# Super Geodesic Witten Diagrams

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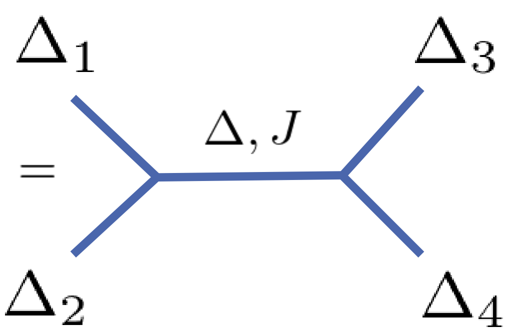
# Introduction

- **The AdS/CFT correspondence** is a conjectured relationship between a gravity theory on AdS spacetime and CFT

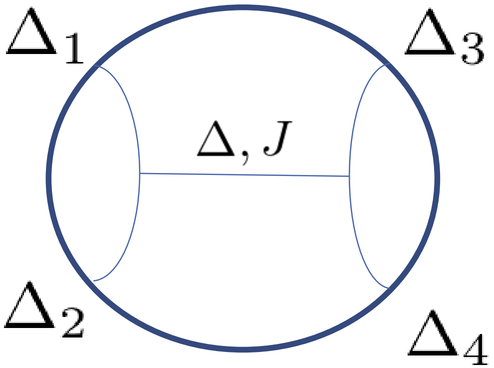
- We are interested in the following kinematical correspondence:

**CFT side** | **AdS side**

$G_{\Delta,J}(u,v) =$




$\Longleftrightarrow$




Conformal block

Geodesic Witten diagram

$$\langle \phi_1\phi_2\phi_3\phi_4\rangle \sim \sum_{\Delta,J} a_{\Delta,J}\, G_{\Delta,J}(u,v)$$

[Hijano-Kraus.-Perlmutter-Snively, 1508.00501]

- In a concrete duality set up, this correspondence is not sufficient.

# Introduction

- The most famous AdS/CFT correspondence

| 4d N=4 SYM | ⟷ | $AdS_5 \times S^5$ superstring |
|---|---|---|

- 4-pt. functions of half BPS operators in N=4 SYM

$$G(x_i, T_i) = \langle \mathcal{O}_{p_1}(x_1; T_1) \mathcal{O}_{p_2}(x_2; T_2) \mathcal{O}_{p_3}(x_3; T_3) \mathcal{O}_{p_4}(x_4; T_4) \rangle$$

**Half-BPS op.**

$$\mathcal{O}_p(x; T) = T_{\hat{A}_1} \ldots T_{\hat{A}_p} \mathrm{Tr}(\phi^{\hat{A}_1}(x) \ldots \phi^{\hat{A}_p}(x)) \quad \phi^{\hat{A}}(\hat{A} = 1, \ldots, 6)$$

- Complex null vector: $T_A \in \mathbb{C}^6 \quad T \cdot T = 0$
- $\Delta_{\mathcal{O}_p} = p$ , SU(4) R-symmetry rep. $[0, p, 0]$

Additional global symmetry

# Introduction

- 4 pt. function is expanded by superconformal blocks (long multiplet)

[Nirschl-Osborn, 0407060]
[Bissi-Lukowski, 1508.02391]

$$G(x_i, T_i) \sim \sum_{\Delta, J, l, s} c(\Delta, J, l, s)\, \check{G}^{a,b}_{\Delta+4,J}(u,v)\, \hat{G}^{a,b}_{l,s}(\sigma,\tau)$$

Superconformal block = **Conformal block** + **R-symmetry block**

cross ratios

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \quad v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2}\,, \quad \sigma = \frac{T_{12}T_{34}}{T_{13}T_{24}}\,, \quad \tau = \frac{T_{23}T_{14}}{T_{13}T_{24}}$$

What is the gravity dual of the R-symmetry block?

# Gravity dual of R-symmetry block

Cf ( Geodesic Witten diagram )

= ( Bulk-to-boundary propagators ) + ( Bulk-to-bulk propagator )

# Embedding formalism

- Embedding formalism

  A useful description of the AdS bulk-to-boundary (bulk) propagator

  In particular, it is easy to generalize them to the sphere case.

| | Bulk | | Boundary | |
|---|---|---|---|---|
| $AdS_{d+1}$ | $\check{X}^2=-1$ | $\check{X}^{\check{A}}\in\mathbb{R}^{d,2}$ | $P\cdot P=0$ | $P^{\check{A}}\in\mathbb{R}^{d,2}$ |
| $S^{d+1}$ | $\hat{X}^2=1$ | $\hat{X}^{\hat{A}}\in\mathbb{R}^{d+2}$ | $T\cdot T=0$ | $T^{\hat{A}}\in\mathbb{C}^{d+2}$ |

Cf. $u=\dfrac{P_{12}P_{34}}{P_{13}P_{24}}$ $v=\dfrac{P_{23}P_{14}}{P_{13}P_{24}}$ $\sigma=\dfrac{T_{12}T_{34}}{T_{13}T_{24}}$ $\tau=\dfrac{T_{23}T_{14}}{T_{13}T_{24}}$

# Bulk-to-boundary propagator

1. Scalar bulk-to-boundary propagator

**AdS** $$K^{\mathsf{a}}_{\Delta,0}(\check{X};P)=(\check{X}\cdot P)^{-\Delta}$$

**Sphere** $$\boxed{K^{\mathsf{s}}_{l,0}(\hat{X};T)=(\hat{X}\cdot T)^{l}}$$

**NOTE**

- $\hat{\nabla}^2 K^{\mathsf{s}}_{l,0}(\hat{X};T)=-l(l+d)K^{\mathsf{s}}_{l,0}(\hat{X};T)$ ➡ R-symmetry rep. $[0,l,0]$
- Half-BPS operator $\mathcal{O}_p(P;T)$ ➡ $K^{\mathsf{a}}_{p,0}(\check{X};P)K^{\mathsf{s}}_{p,0}(\hat{X};T)$

# Split representation of harmonic function

2. Harmonic function satisfying (analog of bulk-to-bulk propagator)

$$(\hat{\nabla}^2 + l(l+d))\hat{\Omega}_{l,0}(\hat{X}, \hat{X}') = 0$$

As in the AdS case, we find the split representation

$$\hat{\Omega}_{l,0}(\hat{X}; \hat{X}') = \int_{\mathbb{C}^{d+2}} D^d T \, K^{\mathsf{s}}_{l,0}(\hat{X}; T) K^{\mathsf{s}}_{\tilde{l},0}(\hat{X}'; T)$$

**NOTE**: $K^{\mathsf{s}}_{\tilde{l},0}(\hat{X}'; T)$ has $\tilde{l} = -d - l$ (negative integer)

➡ an analog of the shadow conformal dim. $\tilde{\Delta} = d - \Delta$

$$\hat{\Omega}_{l,0}(\hat{X}; \hat{X}') = \frac{\mathrm{Vol}(S^d)}{C_l^{(d/2)}(1)} C_l^{(d/2)}(\hat{X} \cdot \hat{X}')$$ Gegenbauer polynomial

# Gravity dual of R-symmetry block

3. R-symmetry block ( A similar expression appears in [Uruchurtu, 1106.0630] )

$$\hat{W}_{l,0}(T_i) = \int_{S^{d+1}} \mathrm{d}^{d+2}\hat{\mathbf{X}} \int_{S^{d+1}} \mathrm{d}^{d+2}\hat{\mathbf{X}}' K^{s}_{l_1,0}(\hat{X};T_1) K^{s}_{l_2,0}(\hat{X};T_2) \times \hat{\Omega}_{l,0}(\hat{X};\hat{X}') K^{s}_{l_3,0}(\hat{X}';T_3) K^{s}_{l_4,0}(\hat{X}';T_4)$$

- Different points

  A) Integration region

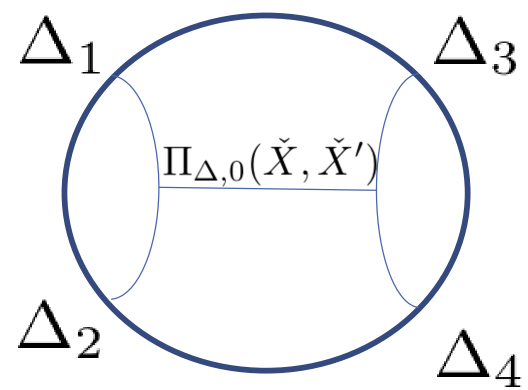
  **AdS** Geodesics connecting the boundary op.

  **Sphere** Whole space

  B) Exchanged sate part

  **AdS** Bulk-to-bulk propagator $(\check{\nabla}^2 - \Delta(\Delta - d))\Pi_{\Delta,0}(\check{X},\check{X}') = -\delta(\check{X},\check{X}')$

  **Sphere** Harmonic function $(\hat{\nabla}^2 + l(l+d))\hat{\Omega}_{l,0}(\hat{X},\hat{X}') = 0$




**AdS side**

- We can explicitly show $\hat{W}_{l,s}(T_i) \sim \hat{G}^{(\hat{a},\hat{b})}_{l,0}(\sigma,\tau)$ (d=4)

# Summery and discussion

# Summery and discussion

- We have developed the embedding formalism of the sphere, and constructed a gravity dual of the R-symmetry block.

- In this talk, we focused on the s=0 case. However, the above discussion can be extended to the more general case $[s, l-s, s]$

$$K^{\mathsf{s}}_{l,s}(\hat{X}, U; T, R) = (\hat{X} \cdot \hat{\mathsf{C}} \cdot U)^s (\hat{X} \cdot T)^{l-s} \quad \text{where} \quad \hat{\mathsf{C}}_{\hat{A}\hat{B}} = T_{\hat{A}} R_{\hat{B}} - R_{\hat{A}} T_{\hat{B}}$$

$$\hat{\Omega}_{l,s}(\hat{X}, U; \hat{X}', U') = \mathrm{Vol}(S^d) \sum_{r=0}^{s} \frac{s!(U \cdot U')^s}{2^{2r}(s-r)!\left(h+\frac{1}{2}\right)_r} \frac{C^{(h+r)}_{l-r}(\hat{X} \cdot \hat{X}')}{C^{(h+r)}_{l-r}(1)} \times r!(h-1+s)_{-r}\, C^{(h-1+s-r)}_r(\hat{X} \cdot \hat{X}')$$

$\hat{W}_{l,s}(T_i) =$ ( Linear combination of Appell function $F_4$ ? ) [work in progress]

This is similar to the AdS case [Heng-Yu Chen-Kyono 1906.03135]

- Can we use our R-symmetry block for other SCFT dual to $AdS_p \times S^q$ ?

Thank you