Weyl Anomaly Induced Current and Fermi Condensation for BCFT

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Related Works

- Chong-Sun Chu, Rong-Xin Miao, “Weyl Anomaly Induced Current in Boundary Quantum Field Theories ”, Phys.Rev.Lett. 121 (2018) no.25, 251602.
- Chong-Sun Chu, Rong-Xin Miao, “Anomalous Transport in Holographic Boundary Conformal Field Theories ”, JHEP 1807 (2018) 005.
- Rong-Xin Miao, Chong-Sun Chu, “Universality for Shape Dependence of Casimir Effects from Weyl Anomaly ”, JHEP 1803 (2018) 046.
- Chong-Sun Chu, Rong-Xin Miao,“Boundary String Current Weyl Anomaly in Six-dimensional Conformal Field Theory ”,JHEP 1907 (2019) 151.
- J.J Zheng, D.Q Li, Y.Q Zeng, R.X Miao,“Anomalous Current Due to Weyl Anomaly for Conformal Field Theory ”, accepted by PLB.
- Works in progress.

Outline

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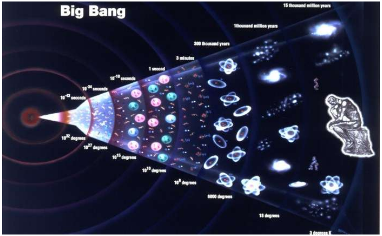
1. Background
   - Why BCFT/dCFT?
   - Review of Weyl anomaly
2. Main Results
   - Current from Weyl anomaly
   - Fermi Condensation from Weyl anomaly
3. Summary and Outlook

Motivation

Since many physical systems have boundaries, it is interesting to study the boundary effects of quantum systems.

- Casimir effects
- Topological Insulator
- Big Bang of the universe
  It implies that there is a boundary of time.
- Cosmological horizon is also a kind of boundary.



BCFT

- Definitions
  BCFT is a conformal field theory defined on a manifold M with a boundary P, where suitable boundary conditions are imposed.
- Example of free BCFT
  - Conformal free scalar field

$$I = -\frac{1}{2}\int_M d^d x\sqrt{g}[(\partial\phi)^2 + \xi R\phi^2] - \xi\int_P d^{d-1}y\sqrt{\sigma}K\phi^2 \qquad (1)$$

    where $\xi = \frac{d-2}{4(d-1)}$, and K is the extrinsic curvature.
  - Conformally invariant boundary conditions

$$\text{Dirichlet BC} : \phi|_P = 0, \qquad (2)$$

$$\text{Robin BC} : (\partial_n + 2\xi K)\phi|_P = 0, \qquad (3)$$

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Weyl anomaly of conformal field theory

Weyl anomaly is the violation of scale invariance by quantum corrections, quantified in renormalization.

- Consider Weyl transformation $g_{\mu\nu} \to e^{2\sigma} g_{\mu\nu}$

$$\delta_\sigma I_{reg} = \int_M \sqrt{g} < T^{\mu\nu} >_{ren} \sigma g_{\mu\nu} + O(\sigma^2) \qquad (4)$$

- Definition I of Weyl anomaly

$$\mathcal{A} = \int_M \sqrt{g} < T^{\mu\nu} >_{ren} g_{\mu\nu} \qquad (5)$$

- Definition II of Weyl anomaly

$$I_{non-ren} = ... + \mathcal{A} \ln \frac{1}{\epsilon} + I_{ren} \qquad (6)$$

where ... denote divergent terms and ϵ is the cutoff.

Equivalence for two definitions of Weyl anomaly

Key Point: the non-renormalized effective action of CFT is conformally invariant.

Consider Weyl transformation $g_{\mu\nu} \to e^{2\sigma} g_{\mu\nu}$ and $\epsilon \to e^{\sigma} \epsilon$ for non-renormalized effective action

$$I_{non-ren} = ... + \mathcal{A} \ln \frac{1}{\epsilon} + I_{ren}$$

$$\delta_\sigma I_{non-ren} = \sigma \left(-\mathcal{A} + \int_M \sqrt{g} < T^{\mu\nu} >_{ren} g_{\mu\nu} \right) + O(\sigma^2) = 0 \qquad (7)$$

where the divergent term ... and Weyl anomaly $\mathcal{A}$ are conformally invariant.

Boundary Weyl anomaly

In the presence of boundary, Weyl anomaly of CFT generally pick up a boundary contribution $\langle T^a_a \rangle_P$ in addition to the usual bulk term $\langle T^i_i \rangle_M$, i.e. $\langle T^i_i \rangle = \langle T^i_i \rangle_M + \delta(x_\perp) \langle T^a_a \rangle_P$.

- Bulk Weyl anomaly

$$\langle T^i_i \rangle_M = \frac{c}{16\pi^2} C^{ijkl} C_{ijkl} - \frac{a}{16\pi^2} E_4 + b\ F_{ij} F^{ij}, \qquad d = 4, \tag{8}$$

- Boundary Weyl anomaly

$$\langle T^a_a \rangle_P = d_1 \mathrm{Tr} \bar{k}^3 + d_2 C^{ac}{}_{bc} \bar{k}^b{}_a, \quad d = 4, \tag{9}$$

  where $\bar{k}_{ab}$ is the traceless part of extrinsic curvature, C_{ijkl} is the Weyl tensor, F_{ij} is the field strength of gauge field.

- a, b, c are the bulk central charges independent of BC.
- d_i are boundary central charges which depend on BC.

Outline

Weyl anomaly can be defined as the difference between the trace of renormalized stress tensor and the renormalized trace of stress tensor.

- Definition I of Weyl anomaly

$$\mathcal{A} = \int_M \sqrt{g}\,[< T^{\mu\nu} >_{ren} g_{\mu\nu} - < T^{\mu\nu} g_{\mu\nu} >_{ren}] \tag{10}$$

- Definition II of Weyl anomaly

$$I_{non-ren} = ... + \mathcal{A} \ln \frac{1}{\epsilon} + I_{ren} \tag{11}$$

- For unitary, renormalizable and gauge invariant QFT

$$\mathcal{A} = \int_M \sqrt{g}[b F_{\mu\nu} F^{\mu\nu} + O(R^2)] + \int_{\partial M} \sqrt{h} O(Rk), \tag{12}$$

where b is beta function.

Current near the boundary

Renormalized current is divergent near the boundary. However, nothing goes wrong since there is boundary current cancel the bulk "divergence".

- Near-boundary current

$$\langle J_\mu \rangle = \frac{1}{x^3} J_\mu^{(3)} + \frac{1}{x^2} J_\mu^{(2)} + \frac{1}{x} J_\mu^{(1)} + \cdots, \qquad x \sim 0, \tag{13}$$

  where x is the proper distance from the boundary, $J_\mu^{(n)}$ depend on only the background geometry, the background gauge field strength.

- Imposing $\nabla_\mu J^\mu = 0$, we get

$$\begin{aligned} &J_\mu^{(3)} = 0, \qquad J_\mu^{(2)} = 0, \\ &J_\mu^{(1)} = \alpha_1 F_{\mu\nu} n^\nu + \alpha_2 \mathcal{D}_\mu k + \alpha_3 \mathcal{D}_\nu k_\mu^\nu + \alpha_4 \star F_{\mu\nu}\, n^\nu \end{aligned} \tag{14}$$

Current from Weyl anomaly

Recall that Weyl anomaly can be obtained as the logarithmic UV divergent term of the effective action.

- Vary the vector and focus on the boundary term

$$(\delta\mathcal{A})_{\partial M} = \delta I_{\text{eff}}\Big|_{\ln 1/\epsilon} = \left(\int_{x\geq\epsilon}\sqrt{g}J^{\mu}\delta A_{\mu}\right)_{\log(1/\epsilon)}, \tag{15}$$

- Variation of Weyl anomaly

$$(\delta\mathcal{A})_{\partial M} = 4b\int_{\partial M}\sqrt{h}F^{b}{}_{n}\,\delta A_{b}. \tag{16}$$

- Variation of effective action

$$\int_{\partial M}\sqrt{h}(\alpha_1 F^{b}{}_{n} + \alpha_2\mathcal{D}^{b}k + \alpha_3\mathcal{D}_j k^{jb} + \alpha_4 \star F^{b}{}_{n})\delta A_b. \tag{17}$$

- Identifying (38) with (39), we get $\alpha_1 = 4b, \quad \alpha_2 = \alpha_3 = \alpha_4 = 0.$

Key result: current from Weyl anomaly

The expectation value of the current take universal form

$$J_a = \frac{4bF_{an}}{x}, \quad x \sim 0, \tag{18}$$

near the boundary.

- The universal law holds for general BQFTs which are covariant, gauge invariant, unitary and renormalizable.
- The current is independent of boundary conditions.
- The magnitude of the induced current is large.

$$J_a = \frac{e^2 c}{\hbar} \frac{4bF_{an}}{x}. \tag{19}$$

- This current comes from the vacuum magnetization near boundary.

Another derivation of current from Weyl anomaly

Consider the Weyl transformation

$$g'_{ij} = e^{2\sigma} g_{ij}, \qquad F'_{ij} = F_{ij} \tag{20}$$

- Variation of effective action

$$\delta_\sigma I_{\text{eff}} = \mathcal{A}\delta\sigma = b\int_M dx^4 \sqrt{g} F^{ij} F_{ij} \delta\sigma(x) + O(R^2, \sigma). \tag{21}$$

- Anomalous action

$$I_{\text{anomalous}} = I(e^{2\sigma} g_{ij}) - I(g_{ij}) = b\int_M dx^4 \sqrt{g'} F'^{ij} F'_{ij} \sigma(x). \tag{22}$$

- Weyl transformation of current

$$J'^i = e^{-4\sigma} J^i + 4b\nabla'_j(F'^{ij}\sigma). \tag{23}$$

Another derivation of current from Weyl anomaly

Key observation: BCFT in the half space

$$ds^2 = dx^2 + dy_a^2, \quad x \geq 0 \tag{24}$$

is conformally equivalent to CFT in the Poincare patch of AdS

$$ds^2 = \frac{dx^2 + dy_a^2}{x^2}, \quad x \geq 0. \tag{25}$$

- Finite current in AdS

$$\delta I_{\text{ren}} = \int_M dx^4 \sqrt{g} J^i \delta A_i = \int_M dx^4 \frac{J^i}{x^4} \delta A_i, \tag{26}$$

- Current in half space from Weyl transformation law (23)

$$J^i_{\text{BCFT}} = \frac{J^i}{x^4} + 4b\, \nabla'_j (F'^{ij} \ln x) = \frac{4b\, F'^{ix}}{x} + O(\ln x, x^0) \tag{27}$$

Finite Total Current

There are boundary current, which exactly cancel the apparent "divergence" in the bulk current and make finite total current.

- Gauge invariance

$$\begin{aligned}\delta_\alpha I &= \int_M \sqrt{g} J^i \delta A_i + \int_{\partial M} \sqrt{h} j^b \delta a_b = 0 \\ &= -\int_M \sqrt{g} \nabla_i J^i \alpha - \int_{\partial M} \sqrt{h} (D_b j^b - J_n) \alpha \end{aligned} \tag{28}$$

- Conservation laws

$$\text{Bulk}: \ \nabla_i J^i = 0 \ \Rightarrow \ J_n = 4b D_a F^a{}_n \ln x + O(1) \tag{29}$$

$$\text{Boundery}: \ D_a j^a = J_n \ \Rightarrow \ j_a = 4b F_{an} \ln \epsilon. \tag{30}$$

- Finite total current

$$J_a = \frac{4bF_{an}}{x} + \delta(x; \partial M) 4b F_{an} \ln \epsilon + O(1). \tag{31}$$

Physical Picture

When there is a boundary, the contribution from source points at $x < 0$ are missing. This leads to a net amount of charge moving to $-y$ direction.

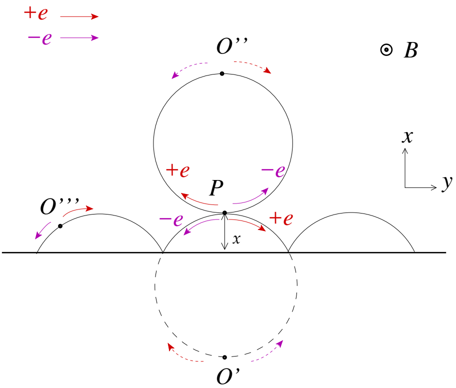

Figure: Induced current from virtual pair creation in presence of boundary.

Outline

Our model

- Fermi Field with a background scalar

$$I = \int_M \sqrt{g}[\bar{\Psi}(\gamma^\mu \nabla_\mu - m)\Psi + \lambda\hat{\phi}\bar{\Psi}\Psi + \mathcal{L}(\hat{\phi}, \nabla\hat{\phi})] \qquad (32)$$

where the scalar can be either Higgs or phonon.

- Weyl anomaly at one loop

$$\mathcal{A} = \frac{1}{32\pi^2}\left(\int_M \sqrt{g}[(\nabla\phi)^2 + \phi^4 + \frac{1}{6}R\phi^2] + \int_{\partial M}\sqrt{h}\frac{1}{3}K\phi^2\right), \qquad (33)$$

where we have redefined $\phi = \lambda\hat{\phi} - m$.

- Fermi Condensation is related to the scalar current

$$<\bar{\Psi}\Psi> = <O> = \frac{1}{\sqrt{g}}\frac{\delta I_{eff}}{\delta\phi}. \qquad (34)$$

Fermi Condensation near the boundary

Scalar current is divergent near the boundary. However, nothing goes wrong since there is boundary current cancel the bulk "divergence".

- Near-boundary current

$$< \bar{\Psi}\Psi >=< O >= \frac{1}{x^3}O^{(3)} + \frac{1}{x^2}O^{(2)} + \frac{1}{x}O^{(1)} + \cdots, \qquad x \sim 0, \quad (35)$$

  where x is the proper distance from the boundary, $O^{(n)}$ depend on only the background geometry and the background scalar.

- For example

$$O^{(3)} = c_1, \; O^{(2)} = c_2\phi + c_3 K,$$
$$O^{(1)} = c_4 n^\mu \nabla_\mu \phi + c_5 K\phi + c_6\phi^2 + ... \quad (36)$$

  where c_i are constants, n is the normal vector.

Fermi Condensation from Weyl anomaly I

Recall that Weyl anomaly can be obtained as the logarithmic UV divergent term of the effective action.

- Vary the scalar and focus on the boundary term

$$(\delta\mathcal{A})_{\partial M} = \delta I_{\text{eff}}\big|_{\ln 1/\epsilon} = \left(\int_{x\geq\epsilon}\sqrt{g}\,O\delta\phi\right)_{\log(1/\epsilon)} \tag{37}$$

- Variation of Weyl anomaly

$$(\delta\mathcal{A})_{\partial M} = \frac{1}{16\pi^2}\int_{\partial M}\sqrt{h}(\nabla_n\phi + \frac{1}{3}K\phi)\,\delta\phi. \tag{38}$$

- Variation of effective action

$$\int_{\partial M}\sqrt{h}[O^{(3)}\delta(\nabla_n^2\phi) + O^{(2)}\delta(\nabla_n\phi) + O^{(1)}\delta\phi] \tag{39}$$

- Identifying (38) with (39), we get $O^{(3)} = O^{(2)} = 0$, $O^{(1)} = \frac{\nabla_n\phi + \frac{1}{3}K\phi}{16\pi^2}$.

Fermi Condensation from Weyl anomaly I

Fermi Condensation takes universal form

$$< \bar{\Psi}\Psi > = < O > = \frac{1}{16\pi^2} \frac{\nabla_n \phi + \frac{1}{3} K\phi}{x}, \quad x \sim 0, \tag{40}$$

near the boundary.

- This result works at one loop.
- For Higgs field, we have

$$< \bar{\Psi}\Psi > \sim \frac{mK}{x} \tag{41}$$

  Recall that $\phi = \lambda\hat{\phi} - m, < \hat{\phi} > = 0$.

- In higher loops, it is expected that

$$< \bar{\Psi}\Psi > \sim \frac{1}{x^3} \tag{42}$$

Fermi Condensation from Weyl anomaly II

Weyl anomaly can induce Fermi Condensation in conformally flat space without boundaries.

- Anomalous action in conformally flat space

$$I_{\text{anomalous}} = I(e^{2\sigma}\eta_{ij}) - I(\eta_{ij}) = \mathcal{A}\sigma + O(\sigma^2) \tag{43}$$

- Fermi Condensation

$$\begin{aligned} <\bar{\Psi}\Psi> &= \frac{\delta I_{\text{anomalous}}}{\delta\phi} \\ &= \frac{1}{16\pi^2}\left(-\nabla(\nabla\phi\sigma) + 2\phi^3\sigma + \frac{1}{3}\sigma\phi R\right) + O(\sigma^2) \end{aligned} \tag{44}$$

- For Higgs field, we have $\phi \sim m$

$$<\bar{\Psi}\Psi> \sim m^3\sigma + O(\sigma^2) \tag{45}$$

Summary and Outlook I

Summary for Current:

- Weyl anomaly induce a current, when external magnetic field parallel to the boundary is applied.
- Near the boundary, the current take universal form for covariant, gauge invariant, unitary and renormalizable QFT.
- The universal law is independent of boundary conditions, temperature and the states of QFT.
- The current is due to vacuum magnetization.

Outlook:

- Generalizations to defect QFT.
- Experimental measurement.

Summary and Outlook II

Summary for Fermi Condensation:

- For a theory with Yukawa coupling, Fermi Condensation is given by the expectation value of scalar current.
- Weyl anomaly can induce Fermi Condensation near the boundary, when there is a background scalar.
- Weyl anomaly can also induce Fermi Condensation in conformally flat space without boundaries.

Outlook:

- Higher-loop Effects for Fermi Condensation.
- Applications in Cosmology and Condensed Matter?

Thank you!