De Sitter Duality and Logarithmic Decay of Dark Energy

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Introduction

- Nontrivial scaling relation has been observed in CMB: n_s~0.97, r < 0.07
- The equation of state w in dark energy: w ~ -1
- Small $g = G_N H^2/\pi$: and large de Sitter Entropy 1/g
- What carries de Sitter entropy
- De Sitter duality: Einstein gravity/inflation theory
- It ensures general covariance of Einstein gravity while restricts inflation theory

Einstein gravity in de Sitter space

$$\frac{1}{\kappa^2}\int d^Dx\sqrt{-g}\big[R-(D-1)(D-2)H^2\big]$$

$$ds^2 = a^2(-d\tau^2 + dx_i^2),$$

$$g_{\mu\nu} = \Omega^2(x)\tilde{g}_{\mu\nu}, \quad \Omega(x) = a(\tau)\phi(x), \quad \phi(x) = e^{\omega(x)},$$

$$a: \ \partial_0^2 a = 2H^2a^3,$$

$$ds^2 = \Big(\frac{1}{-\tau H}\Big)^2(-d\tau^2 + dx_i^2) = -dt^2 + e^{2Ht}dx_i^2.$$

$$h^{00}: \ \partial_0^2 a^2 = 6\partial_0 a\partial_0 a.$$

$$\frac{i}{16\pi G_N}\int d^4x\sqrt{-g}\ 6H^2(\Omega^4 - 2\Omega^2) \ \to \ \frac{\pi}{G_N H^2}(\Omega^4 - 2\Omega^2),$$

De Sitter entropy $\quad \dfrac{\pi}{G_N H^2}.$

Dual Inflation theory

$$\frac{1}{\kappa^2}\int d^4x\sqrt{-g}\left[R-6H^2(\gamma)\exp(-2\Gamma(\gamma)f)-2\Gamma(\gamma)g^{\mu\nu}\partial_\mu f\partial_\nu f\right].$$

We propose IR quantum effects of the Einstein gravity
Is given by the classical evolution of the inflaton

Classical Solutions $\Omega = e^{\omega} = e^{f} = a^{1+\gamma}.$ $\log a_c \;=\; Ht \;=\; \omega_c$

Quantum correction $\exp(-2\gamma\omega_c) = \exp(-2\gamma f_c).$ Inflaton potential

Putting inflaton under the rug by the equality with the conformal mode

$$\frac{1}{\kappa^2}\int d^4x\left[\Omega^2\tilde{R}+(6-2\Gamma)\tilde{g}^{\mu\nu}\partial_\mu\Omega\partial_\nu\Omega-6H^2(\gamma)\Omega^{4(1-\frac{\Gamma}{2})}\right].$$

General covariance is ensured by duality

Quantum IR effects in 4d de Sitter space

- Background gauge calculation for IR logarithms

$$\frac{1}{\kappa^2}\int d^4x\sqrt{-\hat{g}}\big[(\hat{R}-12H^2)\langle 4\omega^2\rangle - 2\hat{R}^{\mu}{}_{\nu}\langle h^{\nu}{}_{\mu}\omega\rangle\big].$$

$$a^2\hat{R}_{\mu\nu}h^{\mu\nu} \simeq a^2(\hat{R}_{00}+\hat{R}_{11})h^{00} = (-2a\partial_0^2 a + 4\partial_0 a\partial_0 a)h^{00},$$

We introduce inflaton f as a counter term to subtract non covariant IR logs

$$-2\Gamma\int d^4x\sqrt{-g}g^{\mu\nu}\partial_\mu f\partial_\nu f - \int d^4x\,\sqrt{-g}6H^2V(f)$$

$6\partial_0 a\partial_0 a - \partial_0^2 a^2 = 2\Gamma(\gamma)a^2\partial_0 f\partial_0 f.$ V(f) = $\exp(-2\Gamma f)$ -1 $e^f = a_c^{1+\gamma} = \phi,$

1 loop effective action

Quantum correction

Counter term

$$\frac{1}{\kappa^2}\int d^4x\sqrt{-\hat{g}}\,(\hat{R}-12H^2)\Big(-\frac{3}{4}\frac{\kappa^2H^2}{4\pi^2}\log a\Big) \quad + \quad \frac{1}{\kappa^2}\int d^4x a_c^{2\gamma}\big[a_c^4R-6a_c^{2\gamma}a_c^4H^2\big] \quad + \quad \text{Inflaton}$$

$$= \frac{1}{\kappa^2}\int d^4x\big[a_c^4R-6a_c^4H^2\exp(-2\gamma f)-2a_c^2\gamma\partial_\mu f\partial^\mu f\big].$$

$$\frac{1}{\kappa^2}\int d^4x\big[a_c^4R-6a_c^4H^2(1-2\gamma\log a_c)\big]$$

$$=\frac{1}{\kappa^2}\int d^4x\big[a_c^4R-6a_c^4H^2a_c^{-2\gamma}\big]$$

Effective action of Einstein gravity is inflation theory

De Sitter duality is proved

De Sitter entropy as von-Neumann entropy of conformal zero modes

- So far our results are local Ht<1. However we need global predictions in cosmology Ht>>1
- We resum all powers of (Ht)^n=(log a)^n to obtain one loop exact results by Fokker-Planck equation.
- In a very reliable gaussian approximation, it is equivalent to solve diffusion equation exactly.
- The distribution function of conformal zero modes

$$\rho(\omega) = \frac{1}{N} \exp\left(-\frac{4\xi}{g}\omega^2\right)$$

- We postulate von-Neumann entropy = de Sitter entropy

$$g = G_N H^2/\pi$$

$$S = -\mathrm{tr}(\rho \log \rho) = \frac{1}{2}(1 + \log \pi g - \log 4\xi).$$

Asymptotic Freedom

- Diffusion equation for standard deviation $$\dot{\xi} = -6H\xi^2. \quad \xi = \frac{1}{1+6Ht}.$$
- De Sitter entropy
$$S(t) = \frac{1}{g} + \frac{1}{2}\big(\log \pi g - \log 4\xi(t)\big).$$
- Bare action
$$S_B = \frac{1}{g(t)} + \frac{1}{2}\log \xi(t).$$
- Beta function in Einstein gravity on de Sitter space
$$\beta(g) = -\frac{1}{2}g^2, \quad \beta(g) \equiv \frac{\partial}{\partial \log(1+6Ht)} g.$$
- Asymptotically free toward future
$$g(t) = \frac{2}{\log(1+6Ht) - \log(1+6Ht_\Lambda)}.$$

Physical Implications
logarithmic decay of Dark Energy

- Although g is 10^{-120}, its beta function has no small number $\beta(g) = -\frac{1}{2}g^2.$
- The Universe is very close to the IR fixed point with a large entropy 1/g
- The present energy contents of the Universe $\Omega_M = 0.3. \quad \Omega_\Lambda = 0.7$

$$\frac{H^2}{H_0^2} = \frac{\Omega_M}{a^3} + \Omega_\Lambda, \qquad \frac{-\dot{H}}{H^2} = \frac{3}{2}\Omega_M \sim \frac{1}{2}.$$

- Our proposal

$$\frac{H^2(z)}{H_0^2} = 0.3(1+z)^3 + 0.7\log\left(e + \log(1+z)\right), \qquad Z< 0.6$$

$$\frac{H^2(z)}{H_0^2} = \Omega_M(1+z)^3 + \begin{cases} \Omega_\Lambda \log\left(e + \log(1+z)\right), & (z \leq 0.6) \\ \Omega_\Lambda \log\left(e + \log(1.6)\right), & (z > 0.6) \end{cases}.$$

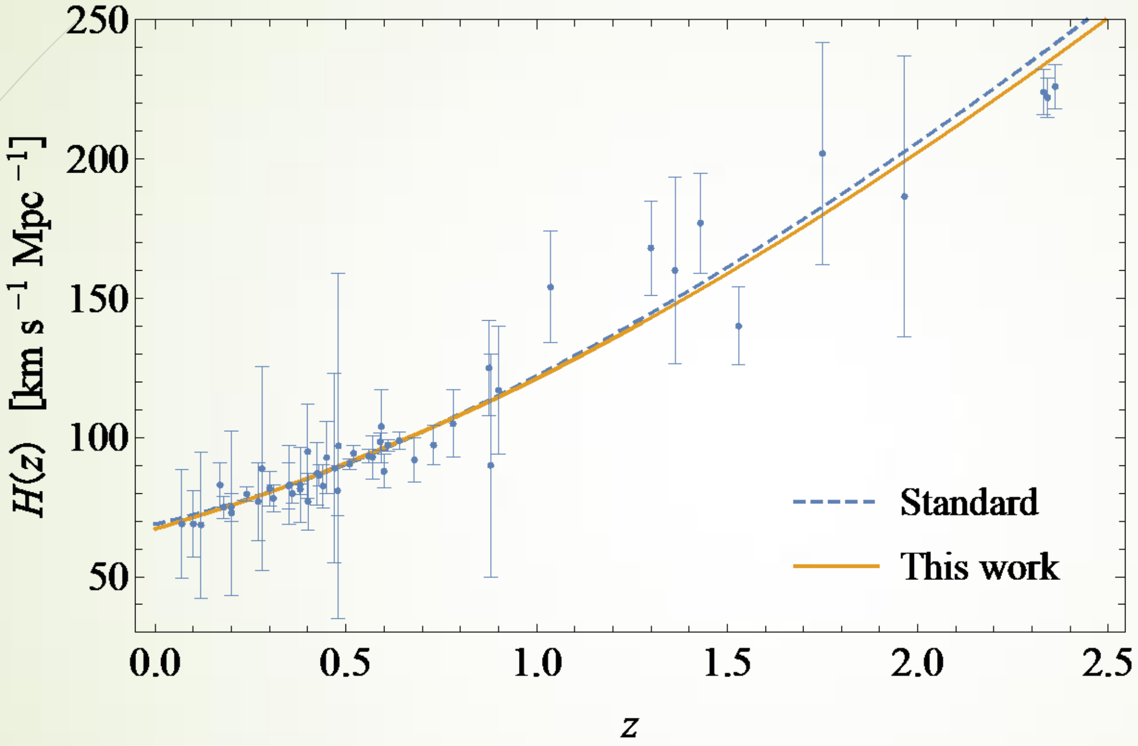

Figure 1: The Hubble parameter measurements and their errors (in units of km s^{-1} Mpc^{-1}) [32] are compared with theoretical predictions.

	H_0 [km s^{-1} Mpc^{-1}]	χ^2/dof
Standard	68.8 ± 2.5	0.739
This work	67.2 ± 2.5	0.623

Table 1:

Equation of states of Dark Energy

We consider z < 0.6 in the accelerated phase

Our characteristic prediction
$$3(1+w) = \frac{1}{\log(e+\log(1+z))}\frac{1}{e+\log(1+z)}.$$

$$\begin{aligned}
&\text{Observation :} && w_0 = -0.91 \pm 0.10, && w_a = -0.39 \pm 0.34, && (5.67)\\
&\text{Standard :} && w_0 = -1, && w_a = 0, && (5.68)\\
&\text{This work :} && w_0 = -0.877\cdots, && w_a = -0.090\cdots. && (5.69)
\end{aligned}$$

Our theory may be testable in the near future

CMB: to B or not to B

- We have generated a linear potential at the one loop

$$\frac{1}{\kappa^2}\int d^4x\sqrt{-g}\big[R-6H_0^2V(f)-\frac{\kappa^2}{2}g^{\mu\nu}\partial_\mu f\partial_\nu f\big].$$

$$V(f)=1+\sqrt{\gamma}\kappa f=1+2\gamma H_0 t=1+3gH_0 t.$$

We obtain logarithmic potential after resummation

$$g(t)=\frac{2}{\log(1+6H_0 t)}.$$

Classical solution of linear potential

- Fokker-Planck equation for time-dependent g(t)

$$\dot{\xi}\frac{\partial}{\partial\xi}\rho = (\frac{1}{2}\frac{\partial}{\partial t}(\log\frac{\xi}{g(t)})\rho - (\frac{\partial}{\partial t}\frac{4\xi}{g(t)})\omega^2\rho),$$

$$\frac{\partial}{\partial t}\log\frac{\xi}{g(t)} = -6H\xi$$

$$\xi = \xi' g = \frac{\sqrt{N}}{N^{3/2}} = \frac{1}{N}$$

$$r = \frac{4}{N} \sim 0.067$$

$$\begin{aligned} 1 - n_s &= \frac{\partial}{\partial N_*}\log\langle\zeta\zeta\rangle \\ &= \frac{\partial}{\partial N_*}\log N^{\frac{3}{2}} \\ &= \frac{3}{2N} \sim 0.025 \end{aligned}$$

Linear potential model under scrutiny

Quantum effects due to logarithmic potential

- One loop renormalization

$$\beta(g) = -\frac{1}{2}g^2, \quad \beta(g) \equiv \frac{\partial}{\partial \log(1+6Ht)} g.$$

. So the total logarithmic factor is $(\sqrt{\log H_0 t})^{DegH}$.

- Loop contribution: We can simply count the net number of H

There arises an IR logarithmic factor $\sqrt{\log N}$ per H

Prediction of a linear potential model with IR quantum effects

- R ratio

$$r = \frac{1}{N\sqrt{\log N}} \sim 0.033$$

- Scalar spectral index

$$\begin{aligned} 1 - n_s &= \frac{\partial}{\partial N_*} \log\langle\zeta\zeta\rangle \\ &= \frac{\partial}{\partial N_*} \log\left\{\frac{3}{N\log^2 N}\right\} \\ &= \frac{3}{2N} + \frac{3}{2N\log N} \sim 0.03 \end{aligned}$$

Discussions

- Quantum corrections revive linear potential model

$$1 - n_s = \frac{\partial}{\partial N_*} \log\langle\zeta\zeta\rangle = \frac{\partial}{\partial N_*} \log\left\{\frac{3}{N\log^2 N}\right\} = \frac{3}{2N} + \frac{3}{2N\log N} \sim 0.03$$

$$r = \frac{1}{N\sqrt{\log N}} \sim 0.033$$

- Log N corrections increase (1-n_s) while suppress r
- The theoretical predictions are not trustable until quantum corrections are under control