Phenomenology of large loop-induced flavor mixing: Majorana fields and CP violation

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Contents

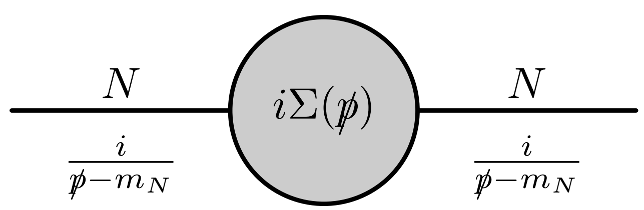
- Single flavor
- Multiple flavors
- Large loop-induced flavor mixing
- Examples
- Application to CP violation

Single flavor

RH Majorana field Yukawa coupling Scalar SU(2) doublet

- Lagrangian $$\mathcal{L} = \frac{1}{2}\overline{N}i\partial\!\!\!/ N - \frac{1}{2}m_N\overline{N}N - f\overline{L}\widetilde{\phi}\mathrm{R}N - f^*\overline{N}\widetilde{\phi}^\dagger\mathrm{L}L$$ Lepton SU(2) doublet
$$+\frac{1}{2}\delta_N\overline{N}i\partial\!\!\!/ N - \frac{1}{2}m_N(\delta_N+\delta_M+\delta_N\delta_M)\overline{N}N - \delta_V f\overline{L}\widetilde{\phi}\mathrm{R}N - \delta_V^* f^*\overline{N}\widetilde{\phi}^\dagger\mathrm{L}L$$

- Self-energy of the RH neutrino: quantum correction to the propagator



Up to the LO

$$i\Sigma(p\!\!\!/) = ip\!\!\!/\Sigma_R(p^2)$$

$$\Sigma_R(p^2) := \frac{|f|^2}{16\pi^2}\left[-\log\left(\frac{p^2}{\mu^2}\right) + i\pi\Theta(p^2)\right]$$

- Resummed propagator: propagator with quantum corrections

$$\begin{aligned} i\Delta(p\!\!\!/) &= \frac{i}{p\!\!\!/-m_N} + \frac{i}{p\!\!\!/-m_N}[i\Sigma(p\!\!\!/)]\frac{i}{p\!\!\!/-m_N} + \frac{i}{p\!\!\!/-m_N}[i\Sigma(p\!\!\!/)]\frac{i}{p\!\!\!/-m_N}[i\Sigma(p\!\!\!/)]\frac{i}{p\!\!\!/-m_N} + \cdots \\ &= \sum_{n=0}^{\infty}\frac{i}{p\!\!\!/-m_N}\left\{[i\Sigma(p\!\!\!/)]\frac{i}{p\!\!\!/-m_N}\right\}^n \\ &= i\{p\!\!\!/-m_N+\Sigma(p\!\!\!/)\}^{-1} = i\{[1+\Sigma_R(p^2)]p\!\!\!/-m_N\}^{-1}. \end{aligned}$$

Resummation by geometric series

$$\Delta(p^2) = Z_N^{-1}Z_M^{-1}\frac{P(p^2)}{m_N}\frac{p\!\!\!/+P(p^2)}{p^2-P^2(p^2)}$$

Single flavor

- Expanding the propagator around the pole

$$
\begin{aligned}
i\Delta_{RR}(p^2) = i\Delta_{LL}(p^2) &= R^{\widehat{N}} \frac{i p_{\widehat{N}}}{p^2 - p^2_{\widehat{N}}} + \cdots , \\
i\not{p}\Delta_{LR}(p^2) = i\not{p}\Delta_{RL}(p^2) &= R^{\widehat{N}} \frac{i\not{p}}{p^2 - p^2_{\widehat{N}}} + \cdots , \\
i\Delta(\not{p}) &= i\mathsf{R}\Delta_{RR}(p^2) + i\mathsf{R}\not{p}\Delta_{RL}(p^2) + i\mathsf{L}\not{p}\Delta_{LR}(p^2) + i\mathsf{L}\Delta_{LL}(p^2) \\
&= R^{\widehat{N}} \frac{i(\not{p} + p_{\widehat{N}})}{p^2 - p^2_{\widehat{N}}} + \cdots ,
\end{aligned}
$$

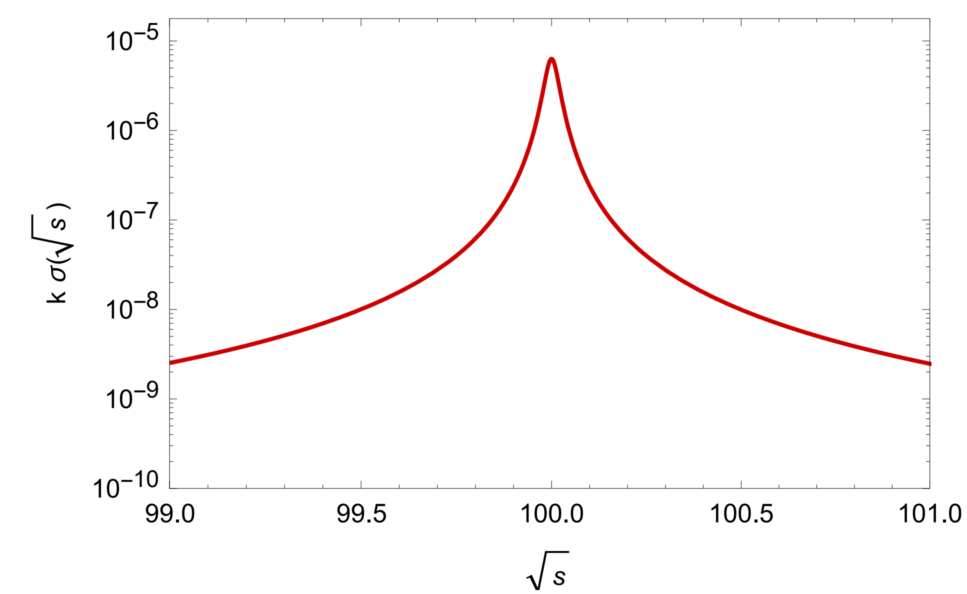
Physical pole

$$p^2_{\widehat{N}} = m^2_{\widehat{N}} - i m_{\widehat{N}} \Gamma_{\widehat{N}}$$

- Physical mass and decay width

$$\sigma_{\mathrm{CM}}(L\phi \to L\phi) \propto |\mathcal{M}(L\phi \to L\phi)|^2$$

$$
\begin{aligned}
i\mathcal{M}(L\phi \to L\phi) &= \overline{u_L}(\mathbf{p}'_L)(-i\widetilde{f}\mathsf{R})[i\Delta(\not{p})](-i\widetilde{f}^*\mathsf{L})u_L(\mathbf{p}_L) \\
&= -i|\widetilde{f}|^2\overline{u_L}(\mathbf{p}'_L)\not{p}\Delta_{RL}(p^2)\mathsf{L}u_L(\mathbf{p}_L) \\
&= -i|\widetilde{f}|^2\overline{u_L}(\mathbf{p}'_L)\frac{R^{\widehat{N}}\not{p}}{p^2 - p^2_{\widehat{N}}}\mathsf{L}u_L(\mathbf{p}_L) + \cdots
\end{aligned}
$$



Breit-Wigner resonance

Multiple flavors

- Kinetic part in the Lagrangian

$$\mathcal{L}_{\rm kin} = \frac{1}{2}\sum_\alpha \overline{N'_{R\alpha}}\partial\!\!\!/ N'_{R\alpha} + \frac{1}{2}\sum_\alpha \overline{N'^c_{R\alpha}}\partial\!\!\!/ N'^c_{R\alpha} - \frac{1}{2}\sum_\alpha (M_N)_{\beta\alpha}\overline{N'^c_{R\beta}}N'_{R\alpha} - \frac{1}{2}\sum_\alpha (M_N)_{\beta\alpha}\overline{N'_{R\beta}}N'^c_{R\alpha}$$

- Diagonalization of the mass matrix

$$N_\alpha = U_{\alpha\beta}N'_\beta, \qquad M_N^{\rm diag} = U^{\sf T} M_N U$$

Unitary matrix

$$\begin{aligned}\mathcal{L}_{\rm kin} &= \frac{1}{2}\sum_\alpha \overline{N_{R\alpha}}i\partial\!\!\!/ N_{R\alpha} + \frac{1}{2}\sum_\alpha \overline{N^c_{R\alpha}}i\partial\!\!\!/ N^c_{R\alpha} - \frac{1}{2}\sum_\alpha m_{N_\alpha}\overline{N^c_{R\alpha}}N_{R\alpha} - \frac{1}{2}\sum_\alpha m_{N_\alpha}\overline{N_{R\alpha}}N^c_{R\alpha} \\ &= \frac{1}{2}\sum_\alpha \overline{N_\alpha}i\partial\!\!\!/ N_\alpha - \frac{1}{2}\sum_\alpha m_{N_\alpha}\overline{N_\alpha}N_\alpha \qquad (4\end{aligned}$$

- Lagrangian

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}\sum_\alpha \overline{N_{R\alpha}}i\partial\!\!\!/ N_{R\alpha} + \frac{1}{2}\sum_\alpha \overline{N^c_{R\alpha}}i\partial\!\!\!/ N^c_{R\alpha} - \frac{1}{2}\sum_\alpha m_{N_\alpha}\overline{N^c_{R\alpha}}N_{R\alpha} - \frac{1}{2}\sum_\alpha m_{N_\alpha}\overline{N_{R\alpha}}N^c_{R\alpha} \\ &- \sum_{i,\alpha}(fC_V^z)_{i\alpha}\overline{L_i}\widetilde{\phi}N_{R\alpha} - \sum_{i,\alpha}(fC_V^z)^*_{i\alpha}\overline{N_{R\alpha}}\widetilde{\phi}^\dagger L_i \\ &+ \frac{1}{2}\sum_{\alpha,\beta}(\delta_N)_{\beta\alpha}\overline{N_{R\beta}}i\partial\!\!\!/ N_{R\alpha} + \frac{1}{2}\sum_{\alpha,\beta}(\delta_N)^*_{\beta\alpha}\overline{N^c_{R\beta}}i\partial\!\!\!/ N^c_{R\alpha} \\ &- \sum_{i,\alpha}(f\delta_V^w C_V^z)_{i\alpha}\overline{L_i}\widetilde{\phi}N_{R\alpha} - \sum_{i,\alpha}(f\delta_V^w C_V^z)^*_{i\alpha}\overline{N_{R\alpha}}\widetilde{\phi}^\dagger L_i \\ = &\frac{1}{2}\sum_\alpha \overline{N_\alpha}i\partial\!\!\!/ N_\alpha - \frac{1}{2}\sum_\alpha m_{N_\alpha}\overline{N_\alpha}N_\alpha - \sum_{i,\alpha}(fC_V^z)_{i\alpha}\overline{L_i}\widetilde{\phi}\mathsf{R}N_\alpha - \sum_{i,\alpha}(fC_V^z)^*_{i\alpha}\overline{N_\alpha}\widetilde{\phi}^\dagger\mathsf{L}L_i \\ &+ \frac{1}{2}\sum_\alpha(\delta_N)_{\beta\alpha}\overline{N_\beta}i\partial\!\!\!/\mathsf{R}N_\alpha + \frac{1}{2}\sum_\alpha(\delta_N)^*_{\beta\alpha}\overline{N_\beta}i\partial\!\!\!/\mathsf{L}N_\alpha \\ &- \sum_{i,\alpha}(f\delta_V^w C_V^z)_{i\alpha}\overline{L_i}\widetilde{\phi}\mathsf{R}N_\alpha - \sum_{i,\alpha}(f\delta_V^w C_V^z)^*_{i\alpha}\overline{N_\alpha}\widetilde{\phi}^\dagger\mathsf{L}L_i,\end{aligned}$$

CHL, arXiv: 1901,xxxxx

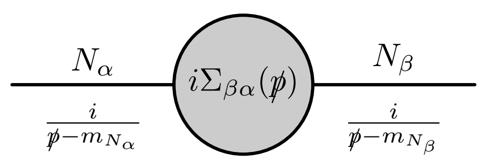
$$Z_N^{\frac{1}{2}} = \bar{Z}_N^{\frac{1}{2}}U^\dagger, \qquad U^\dagger Z_N^{\frac{1}{2}\dagger}Z_N^{\frac{1}{2}}U = \bar{Z}_N^{\frac{1}{2}\dagger}\bar{Z}_N^{\frac{1}{2}} = 1 + \bar{\delta}_N$$

?

Unitary matrix

Multiple flavors

- Self-energy matrix



$$\Sigma(\not{p}) = \not{p}\mathsf{R}\Sigma_R(p^2) + \not{p}\mathsf{L}\Sigma_R^{\mathsf{T}}(p^2)$$

$$(\Sigma_R)_{\beta\alpha}(p^2) := \sum_i \frac{f_{i\beta}^* f_{i\alpha}}{16\pi^2}\left[-\log\left(\frac{p^2}{\mu_{\beta\alpha}^2}\right) + i\pi\Theta(p^2)\right]$$

Non-diagonal in general

- Resummed propagator matrix

$$\Delta(\not{p}) = \mathsf{R}\Delta_{RR}(p^2) + \mathsf{R}\not{p}\Delta_{RL}(p^2) + \mathsf{L}\not{p}\Delta_{LR}(p^2) + \mathsf{L}\Delta_{LL}(p^2).$$

$$\begin{aligned}
\Delta_{RR}(p^2) &= \left\{[1+\Sigma_R^{\mathsf{T}}(p^2)]M_N^{-1}[1+\Sigma_R(p^2)]p^2 - M_N\right\}^{-1},\\
\Delta_{LR}(p^2) &= M_N^{-1}[1+\Sigma_R(p^2)]\Delta_{RR}(p^2),\\
\Delta_{LL}(p^2) &= \left\{[1+\Sigma_R(p^2)]M_N^{-1}[1+\Sigma_R^{\mathsf{T}}(p^2)]p^2 - M_N\right\}^{-1},\\
\Delta_{RL}(p^2) &= M_N^{-1}[1+\Sigma_R^{\mathsf{T}}(p^2)]\Delta_{LL}(p^2).
\end{aligned}$$

Non-diagonal in general

- Diagonalization of the resummed propagator

$$\begin{aligned}
i\mathcal{M}(L_i\phi \to L_j\phi) &= \sum_{\beta,\alpha} \overline{u_{L_j}}(\mathbf{p}_{L_j})(-i\widetilde{f}_{i\beta}\mathsf{R})[i\Delta_{\beta\alpha}(\not{p})](-i\widetilde{f}_{i\alpha}^*\mathsf{L})u_{L_i}(\mathbf{p}_{L_i})\\
&= \sum_{\alpha} \overline{u_{L_j}}(\mathbf{p}_{L_j})(-i\widehat{f}_{i\alpha}\mathsf{R})[i\widehat{\Delta}_{\alpha\alpha}(\not{p})](-i\widehat{f}_{i\alpha}^c\mathsf{L})u_{L_i}(\mathbf{p}_{L_i})
\end{aligned}$$

Resummed Yukawa couplings

Diagonalized resummed propagator

Find mixing matrices that give the resummed propagator of unstable fermion

Multiple flavors

- Diagonalized propagator matrix

$$\Delta_{RR}(p^2) = C_R(p^2)\widehat{\Delta}_{RR}(p^2)C_R^{\mathsf{T}}(p^2) = C_R(p^2)\begin{pmatrix} \frac{P_1(p^2)}{m_{N_1}}\frac{P_1(p^2)}{p^2-P_1^2(p^2)} & 0 \\ 0 & \frac{P_2(p^2)}{m_{N_2}}\frac{P_2(p^2)}{p^2-P_2^2(p^2)} \end{pmatrix} C_R^{\mathsf{T}}(p^2),$$

$$\Delta_{LL}(p^2) = C_L(p^2)\widehat{\Delta}_{LL}(p^2)C_L^{\mathsf{T}}(p^2) = C_L(p^2)\begin{pmatrix} \frac{P_1(p^2)}{m_{N_1}}\frac{P_1(p^2)}{p^2-P_1^2(p^2)} & 0 \\ 0 & \frac{P_2(p^2)}{m_{N_2}}\frac{P_2(p^2)}{p^2-P_2^2(p^2)} \end{pmatrix} C_L^{\mathsf{T}}(p^2),$$

$$\Delta_{LR}(p^2) = C_L(p^2)\widehat{\Delta}_{LR}(p^2)C_R^{\mathsf{T}}(p^2) = C_L(p^2)\begin{pmatrix} \frac{P_1(p^2)}{m_{N_1}}\frac{\not{p}}{p^2-P_1^2(p^2)} & 0 \\ 0 & \frac{P_2(p^2)}{m_{N_2}}\frac{\not{p}}{p^2-P_2^2(p^2)} \end{pmatrix} C_R^{\mathsf{T}}(p^2),$$

$$\Delta_{RL}(p^2) = C_R(p^2)\widehat{\Delta}_{RL}(p^2)C_L^{\mathsf{T}}(p^2) = C_R(p^2)\begin{pmatrix} \frac{P_1(p^2)}{m_{N_1}}\frac{\not{p}}{p^2-P_1^2(p^2)} & 0 \\ 0 & \frac{P_2(p^2)}{m_{N_2}}\frac{\not{p}}{p^2-P_2^2(p^2)} \end{pmatrix} C_L^{\mathsf{T}}(p^2),$$

Mixing matrices

Mixing matrices are **non-unitary.**

$$C_R(p^2) \neq C_L^*(p^2)$$

$$\boxed{\begin{aligned} i(\widehat{\Delta}_{RR})_{\beta\alpha}(p^2) = i(\widehat{\Delta}_{LL})_{\beta\alpha}(p^2) &= \delta_{\beta\alpha}R^{\widehat{N}_\alpha}\frac{ip_{\widehat{N}_\alpha}}{p^2-p^2_{\widehat{N}_\alpha}} + \cdots\,, \\ i\not{p}(\widehat{\Delta}_{LR})_{\beta\alpha}(p^2) = i\not{p}(\widehat{\Delta}_{RL})_{\beta\alpha}(p^2) &= \delta_{\beta\alpha}R^{\widehat{N}_\alpha}\frac{i\not{p}}{p^2-p^2_{\widehat{N}_\alpha}} + \cdots\,, \\ i\widehat{\Delta}_{\beta\alpha}(\not{p}) &= \left[i\mathsf{R}\widehat{\Delta}_{RR}(p^2) + i\mathsf{R}\not{p}\widehat{\Delta}_{RL}(p^2) + i\mathsf{L}\not{p}\widehat{\Delta}_{LR}(p^2) + i\mathsf{L}\widehat{\Delta}_{LL}(p^2)\right]_{\beta\alpha} \\ &= \delta_{\beta\alpha}R^{\widehat{N}_\alpha}\frac{i(\not{p}+p_{\widehat{N}_\alpha})}{p^2-p^2_{\widehat{N}_\alpha}} + \cdots\,, \end{aligned}}$$

Did not introduce any **nontrivial assumptions** for diagonalization!

$$p^2_{\widehat{N}_\alpha} = m^2_{\widehat{N}_\alpha} - i m_{\widehat{N}_\alpha}\Gamma_{\widehat{N}_\alpha}$$

Physical pole

Geometric series: loop-effects + Linear algebra

Diagonalization is **exact** as long as

$$\Sigma(\not{p}) = \not{p}\mathsf{R}\Sigma_R(p^2) + \not{p}\mathsf{L}\Sigma_R^{\mathsf{T}}(p^2)$$

Multiple flavors

- Resumed Yukawa couplings

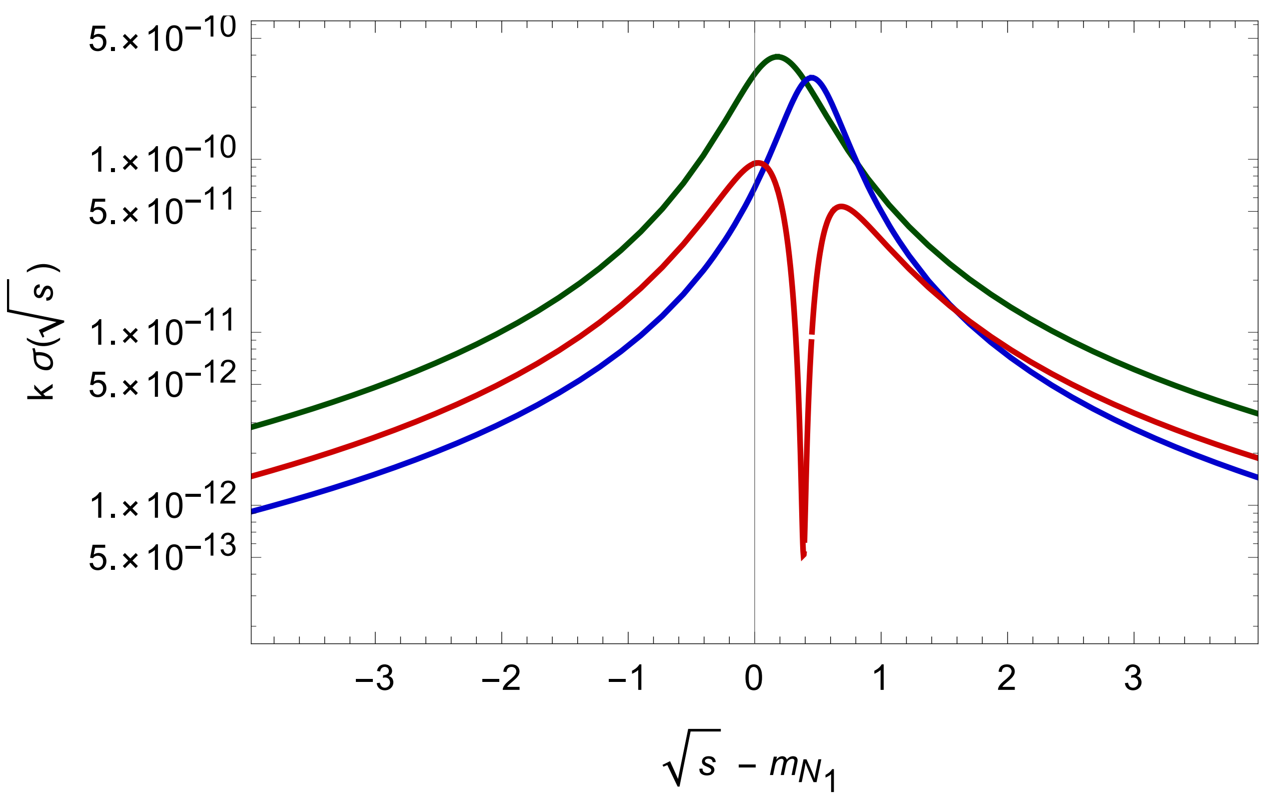
$$C_R^{\widehat{N}_\alpha} := C_R(p^2_{\widehat{N}_\alpha}), \qquad C_L^{\widehat{N}_\alpha} := C_L(p^2_{\widehat{N}_\alpha})$$

$$\widehat{f}_{i\alpha} := (f C_R^{\widehat{N}_\alpha})_{i\alpha}, \qquad \widehat{f}^c_{i\alpha} := (f^* C_L^{\widehat{N}_\alpha})_{i\alpha}.$$

- Physical masses and decay widths

$$p^2_{\widehat{N}_\alpha} = m^2_{\widehat{N}_\alpha} - i m_{\widehat{N}_\alpha} \Gamma_{\widehat{N}_\alpha}$$

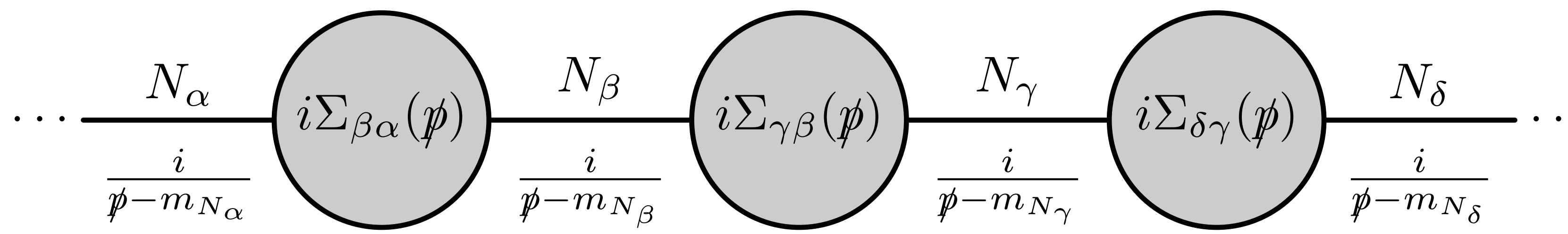
Mass and decay width of an effective particle



Two flavors of RH Majorana neutrinos with an intermediate mass difference

Large loop-induced flavor mixing

- Large flavor mixing



$\Sigma(\not{p}) \sim \mathcal{O}(f^2/16\pi^2)$ $\quad$ $\not{p} - m_{N_\alpha} \sim \not{p} - m_{N_\beta} \sim \mathcal{O}(f^2/16\pi^2)$

Mass differences are small (or intermediate)

Easy to add up by **resummation**, but easy to be **neglected**!

$C_{R,L}(p^2) \neq 1 + \mathcal{O}(f^2/16\pi^2)$

Diagonalization exact at least up to the NLO made the proper analysis of them possible!

Physical particles as asymptotic states **cannot** satisfy Dirac equation even up the LO!

- Physical particles should be interpreted as **quasiparticles**.

Quasiparticle

- Definition

Emergent phenomena that occur when a microscopically complicated system such as a solid behaves **as if** it contained different weakly interacting particles in free space - *Wikipedia*

- Example

A. An **electron traveling through a semiconductor** behaves like an electron with a different mass (**effective mass**) traveling unperturbed through free space - *Wikipedia*

B. Particles in the **early universe** are thought to have been in a thermal environment and have acquired **thermal masses**

Large mass difference

- Large mass difference $$m_{N_2} - m_{N_1} \gg m_{N_1}\mathcal{O}(\Sigma), \qquad m_{N_2} - m_{N_1} \gg m_{N_2}\mathcal{O}(\Sigma).$$
- Perturbatively calculable $\left|\frac{4b_{12}^2}{(a_2-a_1)^2}\right| \ll 1$ Small expansion parameter
- Results

$$C_R(p^2) = M_N^{-\frac{1}{2}} O_R M_N^{\frac{1}{2}} = \begin{pmatrix} 1 & -\frac{m_{N_2}[m_{N_2}(\Sigma_R)_{12}+m_{N_1}(\Sigma_R)_{21}]}{m_{N_2}^2-m_{N_1}^2} \\ \frac{m_{N_1}[m_{N_2}(\Sigma_R)_{12}+m_{N_1}(\Sigma_R)_{21}]}{m_{N_2}^2-m_{N_1}^2} & 1 \end{pmatrix},$$

$$C_L(p^2) = M_N^{-\frac{1}{2}} O_L M_N^{\frac{1}{2}} = \begin{pmatrix} 1 & -\frac{m_{N_2}[m_{N_1}(\Sigma_R)_{12}+m_{N_2}(\Sigma_R)_{21}]}{m_{N_2}^2-m_{N_1}^2} \\ \frac{m_{N_1}[m_{N_1}(\Sigma_R)_{12}+m_{N_2}(\Sigma_R)_{21}]}{m_{N_2}^2-m_{N_1}^2} & 1 \end{pmatrix}.$$

$$i\Delta_{\alpha\alpha}(\not{p})\big|_{p^2\approx p^2_{\widehat{N}_\alpha}} = R^{\widehat{N}_\alpha}\frac{i(\not{p}+p_{\widehat{N}_\alpha})}{p^2-p^2_{\widehat{N}_\alpha}} + \cdots,$$

$$i\Delta_{11}(\not{p}) = \begin{cases} R^{\widehat{N}_1}\dfrac{i(\not{p}+p_{\widehat{N}_1})}{p^2-p^2_{\widehat{N}_1}}, & p^2 \approx p^2_{\widehat{N}_1}, \\ 0, & p^2 \approx p^2_{\widehat{N}_2}, \end{cases}$$

$$\widehat{f}_{i\alpha} = (fC_R^{\widehat{N}_\alpha})_{i\alpha} = f_{i\alpha} + f_{i\beta}\frac{m_{N_\alpha}[m_{N_\beta}(\Sigma_R)_{\alpha\beta}(m^2_{N_\alpha}) + m_{N_\alpha}(\Sigma_R)_{\beta\alpha}(m^2_{N_\alpha})]}{m^2_{N_\beta}-m^2_{N_\alpha}},$$

$$\widehat{f}^c_{i\alpha} = (f^*C_L^{\widehat{N}_\alpha})_{i\alpha} = f^*_{i\alpha} + f^*_{i\beta}\frac{m_{N_\alpha}[m_{N_\beta}(\Sigma_R)_{\beta\alpha}(m^2_{N_\alpha}) + m_{N_\alpha}(\Sigma_R)_{\alpha\beta}(m^2_{N_\alpha})]}{m^2_{N_\beta}-m^2_{N_\alpha}},$$

Identical to the decoupled case

Identical to those obtained by the one-loop calculation

Large mass difference

- In the literature, some **nontrivial assumptions** were always introduced to **diagonalize** the propagator, and those assumptions are in fact valid only when the **mass difference is large**.
- As a result, they could see only the case of a large mass difference, and could **never** see the **non-perturbative effects**.
- **No** need to interpret the effective particles as **quasiparticles**.

Small mass difference

- Small mass difference $m_{N_2} - m_{N_1} \ll m_{N_1}\mathcal{O}(\Sigma), \qquad m_{N_2} - m_{N_1} \ll m_{N_2}\mathcal{O}(\Sigma)$
- Non-perturbative in general
- Consider an extreme case $\boxed{N_1 \neq N_2, \qquad m_{\widehat{N}} := m_{\widehat{N}_1} = m_{\widehat{N}_2}, \qquad m_N := m_{N_1} = m_{N_2}, \qquad f_i := f_{i1} = f_{i2},}$
- Results

$$\boxed{C_R(p^2) = M_N^{-\frac{1}{2}} O_R M_{\widehat{N}}^{\frac{1}{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \qquad C_L(p^2) = M_N^{-\frac{1}{2}} O_L M_{\widehat{N}}^{\frac{1}{2}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.}$$

$$\begin{aligned} i\Delta(\not{p}) &= \frac{P_1(p^2)}{m_{\widehat{N}}} \frac{i[\not{p} + P_1(p^2)]}{p^2 - P_1^2(p^2)} \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} + \frac{P_2(p^2)}{m_{\widehat{N}}} \frac{i[\not{p} + P_2(p^2)]}{p^2 - P_2^2(p^2)} \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \\ &= \begin{cases} R^{\widehat{N}_1} \dfrac{i(\not{p} + p_{\widehat{N}_1})}{p^2 - p_{\widehat{N}_1}^2} \dfrac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} + \cdots, & p^2 \approx p_{\widehat{N}_1}^2, \\ R^{\widehat{N}_2} \dfrac{i(\not{p} + p_{\widehat{N}_2})}{p^2 - p_{\widehat{N}_2}^2} \dfrac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \cdots, & p^2 \approx p_{\widehat{N}_2}^2. \end{cases} \end{aligned}$$

$$i\Delta_{11}(\not{p}) = \begin{cases} \dfrac{1}{2} \dfrac{i(\not{p} + m_{\widehat{N}})}{p^2 - m_{\widehat{N}}^2}, & p^2 \approx m_{\widehat{N}}^2, \\ \dfrac{1}{2} R^{\widehat{N}_2} \dfrac{i(\not{p} + p_{\widehat{N}_2})}{p^2 - p_{\widehat{N}_2}^2}, & p^2 \approx p_{\widehat{N}_2}^2, \end{cases}$$

Equal contributions from two propagators, which clearly shows that the current analysis in the literature is wrong.

$$\boxed{\begin{aligned} p_{\widehat{N}_1} &= P_1(m_{\widehat{N}}^2) = m_N = m_{\widehat{N}} - i\frac{\Gamma_{\widehat{N}_1}}{2}, \\ p_{\widehat{N}_2} &= P_2(m_{\widehat{N}}^2) = m_N[1 - 2\Sigma_R^a(m_{\widehat{N}}^2)] = m_{\widehat{N}} - i\frac{\Gamma_{\widehat{N}_2}}{2}, \end{aligned}}$$

$$\boxed{\begin{aligned} \widehat{f}_{i1} &= (fC_R^{\widehat{N}_1})_{i1} = \frac{1}{\sqrt{2}}(1-1)f_i = 0, & \widehat{f}_{i2} &= (fC_R^{\widehat{N}_2})_{i2} = \frac{1}{\sqrt{2}}(1+1)f_i = \sqrt{2}f_i, \\ \widehat{f}_{i1}^c &= (f^*C_L^{\widehat{N}_1})_{i1} = \frac{1}{\sqrt{2}}(1-1)f_i^* = 0, & \widehat{f}_{i2}^c &= (f^*C_L^{\widehat{N}_2})_{i2} = \frac{1}{\sqrt{2}}(1+1)f_i^* = \sqrt{2}f_i^*. \end{aligned}}$$

Intermediate mass difference

- Intermediate mass difference $m_{N_2} - m_{N_1} \sim m_{N_1}\mathcal{O}(\Sigma), \qquad m_{N_2} - m_{N_1} \sim m_{N_2}\mathcal{O}(\Sigma).$

- Only numerically calculable

$$m_{N_1} = 1000.$$

$$m_{N_2} = 1000.$$

$$\Sigma_R(m_{N_1}^2) = \begin{pmatrix} 0. + 3.24469\times10^{-10}\, i & 6.1477\times10^{-11} - 2.88915\times10^{-10}\, i \\ -6.1477\times10^{-11} - 2.88915\times10^{-10}\, i & 1.11725\times10^{-19} + 2.78145\times10^{-10}\, i \end{pmatrix}$$

$$\Sigma_R(m_{N_2}^2) = \begin{pmatrix} -1.30333\times10^{-19} + 3.24469\times10^{-10}\, i & 6.1477\times10^{-11} - 2.88915\times10^{-10}\, i \\ -6.1477\times10^{-11} - 2.88915\times10^{-10}\, i & 0. + 2.78145\times10^{-10}\, i \end{pmatrix}$$

$$p_{N_1} = P_{11} = 1000. - 3.5515\times10^{-7}\, i$$

$$p_{N_2} = P_{22} = 1000. - 2.47464\times10^{-7}\, i$$

$$f = \begin{pmatrix} 0.0000642625 + 0.0000466894\, i & -0.0000510455 - 0.0000370867\, i \\ 0.0000951057 + 0.0000309017\, i & -0.0000809017 - 0.0000587785\, i \end{pmatrix}$$

$$\hat{f} = \begin{pmatrix} 0.0000687768 + 0.0000965131\, i & -0.0000902131 + 0.0000193361\, i \\ 0.0000949879 + 0.00010018\, i & -0.000111624 + 0.0000163194\, i \end{pmatrix}$$

$$\hat{f}^c = \begin{pmatrix} 0.000113043 - 0.0000355864\, i & -9.48769\times10^{-6} + 0.0000917729\, i \\ 0.000176382 - 1.85878\times10^{-6}\, i & -0.0000492687 + 0.000149514\, i \end{pmatrix}$$

$$C_L(m_{N_1}^2) = \begin{pmatrix} 1.24519 - 0.130381\, i & 0.213356 + 0.760928\, i \\ -0.213356 - 0.760928\, i & 1.24519 - 0.130381\, i \end{pmatrix}$$

$$C_R(m_{N_1}^2) = \begin{pmatrix} 1.24519 - 0.130381\, i & 0.213356 + 0.760928\, i \\ -0.213356 - 0.760928\, i & 1.24519 - 0.130381\, i \end{pmatrix}$$

$$C_R^\dagger(m_{N_1}^2)\, C_R(m_{N_1}^2) = \begin{pmatrix} 2.19202 + 0.\, i & 1.05028\times10^{-10} + 1.95063\, i \\ 1.05028\times10^{-10} - 1.95063\, i & 2.19202 + 0.\, i \end{pmatrix}$$

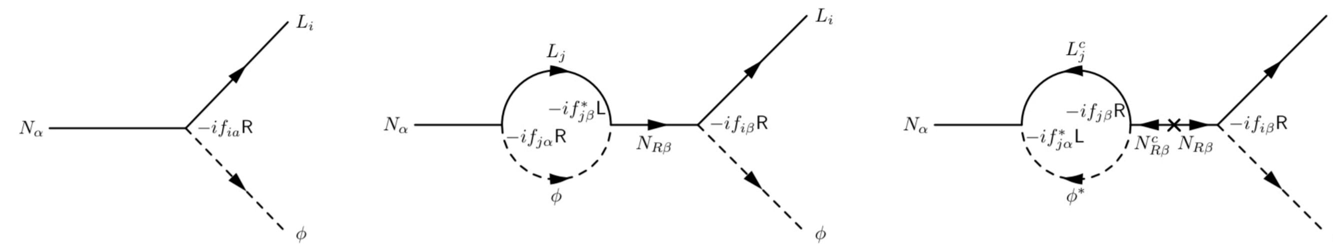
$$2^{-1/2}\, \| C_R^\dagger(m_{N_1}^2)\, C_R(m_{N_1}^2) \|_F = 2.93426$$

$$\|A\|_F := \sqrt{\mathrm{tr}[AA^\dagger]}$$

If *A* is unitary, the norm is 1.

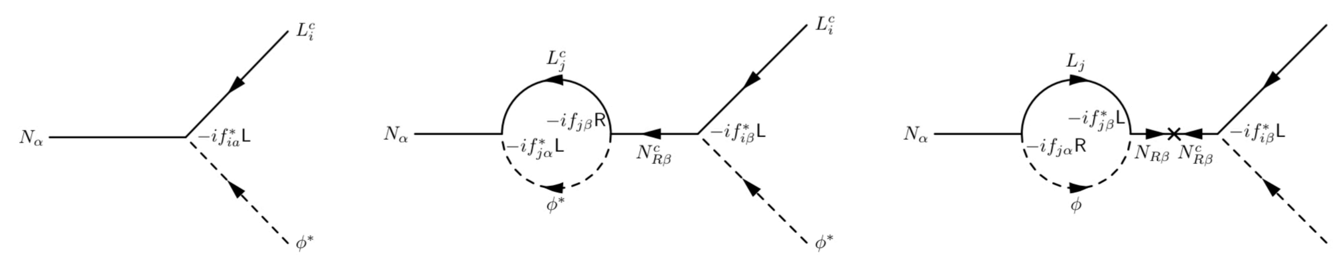
CP violation in the decay of heavy neutrinos

- CP asymmetry



(a) $-if_{i\alpha}\mathsf{R}$ (b) $-i(\delta f^D_{wR})_{i\alpha}\mathsf{R}$ (c) $-i(\delta f^M_{wR})_{i\alpha}\mathsf{R}$

Figure 5. Feynman diagrams that contribute to $N_\alpha \to L_i\phi$ through the wavefunction renormalization.



(a) $-if^*_{i\alpha}\mathsf{L}$ (b) $-i(\delta f^D_{wL})_{i\alpha}\mathsf{L}$ (c) $-i(\delta f^M_{wL})_{i\alpha}\mathsf{L}$

Figure 6. Feynman diagrams that contribute to $N_\alpha \to L^c_i\phi^*$ through the wavefunction renormalization.

One-loop calculation

$$\delta^i_{N_\alpha} := \frac{\Gamma(N_\alpha \to L_i\phi) - \Gamma(N_\alpha \to L^c_i\phi^*)}{\sum_{i=1}^3 \left[\Gamma(N_\alpha \to L_i\phi) + \Gamma(N_\alpha \to L^c_i\phi^*)\right]}$$
$$= \frac{1}{8\pi(f^\dagger f)_{\alpha\alpha}} \sum_{\beta\neq\alpha} \left\{ \mathrm{Im}\big[(f^\dagger f)_{\beta\alpha} f_{i\beta} f^*_{i\alpha}\big] m_{N_\alpha} + \mathrm{Im}\big[(f^\dagger f)_{\alpha\beta} f_{i\beta} f^*_{i\alpha}\big] m_{N_\beta} \right\} \frac{m_{N_\alpha}}{m^2_{N_\alpha} - m^2_{N_\beta}},$$

$$\varepsilon_{N_\alpha} = \delta_{N_\alpha} := \sum_i \delta^i_{N_\alpha} = \frac{1}{8\pi(f^\dagger f)_{\alpha\alpha}} \sum_{\beta\neq\alpha} \mathrm{Im}\left[\left\{(f^\dagger f)_{\alpha\beta}\right\}^2\right] \frac{m_{N_\alpha} m_{N_\beta}}{m^2_{N_\alpha} - m^2_{N_\beta}}$$

Covi, Roulet, Vissani, 1996

Consistent with

$$\widehat{f}_{i\alpha} = (fC^{\widehat{N}_\alpha}_R)_{i\alpha} = f_{i\alpha} + f_{i\beta} \frac{m_{N_\alpha}[m_{N_\beta}(\Sigma_R)_{\alpha\beta}(m^2_{N_\alpha}) + m_{N_\alpha}(\Sigma_R)_{\beta\alpha}(m^2_{N_\alpha})]}{m^2_{N_\beta} - m^2_{N_\alpha}},$$
$$\widehat{f}^c_{i\alpha} = (f^*C^{\widehat{N}_\alpha}_L)_{i\alpha} = f^*_{i\alpha} + f^*_{i\beta} \frac{m_{N_\alpha}[m_{N_\beta}(\Sigma_R)_{\beta\alpha}(m^2_{N_\alpha}) + m_{N_\alpha}(\Sigma_R)_{\alpha\beta}(m^2_{N_\alpha})]}{m^2_{N_\beta} - m^2_{N_\alpha}},$$

CP violation in the decay of heavy neutrinos

- Discrepancy among the expressions of CP asymmetry

$$\delta_{N_i} \approx \varepsilon_{N_i} = \frac{\mathrm{Im}(h^{\nu\dagger}h^{\nu})_{ij}^2}{(h^{\nu\dagger}h^{\nu})_{ii}(h^{\nu\dagger}h^{\nu})_{jj}} \frac{(m_{N_i}^2 - m_{N_j}^2) m_{N_i} \Gamma_{N_j}^{(0)}}{(m_{N_i}^2 - m_{N_j}^2)^2 + m_{N_i}^2 \Gamma_{N_j}^{(0)2}},$$

Pilaftsis, Underwood, 2004

$$\varepsilon_1(\hat{M}_1^2) = \frac{\mathrm{Im}(K_{12}^2)}{8\pi K_{11}} \frac{\hat{M}_1 \hat{M}_2 (\hat{M}_2^2 - \hat{M}_1^2)}{(\hat{M}_2^2 - \hat{M}_1^2 - \frac{1}{\pi}\hat{M}_2\Gamma_2 \ln(\hat{M}_2^2/\hat{M}_1^2))^2 + (\hat{M}_2\Gamma_2 - \hat{M}_1\Gamma_1)^2},$$

$$\varepsilon_2(\hat{M}_2^2) = \frac{\mathrm{Im}(K_{12}^2)}{8\pi K_{22}} \frac{\hat{M}_1 \hat{M}_2 (\hat{M}_2^2 - \hat{M}_1^2)}{(\hat{M}_2^2 - \hat{M}_1^2 - \frac{1}{\pi}\hat{M}_1\Gamma_1 \ln(\hat{M}_2^2/\hat{M}_1^2))^2 + (\hat{M}_2\Gamma_2 - \hat{M}_1\Gamma_1)^2}.$$

Anisimov, Broncano, Plümacher, 2006

- Common error

$$\frac{\Sigma_2 + \Sigma_3\Sigma_4}{A + \Sigma_1} = \frac{\Sigma_2}{A + \Sigma_1}$$

Inconsistent expansion

$$\frac{\Sigma_2 + \Sigma_3\Sigma_4}{A + \Sigma_1} = \frac{\Sigma_2\left(1 + \frac{\Sigma_3\Sigma_4}{\Sigma_2}\right)}{A\left(1 + \frac{\Sigma_1}{A}\right)} = \frac{\Sigma_2}{A}\left(1 + \frac{\Sigma_3\Sigma_4}{\Sigma_2} - \frac{\Sigma_1}{A}\right)$$

Consistent expansion

CP violation in the decay of heavy neutrinos

- Pilaftsis, Underwood, 2004

$$\Delta_{11}\big|_{p^2\approx p^2_{N_1}} = \frac{Z_1}{\not p - p_{N_1}}, \qquad \Delta_{22}\big|_{p^2\approx p^2_{N_2}} = \frac{Z_2}{\not p - p_{N_2}},$$

Implicitly assumed a **large** mass difference

$$\Delta_{11} = \begin{cases} \dfrac{Z_1(\not p + p_{N_1})}{p^2 - p^2_{N_1}} + \cdots, & p^2 \approx p^2_{N_1}, \\ (D_{11})^{-1} D_{12} \dfrac{Z_2(\not p + p_{N_2})}{p^2 - p^2_{N_2}} D_{21}(D_{11})^{-1} + \cdots, & p^2 \approx p^2_{N_2} \end{cases}$$

$$D_{\beta\alpha}(\not p) = \delta_{\beta\alpha}(\not p - m_{N_\alpha}) - \Sigma_{\beta\alpha}(\not p)$$

$$[D_{22}(D_{12})^{-1} - D_{21}(D_{11})^{-1}]_{p^2=p^2_{N_\alpha}} = 0$$

Incorrect beyond the NLO

$$(\bar h^\nu_+)_{l1} = h^\nu_{l1} + i B_{l1} - \frac{i h^\nu_{l2} m_{N_1}(m_{N_1} A_{12} + m_{N_2} A_{21})}{m^2_{N_1} - m^2_{N_2} + 2i A_{22} m^2_{N_1}},$$

$$(\bar h^\nu_+)_{l2} = h^\nu_{l2} + i B_{l2} - \frac{i h^\nu_{l1} m_{N_2}(m_{N_2} A_{21} + m_{N_1} A_{12})}{m^2_{N_2} - m^2_{N_1} + 2i A_{11} m^2_{N_2}},$$

$$(\bar h^\nu_-)_{l1} = h^{\nu *}_{l1} + i B^*_{l1} - \frac{i h^{\nu *}_{l2} m_{N_1}(m_{N_1} A^*_{12} + m_{N_2} A^*_{21})}{m^2_{N_1} - m^2_{N_2} + 2i A_{22} m^2_{N_1}},$$

$$(\bar h^\nu_-)_{l2} = h^{\nu *}_{l2} + i B^*_{l2} - \frac{i h^{\nu *}_{l1} m_{N_2}(m_{N_2} A^*_{21} + m_{N_1} A^*_{12})}{m^2_{N_2} - m^2_{N_1} + 2i A_{11} m^2_{N_2}}.$$

Correct only up to the NLO

Up to the NLO, consistent with

$$\boxed{\begin{aligned} \widehat{f}_{i\alpha} &= (f C_R^{\widehat{N}_\alpha})_{i\alpha} = f_{i\alpha} + f_{i\beta} \frac{m_{N_\alpha}[m_{N_\beta}(\Sigma_R)_{\alpha\beta}(m^2_{N_\alpha}) + m_{N_\alpha}(\Sigma_R)_{\beta\alpha}(m^2_{N_\alpha})]}{m^2_{N_\beta} - m^2_{N_\alpha}}, \\ \widehat{f}^c_{i\alpha} &= (f^* C_L^{\widehat{N}_\alpha})_{i\alpha} = f^*_{i\alpha} + f^*_{i\beta} \frac{m_{N_\alpha}[m_{N_\beta}(\Sigma_R)_{\beta\alpha}(m^2_{N_\alpha}) + m_{N_\alpha}(\Sigma_R)_{\alpha\beta}(m^2_{N_\alpha})]}{m^2_{N_\beta} - m^2_{N_\alpha}}, \end{aligned}}$$

CP violation in the decay of heavy neutrinos

- Anisimov, Broncano, Plümacher, 2006

future use, we introduce here an expansion parameter α related to the largest of the couplings K_{ij},

$$\alpha = \mathrm{Max}\left[\frac{K_{ij}}{16\pi^2}\right]. \tag{19}$$

In the interesting case that the masses of the right-handed neutrinos are quasi-degenerate, i.e., $\hat{M}_2 - \hat{M}_1 \ll \hat{M}_1$, one can define an additional small expansion parameter

$$\Delta \equiv \frac{\hat{M}_2 - \hat{M}_1}{\hat{M}_1}. \tag{20}$$

Our results, to be presented in the following, will only be valid as long as $\Delta \gg \alpha$, since otherwise perturbation theory breaks down.

Explicitly assumed a **large** mass difference

$$\varepsilon_1(\hat{M}_1^2) = \frac{\mathrm{Im}(K_{12}^2)}{8\pi K_{11}} \frac{\hat{M}_1\hat{M}_2(\hat{M}_2^2 - \hat{M}_1^2)}{(\hat{M}_2^2 - \hat{M}_1^2 - \frac{1}{\pi}\hat{M}_2\Gamma_2 \ln(\hat{M}_2^2/\hat{M}_1^2))^2 + (\hat{M}_2\Gamma_2 - \hat{M}_1\Gamma_1)^2},$$

$$\varepsilon_2(\hat{M}_2^2) = \frac{\mathrm{Im}(K_{12}^2)}{8\pi K_{22}} \frac{\hat{M}_1\hat{M}_2(\hat{M}_2^2 - \hat{M}_1^2)}{(\hat{M}_2^2 - \hat{M}_1^2 - \frac{1}{\pi}\hat{M}_1\Gamma_1 \ln(\hat{M}_2^2/\hat{M}_1^2))^2 + (\hat{M}_2\Gamma_2 - \hat{M}_1\Gamma_1)^2}.$$

$$\boxed{\begin{aligned} \widehat{f}_{i\alpha} &= (fC_R^{\widehat{N}_\alpha})_{i\alpha} = f_{i\alpha} + f_{i\beta}\frac{m_{N_\alpha}[m_{N_\beta}(\Sigma_R)_{\alpha\beta}(m^2_{N_\alpha}) + m_{N_\alpha}(\Sigma_R)_{\beta\alpha}(m^2_{N_\alpha})]}{m^2_{N_\beta} - m^2_{N_\alpha}}, \\ \widehat{f}^c_{i\alpha} &= (f^*C_L^{\widehat{N}_\alpha})_{i\alpha} = f^*_{i\alpha} + f^*_{i\beta}\frac{m_{N_\alpha}[m_{N_\beta}(\Sigma_R)_{\beta\alpha}(m^2_{N_\alpha}) + m_{N_\alpha}(\Sigma_R)_{\alpha\beta}(m^2_{N_\alpha})]}{m^2_{N_\beta} - m^2_{N_\alpha}}, \end{aligned}}$$

$$\longrightarrow$$

$$\boxed{\begin{aligned} \widehat{f}_{i\alpha} &= f_{i\alpha} + f_{i\beta}\frac{m_{N_\alpha}[m_{N_\alpha}(\Sigma_R)_{\beta\alpha}(m^2_{N_\alpha}) + m_{N_\beta}(\Sigma_R)_{\alpha\beta}(m^2_{N_\alpha})]}{p^2_{N_\alpha} - p^2_{N_\beta}}, \\ \widehat{f}^c_{i\alpha} &= f^*_{i\alpha} + f^*_{i\beta}\frac{m_{N_\alpha}[m_{N_\alpha}(\Sigma_R)_{\alpha\beta}(m^2_{N_\alpha}) + m_{N_\beta}(\Sigma_R)_{\beta\alpha}(m^2_{N_\alpha})]}{p^2_{N_\alpha} - p^2_{N_\beta}}, \end{aligned}}$$

Difference is irrelevant up to the NLO

Physical implications

- The **intermediate** mass difference allows **maximal CP violation**. Applicable to the resonant leptogenesis. The current analysis should be corrected.
- The **quasiparticles** of heavy neutrinos are **not Majorana** even up to the LO when the mixing is large.
- The correlation between the CP violation and the generation of quasiparticles. (Incomplete)
- **Dirac** fields
  - Implication for the CKM matrix: mixing matrices are non-unitary.
- **Scalar** fields
  - **Meson decay anomaly**: might resolve the discrepancy between the current analysis and experimental results for the B meson.

Summary

- **Loop-induced mixing** generates **quasiparticles** which are effective particles generated by RH Majorana neutrinos interacting with each other through Yukawa interaction.

- In the cases of **intermediate** mass difference, those quasiparticles **cannot** be treated perturbatively or analytically.

- The **large** loop-induced flavor mixing can be properly treated by **carefully diagonalizing** the propagator matrix.

- Correct calculation of the **CP violation** effects is made possible.