Anisotropic Holography and Inverse Catalysis

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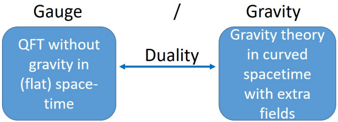
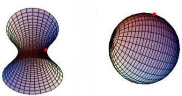
Outline

Briefly on AdS/CFT

- Gauge/Gravity duality: A way to map quantum questions to gravity geometric questions and answer them.



- The initial AdS/CFT correspondence: $\mathcal{N} = 4$ sYM on flat space $\Leftrightarrow AdS_5 \times S^5$, is the harmonic oscillator of the gauge/gravity dualities.



- It is the simplest toy model and very far from describing Nature!

- Since the discovery of the initial correspondence, there is an extensive research aiming to construct more realistic gauge/gravity dualities (confinement, no susy, temperature, quarks...).

✓ This talk: Anisotropic theories in Gauge/Gravity correspondence.

Why? Attempts for a first Realization of Nature

The existence of strongly coupled anisotropic systems.

- The expansion of the plasma along the longitudinal beam axis at the earliest times after the collision results to momentum anisotropic plasmas.
- Strong Magnetic Fields in strongly coupled theories.
- New interesting phenomena in presence on such fiels, i.e. inverse magnetic catalysis.

  eg: (Bali, Bruckmann, Endrodi, Fodor, Katz, Krieg et al. 2011)
- Anisotropic low dimensional materials in condensed matter.

Why? More:

- Weakly coupled vs strongly coupled anisotropic theories.

  (Dumitru, Strickland, Romatschke, Baier,...)

- Top-down supergravity Black hole solutions that are AdS in UV **flowing** to Lifshitz-like in IR :

  ⋆ Fixed scaling parameter z for such anisotropic solutions?

  (Azeyanagi, Li, Takayanagi, 2009; Mateos, Trancanelli, 2011;...)

  ⋆ New flows to alternative IR fixed points?

- Ⓢtriking Features! Several Universality Relations for the isotropic theories are violated in aniso!

  Shear viscosity η over entropy density s: takes parametrically low values wrt degree of anisotropy $\frac{\eta}{s} < \frac{1}{4\pi}$!

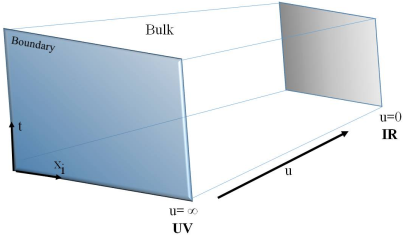
  (Rebhan, Steineder 2011; D.G. 2012; Jain, Samanta, Trivedy 2015; D.G., Gursoy, Pedraza, 2017)

Reminding Slide:

- The anisotropic hyperscaling violation metric

$$ds^2 = u^{-\frac{2\theta}{d}}\left(-u^{2z}\left(dt^2 + dy_i^2\right) + u^2 dx_i^2 + \frac{du^2}{u^2}\right)$$

exhibits a critical exponent z and a hyperscaling violation exponent θ.



- $\theta = 0, \; z = 1 \Rightarrow$ AdS.
- $\theta = 0 \Rightarrow$ scale invariant theory.
- In general no scale invariance.

$$t \to \lambda^z t, \qquad y \to \lambda^z y, \qquad x \to \lambda x, \qquad u \to \frac{u}{\lambda}\,, \qquad ds \to \lambda^{\frac{\theta}{d}} ds\,.$$

A Theory with Phase Transitions in One Page:

- How the Field Theory looks like?
  - ✓ 4d $SU(N)$ Strongly coupled anisotropic gauge theory.
  - ✓ Its dynamics are affected by a scalar operator $\mathcal{O}_\Delta$ ($\sim TrF^2$).
  - ✓ Anisotropy is introduced by another operator $\tilde{\mathcal{O}} \sim \theta(x_3) TrF \wedge F$ with a space dependent coupling.

- The gravity dual theory is an Einstein-Axion-Dilaton theory in 5 dimensions with a non-trivial potential.
  - ✓ A "backreacting" scalar field depending on spatial directions, the axion; and a non-trivial dilaton.
  - ✓ Solutions are non-trivial RG flows: Conformal fixed point in the UV $\Rightarrow$ Anisotropic (Hyperscaling Lifshitz-like) in IR.

- The vacuum state confines color and there exists a phase transition at finite T_c above which a deconfined plasma state arises.

(D.G., Gursoy, Pedraza, 2017)

How is Anisotropy introduced? A Pictorial Representation:

- For the Lifshitz-like IIB Supergravity solutions

$$ds^2 = u^{2z}(dx_0^2 + dx_i^2) + u^2 dx_3^2 + \frac{du^2}{u^2} + ds^2_{S^5}.$$

Introduction of additional branes: *(Azeyanagi, Li, Takayanagi, 2009)*

Spacetime

AdS

	x_0	x_1	x_2	x_3	u	S^5
D3	x	x	x	x		
D7	x	x	x			x

- Which equivalently leads to the following AdS/CFT deformation.

$N = 4\, sYM$ $\longleftrightarrow$ $AdS_5 \; x \; S^5, \; \varphi, \; F_5$

$$\tau = \frac{\vartheta}{4\pi} + \frac{4\pi i}{g_{ym}^2} = \chi + i e^{-\phi}$$

$S = S_{N=4} + \delta S$

$\delta S = \int \theta(x_3)\, Tr\, F \wedge F$ $\longleftrightarrow$ $\overline{AdS}_5 \; x \; S^5, \; \varphi, \; \chi(x_3), \; F_5$

- $dC_8 \sim \star d\chi$ with the non-zero component $C_{x_0 x_1 x_2 S^5}$.

An Anisotropic Theory

The generalized Einstein-Axion-Dilaton action with a potential for the dilaton and an arbitrary coupling between the axion and the dilaton:

$$S=\frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left[R-\frac{1}{2}(\partial\phi)^2+V(\phi)-\frac{1}{2}Z(\phi)(\partial\chi)^2\right].$$

The eoms read

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=\frac{1}{2}\partial_\mu\phi\partial_\nu\phi+\frac{1}{2}Z(\phi)\partial_\mu\chi\partial_\nu\chi-\frac{1}{4}g_{\mu\nu}(\partial\phi)^2-\frac{1}{4}g_{\mu\nu}Z(\partial\chi)^2+\frac{1}{2}g_{\mu\nu}V(\phi)\ ,$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right)=\frac{1}{2}\partial_\phi Z(\phi)(\partial\chi)^2-V'(\phi)\ ,$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\chi\right)=0\ .$$

Where

$$V(\phi)=12\cosh(\sigma\phi)+\left(\frac{m(\Delta)^2}{2}-6\sigma^2\right)\phi^2,\qquad Z(\phi)=e^{2\gamma\phi}\ .$$

(Gursoy, Kiritsis, Nitti, 2007; (Gubser, Nellore), Pufu, Rocha 2008a,b)

Remark: For $\sigma=0,\gamma=1,m(\Delta)=0$ the action and the solution of eoms, are reduced of IIB supergravity.

A Solution : The RG Flow

$$ds^2 = \frac{1}{u^2}\left(-\mathcal{F}(u)\mathcal{B}(u)\,dt^2 + dx_1^2 + dx_2^2 + \mathcal{H}(u)dx_3^2 + \frac{du^2}{\mathcal{F}(u)}\right),$$

$$\chi = \alpha x_3\,, \qquad \phi = \phi(u)\,, \qquad \mathcal{F}(u_h) = 0\ .$$

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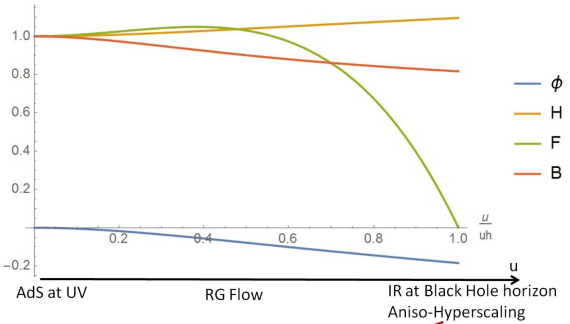


$$ds^2 = u^{-\frac{2\theta}{3}}\left(-u^{2z}\left(f(u)dt^2 + dx_{1,2}^2\right) + \tilde{\alpha}\,u^2 dx_3^2 + \frac{du^2}{f(u)u^2}\right)$$

Axion-Dilaton Coupling and Potential, rule the Scaling Coefficients

- The values of (θ, z) dependence on (γ, σ)

$$z = \frac{4\gamma^2 - 3\sigma^2 + 2}{2\gamma(2\gamma - 3\sigma)} , \qquad \theta = \frac{3\sigma}{2\gamma} .$$

- Special case: $(\sigma = 0,\ \gamma = 1)$ supergravity truncated action with a single solution $(\theta = 0,\ z = 3/2)$. *(Mateos, Trancanelli, 2011)*
- The scaling factors z and θ are determined by the constants in the Axion-Dilaton Coupling and the Potential. This is the reason that in the particular setup the supergravity solutions have them fixed.

We have obtained the theories, are they physical and stable?

$$\Downarrow$$
$$\Downarrow$$

✓ Energy Conditions Analysis: $T_{\mu\nu} N^\mu N^\nu \geq 0 \;, \quad N^\mu N_\mu = 0 \;.$

AND

✓ Local Thermodynamical Stability Analysis: Specific Heat, Chemical potential...

$$\Downarrow$$
$$\Downarrow$$

YES!

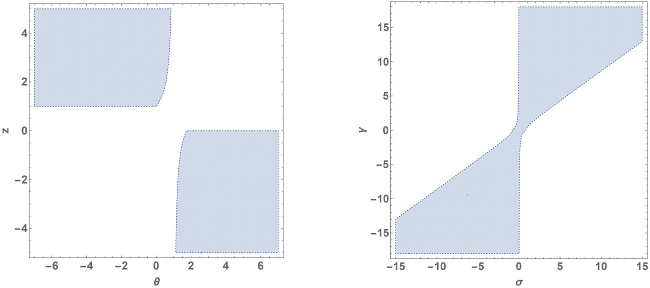
Three conditions that constrain (z,θ) and as a result (γ,σ).

$$(z-1)(1-\theta+3z)\geq 0\ , \qquad z=\frac{2+4\gamma^2-3\sigma^2}{2\gamma(2\gamma-3\sigma)}\ ,$$

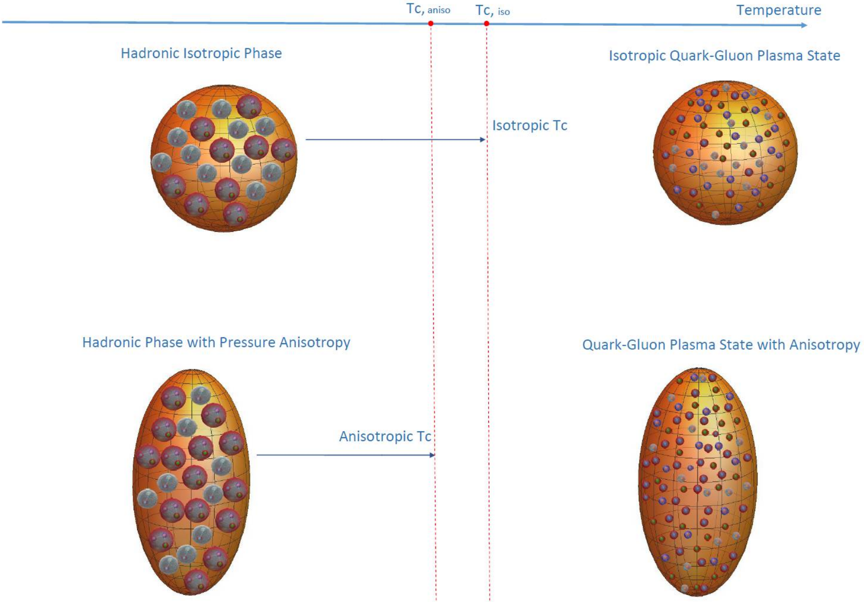
$$\theta^2-3+3z(1-\theta)\geq 0\ , \qquad \theta=\frac{3\sigma}{2\gamma}\ ,$$

$$1-\theta+2z\geq 0\ .$$

The blue region is the acceptable for the theory parameters.

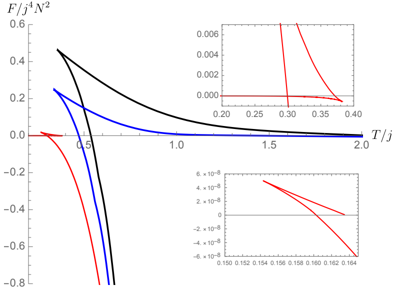


Catalysis: Lower Phase Transition Temperature
due to any Source of Anisotropy.



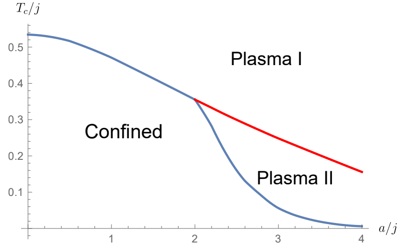
Confinement/Deconfinement Phase transitions

- Competition for dominance between different gravitational backgrounds.
- The free energy of the theories vs the temperature T for different anisotropy (α/j=0,1,3):



- Horizontal Axis: Confining Phase.
- Upper Branch: Black hole A:Deconfining Plasma Phase.
- Lower Branch: Black hole B:Deconfining Plasma Phase.
- $\alpha/j \simeq 2$: A critical value above which a richer structure in the phase diagram exist.

- The Critical Temperature of the theories vs the anisotropy gives:



- The T_c is reduced in presence of anisotropies of the theory.

A Proposal

- The $Tc(\alpha)$ decrease with α, resembling the phenomenon of inverse magnetic catalysis where the confinement-deconfinement temperature decreases with the magnetic field B.
- No charged fermionic degrees of freedom in our case; our plasma is neutral.
- Our findings suggest that the anisotropy could be a cause of lower T_c, together with the charge dynamics caused by the magnetic field in inverse magnetic catalysis.

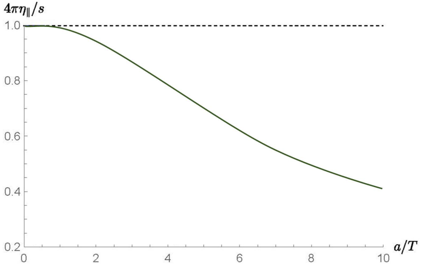
Universal Results: η/s in Theories with Broken Symmetry

Consider a finite T theory:

$$ds^2 = g_{tt}(u)dt^2 + g_{11}(u)(dx_1^2 + dx_2^2) + g_{33}(u)dx_3^2 + g_{uu}(u)du^2$$

- The Shear Viscosity is obtained by the two-point function of the energy momentum tensor.
- The anisotropic viscosity violates the isotropic “bound” of $1/4\pi$ **:**

Frictionless Anisotropic Plasma.

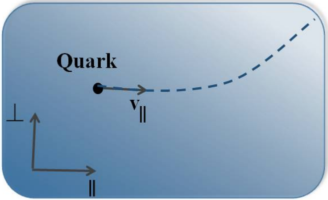


- The Ratio:

$$4\pi\frac{\eta_\parallel}{s} = \frac{g_{11}}{g_{33}}\bigg|_{u=u_h} \sim \left(\frac{T}{\alpha}\right)^p , \qquad p = 2 - \frac{2}{z} \sim [0,\infty) , \quad \alpha \gg T .$$

Langevin Dynamics and Brownian Motion

Langevin coefficients κ: Consider a heavy quark ($M \gg T$) moving along the "$\|$" direction in a strongly coupled plasma.



The Macroscopic Langevin equation:

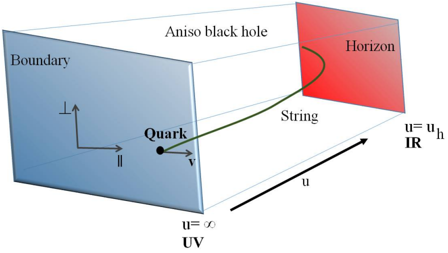
$$\dot{p}_i(t) = -\eta_D p_i(t) + \xi_i(t) \ ,$$

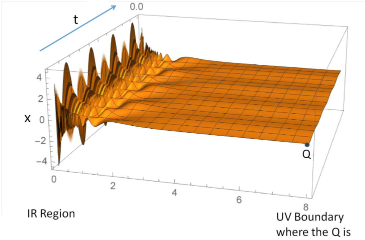
p: the momentum of the particle, η_D: the friction coefficient, ξ: the random force.

$$\langle \xi_{\|,\perp}(t) \rangle = 0 \ , \qquad \langle \xi_{\|,\perp}(t) \xi_{\|,\perp}(t') \rangle = \kappa_{\|,\perp} \delta(t-t') \ , \qquad \left\langle p^2_{\|,\perp} \right\rangle = 2\kappa_{\|,\perp} \mathcal{T}$$

- For any holographic theory in the deconfined phase the stochastic nature of the particle obtained by the string Fluctuations and then Quantization in the holographic space:

(deBoer, Hubeny, Rangamani, Shigemori, 2009; Tong, Wong 2013)





A Universal Inequality for Isotropic Theory:

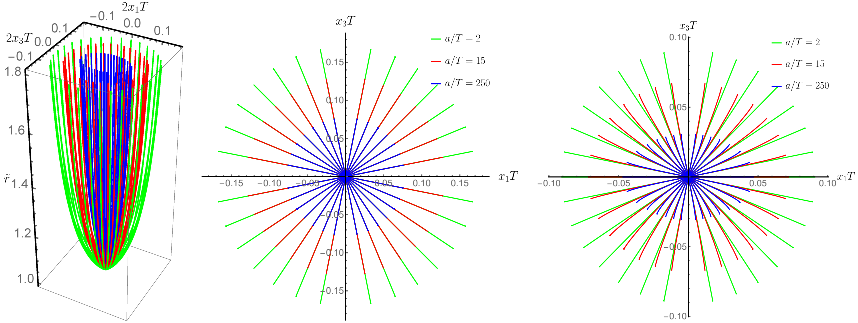
$\kappa_{\parallel} \geq \kappa_{\perp}$ for any isotropic strongly coupled plasma!

Can be inverted in the anisotropic theories: $\kappa_{\parallel} \geq < \kappa_{\perp}$.

(Gursoy, Kiritsis, Mazzanti, Nitti, 2010; D.G, Soltanpanahi, 2013a,b; D.G., Lee, Yeh 2018)

Baryons in Theories with External Fields

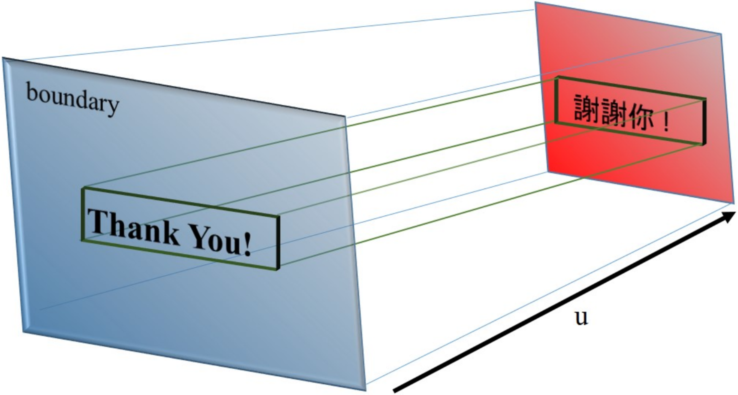
- The quark distribution for baryons in theories with anisotropic dynamics:



- Baryon on the transverse plane and Baryon on the plane that the field lies. (D.G. 2018)
- System of fundamental F1 strings with a vertex Dp-brane, in an anisotropic gravity theory.
- Similar effect on Q-distribution, for speeding baryons in strong coupled isotropic plasma. (Athanasiou, Liu, Rajagopal 2008)

Conclusions

- ✓ Observation: In strongly coupled theories many phenomena are more sensitive to the presence of the anisotropy than the source that triggers it.
- ✓ Confining Anisotropic theories with confinement /deconfinement phase transition. (1st construction in the literature)
- ✓ The phase transitions occur at lower critical Temperature as the anisotropy is increased = Inverse Anisotropic Catalysis!
- ✓ Several Universal Isotropic relations are anisotropically violated. E.g. The shear viscosity over entropy density ratio, takes values parametrically lower than $1/4\pi$, as $\sim (T/\alpha)^{2-2/z}$.


boundary
Thank You!
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Special case: IIB Supergravity

Remark:
The ten dimensional action gives our generalized model, when the internal space is an S^5 supported by fluxes and $\sigma = 0, \gamma = 1, \Delta = 4$:

$$S = \frac{1}{2\kappa_{10^2}} \int d^{10}x \sqrt{-g} \left[R + 4\partial_M\phi\partial^M\phi - e^{2\phi}\left(\frac{1}{2}F_1^2 + \frac{1}{4\cdot 5!}F_5^2\right) \right], \; F_1 := d\chi \,.$$

where $M = 0, \ldots, 9$ and F_1 is the axion field-strength. The equations of motion for the background are:

$$R + 4g^{MN}\left(\nabla_M\nabla_N\phi - \partial_M\phi\partial_N\phi\right) = 0\,,$$
$$R_{MN} + 2\nabla_M\nabla_N\phi + \frac{1}{4}g_{MN}e^{2\phi}\partial_P\chi\partial^P\chi - \frac{1}{2}e^{2\phi}\left(F_MF_N + \frac{1}{48}F_{MABCD}F_N{}^{ABCD}\right) = 0\,.$$

plus the Bianchi identities and self duality constraints. The axion field equation is satisfied trivially for linear axion.

An exact solution

The potential and the axion-dilaton coupling

$$V(\phi) = 6e^{\sigma\phi}, \qquad Z(\phi) = e^{2\gamma\phi}.$$

A Lifshitz-like anisotropic hyperscaling violation background which may accommodate a black hole

$$ds_s^2 = \alpha^2 C_R e^{\frac{\phi(u)}{2}} u^{-\frac{2\theta}{3z}} \left(-u^2 \left(f(u) dt^2 + dx_i^2 \right) + C_Z u^{\frac{2}{z}} dx_3^2 + \frac{du^2}{f(u)\alpha^2 u^2} \right) ,$$

where

$$f(u) = 1 - \left(\frac{u_h}{u} \right)^{3+(1-\theta)/z} , \qquad e^{\frac{\phi(u)}{2}} = u^{\frac{\sqrt{\theta^2+3z(1-\theta)-3}}{\sqrt{6}z}} ,$$

$$C_R = \frac{(3z-\theta)(1+3z-\theta)}{6z^2} , \quad C_Z = \frac{z^2}{2(z-1)1+3z-\theta} ,$$

$$z = \frac{4\gamma^2 - 3\sigma^2 + 2}{2\gamma(2\gamma - 3\sigma)} , \qquad \theta = \frac{3\sigma}{2\gamma} .$$

Null Energy Condition

- The averaged radial acceleration between two null geodesics is

$$A_r = -4\pi T_{\mu\nu} N^\mu N^\nu \ ,$$

if it is negative the null geodesics observe a non-repulsive gravity on nearby particles along them.

- This imposes the Null Energy Condition

$$T_{\mu\nu} N^\mu N^\nu \geq 0 \ , \quad N^\mu N_\mu = 0 \ ,$$

leading to the following constrains:

- For the Lifshitz-like space $z \geq 1$.
- For the Hyperscaling violation anisotropic metric in 3+1-dim spacetime and anisotropic in 1-dim reads

$$(z-1)(1-\theta+3z) \geq 0 \ ,$$
$$\theta^2 - 3 + 3z(1-\theta) \geq 0 \ .$$

Additional conditions from thermodynamics?

Local Thermodynamic Stability

- The necessary and sufficient conditions for local thermodynamical stability in the canonical ensemble are

$$c_\alpha = T\left(\frac{\partial S}{\partial T}\right)_\alpha \geq 0\ , \qquad \Phi' = \left(\frac{\partial \Phi}{\partial \alpha}\right)_T \geq 0$$

  c_α is the specific heat: increase of the temperature leads to increase of the entropy.
  Φ' is derivative of the potential: the system is stable under infinitesimal charge fluctuations.

- In the GCE these conditions should be equivalent of having no positive eigenvalues of the Hessian matrix of the entropy with respect to the thermodynamic variables. *(Gubser, Mitra 2001)*

- In the IR the positivity of the specific heat imposes

$$c_\alpha = 1 - \theta + 2z \geq 0$$

η/s for our theories

- Shear Viscosity over Entropy Density

$$\eta_{ij,kl} = -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\int dtdxe^{i\omega t}\langle T_{ij}(t,x),\, T_{kl}(0,0)\rangle$$
$$s = \frac{2\pi}{\kappa^2}A\ .$$

The two-point function is obtained by calculating the response to turning on suitable metric perturbations in the bulk.

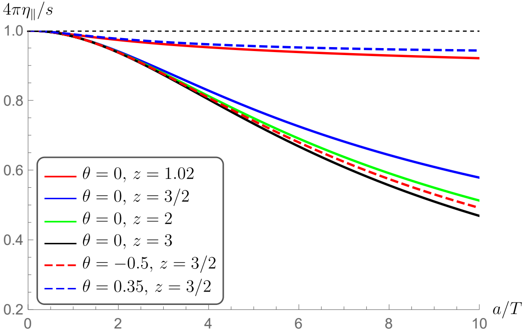
- The relevant part of the perturbed action is mapped to a Maxwell system with a mass term.

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(-\frac{1}{4g_{eff}^2}F^2 - \frac{1}{4}m^2(u)A^2\right)\ ,$$

where

$$m^2(u) = Z(\phi + \frac{1}{4}\log g_{33})\alpha^2\ , \quad \frac{1}{g_{eff}^2} = g_{33}^{3/2}(u)\ , \quad A_\mu = \frac{\delta g_{\mu 3}}{g_{33}}$$

- The shear viscosity over entropy ratio for arbitrary (z, θ).



- The ratio depends on the temperature at $\alpha/T \gg 1$ as

$$4\pi \frac{\eta_\parallel}{s} = \frac{g_{11}}{g_{33}} \sim \left(\frac{T}{\tilde{\alpha}|1 + 3z - \theta|} \right)^{2-\frac{2}{z}} .$$

- The range of the temperature power is $[0, \infty)$.

Probing the Theory: Energy Loss of Heavy Quark

- A Heavy Quark with mass $M \gg T$ undergoes a Brownian motion in the plasma.
  (deBoer, Hubeny, Rangamani, Shigemori, JHEP 2009; DG, Lee, Yeh, JHEP 2018)
- The momentum evolves according to the macroscopic Langevin equations

$$\dot{p}_i(t) = -\eta_D p_i(t) + \xi_i(t) \ ,$$

p: the momentum of the particle, η_D: the friction coefficient, ξ: the random force:

$$\langle \xi_{\parallel,\perp}(t) \rangle = 0 \ , \quad \langle \xi_{\parallel,\perp}(t) \xi_{\parallel,\perp}(t') \rangle = \kappa_{\parallel,\perp} \delta(t-t') \ , \quad \left\langle p^2_{\parallel,\perp} \right\rangle = 2\kappa_{\parallel,\perp} \mathcal{T}$$

$(\parallel, \perp)$ = (direction of the quark motion, transverse plane).
κ = mean squared momentum per unit of time $\mathcal{T}$.

Brownian Motion of Heavy Quarks

- The response function $\chi(\omega)$ of the quark to an external force $\propto$ two point correlator of a string fluctuations divided by the applied force.
- For any theory of the form

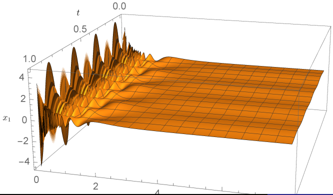
$$ds^2 = u^{a_0} f(u) + u^{a_i} dx_i^2 + \frac{du^2}{u^{a_u} f(u)} \,,$$

the string fluctuations along x_1 close to the boundary are

$$\frac{\partial}{\partial u}\left(\frac{g_{11}\sqrt{-g_{00}}}{\sqrt{g_{uu}}}\delta x_1'\right) - \frac{g_{11}\sqrt{g_{uu}}}{\sqrt{g_{00}}}\delta \ddot{x}_1^2 = 0 \,.$$

and can be found by the monodromy patching method

$$\delta x_{1\omega}(u) = c_1\left(1 + i\omega c_0 g_{11}(u_h) + \frac{i\omega g_{11}(u_h)}{2\kappa\nu} u^{-2\kappa\nu}\right) \,, \quad \nu := \frac{a_0 + 2a_1 + a_u - 2}{2(a_0 + a_u - 2)} .$$



- The reason for the unified formula is that the solution of fluctuations is of Bessel type with order ν.
- The diffusion coefficient

$$D = T \lim_{\omega \to 0} \left(-i\, \omega \chi(\omega)\right) \sim T^{2(1-\nu)} \, .$$

- Fluctuation-Dissipation theorem holds along each direction;The noise is white; Self energy and thermal mass of the particle depend on the properties and direction of the system... (D.G., Lee, Yeh 2018)

Parts of the Theory Timeline-Related bibliography:

Non-Confining Anisotropic Theories:

(Azeyanagi, Li, Takayanagi, 2009; Mateos, Trancanelli, 2011; Jain, Kundu, Sen, Sinha, Trivedi, 2015;...)

Confining Anisotropic Theories:

(D.G., Gursoy, Pedraza, 2017)

Similar ideas in different context. For example:

(Gaiotto, Witten 2008; Chu, Ho, 2006; Choi, Fernadez, Sugimoto 2017;...)