Exploring self-dual string theories in 6-dimensions

Jaemo Park

Postech, South Korea

National Center for Theoretical Sciences, Hsinchu
2019. 10. 30

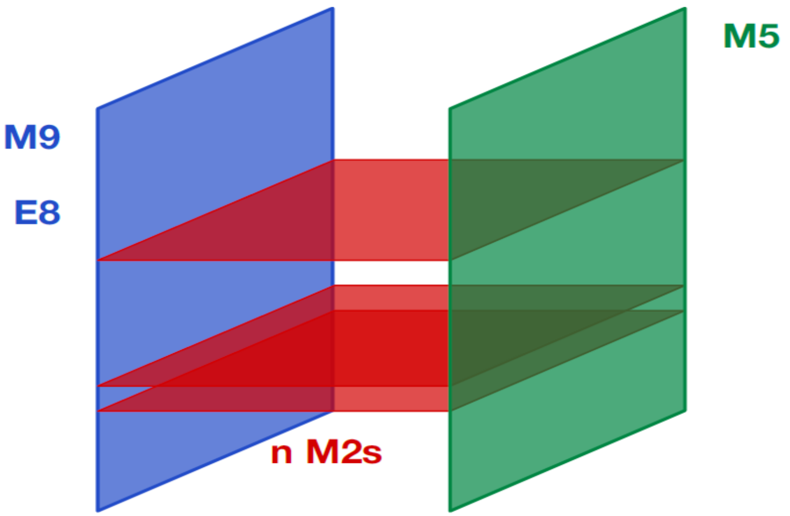
- Based on following papers
- Elliptic genus of E-strings
  by Joonho Kim (KIAS), Seok Kim (Seoul National University), Kimyeong Lee (KIAS), JP, Cumrun Vafa(Harvard)
  arXiv:1411.2324
- 6d string theories from new chiral gauge theories
  by Hee-Cheol Kim (Postech), Seok Kim, JP
  arXiv:1608.03919
- On elliptic genera of 6d string theories
  by Joonho Kim, Kimyeong Lee, JP
  arXiv:1801.01631
- 6d strings and exceptional instantons
  by Hee-Cheol Kim, Joonho Kim, Seok Kim, Ki-Hong Lee (Seoul National University), JP
  arXiv:1801.03579

Contents

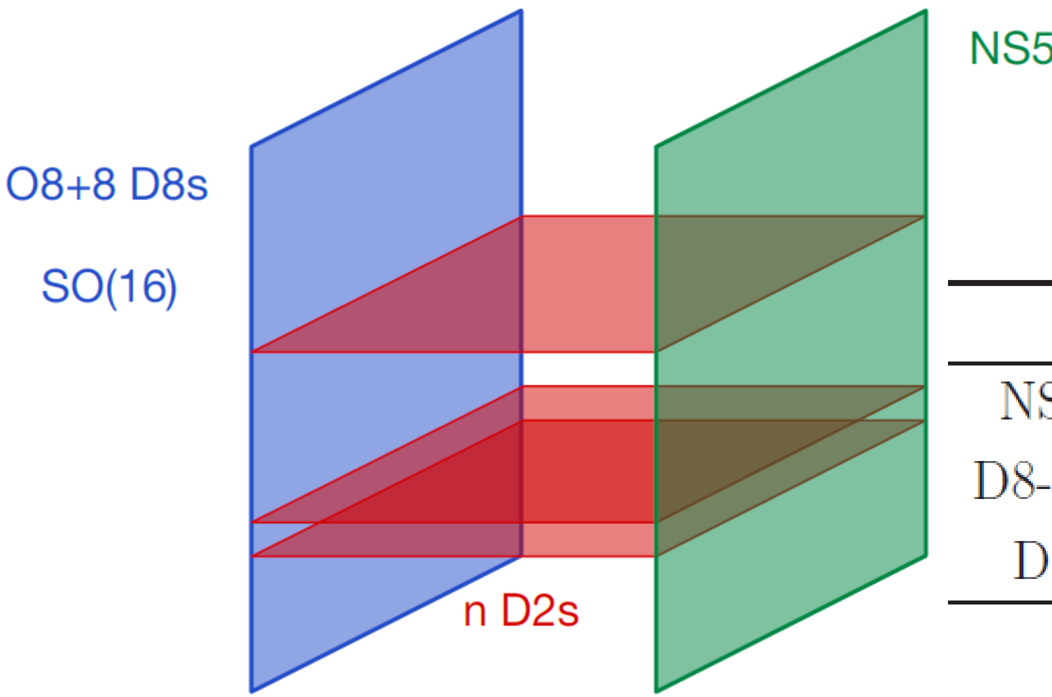
- 2d gauge theory description of self-dual string theories
- more self-dual string theories and instantonic strings
- Exceptional instantonic strings
- Structures of 6d strings and the elliptic genera bootstrap
- Conclusions and future directions

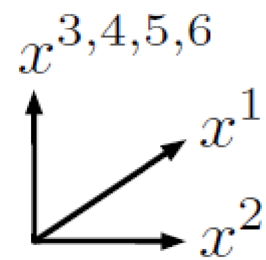
Self-dual string

- Many interesting (0,1) SCFTs arise in the M theory or F-theory setting
- Focus on (0,1) E_8 theory (Witten; Ganor, Hanany; Seiberg, Witten; Klemm, Mayr, Vafa; circa mid 90s + many more ....)
- Coincidence limit of M5 and M9 produces the tensionless self-dual string
- which is an interacting 2D SCFT
- Long standing puzzle: How can we describe such self-dual string theories?



- Use the related gauge theory configuration
- and take $g_{2YM} \to \infty$ limit to obtain the 2d SCFT
- which corresponds to the decompactifying limit in Type IIA setup





	0	1	2	3	4	5	6	7	8	9
NS5	•	•		•	•	•	•			
D8-O8	•	•		•	•	•	•	•	•	•
D2	•	•	•							

Self-dual string

- The resulting 2D theory has (0,4) world-sheet SUSY
- The symmetry is
  $SO(4)_{3456} \times SO(3)_{789} \sim SU(2)_L \times SU(2)_R \times SU(2)_I$
  doublet indices α, β $\quad \dot{\alpha}, \dot{\beta}$ $\quad A, B$
- matter content

$$\begin{aligned} \text{vector} &: \quad O(n) \text{ antisymmetric} \quad (A_\mu,\ \lambda_+^{\dot{\alpha}A}) \\ \text{hyper} &: \quad O(n) \text{ symmetric} \quad (\varphi_{\alpha\dot{\beta}},\ \lambda_-^{\alpha A}) \\ \text{Fermi} &: \quad O(n) \times SO(16) \text{ bifundamental} \quad \Psi_l \end{aligned}$$

- interactions determined by (0, 4) SUSY

- We work out the elliptic genus as an evidence for the proposal (following the work by Gadde, Gukov and Benini, Eager, Hori, Tachikawa)
- With a choice of (0,2) worldsheet SUSY, the elliptic genus is given by

$$Z_n(q, \epsilon_{1,2}, m_l) = \mathrm{Tr}_{\mathrm{RR}}\left[(-1)^F q^{H_L} \bar{q}^{H_R} e^{2\pi i \epsilon_1 (J_1 + J_I)} e^{2\pi i \epsilon_2 (J_2 + J_I)} \prod_{l=1}^{8} e^{2\pi i m_l F_l}\right]$$

- J_1, J_2 Cartans of $SO(4)_{3456}$, J_I Cartans of $SU(2)_{789}$, F_l are the Cartans of $SO(16)$
- elliptic genus gives the BPS spectrum of 6d theory and all-genus topological string amplitudes on related CY_3 (F-theory, M-theory setup)
- important check: E_8 symmetry in IR

single string

- $O(1) \sim Z_2$ and we have four discrete holonomies on T^2

$$Z_1 = \sum_{i=1}^{4} \frac{Z_{1(i)}}{2} = -\frac{\Theta(q, m_l)}{\eta^6 \theta_1(\epsilon_1)\theta_1(\epsilon_2)}$$

- where $\Theta(q, m_l)$ is the E_8 theta function
- the sum over discrete holonomies is the same as that of R, NS sectors with GSO projections to obtain $E_8(\times E_8)$ heterotic string out of the free fermion formalism

- We call it E-string

Higher E-string

- Two E-string: $O(2)$ gauge theory has 7 holonomy sectors

$$Z_2(\tau,\epsilon_{1,2},m_l) = \frac{1}{2}Z_{2(0)} + \frac{1}{4}\sum_{a=1}^{6} Z_{2(a)}$$

$$Z_{2(0)} = \frac{1}{2\eta^{12}\theta_1(\epsilon_1)\theta_1(\epsilon_2)}\sum_{i=1}^{4}\left[\frac{\prod_{l=1}^{8}\theta_i(m_l\pm\frac{\epsilon_1}{2})}{\theta_1(2\epsilon_1)\theta_1(\epsilon_2-\epsilon_1)} + \frac{\prod_{l=1}^{8}\theta_i(m_l\pm\frac{\epsilon_2}{2})}{\theta_1(2\epsilon_2)\theta_1(\epsilon_1-\epsilon_2)}\right]$$

$$Z_{2(1)} = \frac{\theta_2(0)\theta_2(2\epsilon_+)\prod_{l=1}^{8}\theta_1(m_l)\theta_2(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_2(\epsilon_1)\theta_2(\epsilon_2)}\ ,\quad Z_{2(2)} = \frac{\theta_2(0)\theta_2(2\epsilon_+)\prod_{l=1}^{8}\theta_3(m_l)\theta_4(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_2(\epsilon_1)\theta_2(\epsilon_2)}$$

$$Z_{2(3)} = \frac{\theta_4(0)\theta_4(2\epsilon_+)\prod_{l=1}^{8}\theta_1(m_l)\theta_4(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_4(\epsilon_1)\theta_4(\epsilon_2)}\ ,\quad Z_{2(4)} = \frac{\theta_4(0)\theta_4(2\epsilon_+)\prod_{l=1}^{8}\theta_2(m_l)\theta_3(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_4(\epsilon_1)\theta_4(\epsilon_2)}$$

$$Z_{2(5)} = \frac{\theta_3(0)\theta_3(2\epsilon_+)\prod_{l=1}^{8}\theta_1(m_l)\theta_3(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_3(\epsilon_1)\theta_3(\epsilon_2)}\ ,\quad Z_{2(6)} = \frac{\theta_3(0)\theta_3(2\epsilon_+)\prod_{l=1}^{8}\theta_2(m_l)\theta_4(m_l)}{\eta^{12}\theta_1(\epsilon_1)^2\theta_1(\epsilon_2)^2\theta_3(\epsilon_1)\theta_3(\epsilon_2)}$$

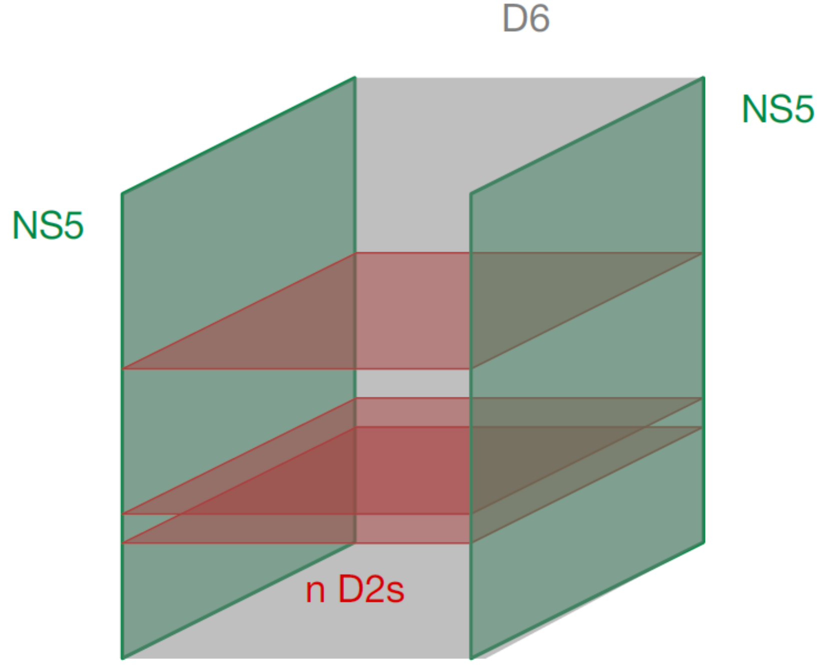
- Coincides with the previous result of Haghighat, Lockhart, Vafa obtained using the E_8 symmetry with low genus expansion, where E_8 symmetry is manifest

$$Z_2 = \frac{1}{576\eta^{12}\theta_1(\epsilon_1)\theta_1(\epsilon_2)\theta_1(\epsilon_2-\epsilon_1)\theta_1(2\epsilon_1)}\Big[4A_1^2(\phi_{0,1}(\epsilon_1)^2-E_4\theta_{-2,1}(\epsilon_1)^2)$$
$$+3A_2(E_4^2\phi_{-2,1}(\epsilon_1)^2-E_6\phi_{-2,1}(\epsilon_1)\phi_{0,1}(\epsilon_1))+5B_2(E_6\phi_{-2,1}(\epsilon_1)^2-E_4\phi_{-2,1}(\epsilon_1)\phi_{0,1}(\epsilon_1))\Big]$$
$$+(\epsilon_1\leftrightarrow\epsilon_2)$$

- Higher string results coincide with the known results of topological string amplitudes

More self-dual strings

- M strings: M2 branes between two M5 branes worked out by Haghighat, Kozcaz, Lockhart, Vafa before E string
- Type IIA description



- 2D (0,4) theory with U(n) vector multiplet, one adjoint hyper, one fundamental hyper, one fundamental Fermi

More self-dual strings

- F theory (Generalization of Type IIB compactification) on ellptic CY_3 with the base $O(-n) \to P^1$
- Self-dual string is D3 wrapping on a shrinking 2-cycle
- $n = 1$ E-string $n = 2$ M-string (Witten 1996)
- $n = 4$ admits perturbative IIB orientifold description (Bershadsky, Vafa 1997) on C^2/Z_2 with $\Omega\Pi$ where $\Pi : z_1 \to z_1 \;\; z_2 \to -z_2$
- The resulting (0,4) 2D theory is $Sp(k)$ gauge theory with $Sp(k)$ vector multiplets, $Sp(k) \times SO(8+2p)$ bifundamental hyper, $Sp(k) \times Sp(p)$ bifundamental Fermi (Haghighat, Klemm, Lockhart, Vafa

More self-dual strings and instantonic string

- F-theory can develop the enhanced gauge group
- In this case the self-dual string can be described as the instantonic string
  $dH = d * H = TrF \wedge F$
- the above n=4 case is one example. More intersting case is n=3 SU(3)case.
- The usual ADHM constrution of SU(3) case leads to anomalous 2d gauge theory. Thus we need alternative description of SU(3) instanton moduli space.
- The construction is given by Hee-cheol Kim, JP, Seok Kim

Exceptional Instantonic Strings of $SU(3)$

- Usual ADHM description k instanton of $SU(N)$ gauge theory
  $U(k)$ vector, $(k, \bar{N})$ hyper, (adj, 1) hyper : anomalous
- anaomaly free instantonic string theory

$$
\begin{aligned}
(A_\mu, \lambda_0, \bar{\lambda}_0, D) + (\lambda, G_\lambda) &: \quad U(k) \text{ vector multiplet} + \text{complex adjoint Fermi} \\
(q, \psi_+) + (\tilde{q}, \tilde{\psi}_+) &: \quad \text{chiral multiplets in } (\mathbf{k}, \overline{\mathbf{3}}) + (\overline{\mathbf{k}}, \mathbf{3}) \\
(a, \Psi_+) + (\tilde{a}, \tilde{\Psi}_+) &: \quad \text{chiral multiplets in } (\mathbf{adj}, \mathbf{1}) + (\mathbf{adj}, \mathbf{1}) \ .
\end{aligned}
$$

$$
\begin{aligned}
(\phi, \chi) &: \quad \text{chiral multiplet in } (\overline{\mathbf{k}}, \overline{\mathbf{3}}) \\
(b, \xi) + (\tilde{b}, \tilde{\xi}) &: \quad \text{two chiral multiplet in } (\overline{\mathbf{anti}}, \mathbf{1}) \\
(\hat{\lambda}, \hat{G}) &: \quad \text{complex Fermi multiplet in } (\mathbf{sym}, \mathbf{1}) \\
(\check{\lambda}, \check{G}) &: \quad \text{complex Fermi multiplet in } (\mathbf{sym}, \mathbf{1}) \\
(\zeta, G_\zeta) &: \quad \text{complex Fermi multiplet in } (\overline{\mathbf{k}}, \mathbf{1}) \ .
\end{aligned}
$$

Further constructions of exceptional instantonic string

- The above construction can be generalized to the self-dual string theories of 6-d (0,1) theory with matters
- The above $SU(3)$ theory is related by Higgsing to $SO(7)$ with 2 spinor representation 8 -> G_2 with 7 -> $SU(3)$
- The gauge group construction of the self-dual string theories were worked out by de-Higgsing procedure. (Kim, Kim, Kim, Lee, Park)
- This construction gives detailed information about the instanton moduli space of SO gauge group with spinor representation and G_2 gauge group! (cf. Hayashi, Kim, Lee, Yagi)
- These theories are important since G_2 and $SO(7)$ theories are part of the atomic constituents of 6d Superconformal field theories.
  (Heckman, Morrison, Vafa; Heckman, Morrison, Rudelius, Vafa)

Atomic constituents of 6d SCFTs

n	1	2	3	4	5	6	7	8	12
gauge symmetry	-	-	$SU(3)$	$SO(8)$	F_4	E_6	E_7	E_7	E_8
global symmetry	E_8	$SO(5)_R$	-	-	-	-	-	-	-
matter			-	-	-	-	$\frac{1}{2}\mathbf{56}$	-	-

Table 3: Symmetries/matters of SCFTs with rank 1 tensor branches

base	3, 2	3, 2, 2	2, 3, 2
gauge symmetry	$G_2 \times SU(2)$	$G_2 \times SU(2) \times \{\ \}$	$SU(2) \times SO(7) \times SU(2)$
matter	$\frac{1}{2}(\mathbf{7}+\mathbf{1},\mathbf{2})$	$\frac{1}{2}(\mathbf{7}+\mathbf{1},\mathbf{2})$	$\frac{1}{2}(\mathbf{2},\mathbf{8},\mathbf{1})+\frac{1}{2}(\mathbf{1},\mathbf{8},\mathbf{2})$

Table 4: Non-Higgsable atomic SCFTs with higher rank tensor branches

Relation between partition function of 6-d SCFT and elliptic genus of self-dual strings

- It's known that supersymmetric partition functions on Omega-deformed $R^4 \times T^2$ partition function of 6d SCFTs can be written in terms of elliptic genus of self-dual strings

$$Z_{6d} = \mathcal{I}_0 \cdot \left(1 + \sum_k \mathfrak{n}^k \cdot \mathcal{I}_k\right)$$

- I_0 is the Witten index of pure momentum states
- Thus by working out the elliptic genus of self-dual strings of arbitrary string numbers (related to the rank of the gauge group),one can construct the partition function of 6d SCFTs.

Modular properties of the elliptic genus of self-dual string theories

The elliptic genus $\mathcal{I}_k$ is a weak Jacobi form of weight 0 and index $\mathrm{i}(z)$

$$\mathcal{I}(\tau, z) \longrightarrow \mathcal{I}\left(\frac{a\tau + b}{c\tau + d}, \frac{z}{c\tau + d}\right) = \varepsilon(a,b,c,d) \exp\left(\frac{-\pi i c \cdot \mathrm{i}(z)}{c\tau + d}\right) \mathcal{I}(\tau, z)$$

z denotes collectively chemical potentials of 6d symmetry group

index $\mathrm{i}(z)$ is completely determined by the worldsheet chiral anomaly

- Using this fact and introducing suitable basis for symmetry group, one can completely determine the expression I_k, given a finite input of coefficients (del Zoto, Lockhart; Kim, Lee, Park; del Zoto, Gu, Huang, Kashani-Poor, Klemm, Lockart; Gu, Haghighat, Sun, Wang; Gu, Klemm, Sun, Wang)

Conclusion and future directions

- We define the tensionless self-dual string theories as IR limit of certain 2d gauge theories and give evidences for the proposals such as computing moduli space and elliptic genus
- Many of the atomic constituents were already worked out in terms of gauge theories. It would be interesting to workout $n > 4$ case for 6d SCFTs arise from the F theory with base $O(-n)- > P^1$ $n = 3$ construction is an important step since $n > 4$ cases have no brane constructions.
- It appears that more systematic methods are needed.