Non-relativistic hybrid geometry and gravitational gauge fixing term

East Asia Joint Workshop on Fields and Strings 2019
12th Taiwan String Theory Workshop

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Shuichi Yokoyama

Yukawa Institute for Theoretical Physics

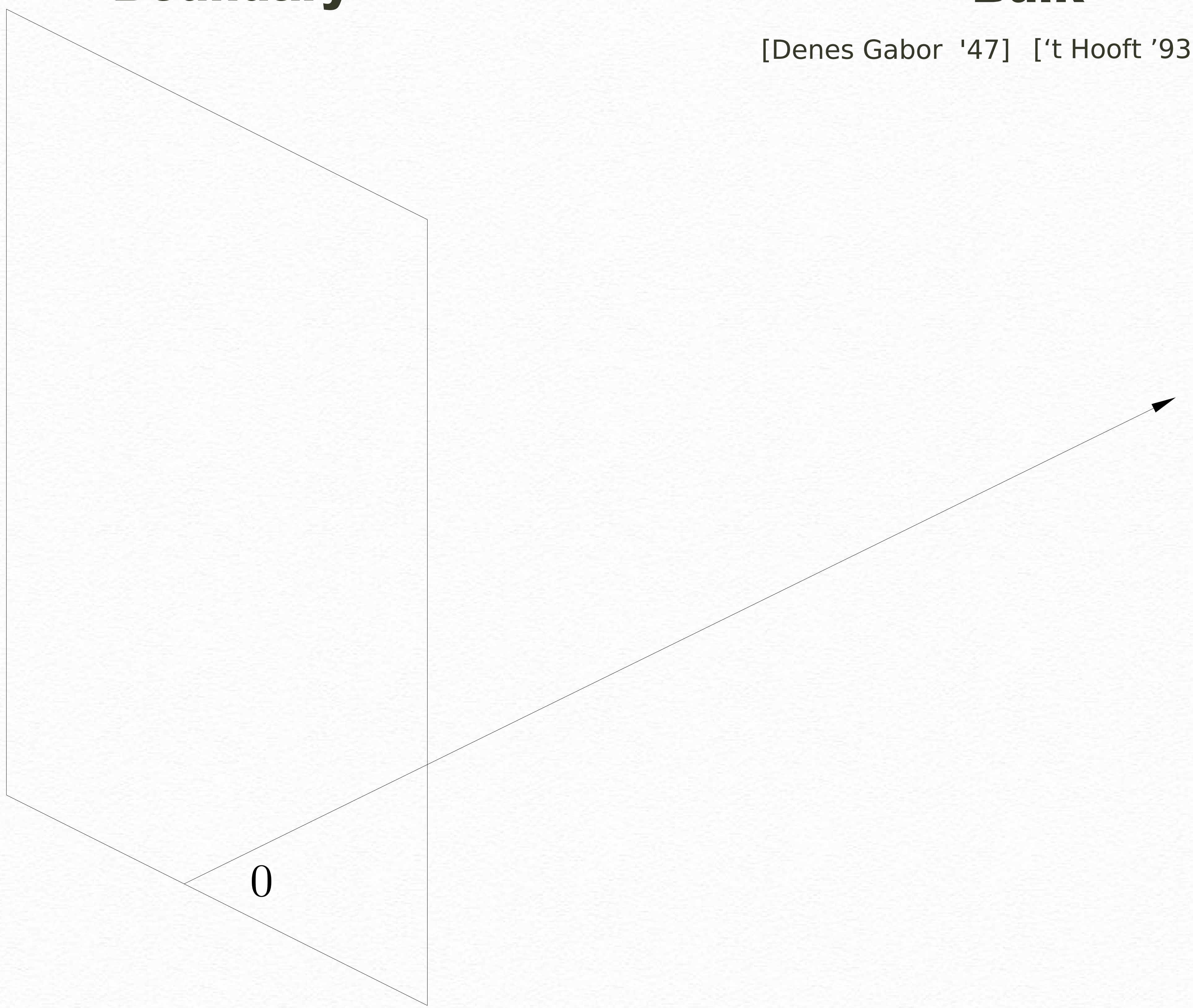
Ref. Aoki-SY-Yoshida PRD (2019) no.12, 126002 arXiv:1902.02578

Aoki-Balog-SY-Yoshida arXiv:1910.11032

Holography

Boundary ⟷ **Bulk**

[Denes Gabor '47] [‘t Hooft ’93, Susskind ‘94]



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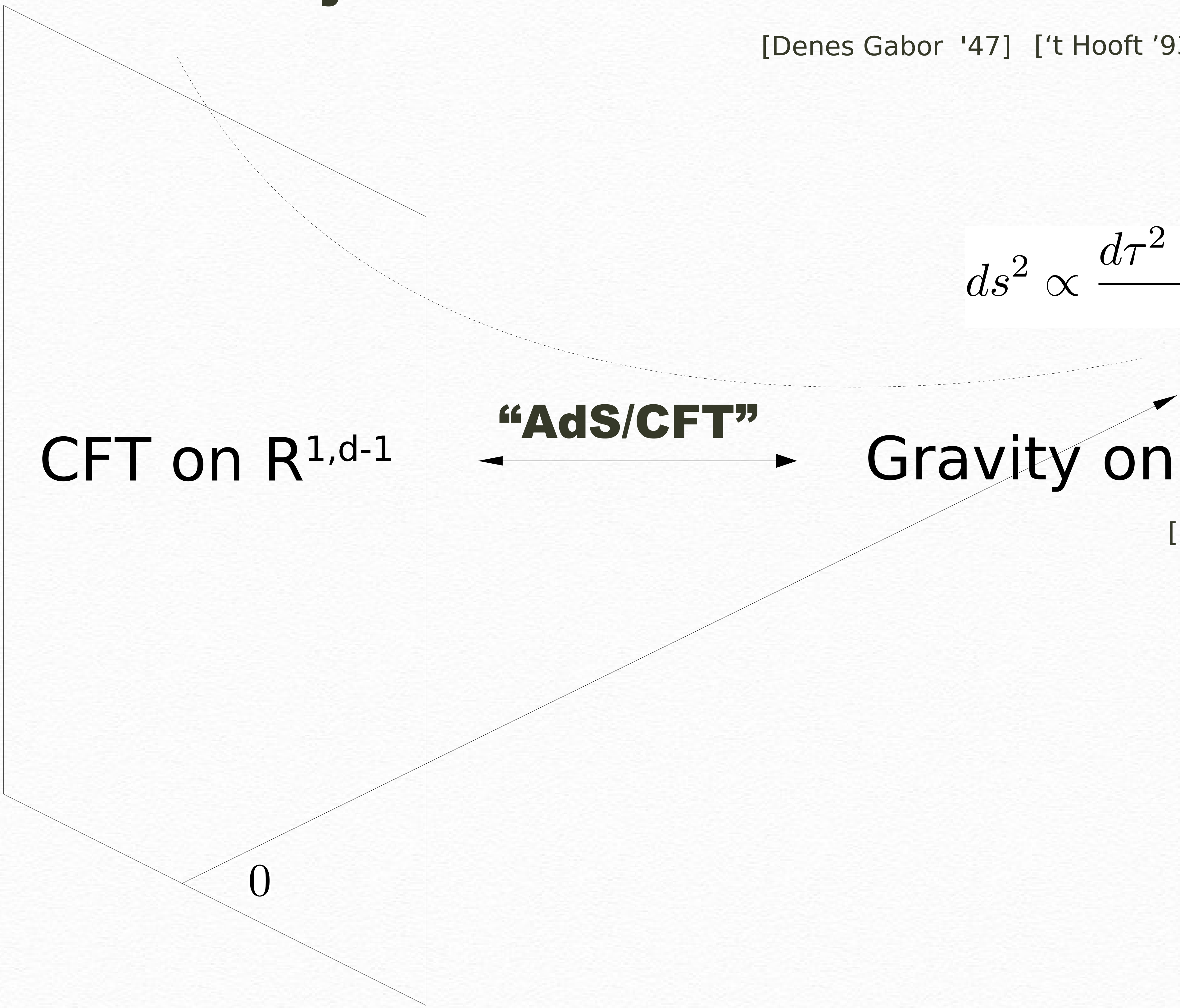
$$ds^2 \propto \frac{d\tau^2 + (dx^\mu)^2}{\tau^2}$$



CFT on $R^{1,d-1}$ **“AdS/CFT”** ⟷ Gravity on AdS_{d+1}

[Maldacena ’97]

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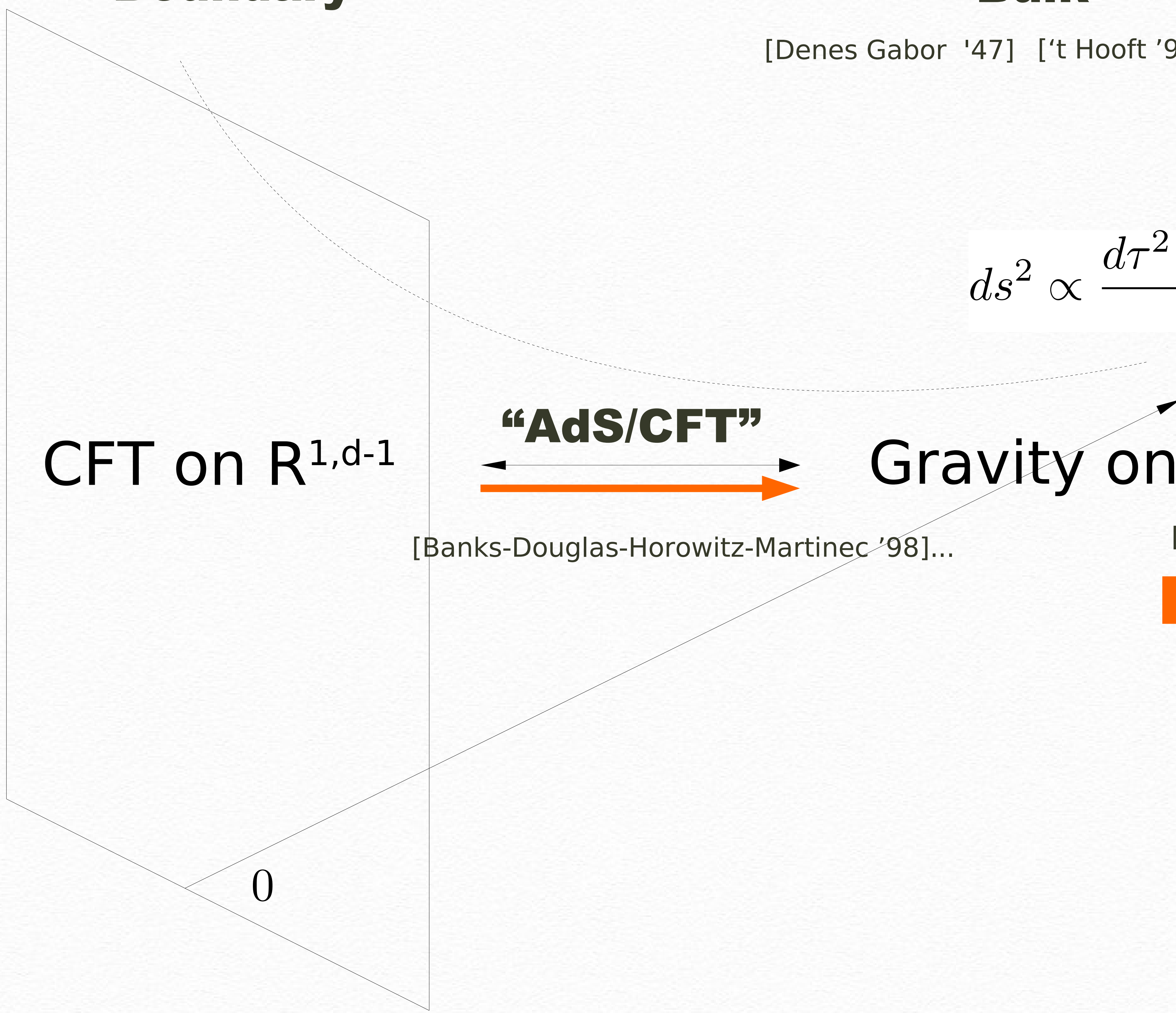

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τ

CFT on $R^{1,d-1}$ **"AdS/CFT"** Gravity on AdS_{d+1}

[Banks-Douglas-Horowitz-Martinec '98]...

[Maldacena '97]

How?

Holography

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"AdS/CFT"

CFT on $R^{1,d-1}$ ⟷ Gravity on AdS_{d+1}

[Banks-Douglas-Horowitz-Martinec '98]...

[Maldacena '97]

0

How?

- Relevant RG defromation

[Girardello-Petrini-Porrati-Zaffaroni '98] [Distler-Zamora '98]

- Wilsonian RG scale

[Heemskerk-Polchinski '10]

- Relative coordinate in bilocal field

[Das-Jevicki '03]

- entanglement entropy, cMERA

[Ryu-Takayanagi '06], [Swingle '09]

Holography

Boundary $\longleftrightarrow$ **Bulk**

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[Maldacena ’97]

NR CFT on $R^{1,d-1}$ $\longleftrightarrow$ Gravity on Sch_{d+2}

“AdS/CMP”

[Son ’08, Balasubramanian-McGreevy ‘08]

Lifshitz FT on $R^{1,d-1}$ $\longleftrightarrow$ Gravity on $Lifs_{d+1}$

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[Kachru-Liu-Mulligan ‘08]

Anistoropic scaling -> cf. Trapped cold atom system

Holography

Boundary ⟷ **Bulk**

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Q: Possible to realize?

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Q: Possible to realize?

A: Yes, by flow equation approach!

τ

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Flow equation

(Gradient) flow equation

1. was introduced to help numerics of lattice QCD. cf. def of stress energy tensor

[Albanese et al. (APE) '87] [Narayanan-Neuberger '06] [Luscher '10,'13]

2. describes a **non-local course-graining of an operator**.

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2. describes a **non-local course-graining of an operator**.

Consider a CFT_d which contains a primary scalar φ $\langle \phi(x_1)\phi(x_2)\rangle = \frac{1}{x_{12}^{2\Delta}}$

$x_{12} := x_1 - x_2$

General flow equation

$$\boxed{\frac{\partial \phi(x;\eta)}{\partial \eta} = -\left.\frac{\delta S_f(\phi)}{\delta \phi(x)}\right|_{\phi(x)\to\phi(x;\eta)} \qquad \phi(x;0) = \phi(x)}$$

NOTE: If S_f coincide with the action of the theory, the flow equation is called the **gradient flow**.

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Free flow equation

$$\frac{\partial \phi(x;\eta)}{\partial \eta} = \partial^2 \phi(x;\eta). \quad \phi(x;0) = \phi(x)$$

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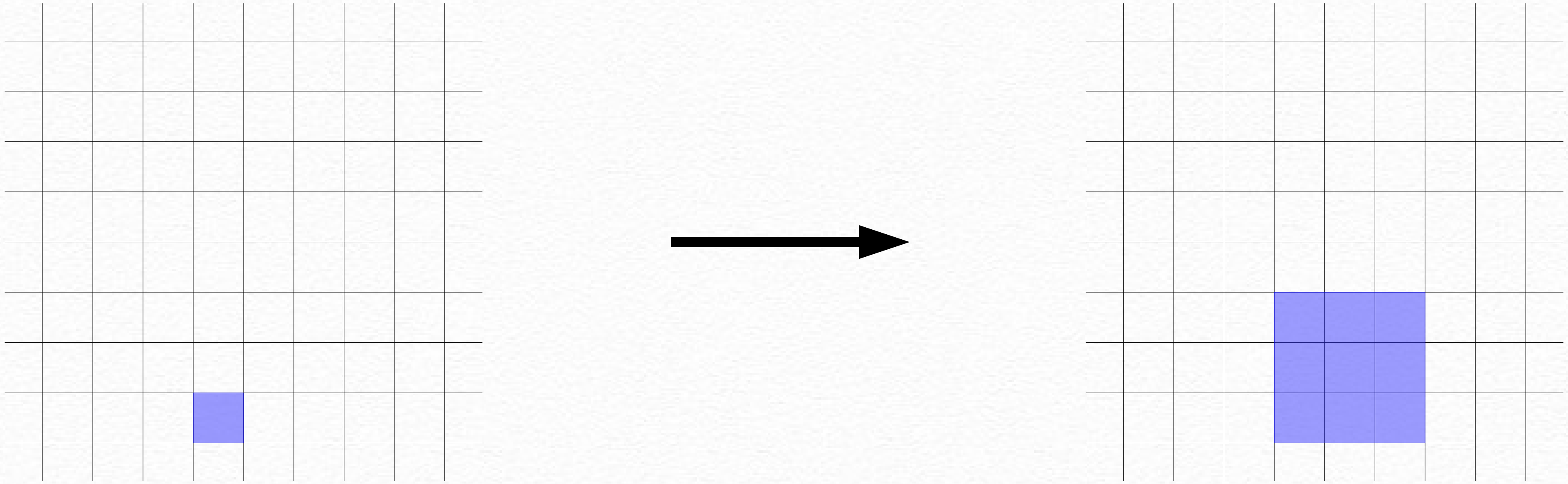
Free flow equation

$$\boxed{\frac{\partial\phi(x;\eta)}{\partial\eta} = \partial^2\phi(x;\eta). \quad \phi(x;0) = \phi(x)}$$

The solution: $\phi(x;\eta) = \int d^d y\, K(x-y;\eta)\phi(y).$ $\quad K(x-y;\eta) = \frac{e^{-(x-y)^2/4\eta}}{(4\pi\eta)^{d/2}}$

➡ Reminiscent of the block spin transformation!?

Block spin v.s. Flowed operator



s_i

$$S_I = \frac{1}{\langle S_I \rangle} \sum_{i \in I} s_i$$

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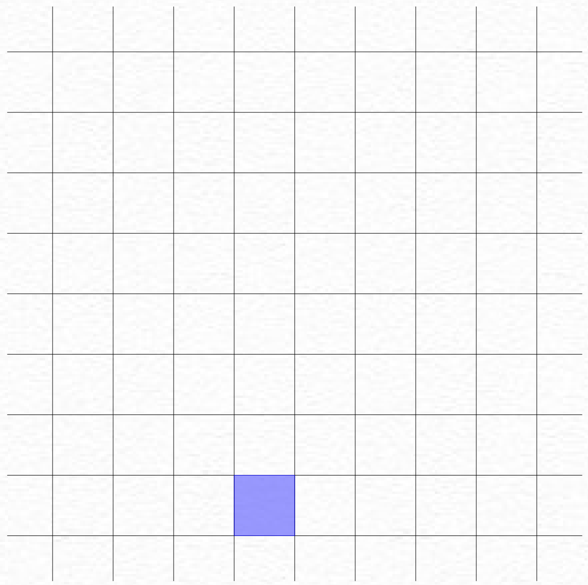

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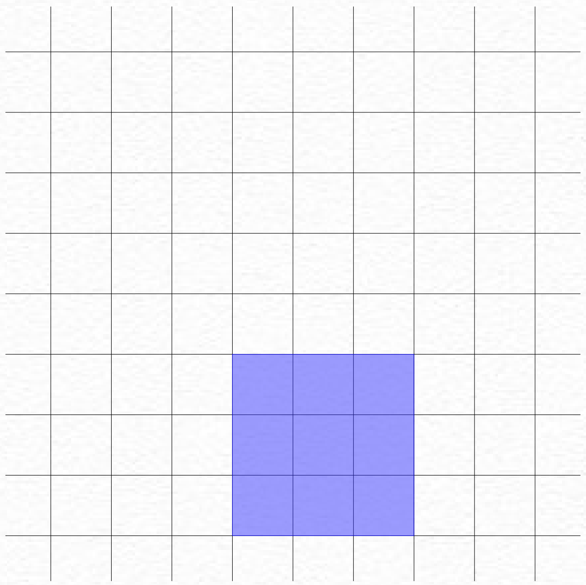

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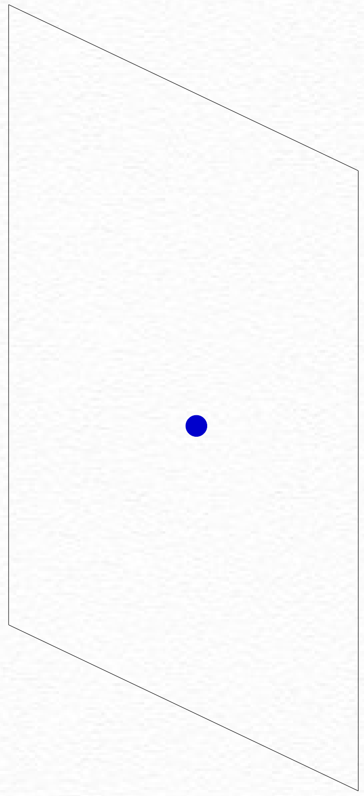
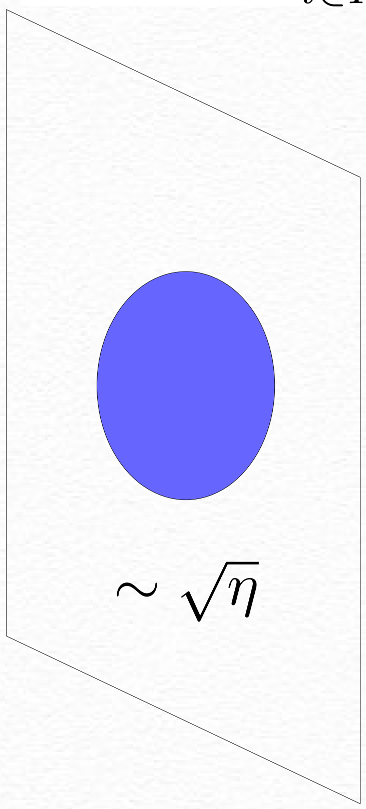


$\phi(x)$



$$\phi(x;\eta) = \int d^d y\, K(x-y;\eta)\phi(y).$$

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Claim: **Contact singularity in 2pt function is resolved**.

[Albanese et al. (APE) '87] [Narayanan-Neuberger '06]

$\because$

$$\langle\phi(x_1;\eta_1)\phi(x_2;\eta_2)\rangle = \frac{1}{\eta_+^{\Delta}} F\left(\frac{x_{12}^2}{\eta_+};1\right)$$

$\eta_+ := \eta_1 + \eta_2$

$$F(v;1) = \frac{1}{(4\pi)^{\frac{d}{2}}}\int_0^1 du(1-u)^{d/2-\Delta-1}e^{-vu/4}u^{\Delta-1}$$

$$\frac{d-2}{2} \le \Delta < \frac{d-1}{2}$$

Construction of holographic space

[Aoki-Kikuchi-Onogi ’15]

[Aoki-Balog-Onogi-Weisz ’16,'17]

[Aoki-SY ’17]

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Def. (Dimensionless normalized operator)

$$\sigma(x;\eta) := \frac{\phi(x;\eta)}{\sqrt{\langle \phi(x;\eta)^2 \rangle}}$$

“Operator renormalization”

NOTE: $\langle \sigma(x;\eta)\sigma(x;\eta) \rangle = 1$

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Def. (Metric operator and induced metric)

$$\hat{g}_{MN}(x;\eta) := \frac{\partial\sigma(x;\eta)}{\partial z^M}\frac{\partial\sigma(x;\eta)}{\partial z^N}$$

$$g_{MN}(z) := \langle \hat{g}_{MN}(x;\eta) \rangle_{CFT}$$

$$z^M = (x^\mu, \tau) \text{ with } \tau \propto \sqrt{\eta}$$

Comment: An induced metric can be interpreted as the **information metric**.

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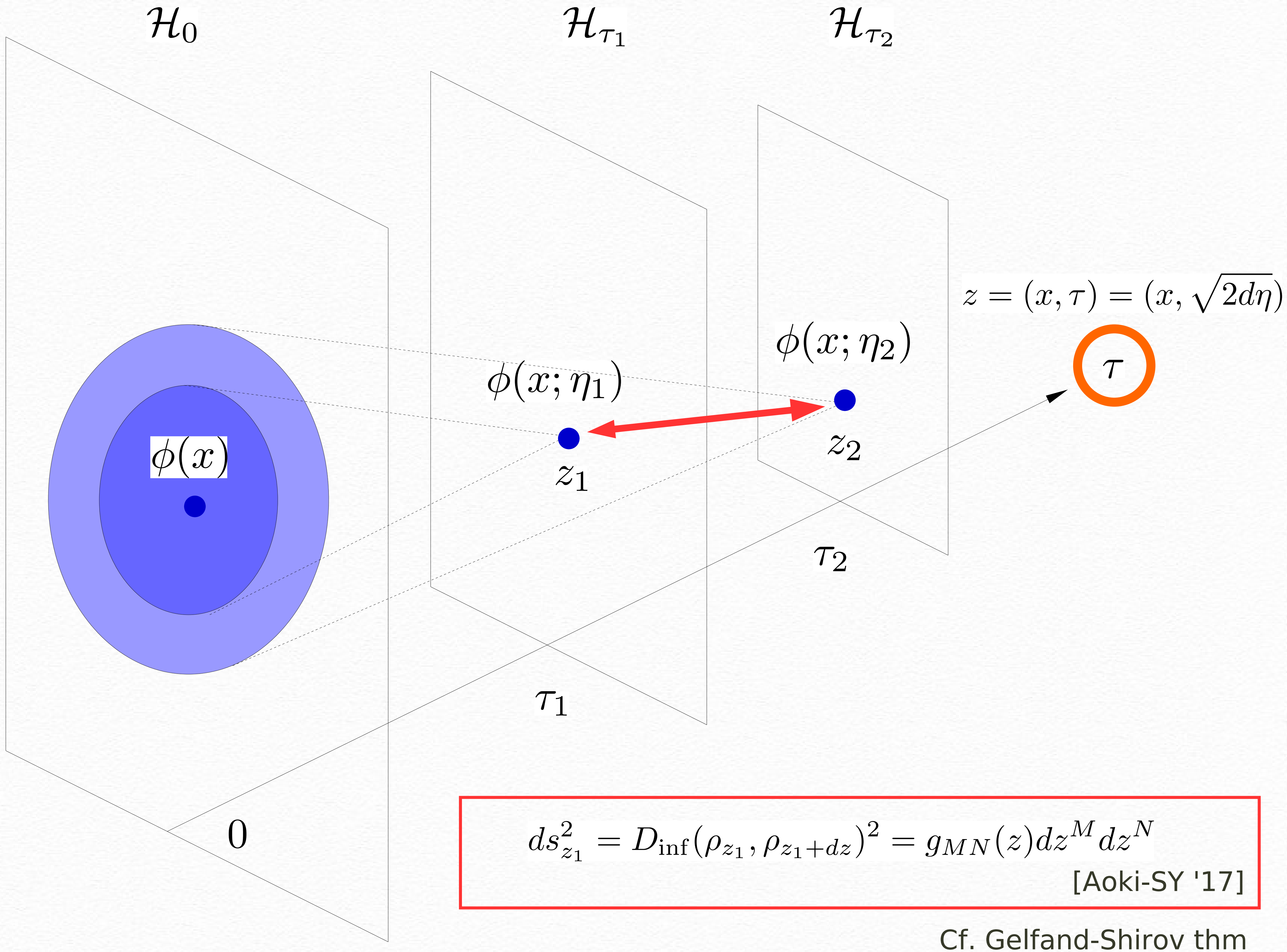
In the current case,

$$\langle \sigma(x_1;\eta_1)\sigma(x_2;\eta_2)\rangle = \left(\frac{2\sqrt{\eta_1\eta_2}}{\eta_+}\right)^\Delta G\left(\frac{x_{12}^2}{\eta_+}\right), \qquad G(u) := F(u;1)/F(0;1)$$

$$g_{\mu\nu}(z) = \delta_{\mu\nu}\frac{\Delta}{\tau^2}, \quad g_{\tau\tau}(z) = \frac{\Delta}{\tau^2} \qquad \tau := \sqrt{-\Delta\eta/G'(0)}$$

➡ $$ds^2 = \Delta\frac{dx^2 + d\tau^2}{\tau^2}.$$

Smearing = Extra direction



$$ds^2_{z_1} = D_{\inf}(\rho_{z_1}, \rho_{z_1+dz})^2 = g_{MN}(z)dz^M dz^N$$

[Aoki-SY '17]

Cf. Gelfand-Shirov thm

Non-relativistic Hybrid geometry

[Aoki-SY-Yoshida '19]

PRD (2019) no.12, 126002

Non-relativistic flow equation

Consider NRCFT$_d$ with a complex scalar primary field O(x,t)

$$\langle O(\vec{x}_1, t_1) O^\dagger(\vec{x}_2, t_2) \rangle = \frac{1}{t_{12}^{\Delta_{\mathcal{O}}}} f\left(\frac{\vec{x}_{12}^2}{2t_{12}}\right),$$

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Introduce one extra direction x^-

$$\langle O(\vec{x}_1,x_1^+,x_1^-)O^\dagger(\vec{x}_2,x_2^+,x_2^-)\rangle = \frac{1}{(x_{12}^+)^{\Delta_{\mathcal{O}}}} f\left(x_{12}^- + \frac{\vec{x}_{12}^2}{2x_{12}^+}\right). \qquad x^+ = t$$

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Smear this operator O by a free **NR flow equation**:

$$\boxed{\frac{\partial\phi(x;\eta)}{\partial\eta} = (2i\bar{m}\partial_- + 2\partial_+\partial_- + \vec{\partial}^2)\phi(x;\eta), \quad \phi(x;0) = O(x)}$$

$$x := (x^\mu) = (\vec{x}, x^+, x^-)$$

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$$x := (x^\mu) = (\vec{x}, x^+, x^-)$$

$$\rightarrow \quad \phi(x;\eta) = \int d^D y K(x-y;\eta)O(y) \qquad K(x;\eta) = \exp\left[-i\bar{m}x^-\right]\frac{e^{\frac{-2x^+x^- - \vec{x}^2}{4\eta}}}{\sqrt{4\pi\eta}^D}$$

NOTE: Contact singularity is generically resolved thanks to $\boxed{\text{a new term.}}$

Holographic geometry for NR CFT

Def. Induced (holographic) metric

$$ds^2 = \frac{1}{2}(\langle \partial_M \sigma^\dagger \partial_N \sigma \rangle + \langle \partial_M \sigma \partial_N \sigma^\dagger \rangle) dx^M dx^N$$

where σ is the normalized flowed field whose 2pt function is given by

$$\langle \sigma(x_1;\eta_1)\sigma^\dagger(x_2;\eta_2)\rangle = \left(\frac{4\eta_1\eta_2}{\eta_+^2}\right)^{\Delta_{\mathcal{O}}/2} G\left(\frac{2(x_{12}^+ + 2i\bar{m}\eta_+)x_{12}^- + (\vec{x}_{12})^2}{\eta_+}, \frac{x_{12}^+}{\eta_+}\right)$$

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RESULT

$$\boxed{\begin{aligned} ds^2 =& \frac{-G^{(0,2)}(\vec{0})}{4\eta^2}(dx^+)^2 + 2(\frac{-G^{(1,0)}(\vec{0}) - 2i\bar{m}G^{(1,1)}(\vec{0})}{\eta})dx^+dx^- + 4\bar{m}^2 G^{(2,0)}(\vec{0})(dx^-)^2 \\ &+ \frac{\Delta_{\mathcal{O}}}{4\eta^2}d\eta^2 + \frac{-\delta_{ij}G^{(1,0)}(\vec{0})}{\eta}dx^i dx^j \end{aligned}}$$

$$\left(-\Delta_{\mathcal{O}} = (2d+2)G^{(1,0)}(\vec{0}) + 8i\bar{m}G^{(1,1)}(\vec{0}) + 2i\bar{m}G^{(0,1)}(\vec{0}), \right)$$

1) If G is a general function → **Lifshitz**$_{d+1}$ x **R** with Z=2

2) $G^{(2,0)}(\vec{0}) = G^{(1,1)}(\vec{0}) = 0$ → **Schrodinger**$_{d+2}$

→ **NR hybrid geometry**

Q: GR system for NR hybrid geometry?

[Aoki-Balog-SY-Yoshida]

arXiv:1910.11032

GR system to realize NR hybrid?

$$ds^2 = \Delta_{\mathcal{O}}\left[- \alpha \frac{(dx^+)^2}{\tau^4} + \frac{d\tau^2 + d\vec{x}^2 + 2(1+\beta)dx^+ dx^-}{\tau^2} + \gamma (dx^-)^2 \right].$$

$$\alpha = \frac{\Delta_{\mathcal{O}} G^{(0,2)}(\vec{0})}{4 G^{(1,0)}(\vec{0})^2}, \quad \beta = \frac{2i\bar{m} G^{(1,1)}(\vec{0})}{G^{(1,0)}(\vec{0})}, \quad \gamma = \frac{4\bar{m}^2 G^{(2,0)}(\vec{0}))}{\Delta_{\mathcal{O}}}$$

Field contents?

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Boundary **Field contents?** **Bulk**

U(1)$_F$ global action

$$\sigma(x;\eta) \to \sigma(x;\eta)' = e^{i\lambda}\sigma(x;\eta)$$

U(1)$_F$ invariant operators

$$\exists\ \hat{A}_M := \frac{1}{2}(i\partial_M\sigma^\dagger\sigma - i\sigma^\dagger\partial_M\sigma)$$

$$\left(A_-(z) = 4\bar{m}G^{(1,0)}(\vec{0}), \quad A_+(z) = \frac{i\Delta G^{(0,1)}(\vec{0})}{2G^{(1,0)}(\vec{0})\tau^2},\right)$$

$$\hat{g}_{MN} = \frac{1}{2}(\partial_M\sigma^\dagger\partial_N\sigma + \partial_N\sigma^\dagger\partial_M\sigma)$$

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Boundary	**Field contents?**	**Bulk**
U(1)$_F$ global action	$\longrightarrow$ [Witten '98]	U(1)$_g$ gauge action
$\sigma(x;\eta) \to \sigma(x;\eta)' = e^{i\lambda}\sigma(x;\eta)$	$\longrightarrow$	$\Phi(X)$
		$\Phi \to e^{i\lambda}\Phi$
U(1)$_F$ invariant operators		
$\exists\ \hat{A}_M := \frac{1}{2}(i\partial_M\sigma^\dagger\sigma - i\sigma^\dagger\partial_M\sigma)$	$\longrightarrow$	$A_M(X)$
$\left(A_-(z) = 4\bar{m}G^{(1,0)}(\vec{0}),\quad A_+(z) = \frac{i\Delta G^{(0,1)}(\vec{0})}{2G^{(1,0)}(\vec{0})\tau^2},\right)$		$A_M \to A_M - i\partial_M\lambda$
$\hat{g}_{MN} = \frac{1}{2}(\partial_M\sigma^\dagger\partial_N\sigma + \partial_N\sigma^\dagger\partial_M\sigma)$	$\longrightarrow$	$g_{MN}(X)$

cf. [Balasubramanian-McGreevy '10]

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$\hat{g}_{MN} = \frac{1}{2}(\partial_M\sigma^\dagger\partial_N\sigma + \partial_N\sigma^\dagger\partial_M\sigma)$	→	$g_{MN}(X)$

cf. [Balasubramanian-McGreevy '10]

→ **Einstein-Maxwell-Higgs system**

$$S = \int d^{D+1}x\sqrt{-g}\left(\frac{1}{2\kappa^2}(R - 2\Lambda) - \frac{1}{4}g^{\mu\nu}g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} - g^{\mu\nu}D_\mu\Phi^\dagger D_\nu\Phi - V(|\Phi|)\right) \quad \textbf{?}$$

GR system to realize NR hybrid?

$$ds^2 = \Delta_{\mathcal{O}}\left[-\alpha\frac{(dx^+)^2}{\tau^4} + \frac{d\tau^2 + d\vec{x}^2 + 2(1+\beta)dx^+dx^-}{\tau^2} + \gamma(dx^-)^2\right].$$

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Boundary	**Field contents?**	**Bulk**
U(1)$_F$ global action	$\longrightarrow$ [Witten '98]	U(1)$_g$ gauge action
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[Taylor '08] cf. [Kachru-Liu-Mulligan 08]

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choice of smearing $\longleftrightarrow$ **!?** **choice of gauge**

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