HAWKING RADIATION AS STIMULATED EMISSION

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Thermal v.s. non-thermal (featured)

- Parikh-Wilczek regarded Hawking radiation as a tunneling process (with back reaction or energy conservation) and derived the tunneling rate: [PW, PRL 2000]

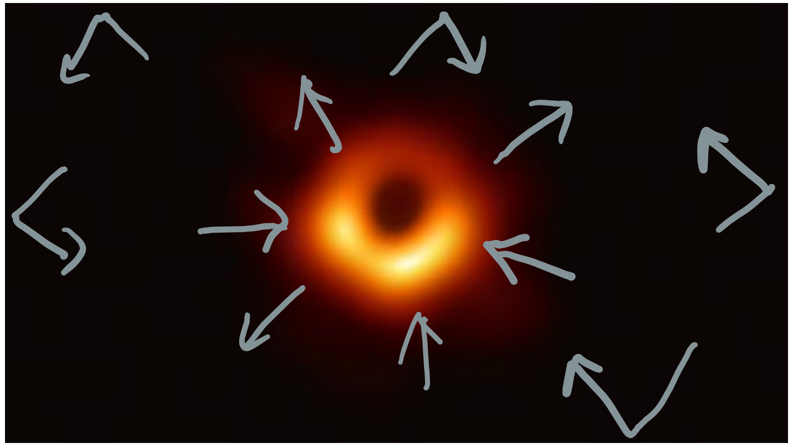
$$\Gamma \sim e^{-8\pi\omega(M-\omega/2)} = \underbrace{e^{-\omega/T_H}}_{\text{Boltzmann factor}} \quad e^{+4\pi\omega^2}$$

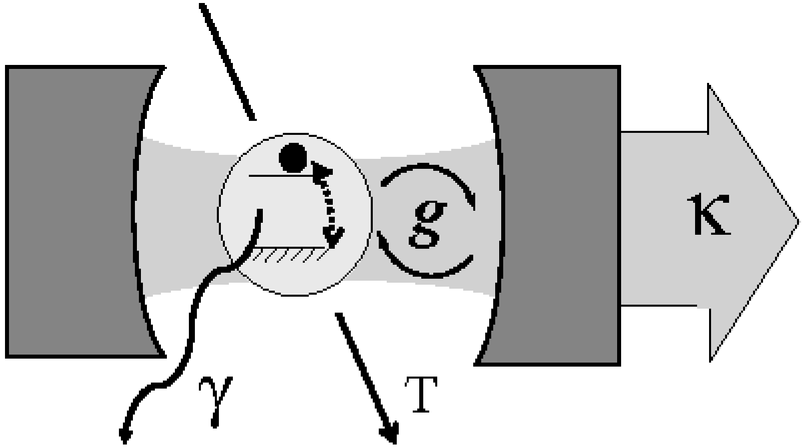
- The tunneling rate is composed of a thermal part and non-thermal part. This suggests radiation contains more features than just temperature (determined by mass M).

Energy Conservation = no information loss

- The exponent is simply the change of Bekenstein-Hawking entropy as the (Schwarzschild) black hole loses a bit of energy/mass ω via tunneling. The conservation of entropy/information may help resolve the notorious information loss paradox [Zhang-Cai-You-Zhan, PLB 2009; Kyung Kiu Kim-W, PLB 2014; Kuwakino-W, JHEP 2015] or reveal the existence of remnant [Li Xiang, PLB 2007; Yi-Xin Chen, Kai-Nan Shao, PLB 2009]
- Or not [Mathur, CQG 2009], see also Firewall [AMPS, JHEP 2013]
- We have two observations here:
    1. Microscopic degrees of freedom to carry information are still unclear.
    2. PW tunneling rate can be derived without concept of spacetime [Braunstein-Patra, PRL 2011]
- Is a quantum mechanical model of Hawking radiation with PW tunneling feature possible?

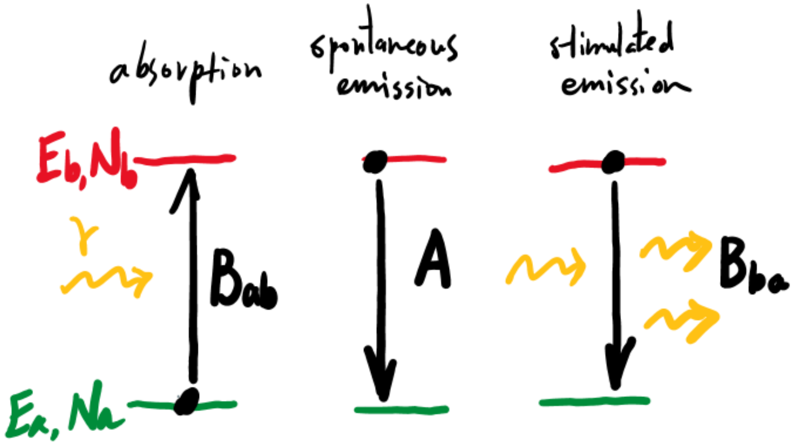
Black hole in thermal bath ~ atom in cavity (1)





Trapped atoms in cavity QED: coupling quantized light and matter
Reinhard Miller, Tracy E. Northup, Kevin M. Birnbaum, Alessandra Del Boca, A. D. Boozer, H. Jeff Kimble

Black hole's stimulated emission



$$\rho(\omega) = \frac{A}{(N_a/N_b)B_{ab} - B_{ba}}$$

(a) Each state has degeneracy $g_{a(b)} \sim e^{\frac{\alpha}{4}\mathcal{A}(\beta M_{a(b)})}$

(b) Dof are located somewhere at or outside horizon

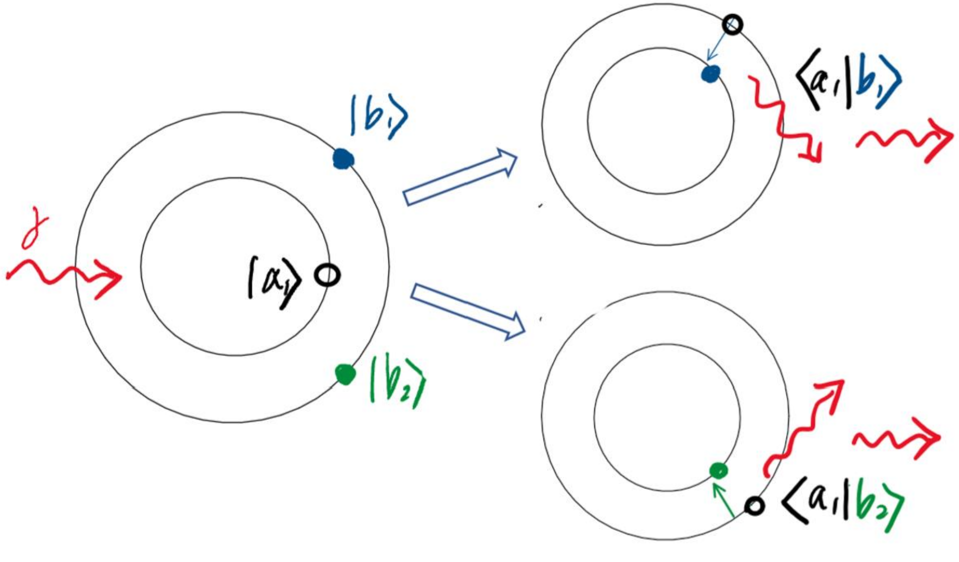


FIG. 1. (Left) Degenerate excited states $|b_1\rangle$ and $|b_2\rangle$ are stimulated by a photon in the cavity. (Right) The stimulated emission may have different feature depending on which transition $\langle a_1|b_1\rangle$ or $\langle a_1|b_2\rangle$ occurs.

Stimulated emission = PK tunneling rate

The proportionality coefficients α and β will be determined shortly. We are looking for large black hole limit where $1/M \ll \omega \ll M$, then equation (2) can be cast into

$$\rho(\omega) \simeq (A/B_{ba})e^{-\omega/T_H} e^{\alpha \frac{\pi}{4} C(M,\omega)}, \tag{4}$$

where

$$C(M,\omega) = (\frac{2}{\beta^2} + \frac{8}{\beta} - 32\beta - 32\beta^2)M\omega + (\frac{3}{\beta^2} + \frac{8}{\beta} + 16\beta^2)\omega^2 + \mathcal{O}(\omega^3). \tag{5}$$

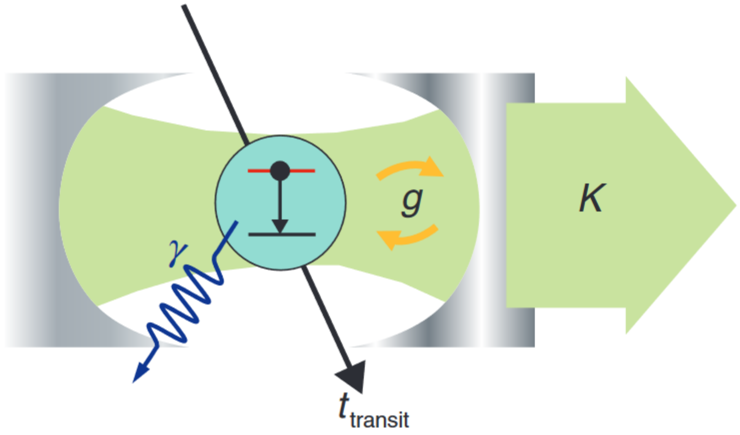
We remark the choices for coefficients α and β as follows:

- To recover the Boltzmann factor, we choose $\beta = 1/2$ such that the leading term in function $C(M,\omega)$ vanishes. This suggests those degrees of freedom are seated at the horizon [2].
- To reproduce the Parikh-Wilczek nonthermal spectrum, we further choose $\alpha = 2$. This implies that the degeneracy at each energy level is twice amount of the black hole entropy, for $S_{BH} = A/4$.

2* Isotropic metric is used to calculate area

$$\boxed{ds^2 = -\frac{(1-M/2r)^2}{(1+M/2r)^2}dt^2 + (1+M/2r)^4(dr^2 + r^2 d\Omega^2).}$$

Jaynes-Cummings model of cavity-QED



$$\hat{H}_{JC} = \boxed{\frac{1}{2}\hbar\omega\hat{\sigma}_z} + \boxed{\hbar\omega_0\hat{a}^+\hat{a}} + \boxed{\hbar g\,(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^+)}$$

Two-level atom　　Electromagnetic field　　Interaction

. Two-level atom interacting with a photon in a single-mode cavity and Jaynes-Cummings Hamiltonian.

Jaynes-Cummings model of cavity-black holes

$$\mathcal{H} = \underbrace{\sum_i M_a |a_i\rangle\langle a_i| + \sum_j M_b |b_j\rangle\langle b_j|}_{\text{black holes energy}} + \underbrace{\omega \hat{\alpha}^\dagger \hat{\alpha}}_{\text{photon energy}} + \underbrace{\sum_i \sum_j g_{ij} (\hat{\alpha} |b_j\rangle\langle a_i| + \hat{\alpha}^\dagger |a_i\rangle\langle b_j|)}_{\text{interaction}} \quad (6)$$

where indices i, j label each degeneracy state. $\hat{\alpha}$ and $\hat{\alpha}^\dagger$ are annihilation and creation operators of photons. The couplings g_{ij} are responsible for emission and absorption.

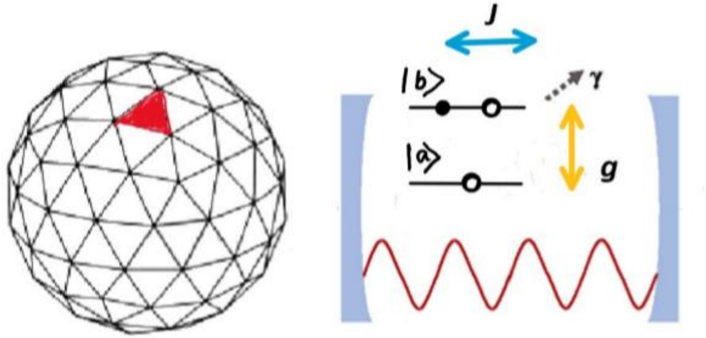


FIG. 2. (Left) Degenerate microstates of Planck size are seated at horizon. (Right) A black hole in the cavity is modeled by a two-level atom in the Jaynes-Cummings model. Coupling g is the transition strength between excited state $|b\rangle$ and ground state $|a\rangle$, while J is the hopping strength among degenerate excited states.

Reversible featured emission = write a qubit

First a qubit is prepared at ground state $\psi(0) = |n, a_1\rangle$. Given time, it will evolve accordingly,

$$
\begin{aligned}
\psi(t) &= (2\sqrt{1+\delta^2})^{-1}\Big(|E_+\rangle e^{-iE_+t} - |E_-\rangle e^{-iE_-t}\Big) \\
&= e^{-in\omega t}\Big\{cos\Omega t|n, a_1\rangle - isin\Omega t\frac{\delta}{\sqrt{1+\delta^2}}\Big(\delta|n-1, b_1\rangle + |n-1, b_2\rangle\Big)\Big\},
\end{aligned} \tag{7}
$$

where we regard the coupling strength ratio $\delta = g_{11}/g_{12}$ as a controllable parameter. We remark that at late time $t_r = \pi/2\Omega$, a superposition of excited states $\psi(t_r)$ is created

the analytic continuation $\Omega \to i\Omega'$ will bring equation (7) to following form

$$
\psi(t) \sim \frac{1}{2\sqrt{1+\delta^2}}\Big(|E_+\rangle e^{\Omega' t} - |E_-\rangle e^{-\Omega' t}\Big), \tag{8}
$$

in comparison to the Goldstone boson mode [4]

[Hwaking-Perry-Strominger PRL, 2016; Maitra-Maity-Majhi, 1906.04489; Chu-Koyama, JHEP 2018]

$$
\hat{F}(\nu, \theta, \phi) \sim \sum_{lm} c_{lm} Y_{lm}(\theta, \phi)\Big(\hat{A}_+ e^{\tilde{\Omega}\nu} - \hat{A}_- e^{-\tilde{\Omega}\nu}\Big) \tag{9}
$$

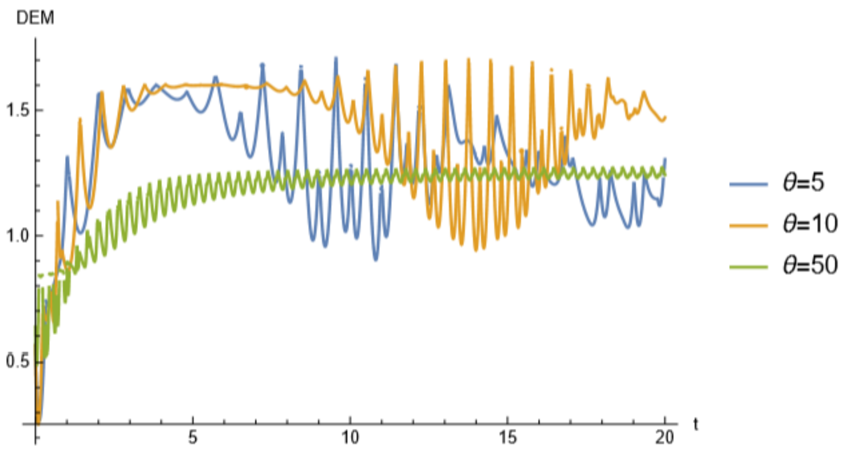
Entanglement sustained by photon interaction

prepare the initial state

$$\rho(0) = \lambda_1 |n-1, b_1\rangle\langle n-1, b_1| + \lambda_2 |n-1, b_2\rangle\langle n-1, b_2|$$

Then the DEM is computed as

$$\mathcal{I}_\rho(\rho(t)^A, \rho(t)^F) = -c_{11}\log c_{11} - c_{22}\log c_{22} - c_{33}\log c_{33}$$
$$+ c_{12}\log c_{12} + c_{21}\log c_{21} + c_{23}\log c_{23} + c_{32}\log c_{32} + c_{13}\log c_{13} + c_{31}\log c_{31}$$



$$c_{11} = \frac{1}{2}\exp\big(-|\theta|^2\big)(\lambda_1+\lambda_2)\sum_n \frac{|\theta|^{2n}}{n!}\sin^2\Omega t$$
$$c_{22} = \frac{1}{4}\exp\big(-|\theta|^2\big)\sum_n \frac{|\theta|^{2n}}{n!}\{\lambda_1(1+\cos\Omega t)^2 + \lambda_2(\cos\Omega t - 1)^2\} = c_{33}$$

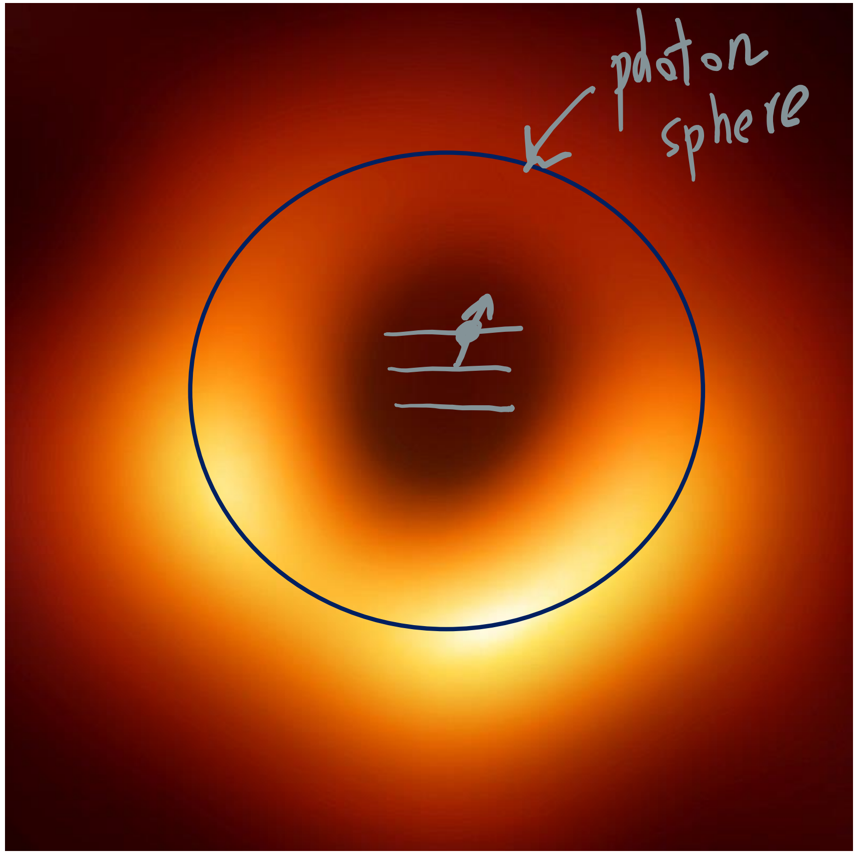
$$c_{12} = \frac{-i}{2\sqrt{2}}\exp\big(-|\theta|^2\big)\sum_n \frac{|\theta|^{2n}}{n!}\sin\Omega t\{\lambda_1(1+\cos\Omega t) + \lambda_2(\cos\Omega t - 1)\} = c_{21}^*$$
$$c_{13} = \frac{-i}{2\sqrt{2}}\exp\big(-|\theta|^2\big)\sum_n \frac{|\theta|^{2n}}{n!}\sin\Omega t\{\lambda_1(\cos\Omega t - 1) + \lambda_2(1+\cos\Omega t)\} = c_{31}^*$$
$$c_{23} = \frac{1}{4}\exp\big(-|\theta|^2\big)\sum_n \frac{|\theta|^{2n}}{n!}(-\sin^2\Omega t)\{\lambda_1+\lambda_2\} = c_{32}^*$$

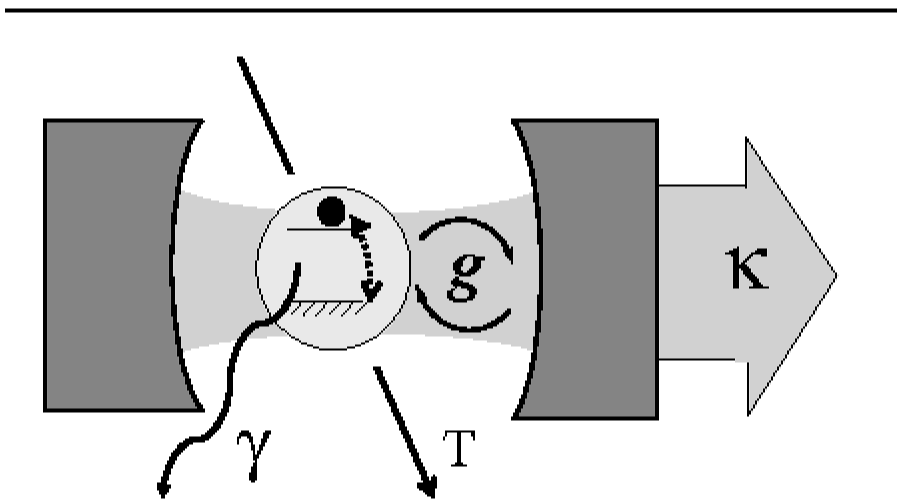
FIG. 3. Evolution of degree of entanglement due to mutual entropy (DEM) in our toy model. We plot for different average photon numbers $\theta = 5, 10, 50$ given $\lambda_1 = 0.25, \lambda_2 = 0.75$. The time evolution shows periodic Rabi oscillation as usual JC model. For more photons in the cavity, the model achieves its strongest entanglement at later time but also lasts longer.

Lessons learned so far…

- We regard PK tunneling picture of Hawking radiation as Einstein's model of stimulated emission, under the assumption that microstates with double degeneracy were seated at horizon. This could closely relate to Bondi-Metzner-Sachs (BMS) symmetry of soft gravitons which claim black hole information. [Averin-Dvali-Gomez-Lust, MPLA, 2016; Eling-Oz, JHEP 2016]
- We demonstrate how to write a qubit via unequal coupling *g* in JC model. In a black hole, featured (angle-dependent) coupling *g* (after analytic continuation) is related to the Goldstone boson generated by BMS transformation.
- The black hole information can be stored in entangled microstates. Though it will be dephased in an open system via interaction, but evolution of DEM suggests it may survive long enough before carried away by featured radiation. Photon sphere may play important role of the cavity to preserve such information.

Black hole in photon sphere ~ atom in cavity (2)





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Thank You and Enjoy Your Stay