6d superconformal Cardy formulas

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Cardy formula

- Cardy formulas of CFTs in $d \geq 2$
  - Asymptotic free energy of CFT on $S^{d-1} \times \mathbb{R}$ in the Cardy limit
  - Cardy limit: angular momenta on $S^{d-1} \gg$ central charge
  - $d = 2$ [Cardy] & $3, 4, 5$ [S.Choi, J.Kim, S.Kim, **JN**][S.Choi, S.Kim][S.Choi, C.Hwang, S.Kim]

    $\Rightarrow$ Entropy of BPS black holes in AdS_{d+1} [Hosseini, Hristov, Zaffaroni]

- Cardy formulas of 6d SCFTs [Di Pietro, Komargodski]
  - Global symmetry: $SO(2,6)$ conformal & $SU(2)$ R-symmetry + (flavor)
  - Superconformal index in the modified basis: $\Delta_R - \omega_1 - \omega_2 - \omega_3 = \boxed{2\pi i}$

$$I(\Delta_R, \omega) = \mathrm{Tr}\left[e^{-\beta(E-4Q_R-J_1-J_2-J_3)} \cdot \underbrace{e^{-\Delta_R Q_R}}_{SU(2)_R} \cdot \underbrace{e^{-\omega_1 J_1 - \omega_2 J_2 - \omega_3 J_3}}_{SO(6)}\right]$$

  - Cardy limit: $|\omega_i| \ll 1$ ($\sim J_i \gg N^3$)

Background field analysis

- Background fields
  - 6d SCFT on $S^5 \times S^1$ with $\beta \to 0$: integrating out KK modes on S^1

    $\Rightarrow$ Effective action W_{eff} of the background fields on S^5
  - Background metric $g_{\mu\nu}$ depends on the chemical potential $\omega_{1,2,3}$
  - Bkgd $SU(2)_R$ gauge field A_μ depends on the chemical potential Δ_R

$$I(\omega_i, \Delta_R) = \exp\Big[-W_{\text{eff}}(g_{\mu\nu}, A_\mu) + (\text{0-modes}) \Big], \qquad \boxed{\delta W_{\text{eff}} \leftarrow \text{t' Hooft anomaly}}$$

- Chern-Simons terms and the anomaly
  - $W_{\text{eff}} \ni$ infinitely many terms arranged in a derivative expansion
  - ✓ CS terms can be fully determined from the anomaly polynomial [Banerjee, Bhattacharya, Bhattacharya, Jain, Minwalla, Sharma], [Jensen, Loganayagam, Yarom], [di Pietro, Komargodski]
  - Non-CS terms & zero-mode contribution are subleading in the Cardy limit

6d (2,0) Cardy formulas

- 6d (2,0) SCFT of type $G \in \{A_N,\ D_N,\ E_N\}$
  - Global symmetries: $SO(5)$ R-symmetry & $SO(6)$ diffeomorphism
  - Chemical potentials: $\Delta_{1,2}$ for R-charge & $\omega_{1,2,3}$ for angular momenta
  - Cardy free energy ($\Delta_1 + \Delta_2 - \omega_1 - \omega_2 - \omega_3 = 2\pi i$)

$$\log I = -\frac{h_G^\vee d_G + r_G}{24}\frac{\Delta_1^2\Delta_2^2}{\omega_1\omega_2\omega_3} - r_G\frac{((\Delta_1+\Delta_2)^2+4\pi^2)((\Delta_1-\Delta_2)^2+4\pi^2)}{192\omega_1\omega_2\omega_3} + r_G\frac{\Delta_1^2+\Delta_2^2-4\pi^2}{96}\frac{\omega_1^2+\omega_2^2+\omega_3^2}{\omega_1\omega_2\omega_3} + \mathcal{O}(\log\omega)$$

- BPS black holes in AdS_7
  - Large N free energy of $A_N(D_N)$ SCFT ($h_G^\vee d_G \propto N^3$)

    $\Rightarrow$ Entropy function of BPS black holes in $AdS_7 \times S^4(/\mathbb{Z}_2)$

$$G_7 \propto N^{-3}$$

6d (2,0) Cardy formulas

$$W_{\mathsf{CS}} = \int_{S^5}\Bigg[\frac{i\beta(4h^\vee_G d_G + 3r_G)}{768\pi^3}\Big((A_0^1)^2(A_0^2)^2\mathbf{a}d\mathbf{a}d\mathbf{a}+\Big(2A_0^1(A_0^2)^2\hat{\mathbf{A}}^1 + 2A_0^2(A_0^1)^2\hat{\mathbf{A}}^2\Big)d\mathbf{a}d\mathbf{a}$$

$$+\Big((A_0^2)^2\hat{\mathbf{A}}^1 d\hat{\mathbf{A}}^1 + 4A_0^1A_0^2\hat{\mathbf{A}}^1 d\hat{\mathbf{A}}^2 + (A_0^1)^2\hat{\mathbf{A}}^2 d\hat{\mathbf{A}}^2\Big)d\mathbf{a}+\Big(2A_0^2\hat{\mathbf{A}}^2 d\hat{\mathbf{A}}^1 d\hat{\mathbf{A}}^1 + 2A_0^1\hat{\mathbf{A}}^1 d\hat{\mathbf{A}}^2 d\hat{\mathbf{A}}^2\Big)\Big)$$

$$+\frac{i\beta r_G}{1536\pi^3}\Bigg\{\sum_{I=1}^{2}\Big((A_0^I)^4\mathbf{a}d\mathbf{a}d\mathbf{a} + 4(A_0^I)^3\hat{\mathbf{A}}^I d\mathbf{a}d\mathbf{a} + 6(A_0^I)^2\hat{\mathbf{A}}^I d\hat{\mathbf{A}}^I d\mathbf{a} + 4A_0^I\hat{\mathbf{A}}^I d\hat{\mathbf{A}}^I d\hat{\mathbf{A}}^I\Big)$$

$$+\Big(\Big((A_0^1)^2 + (A_0^2)^2\Big)\mathrm{tr}[\Gamma_0^2]\mathbf{a}d\mathbf{a}d\mathbf{a} + 2\Big((A_0^1 d\hat{\mathbf{A}}^1 + A_0^2 d\hat{\mathbf{A}}^2)\mathrm{tr}[\Gamma_0^2] + ((A_0^1)^2 + (A_0^2)^2)\mathrm{tr}[\Gamma_0\hat{\mathbf{R}}]\Big)\mathbf{a}d\mathbf{a}$$

$$+\Big(((d\hat{\mathbf{A}}^1)^2 + (d\hat{\mathbf{A}}^2)^2)\mathrm{tr}[\Gamma_0^2] + ((A_0^1)^2 + (A_0^2)^2)\mathrm{tr}[\hat{\mathbf{R}}^2] + 4(A_0^1 d\hat{\mathbf{A}}^1 + A_0^2 d\hat{\mathbf{A}}^2)\mathrm{tr}[\Gamma_0\hat{\mathbf{R}}]\Big)\mathbf{a}$$

$$+2\Big(A_0^1\hat{\mathbf{A}}^1 + A_0^2\hat{\mathbf{A}}^2\Big)\mathrm{tr}[\hat{\mathbf{R}}^2] + 2\Big(\hat{\mathbf{A}}^1 d\hat{\mathbf{A}}^1 + \hat{\mathbf{A}}^2 d\hat{\mathbf{A}}^2\Big)\mathrm{tr}[\Gamma_0\hat{\mathbf{R}}]\Big)$$

$$+\Big(\Big(\mathrm{tr}[\Gamma_0^4] - \frac{1}{4}\mathrm{tr}[\Gamma_0^2]^2\Big)\mathbf{a}d\mathbf{a}d\mathbf{a}+\Big(4\mathrm{tr}[\Gamma_0^3\hat{\mathbf{R}}] - \mathrm{tr}[\Gamma_0^2]\mathrm{tr}[\Gamma_0\hat{\mathbf{R}}]\Big)\mathbf{a}d\mathbf{a}$$

$$+\Big(6\mathrm{tr}[\Gamma_0^2\hat{\mathbf{R}}^2] - \mathrm{tr}[\Gamma_0\hat{\mathbf{R}}]^2 - \frac{1}{2}\mathrm{tr}[\Gamma_0^2]\mathrm{tr}[\hat{\mathbf{R}}^2]\Big)\mathbf{a} - \mathrm{tr}[\Gamma_0\hat{\mathbf{R}}]\mathrm{tr}\Big(\hat{\mathbf{\Gamma}}d\hat{\mathbf{\Gamma}} + \frac{2}{3}\hat{\mathbf{\Gamma}}^3\Big)\Big)\Bigg\}\Bigg]$$

$$+\frac{ir_G}{384\pi}\int_{S^5}\Big(-\frac{4\pi^2}{\beta^3}\mathbf{a}d\mathbf{a}d\mathbf{a} + \frac{2}{\beta}\Big(\hat{\mathbf{A}}^1 d\hat{\mathbf{A}}^1 + \hat{\mathbf{A}}^2 d\hat{\mathbf{A}}^2\Big)d\mathbf{a} - \frac{1}{\beta}\mathbf{a}\mathrm{tr}[\hat{\mathbf{R}}^2]\Big)$$

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$$\begin{aligned}\log I = &- \frac{h_G^\vee d_G + r_G}{24} \frac{\Delta_1^2 \Delta_2^2}{\omega_1\omega_2\omega_3} - r_G \frac{((\Delta_1+\Delta_2)^2 + 4\pi^2)((\Delta_1-\Delta_2)^2+4\pi^2)}{192\omega_1\omega_2\omega_3} \\ &+ r_G \frac{\Delta_1^2 + \Delta_2^2 - 4\pi^2}{96} \frac{\omega_1^2+\omega_2^2+\omega_3^2}{\omega_1\omega_2\omega_3} + \mathcal{O}(\log \omega)\end{aligned}$$

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6d (1,0) Cardy formulas

- **6d (1,0) SCFT**
  - Anomaly polynomial of $SU(2)$ R-symmetry & $SO(6)$ diffeomorphism

$$P_8 = \frac{1}{4!}\Big(\mathfrak{a}\cdot c_2(R)^2 + \mathfrak{b}\cdot c_2(R)p_1(T) + \mathfrak{c}\cdot p_1(T)^2 + \mathfrak{d}\cdot p_2(T)\Big)$$

  - Cardy free energy ($\Delta_R - \omega_1 - \omega_2 - \omega_3 = 2\pi i$)

$$\log I = -\left(\frac{\mathfrak{a}\Delta_R^4}{384} + \frac{\mathfrak{b}\pi^2\Delta_R^2}{24} + \frac{2\mathfrak{c}\pi^4}{3}\right)\frac{1}{\omega_1\omega_2\omega_3} + \left(\frac{\mathfrak{b}\Delta_R^2}{96} + \frac{(2\mathfrak{c}+\mathfrak{d})\pi^2}{6}\right)\frac{\sum \omega_i^2}{\omega_1\omega_2\omega_3} + \mathcal{O}(\log\omega)$$

- **Bound on the anomaly coefficients**
  - Asymptotic entropy in the Cardy limit ($\omega_i = \omega$)

$$\text{Re}[S] = \frac{\pi^4}{12\sqrt{2}\,\text{Re}[\omega]^3}\Big(\mathfrak{a} - 4\mathfrak{b} + 16\mathfrak{c}\Big) + \mathcal{O}(\omega^{-2})$$

  - Entropic bound: $\boxed{\mathfrak{a} - 4\mathfrak{b} + 16\mathfrak{c} \geq 0}$ $\leftarrow$ SUSY Renyi entropy [Yankielowicz, Zhou]

Concluding remarks

- **Matching with the known results**
  - Tested with the SCIs whose closed form is known

    e.g. (1,0) free multiplets, 1 M5-brane on $\mathbb{C}^2/\mathbb{Z}_k$ [Benvenuti, Boneli, Ronzani, Tanzini]
  - Can be applied to 4d & match with 4d $\mathcal{N}=1$ Cardy formula [J.Song, J.Kim, S.Kim], [Cabo-Bizet, Cassani, Martelli, Murthy]
  - Can be applied to $\mathbb{R}^4_\Omega \times T^2 \Rightarrow$ match with the Cardy index from the elliptic genus of the self-dual strings [work in progress]

- **Conclusions**
  - The Cardy limit of the superconformal indices can be determined from the 't Hooft anomaly of 6d SCFTs
  - It counts spectral part & different from the Casimir energy
  - Explicitly sees $N^3 \Rightarrow$ entropy of BPS BHs in AdS_7

Thank you!