Comments on Trapping Horizon

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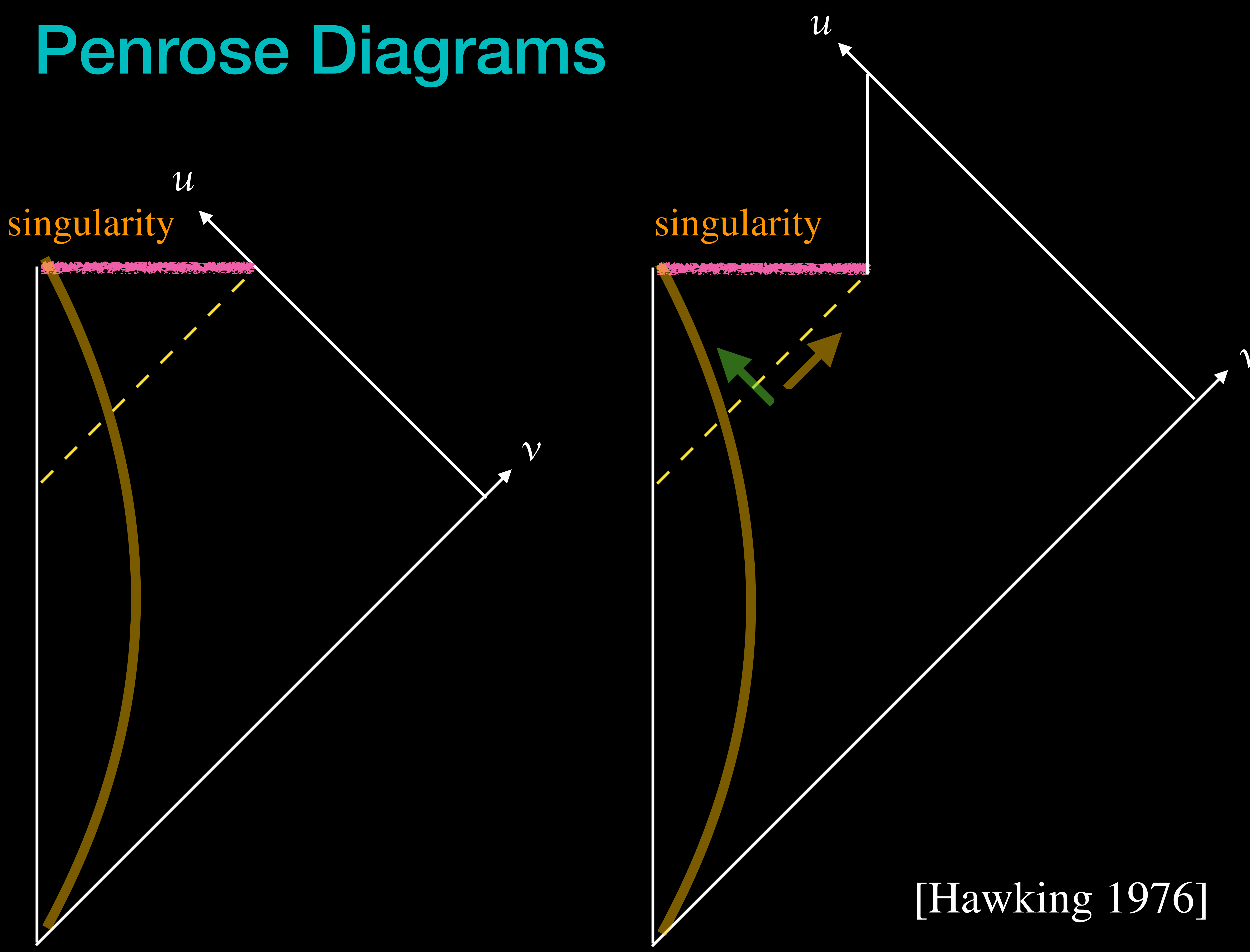
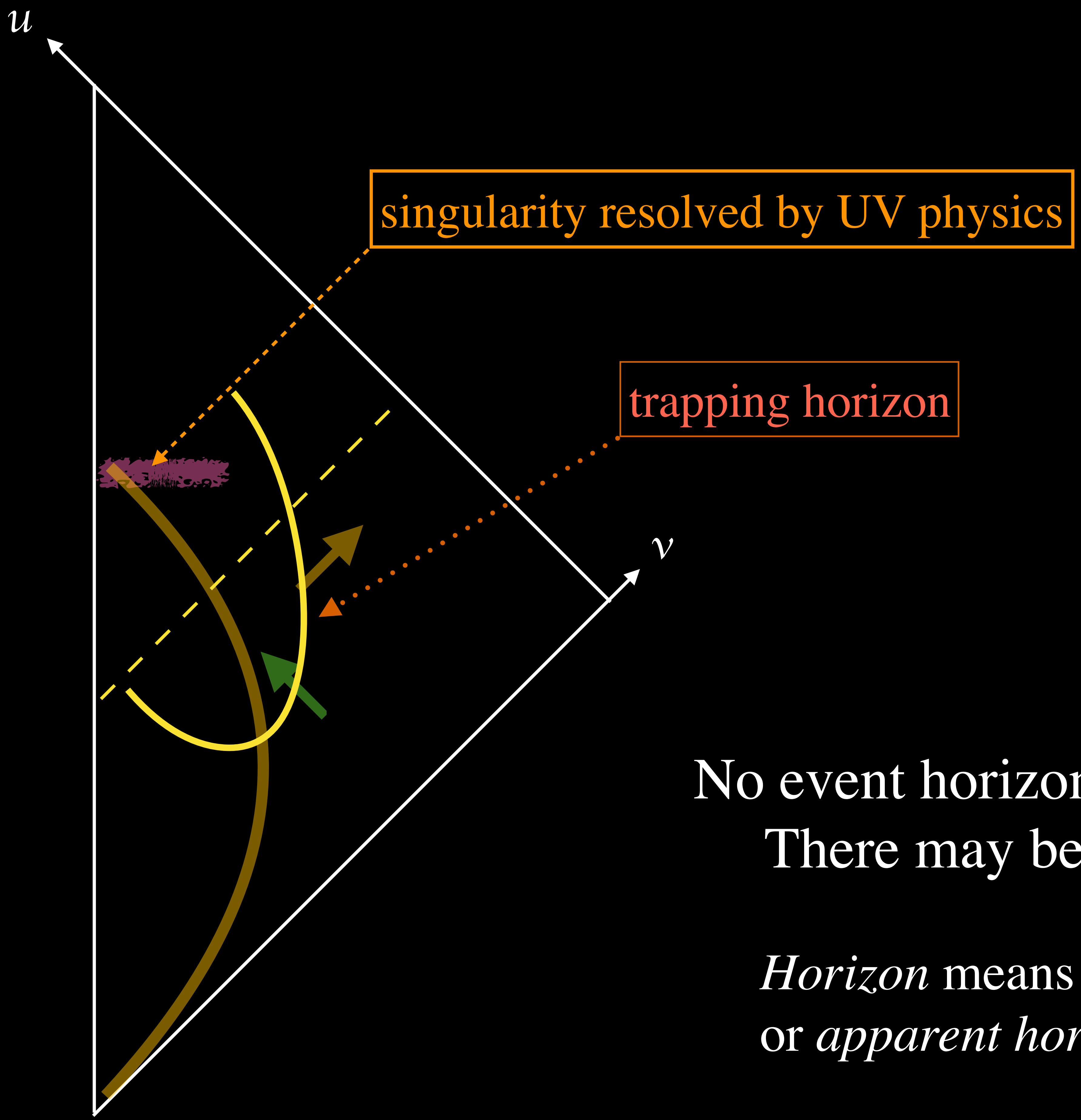
Penrose Diagrams



[Hawking 1976]



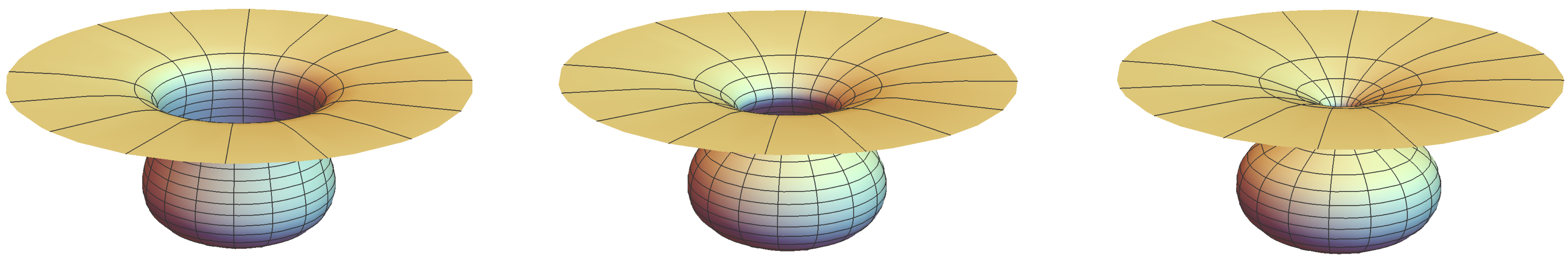
No event horizon if no singularity.
There may be trapping horizon.

Horizon means *trapping horizon*
or *apparent horizon* from now on.

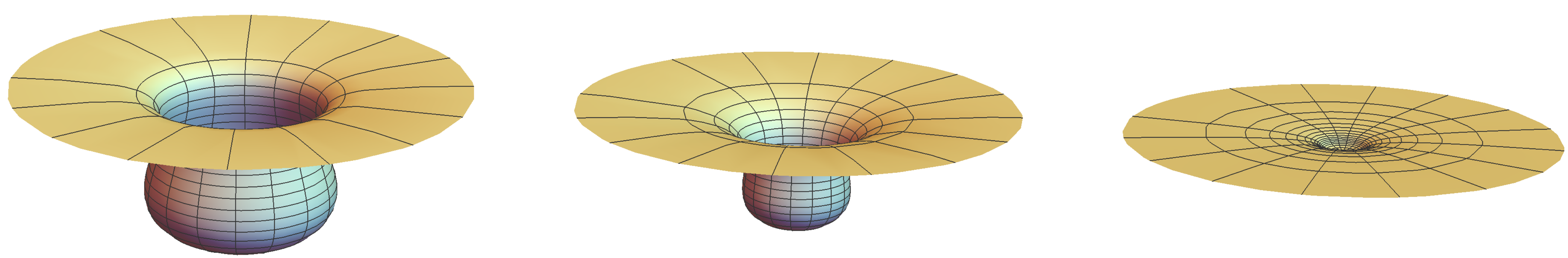
Black-Hole Geometry

[Ho-Matsuo, JHEP 1807]

“Wheeler’s Bag of Gold” as a remnant:



No interaction between matter and vacuum on the neck.
Need “Drama” at horizon for complete evaporation:



Information Paradox

According to the conventional model:

- Matter falls inside the “bag”.
- Macroscopic vacuum energy under horizon.
- No high energy event around the horizon. (?)
- No chance to bring out the information of matter.
- Remnant?
- Information paradox is not just about information.
  (low-energy effective theories, holography, …)

Basic Assumptions

- Semiclassical Einstein equation:

$$G_{\mu\nu} = \kappa T^{class}_{\mu\nu} + \kappa \langle \hat{T}_{\mu\nu} \rangle$$

for a quantum state

- Spherical symmetry

$$ds^2 = -C(u,v)dudv + r^2(u,v)d\Omega^2$$

areal radius

$C \to 1$ when $r \to \infty$

- *Trapping horizon*

Trapping Horizon

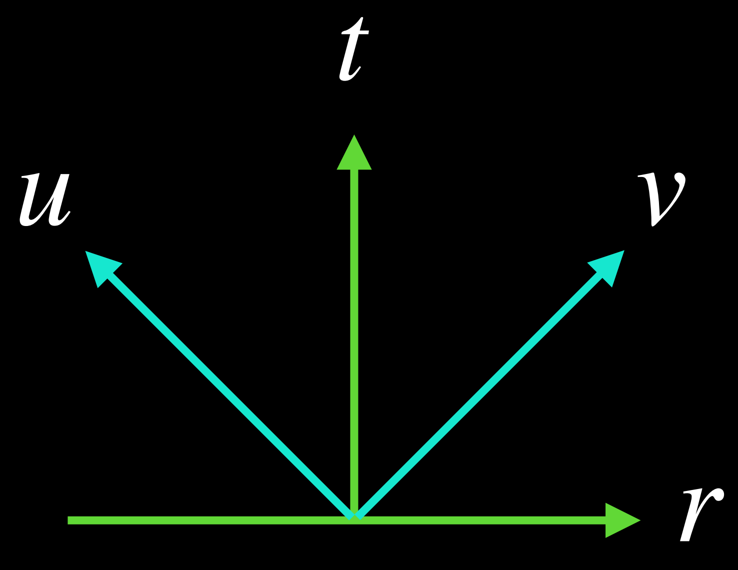
Spherical Symmetry

$$ds^2 = -C(u,v)dudv + r^2(u,v)(d\theta^2 + \sin^2\theta d\phi^2)$$

Trapping Horizon

Minkowski space

$$\partial_u r < 0, \quad \partial_v r > 0$$



Trapped region

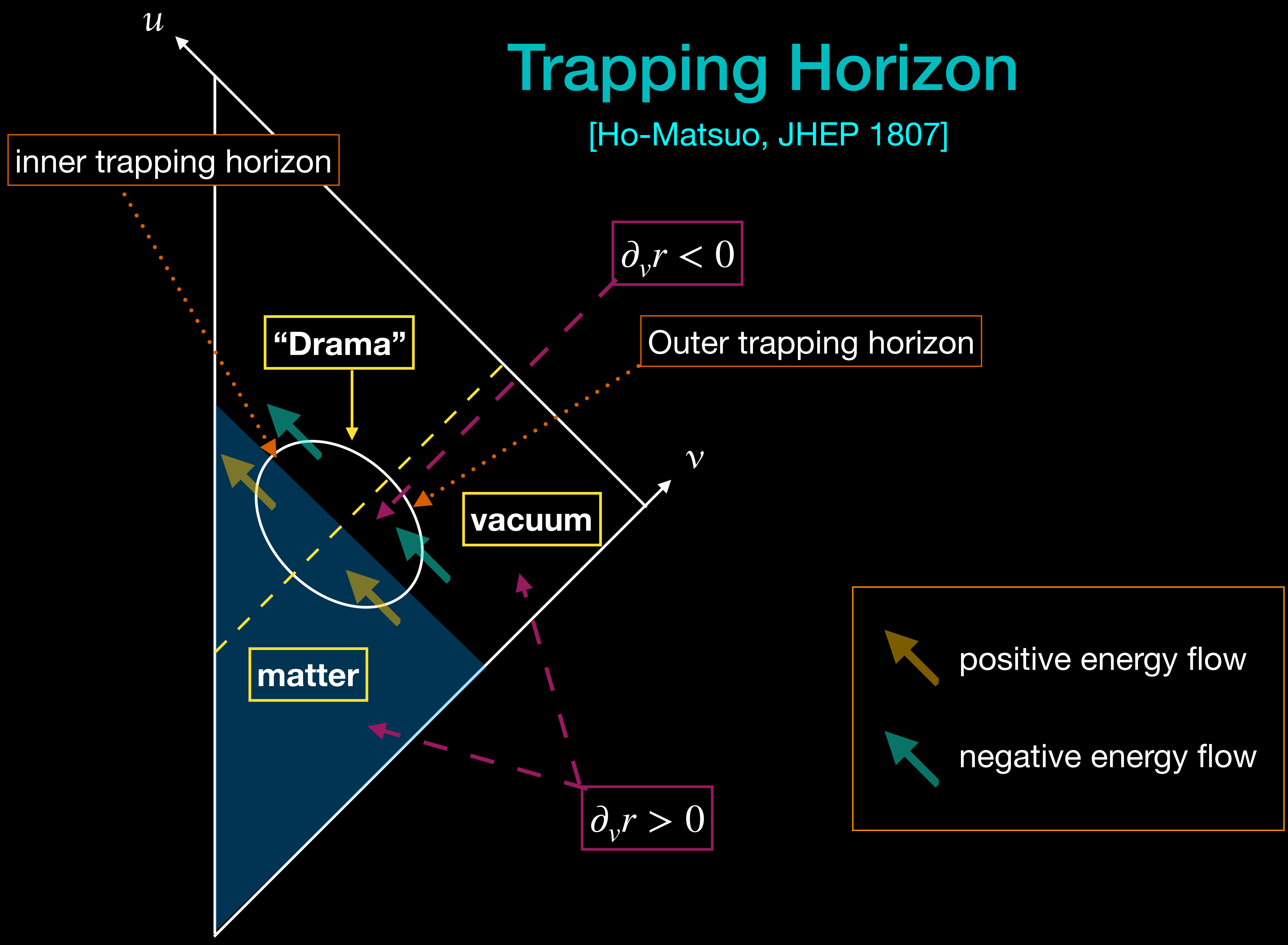
$$\partial_u r < 0, \quad \partial_v r < 0$$

Trapping horizon = *boundary* of trapped region

$$\partial_v r = 0, \quad (\partial_v^2 r > 0, \quad \partial_v^2 r < 0?)$$

Trapping Horizon

[Ho-Matsuo, JHEP 1807]



Comment on Trapping Horizon #1

[Ho-Matsuo, JHEP 1807]

Negative energy always necessary.

Drama at horizon needed if no singularity/ remnant.

Assuming no drama at horizon, there is a Planck-scale gauge-invariant quantity on the trapping horizon.

(“Drama” at horizon?)

Semi-Classical Einstein Equation

[Ho-Matsuo, JHEP 1807]

$\partial_v r = 0$
@ trapping horizon

$$G_{uu} \equiv \frac{2\partial_u C \partial_u r}{Cr} - \frac{2\partial_u^2 r}{r} = \kappa T_{uu}$$

$$G_{vv} \equiv \frac{2\partial_v C \partial_v r}{Cr} - \frac{2\partial_v^2 r}{r} = \kappa T_{vv}$$

$$G_{uv} \equiv \frac{C}{2r^2} + \frac{2\partial_u r \partial_v r}{r^2} + \frac{2\partial_u \partial_v r}{r} = \kappa T_{uv}$$

$$G_{\theta\theta} \equiv \frac{2r^2}{C^3}\left(\partial_u C \partial_v C - C\partial_u \partial_v C\right) - \frac{4r}{C}\partial_u \partial_v r = \kappa T_{\theta\theta}$$

Energy-Momentum Tensor

around Trapping Horizon at $v = v_0(u)$

$$C(u,v) = C_0(u) + C_1(u)(v - v_0(u)) + \cdots$$

$$r(u,v) = a(u) + \frac{1}{2} r_2(u)(v - v_0(u))^2 + \cdots$$

Einstein Eq. ⇒

$$\boxed{\begin{aligned} r_2(u) &= -\frac{1}{2}\ell_p^2 a(0) T_{vv}^{(0)}(u) \\ \dot{v}_0(u) &= \frac{C_0(u) - 2\ell_p^2 a^2(u) T_{uv}^{(0)}}{-2\ell_p^2 a^2(u) T_{vv}^{(0)}} \end{aligned}}$$

$$\ell_p^2 \equiv 8\pi G_N$$

$$\hbar = c = 1$$

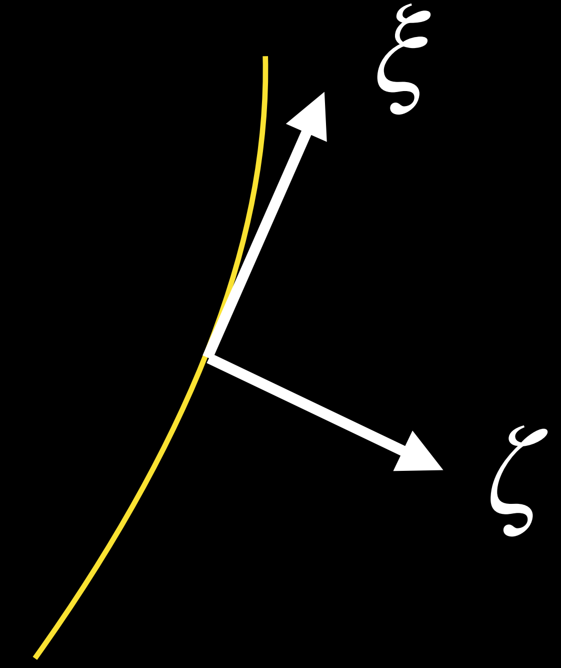
Trapping Horizon in vacuum demands Tvv < 0.
Time-like horizon needs "Drama" to turn space-like.

EMT @ Outer Trapping Horizon in Vacuum

[Ho-Matsuo, JHEP 1807]

$$\mathscr{E} \equiv T_{\xi\xi} \simeq -T_{\xi\zeta} \simeq T_{\zeta\zeta}$$

$$\mathscr{P} \equiv 4\pi a^2(u)\mathscr{E}$$



Universal result:

$$\boxed{\mathscr{E} \simeq -\frac{1}{2\ell_p^2 a^2} \qquad \mathscr{P} \simeq -\frac{2\pi}{\ell_p^2}}$$

$$\ell_p^2 \equiv 8\pi G_N$$
$$\hbar = c = 1$$

Black Hole's mass decreases because of this,
not Hawking Radiation.

Comment on Trapping Horizon #2

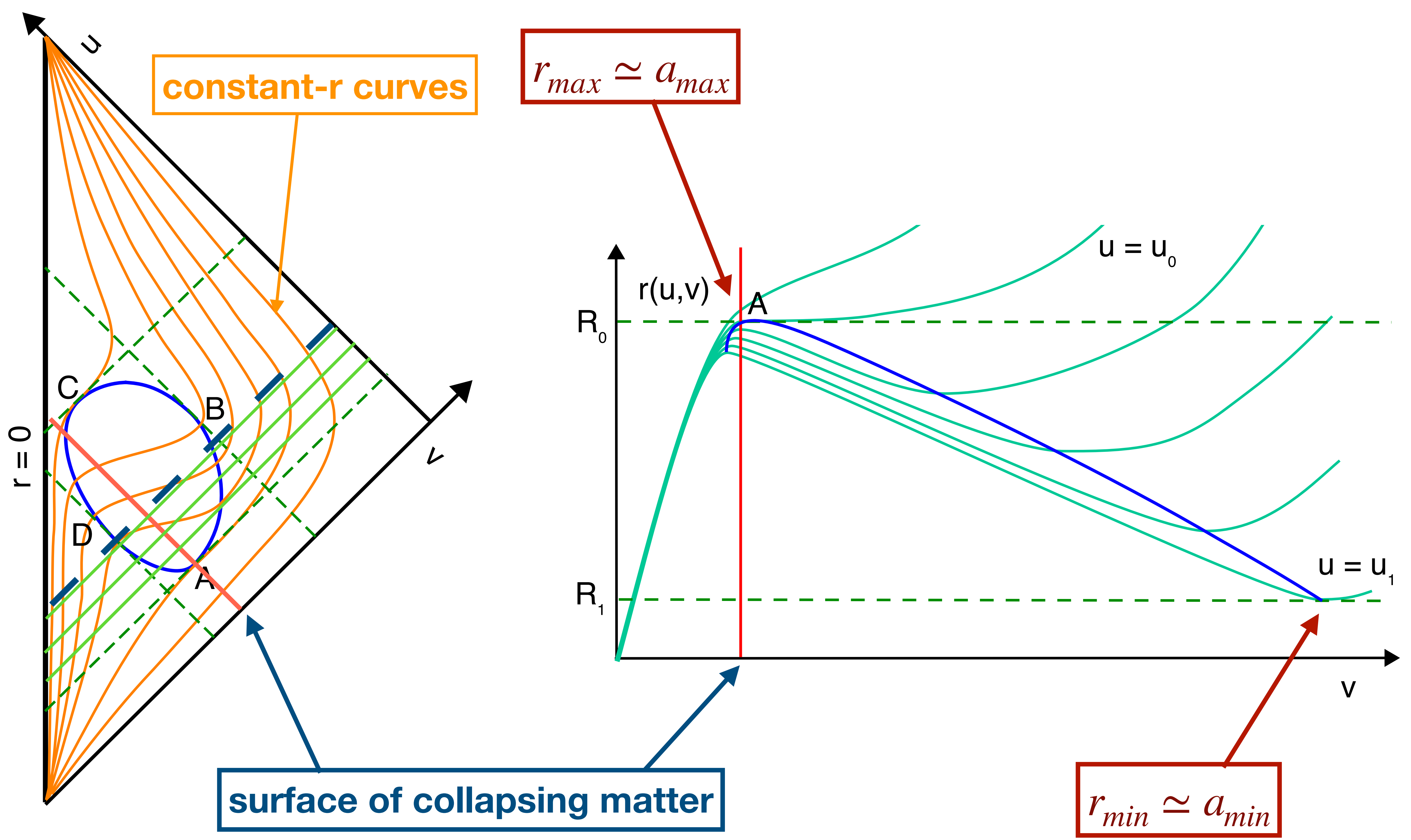
[Ho-Matsuo-Yokokura 19, to appear]

The distance from the trapping horizon

to the collapsing matter is $\mathcal{O}(\ell_p)$ until

the black hole is evaporated to a fraction

$\mathcal{O}\left(\frac{\ell_p^{2/3}}{a^{2/3}}\right)$ of its initial mass.



constant-r curves
$r_{max} \simeq a_{max}$
u
r = 0
C
B
D
A
v
R0
R1
r(u,v)
A
u = u0
u = u1
v
surface of collapsing matter
$r_{min} \simeq a_{min}$

Einstein Eq. → $\mathbf{C(u, v)}$

[Ho-Matsuo-Yokokura 2019]

Assuming "No Drama" at horizon:

$$\langle T^{\mu}{}_{\mu} \rangle, \ \ \langle T^{\theta}{}_{\theta} \rangle \lesssim \mathcal{O}\left(\frac{1}{\ell_p^2 a^2} \right)$$

Schwarzschild approximation at $\ r - a \gg \mathcal{O}\left(\frac{\ell_p^2}{a} \right)$

Around and inside the horizon, $C(u, v)$ can be solved from the semi-classical Einstein equations.

- Semi-classical Einstein equation:

$$G^{\mu}{}_{\mu} - 6G^{\theta}{}_{\theta} = 8\pi G_N \left(\langle T^{\mu}{}_{\mu} \rangle - 6\langle T^{\theta}{}_{\theta} \rangle \right)$$

$$\Rightarrow \quad \partial_u \partial_v \Sigma(u,v) = \frac{C(u,v)}{4r^2(u,v)} + \frac{8\pi G_N C(u,v)}{8} \left(\langle T^{\mu}{}_{\mu} \rangle - 6\langle T^{\theta}{}_{\theta} \rangle \right)$$

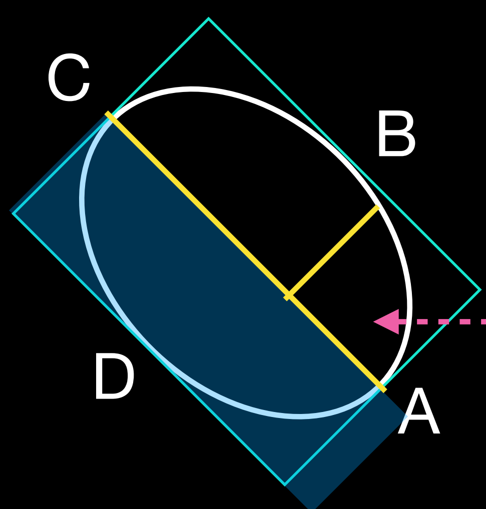
where $\quad C(u,v) = \dfrac{1}{r(u,v)} \exp\left(\Sigma(u,v)\right)$

- Schwarzschild metric near horizon:

$$C(u,v) = 1 - \frac{a}{r(u,v)} \simeq \frac{a}{r(u,v)} \ \exp\left(\frac{v-u-2a}{2a} \right)$$

$C(u,v) \sim \mathcal{O}\left(\dfrac{\ell_p^2}{a^2} \right)$ near horizon

$$C(u,v) \simeq C(u_*,v_*)\frac{r(u_*,v_*)}{r(u,v)}e^{-\int_{u_*}^{u}\frac{du'}{2a(u')}}e^{-\int_{v}^{v_*}\frac{dv'}{2\bar{a}(v')}}$$



vacuum in conventional model

$$C(u_*,v_*) \sim \mathcal{O}(\ell_p^2/a^2)$$

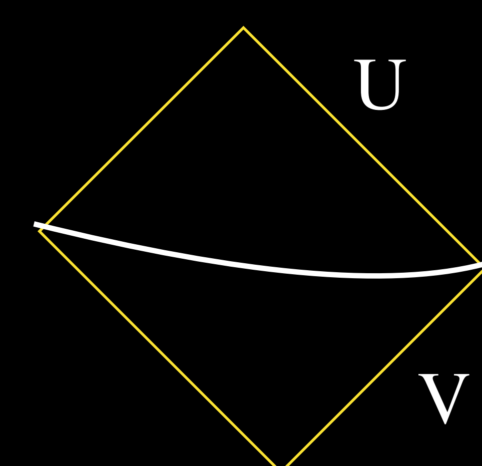
[Ho-Matsuo-Yokokura 2019]

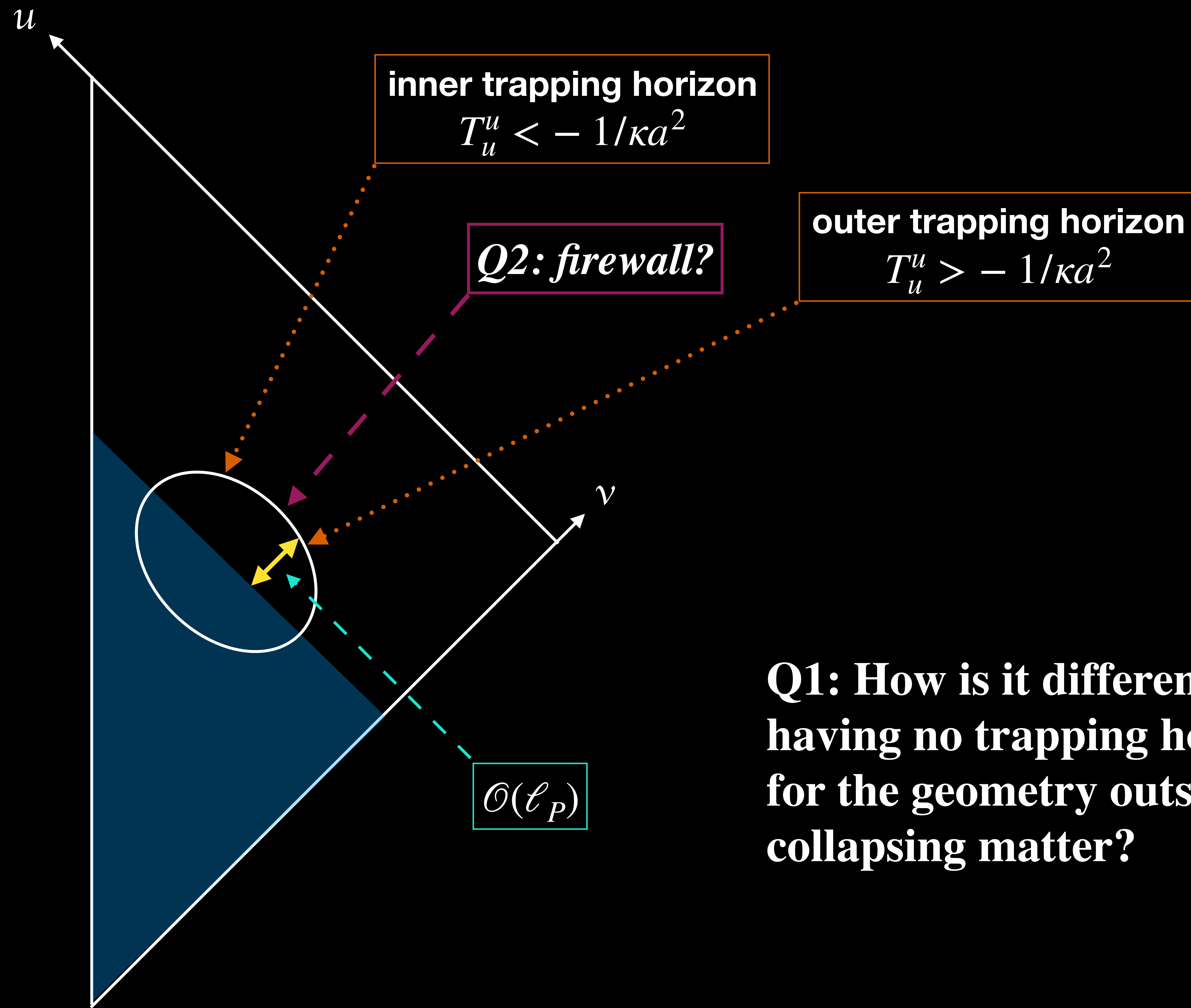
The proper distance between any two points in this region is never larger than the Planck length scale.

$$d\ell^2 = C(u,v)dudv \simeq C_* dUdV$$

$$dU \simeq e^{-\int_{u_*}^{u}\frac{du'}{2a(u')}}\,du$$

$$dV \simeq e^{-\int_{v}^{v_*}\frac{dv'}{2\bar{a}(v')}}\,dv$$





Q1: How is it different from having no trapping horizon for the geometry outside the collapsing matter?

Comment on Trapping Horizon #3

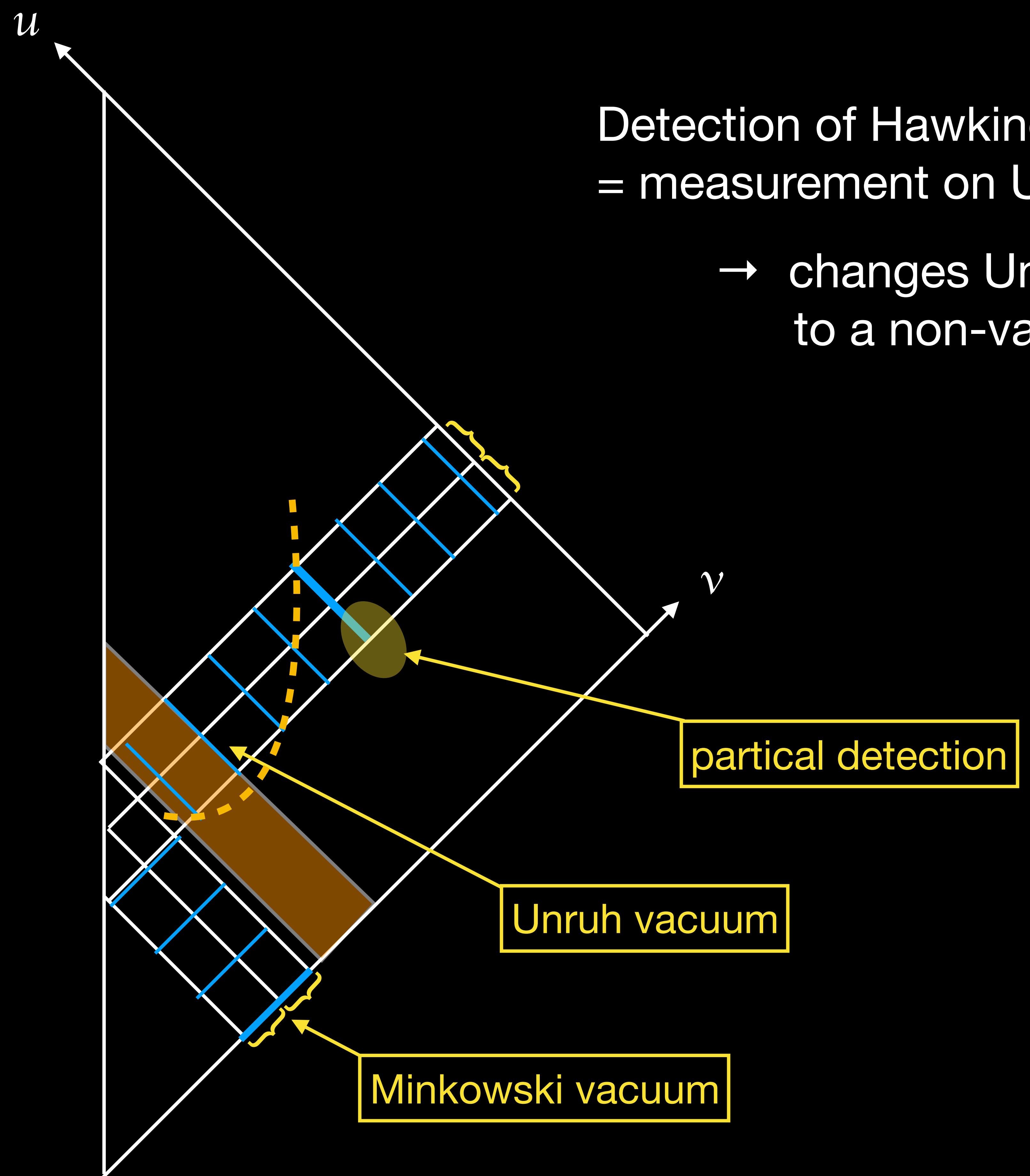
[Ho-Yokokura 19, to appear]

Observations of Hawking particles outside the horizon changes the Unruh vacuum to states with induced particles in the near-horizon region. (firewall) [AMPS 2013]

→ Planck-scale scatterings.

Detection of Hawking particles
= measurement on Unruh vacuum

→ changes Unruh vacuum
to a non-vacuum state.



Unruh vacuum

- Ingoing vacuum of the infinite past (annihilated by b_ω) becomes the Unruh vacuum $|0\rangle_U$ at horizon.
- $|0\rangle_U$ does not interact with particle $b_\omega^\dagger|0\rangle_U$.
- $|0\rangle_U$ is not the same as $|0\rangle_M$.

$(b_\omega, b_\omega^\dagger)$ are operators for $|0\rangle_U$,
$(a_\omega, a_\omega^\dagger)$ are operators for $|0\rangle_M$.

Measurement changes state

[Ho-Yokokura 2019]

For a 2-state detector $\{\,|1\rangle, |2\rangle\,\}$,

the time evolution of the composite system is

$$U = e^{ig\left(\sigma_+ \otimes a_\omega + \sigma_- \otimes a_\omega^\dagger\right)}$$

A particle is “detected” if the detector $|1\rangle \to |2\rangle$

$$\left(\langle 2| \otimes \mathbf{I}\right)\, U\, \left(|1\rangle \otimes |\cdots\rangle\right) \simeq ig\, a_\omega |\cdots\rangle$$

Unruh vacuum no more

[Ho-Yokokura 2019]

- Bogoliubov transformation

$$a_\omega = A_{\omega\omega'} b_{\omega'} + B_{\omega\omega'} b^\dagger_{\omega'} \quad \leftarrow \quad \frac{dU}{du} \sim e^{-\int_{u_*}^{u} \frac{du'}{2a(u')}}$$

- Detection of 1 particle at distance:

$$|0\rangle_U \quad \to \quad |\Psi\rangle \propto \; a_\omega |0\rangle_U = B_{\omega\omega'} b^\dagger_{\omega'} |0\rangle_U$$

- Detection of N particles:

$$\to \to \;\; \cdots \;\; \to \quad b^\dagger_1 b^\dagger_2 \cdots b^\dagger_N |0\rangle_U$$

Induced Particles

[Ho-Yokokura 2019]

- $|0\rangle_U \quad \to \quad b_1^\dagger b_2^\dagger \cdots b_N^\dagger |0\rangle_U$
- Distribution of the induced particles can be calculated.
- Mostly well inside near-horizon region
  to come to large distances at late times.
- This is essentially *firewall*. [AMPS 2013]
- Planck-scale scattering with collapsing matter.

Conclusion

- There are gauge-invariant quantities at Planck-scale.

  Low-energy effective theory modified?

- Trapped region in vacuum is as thin as Planck length.

  How is it different from the KMY model with no horizon?

- Radiation outside horizon entangled with environment → High-energy particles at horizon.

  Information?

Thank you!