2\Delta_A(3 - 2\theta t_{2\theta}^{-1}) + v^2 A(5 - 2\theta t_{2\theta}^{-1})]s_\theta^{-2}$$

$$\theta \equiv \frac{v}{\sqrt{2} f s_\beta c_\beta} \qquad A \equiv \frac{3}{16\pi^2}\left[\lambda_t^4 - \frac{g^4}{8} - \frac{g^4}{16}(1 + t_W^2)^2\right]$$

$$\Delta_A \equiv \frac{3}{16\pi^2}\left[\lambda_t^4 \ln\frac{M_T^2}{m_t^2} - \frac{g^4}{8}\ln\frac{M_X^2}{m_W^2} - \frac{g^4}{16}(1 + t_W^2)^2 \ln\frac{M_{Z'}^2}{m_Z^2}\right]$$

- The mass relation can be viewed as a zeroth-order natural relation.

Other Constraints

- Lower bound on M_T from $M_T^2 \equiv (\lambda_1^{t2} c_\beta^2 + \lambda_2^{t2} s_\beta^2) f^2$

$$M_T \geq \sqrt{2}\frac{m_t}{v} f s_{2\beta} \approx f s_{2\beta}$$

- Constraint from partial wave unitarity: using methods from S. Chang & H-J. He, PLB 586(2004)95. As a result we require

$$M_{Z'} \leq \sqrt{8\pi} f c_\beta \qquad M_T \leq \sqrt{8\pi} f c_\beta$$

- Lower bound on Z' mass from LHC constraint: taken as $f > 7.5\ \text{TeV}$.

- Accordingly, for f~7.5 TeV, we obtain an absolute lower bound on top partner mass at about 1.8 TeV, which is much more stringent than current top partner direct search limit from LHC.

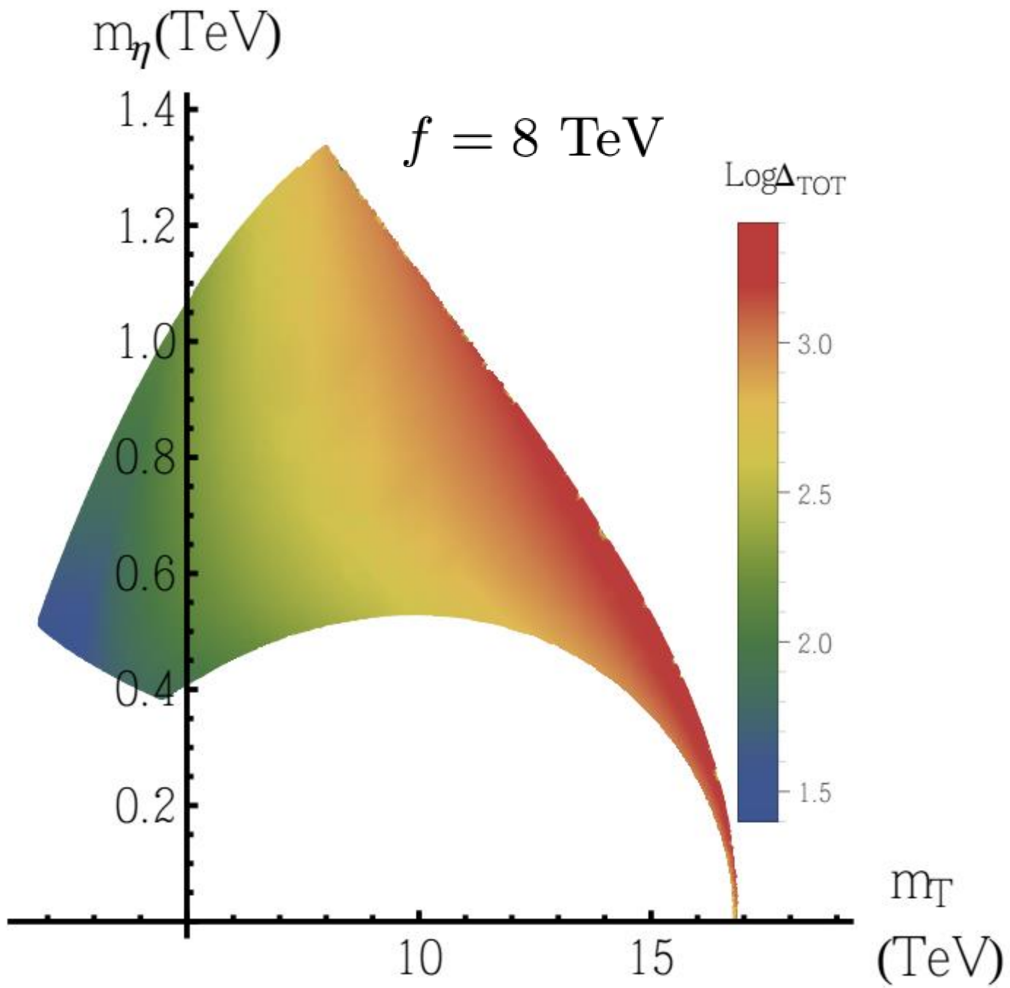
Characterization of the Parameter Space

$$\Delta_{\rm TOT} = \max\{\Delta^{\mu^2}_{\rm TOT}, \Delta^{\lambda}_{\rm TOT}\}$$

$$\Delta^{\lambda}_{\rm TOT} \equiv \left| \frac{\lambda_U}{m_h^2} \frac{\partial m_h^2}{\partial \lambda_U} \right|$$

$$\Delta^{\mu^2}_{\rm TOT} \equiv \left| \frac{\mu_U^2}{m_h^2} \frac{\partial m_h^2}{\partial \mu_U^2} \right|$$

Most natural region is featured by mT~2TeV, and pseudo-axion mass around 500 GeV, with tuning at a few percent level. Small pseudo-axion mass region is unnatural.



Log means $\log_{10}$.

Summary

1. I have presented an analysis of the SLH scalar effective potential without setting the UV contribution at NDA cutoff to zero (which is a widely used but ad hoc assumption). This allows me to identify a hidden mass relation to be the definite prediction of the SLH theory, which has been overlooked in the literature.

2. The analysis presented here could also be relevant for some other models in which Higgs arises as a pNGB.

Thank you!