Exponentially Suppressed Cosmological Constant with Enhanced Gauge Symmetry in Heterotic Interpolating Models

done with S. Nakajima

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arXiv: 1905.10745, PTEP to appear

also H. I., Taylor PLB (1987)

I) Introduction

- SO(16) × SO(16) nonsupersymmetric heterotic string in 10d is receiving revived attention as
  - i) there appears to be no SUSY in multi TeV scale according to the LHC experiment
  - ii) this string model is the only model which is tachyon free and non-supersymmetric in 10 dimensions

    Dixon-Harvey 1986, Alvarez-Gaume et al. 1986

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- Compactification on a twisted circle (in general twisted tori) gives an interpolation of supersymmetric & nonsupersymmetric heterotic vacua. Moreover, the cosmological constant obeys the following formula in SUSY restoring region:

$$\mathcal{V}_D \simeq (n_F^0 - n_B^0) a^D \xi + \mathcal{O}(e^{-\tilde{a}}) \quad \text{as } \tilde{a} \to \infty$$

($a = \sqrt{\alpha'}/R,\ \tilde{a} = 1/a,\ \xi > 0,\ n_F^0, n_B^0$: # of massless fermions bosons)

H.I., T.R.Taylor, Phys. Lett. B186 (1987)

In this work, we consider two parameter extension of this prob. & examine interrelationship among twisted comp. gauge sym. enhancement and cosmol. const.

Contents

For illustration, SO(32) case only today, $E_8 \times E_8$ case not included

More on the exponential suppression in the SUSY restoring $\tilde{a}$ large region

- The integrand of the τ_2 integration involves

$$(*) = \tau_2^{\#}(\Lambda_{0,0} - \Lambda_{1/2,0})e^{-m\pi\tau_2}$$

SUSY restoring factor generic level from $\prod(\text{characters})$
mass splitting $\alpha' M_s^2 = 1/\tilde{a}^2$

- apply the Jacobi imaginary transf.

$$(*) = (\text{const})\tau_2^{\#'} \sum_{n=1}^{\infty} e^{-\frac{1}{4\tau_2}(2n-1)^2\pi\tilde{a}^2 - m\pi\tau_2}$$

i) $m \neq 0$: the sum bdd at least by $\dfrac{e^{-\pi\tilde{a}\sqrt{m}}}{1 - e^{-2\pi\tilde{a}\sqrt{m}}}$

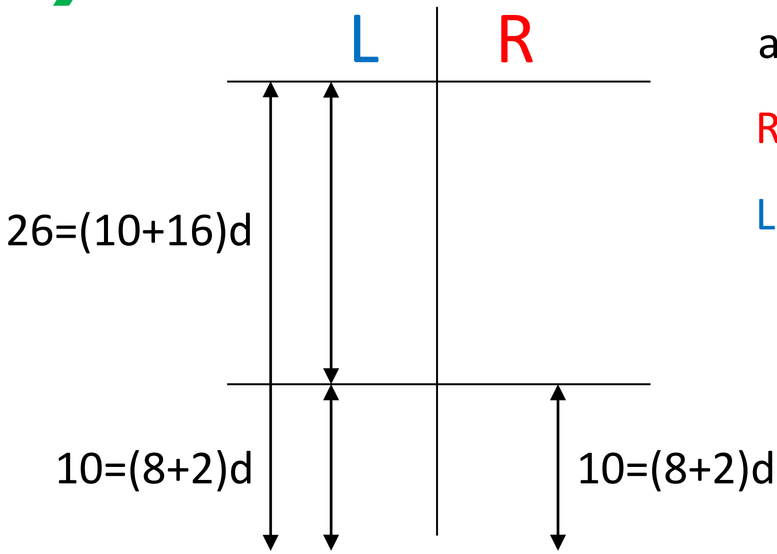
& can integrate over $[1, \infty]$

ii) $m = 0$: term by term integ. over $[1, \infty]$ and resum to get $\zeta(10)$

$\Rightarrow$ the first (dominant) term up to exp. accuracy

- contribution from $\tau_2 < 1$, exp. suppressed.

II) ● Idea of Heterotic strings



adopt the lightcone coordinates

Right mover: 10d superstring $\bar{X}_R^i(\tau-\sigma),\ \bar{\psi}^i(\tau-\sigma)$

Left mover: 26d bosonic string out of which

internal 16d realize rank 16 current algebra

$X_L^i(\tau+\sigma),\ X_L^I(\tau+\sigma)$ (or fermions)

● State counting & characters

- $\mathrm{Tr} q^{L_0}\bar{q}^{\bar{L}_0}$ counts #(states) at level m as coeff. in $q(\bar{q})$ expansion
- It takes the form of $\sum_{i,j}\bar{\chi}_i^{\mathrm{Vir}}(\bar{q})X_{ij}\chi_j^{\mathrm{Vir}}(q)$ and involves spacetime &

  internal SO(2n), n=4, 8 characters $\mathrm{ch(rep)}=O_{2n},V_{2n},S_{2n},C_{2n}$ expressible in terms

  of the four theta constants and the Dedekind eta fn

$$\eta(\tau)=q^{-1/24}\prod_{n=1}^{\infty}(1-q^n)$$

- e.g. SO(32) hetero $Z_B^{(8)}\left(\bar{V}_8-\bar{S}_8\right)\left(O_{16}O_{16}+V_{16}V_{16}+S_{16}S_{16}+C_{16}C_{16}\right)$

The construction of interpolating models

Start from a flat ten-dimensional closed string model M_1 whose partition function is

$$Z_{M_1} = Z_B^{(8)} Z_+^+,$$

Z_+^+ the contribution from the fermionic string $Z_B^{(n)}$ from the bosonic string:

$$Z_B^{(n)} = \tau_2^{-n/2} (\eta\bar{\eta})^{-n}$$

Consider the circle compactification:

$$X^9 \sim X^9 + 2\pi R.$$

The left- and right-moving momenta along the compactified dimension

$$p_L = \frac{1}{\sqrt{2\alpha'}}\left(na + \frac{w}{a}\right), \quad p_R = \frac{1}{\sqrt{2\alpha'}}\left(na - \frac{w}{a}\right),$$

for $n, w \in \boldsymbol{Z}$. After the circle compactification, the partition function of model M_1

$$Z_+^{(9)+} = \left((\eta\bar{\eta})^{-1} \sum_{n,w\in\boldsymbol{Z}} q^{\frac{\alpha'}{2}p_L^2} \bar{q}^{\frac{\alpha'}{2}p_R^2} \right) Z_B^{(7)} Z_+^+.$$

Compactification on a twisted circle

We choose $\mathcal{T}Q$ as the $\boldsymbol{Z}_2$ twist where $\mathcal{T}$ acts on the compactified circle as a half translation:

$$\mathcal{T} : \tilde{X}^9 \to \tilde{X}^9 + \pi \tilde{R}.$$

$\tilde{X}^9$ is the T-dualized coordinate for the compactified dimension and $\tilde{R} = \alpha'/R$ is the T-dualized radius. Q is a $\boldsymbol{Z}_2$ action that acts on the internal part of the string and that determines the two ten-dimensional models at the limits.

Because the $\boldsymbol{Z}_2$ twist contains $\mathcal{T}$, the partition function of the interpolating model contains a set of four momentum lattices:

$$\Lambda_{\alpha,\beta} \equiv (\eta\bar{\eta})^{-1} \sum_{n \in \boldsymbol{Z}+\alpha,\ w \in 2(\boldsymbol{Z}+\beta)} q^{\frac{\alpha'}{2} p_L^2} \bar{q}^{\frac{\alpha'}{2} p_R^2}$$
$$= (\eta\bar{\eta})^{-1} \sum_{n,w \in \boldsymbol{Z}} \exp\left[-\pi \left\{\tau_2 \left(a^2 (n+\alpha)^2 + 4a^{-2}(w+\beta)^2\right) - 4i\tau_1 (n+\alpha)(w+\beta)\right\}\right].$$

α and β are 0 or 1/2, and $\alpha = 0\ (1/2)$ and $\beta = 0\ (1/2)$ imply the integer (half-integer) momenta and the even (odd) winding numbers.

Adding twisted sector

Restart as

$$Z_+^{(9)+} = \left(\Lambda_{0,0} + \Lambda_{0,1/2}\right) Z_B^{(7)} Z_+^+,$$

An interpolating model is obtained from $Z_+^{(9)+}$ by orbifolding with the $\boldsymbol{Z}_2$ action $\mathcal{T}Q$.
A half translation $\mathcal{T}$ affects the lattices $\Lambda_{\alpha,\beta}$ and acts such that only the states with even winding numbers survive:

$$\mathcal{T}Q: \; Z_+^{(9)+} \to Z_-^{(9)+} = \left(\Lambda_{0,0} - \Lambda_{0,1/2}\right) Z_B^{(7)} Z_-^+,$$

where Z_-^+ is defined as the Q-action of Z_+^+.

$$\boldsymbol{S}: \; Z_-^{(9)+} \to Z_+^{(9)-} = \left(\Lambda_{1/2,0} + \Lambda_{1/2,1/2}\right) Z_B^{(7)} Z_+^-,$$

where $Z_-^+(-1/\tau) \equiv Z_+^-(\tau)$. Furthermore, when $\mathcal{T}Q$ acts on $Z_+^{(9)-}$,

$$\mathcal{T}Q: \; Z_+^{(9)-} \to Z_-^{(9)-} = \left(\Lambda_{1/2,0} - \Lambda_{1/2,1/2}\right) Z_B^{(7)} Z_-^-,$$

where Z_-^- is defined as the Q-action of Z_+^-. As a result, the total partition function

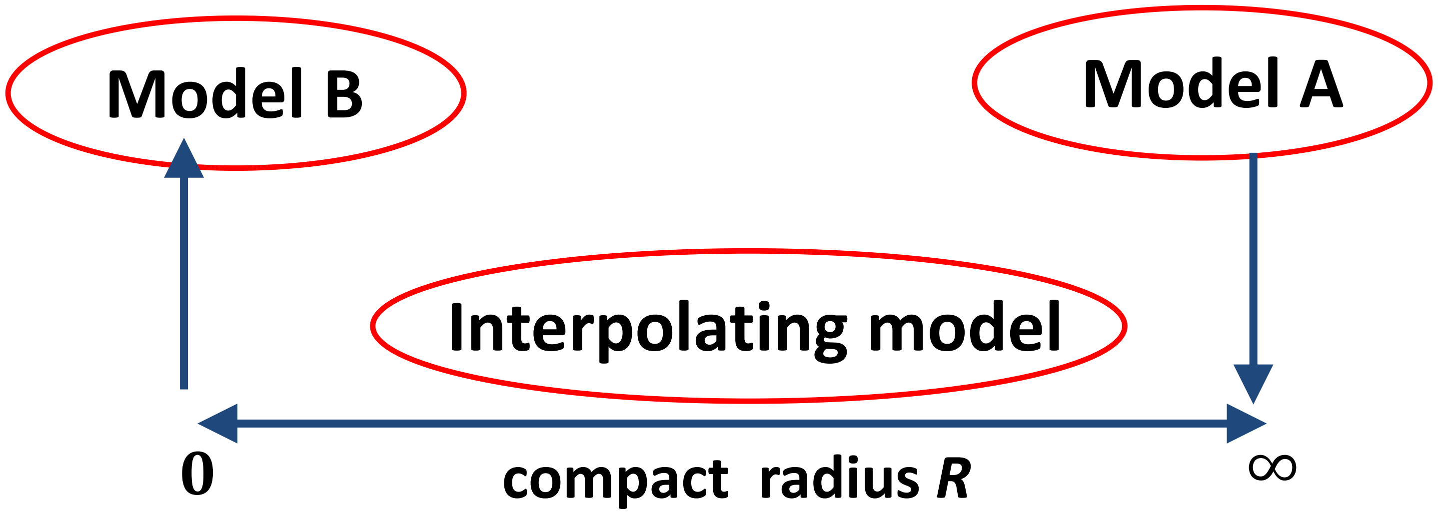
$$\begin{aligned} Z_{\text{int}}^{(9)} &= \frac{1}{2}\left(Z_+^{(9)+} + Z_-^{(9)+} + Z_+^{(9)-} + Z_-^{(9)-}\right) \\ &= \frac{1}{2} Z_B^{(7)} \left\{\Lambda_{0,0}\left(Z_+^+ + Z_-^+\right) + \Lambda_{0,1/2}\left(Z_+^+ - Z_-^+\right)\right. \\ &\qquad \left. + \Lambda_{1/2,0}\left(Z_+^- + Z_-^-\right) + \Lambda_{1/2,1/2}\left(Z_+^- - Z_-^-\right)\right\}. \end{aligned}$$

In $a \to 0$ limit, $Z_{\text{int}}^{(9)}$ produces model M_2 whose partition function is

$$Z_{M_2} = Z_B^{(8)}\left(Z_+^+ + Z_-^+ + Z_+^- + Z_-^-\right).$$

model M_2 is obtained by Q-twisting model M_1

III) Interpolating models with no WL



<u>e.g.</u> **Model A : SUSY SO(32), Model B : SO(16)× SO(16)**

$$Z^{(9)}_{\rm int} = Z^{(7)}_{\rm boson}\left\{\Lambda_{0,0}\left[\bar{V}_8\left(O_{16}O_{16}+S_{16}S_{16}\right)-\bar{S}_8\left(V_{16}V_{16}+C_{16}C_{16}\right)\right]\right.$$
$$+\Lambda_{0,1/2}\left[\bar{O}_8\left(V_{16}C_{16}+C_{16}V_{16}\right)-\bar{C}_8\left(O_{16}S_{16}+S_{16}O_{16}\right)\right]$$
$$+\Lambda_{1/2,0}\left[\bar{V}_8\left(V_{16}V_{16}+C_{16}C_{16}\right)-\bar{S}_8\left(O_{16}O_{16}+S_{16}S_{16}\right)\right]$$
$$\left.+\Lambda_{1/2,1/2}\left[\bar{O}_8\left(O_{16}S_{16}+S_{16}O_{16}\right)-\bar{C}_8\left(V_{16}C_{16}+C_{16}V_{16}\right)\right]\right\}$$

$$Z^{(n)}_{\rm boson} = \tau_2^{-n/2}\left(\bar{\eta}\eta\right)^{-n}$$

$$\Lambda_{\alpha,\beta} = \left(\bar{\eta}\eta\right)^{-1}\sum_{n,w}\bar{q}^{\alpha' p_R^2/2}q^{\alpha' p_L^2/2} = \left(\bar{\eta}\eta\right)^{-1}\sum_{n,w}\exp\left[2\pi i n w\tau_1 - \pi\tau_2\left(n^2a^2+w^2/a^2\right)\right]$$

$$n\in 2(\boldsymbol{Z}+\alpha),\quad w\in\boldsymbol{Z}+\beta$$

- $\boldsymbol{R}\to\infty$: **contribution from the zero winding # only**

$$\Lambda_{\alpha,0}\to(2a)^{-1}Z^{(1)}_{\rm boson},\quad \Lambda_{\alpha,1/2}\to 0$$

- $\boldsymbol{R}\to\boldsymbol{0}$: **contribution from the zero momentum only**

$$\Lambda_{0,\beta}\to aZ^{(1)}_{\rm boson},\quad \Lambda_{1/2,\beta}\to 0$$

e.g. Model A : SUSY SO(32), Model B : SO(16)× SO(16)

- $R \to \infty$: $\Lambda_{\alpha,0} \to (2a)^{-1} Z^{(1)}_{\text{boson}}, \ \Lambda_{\alpha,1/2} \to 0$

$$Z^{(9)}_{\text{int}} = Z^{(7)}_{\text{boson}} \{\Lambda_{0,0} [\bar{V}_8 (O_{16}O_{16} + S_{16}S_{16}) - \bar{S}_8 (V_{16}V_{16} + C_{16}C_{16})]$$
$$+\Lambda_{0,1/2} [\bar{O}_8 (V_{16}C_{16} + C_{16}V_{16}) - \bar{C}_8 (O_{16}S_{16} + S_{16}O_{16})]$$
$$+\Lambda_{1/2,0} [\bar{V}_8 (V_{16}V_{16} + C_{16}C_{16}) - \bar{S}_8 (O_{16}O_{16} + S_{16}S_{16})]$$
$$+\Lambda_{1/2,1/2} [\bar{O}_8 (O_{16}S_{16} + S_{16}O_{16}) - \bar{C}_8 (V_{16}C_{16} + C_{16}V_{16})]\}$$

one-loop partition function of SO(32) supersymmetric heterotic string, which is vanishing

SUSY restored in $R \to \infty$

- $R \to 0$: $\Lambda_{0,\beta} \to a Z^{(1)}_{\text{boson}}, \ \Lambda_{1/2,\beta} \to 0$

$$Z^{(9)}_{\text{int}} = Z^{(7)}_{\text{boson}} \{\Lambda_{0,0} [\bar{V}_8 (O_{16}O_{16} + S_{16}S_{16}) - \bar{S}_8 (V_{16}V_{16} + C_{16}C_{16})]$$
$$+\Lambda_{0,1/2} [\bar{O}_8 (V_{16}C_{16} + C_{16}V_{16}) - \bar{C}_8 (O_{16}S_{16} + S_{16}O_{16})]$$
$$+\Lambda_{1/2,0} [\bar{V}_8 (V_{16}V_{16} + C_{16}C_{16}) - \bar{S}_8 (O_{16}O_{16} + S_{16}S_{16})]$$
$$+\Lambda_{1/2,1/2} [\bar{O}_8 (O_{16}S_{16} + S_{16}O_{16}) - C_8 (V_{16}C_{16} + C_{16}V_{16})]\}$$

1–loop partition function of SO(16)× SO(16) heterotic string

$Z^{(9)}_{\text{int}}$ realizes $R \to \infty$, SUSY SO(32), $R \to 0$ SO(16)×SO(16).

Massless spectrum   at generic *R*, comes from *n=w=0* part

$$
\begin{aligned}
Z_{\rm int}^{(9)} = Z_{\rm boson}^{(7)} \{ & \Lambda_{0,0} \left[\bar{V}_8 \left(O_{16}O_{16} + S_{16}S_{16} \right) - \bar{S}_8 \left(V_{16}V_{16} + C_{16}C_{16} \right) \right] \\
& + \Lambda_{0,1/2} \left[\bar{O}_8 \left(V_{16}C_{16} + C_{16}V_{16} \right) - \bar{C}_8 \left(O_{16}S_{16} + S_{16}O_{16} \right) \right] \\
& + \Lambda_{1/2,0} \left[\bar{V}_8 \left(V_{16}V_{16} + C_{16}C_{16} \right) - \bar{S}_8 \left(O_{16}O_{16} + S_{16}S_{16} \right) \right] \\
& + \Lambda_{1/2,1/2} \left[\bar{O}_8 \left(O_{16}S_{16} + S_{16}O_{16} \right) - \bar{C}_8 \left(V_{16}C_{16} + C_{16}V_{16} \right) \right] \}
\end{aligned}
$$

massless states at generic R

Massless bosons:

- $g_{\mu\nu}, \;\; B_{\mu\nu}, \;\; \phi$
- gauge bosons in adjoint rep of $\mathrm{SO(16)\times SO(16)\times U(1)^2}$

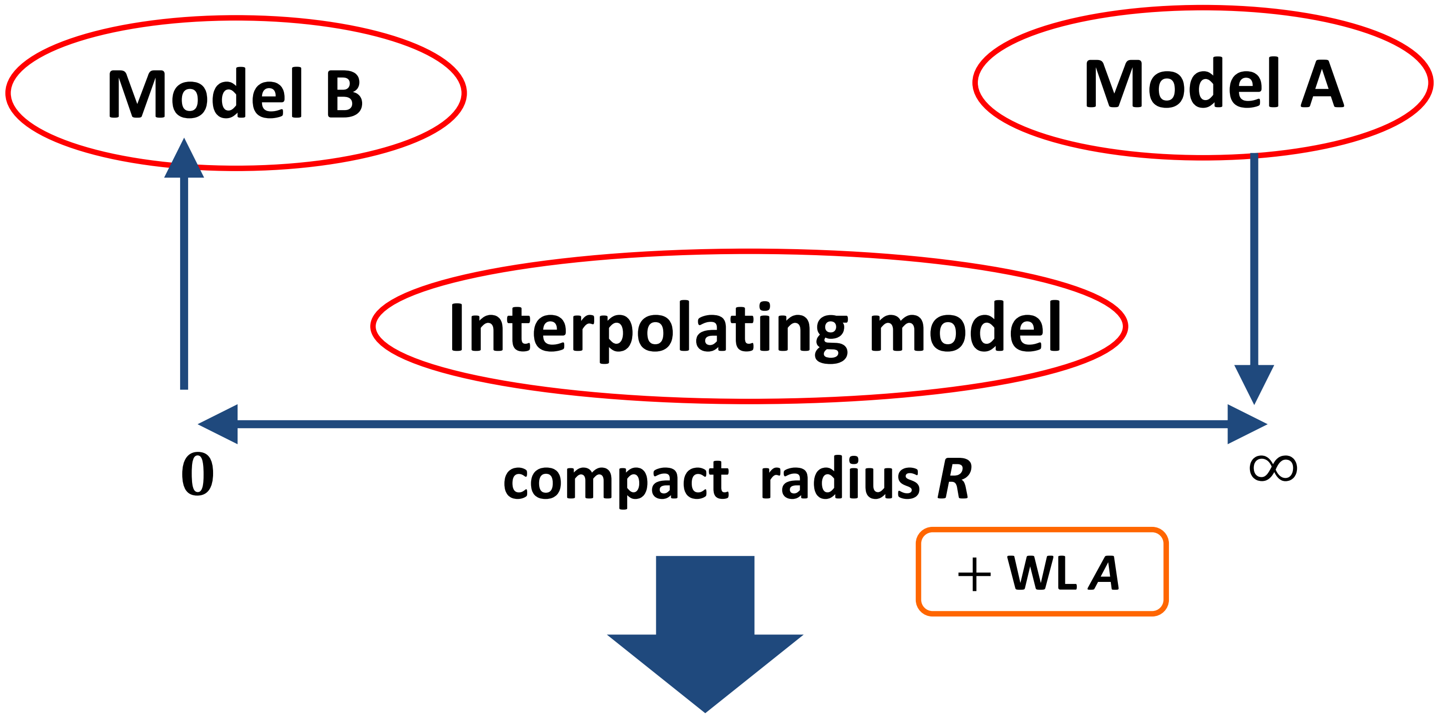
Massless fermions:

- $\mathbf{8}_S \otimes (\mathbf{16}, \mathbf{16})$

$$\longrightarrow \quad n_F^0 - n_B^0 = 64$$

IV)

Interpolating models with WL



#(parameters) = 1 : radius R

#(parameters) = 2 : radius R, WL A

add $A\int d^2z\bar{\partial}X^{\mu=9}\partial X_L^{I=1}$ **to the worldsheet action**

$\longrightarrow$ **momentum lattice is boosted**

boost and rotation

$$\begin{cases} l_L = \frac{1}{\sqrt{\alpha'}}m \\ p_L = \frac{1}{\sqrt{2\alpha'}}\left(an + \frac{w}{a}\right) \\ p_R = \frac{1}{\sqrt{2\alpha'}}\left(an - \frac{w}{a}\right) \end{cases} \Longrightarrow \begin{cases} l'_L = \frac{1}{\sqrt{2\alpha'}}\left(\sqrt{2}m - 2\frac{A}{\sqrt{1+A^2}}\frac{w}{a}\right) \\ p'_L = \frac{1}{\sqrt{2\alpha'}}\left(\sqrt{2}Am + \sqrt{1+A^2}an - \frac{1-A^2}{\sqrt{1+A^2}}\frac{w}{a}\right) \\ p'_R = \frac{1}{\sqrt{2\alpha'}}\left(\sqrt{2}Am + \sqrt{1+A^2}an - \sqrt{1+A^2}\frac{w}{a}\right) \end{cases}$$

the effective change in the 1-loop partition function

introduction of WL

$$\Lambda_{\alpha,\beta}\left(\frac{\vartheta\begin{bmatrix}\gamma\\ \delta\end{bmatrix}}{\eta}\right)^8 \Longrightarrow \Lambda^{(\alpha,\beta)}_{(\gamma,\delta)}\left(\frac{\vartheta\begin{bmatrix}\gamma\\ \delta\end{bmatrix}}{\eta}\right)^7$$

$$\Lambda^{(\alpha,\beta)}_{(\gamma,\delta)} = (\bar{\eta}\eta)^{-1}\eta^{-1}\sum_{n,w,m}(-1)^{2\delta m}q^{\frac{\alpha'}{2}(l'^2_L + p'^2_L)}\bar{q}^{\frac{\alpha'}{2}p'^2_R}$$

$$n\in 2(\boldsymbol{Z}+\alpha),\quad w\in\boldsymbol{Z}+\beta,\quad m\in\boldsymbol{Z}+\gamma$$

- **SUSY SO(32) —SO(16)×SO(16) interpolating model with WL**

<u>1-loop partition function</u>

$$Z_{\rm int}^{(9)} = Z_{\rm boson}^{(7)} \Big\{ \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(V_{16}^{(0,0)} V_{16} + C_{16}^{(0,0)} C_{16} \right)$$
$$+ \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(O_{16}^{(0,1/2)} S_{16} + S_{16}^{(0,1/2)} O_{16} \right)$$
$$+ \bar{V}_8 \left(V_{16}^{(1/2,0)} V_{16} + C_{16}^{(1/2,0)} C_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right)$$
$$+ \bar{O}_8 \left(O_{16}^{(1/2,1/2)} S_{16} + S_{16}^{(1/2,1/2)} O_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}$$

$$O_{16}^{(\alpha,\beta)} = \frac{1}{2\eta^7} \left(\Lambda_{(0,0)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 0 \\ 0 \end{bmatrix}^7 + \Lambda_{(0,1/2)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}^7 \right) \qquad V_{16}^{(\alpha,\beta)} = \frac{1}{2\eta^7} \left(\Lambda_{(0,0)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 0 \\ 0 \end{bmatrix}^7 - \Lambda_{(0,1/2)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 0 \\ 1/2 \end{bmatrix}^7 \right)$$

$$C_{16}^{(\alpha,\beta)} = \frac{1}{2\eta^7} \left(\Lambda_{(1/2,0)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}^7 - \Lambda_{(1/2,1/2)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}^7 \right) \qquad S_{16}^{(\alpha,\beta)} = \frac{1}{2\eta^7} \left(\Lambda_{(1/2,0)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}^7 + \Lambda_{(1/2,1/2)}^{(\alpha,\beta)} \vartheta \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}^7 \right)$$

First examine $R \to \infty$, $R \to 0$ limit for an arbitrary value of WL

- **SUSY SO(32) —SO(16)×SO(16) interpolating model with WL**

***R* → ∞:** $\left(O_{16}^{(\alpha,\beta)}, V_{16}^{(\alpha,\beta)}, S_{16}^{(\alpha,\beta)}, C_{16}^{(\alpha,\beta)}\right) \longrightarrow \frac{\sqrt{\alpha'}}{2r_\infty} Z_{\rm boson}^{(1)} \left(O_{16}, V_{16}, S_{16}, C_{16}\right)\delta_{\beta,0}$

$$
\begin{aligned}
Z_{\rm int}^{(9)} = Z_{\rm boson}^{(7)} \Big\{ & \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16}\right) - \bar{S}_8 \left(V_{16}^{(0,0)} V_{16} + C_{16}^{(0,0)} C_{16}\right) \\
& + \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16}\right) - \bar{C}_8 \left(O_{16}^{(0,1/2)} S_{16} + S_{16}^{(0,1/2)} O_{16}\right) \\
& + \bar{V}_8 \left(V_{16}^{(1/2,0)} V_{16} + C_{16}^{(1/2,0)} C_{16}\right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16}\right) \\
& + \bar{O}_8 \left(O_{16}^{(1/2,1/2)} S_{16} + S_{16}^{(1/2,1/2)} O_{16}\right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16}\right) \Big\}
\end{aligned}
$$

SO(32) supersymmetric heterotic string 1-loop partition function, which is vanishing

SUSY restored in *R* → ∞

- **SUSY SO(32) —SO(16)×SO(16) interpolating model with WL**

$R \to 0$: $\left(O_{16}^{(\alpha,\beta)}, V_{16}^{(\alpha,\beta)}, S_{16}^{(\alpha,\beta)}, C_{16}^{(\alpha,\beta)}\right) \longrightarrow \sqrt{\alpha'} r_0 Z_{\text{boson}}^{(1)} \left(O_{16}, V_{16}, S_{16}, C_{16}\right) \delta_{\alpha,0}$

$$
\begin{aligned}
Z_{\text{int}}^{(9)} = Z_{\text{boson}}^{(7)} \Big\{ & \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16}\right) - \bar{S}_8 \left(V_{16}^{(0,0)} V_{16} + C_{16}^{(0,0)} C_{16}\right) \\
& + \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16}\right) - \bar{C}_8 \left(O_{16}^{(0,1/2)} S_{16} + S_{16}^{(0,1/2)} O_{16}\right) \\
& + \bar{V}_8 \left(V_{16}^{(1/2,0)} V_{16} + C_{16}^{(1/2,0)} C_{16}\right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16}\right) \\
& + \bar{O}_8 \left(O_{16}^{(1/2,1/2)} S_{16} + S_{16}^{(1/2,1/2)} O_{16}\right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16}\right) \Big\}
\end{aligned}
$$

SO(16)× SO(16) heterotic string

1 − loop partition function

Realizes SUSY SO(32) as $R \to \infty$

SO(16) × SO(16) as $R \to 0$

for an arbitrary value of WL

- **SUSY SO(32) —SO(16)×SO(16) interpolating model with WL**

Massless spectrum, at generic *R, A,* comes from *n=w=m=0* part

$$Z_{\rm int}^{(9)} = Z_{\rm boson}^{(7)} \Big\{ \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(V_{16}^{(0,0)} V_{16} + C_{16}^{(0,0)} C_{16} \right)$$
$$+ \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(O_{16}^{(0,1/2)} S_{16} + S_{16}^{(0,1/2)} O_{16} \right)$$
$$+ \bar{V}_8 \left(V_{16}^{(1/2,0)} V_{16} + C_{16}^{(1/2,0)} C_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right)$$
$$+ \bar{O}_8 \left(O_{16}^{(1/2,1/2)} S_{16} + S_{16}^{(1/2,1/2)} O_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}$$

massless states at generic *A, R*

Massless bosons:
- $g_{\mu\nu}, \;\; B_{\mu\nu}, \;\; \phi$
- gauge bosons in adjoint rep of $\mathrm{SO}(16)\times\mathrm{SO}(14)\times\mathrm{U}(1)\times\mathrm{U}(1)^2$

Massless fermions:
- $\mathbf{8}_S \otimes (\mathbf{16}, \mathbf{14})$

$$\longrightarrow \quad n_F^0 - n_B^0 = 32$$

- **SUSY SO(32) —SO(16)×SO(16) interpolating model with WL**

Massless spectrum $^{\exists}$**a few conditions under which the gauge group gets enhanced**

$$Z_{\rm int}^{(9)} = Z_{\rm boson}^{(7)} \Big\{ \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(V_{16}^{(0,0)} V_{16} + C_{16}^{(0,0)} C_{16} \right)$$
$$+ \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(O_{16}^{(0,1/2)} S_{16} + S_{16}^{(0,1/2)} O_{16} \right)$$
$$+ \bar{V}_8 \left(V_{16}^{(1/2,0)} V_{16} + C_{16}^{(1/2,0)} C_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right)$$
$$+ \bar{O}_8 \left(O_{16}^{(1/2,1/2)} S_{16} + S_{16}^{(1/2,1/2)} O_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}$$

condition ① $\sqrt{2}A + \sqrt{1+A^2}\,a n_1 = 0, \quad n_1 \in 2Z$

new massless state : • **two** $\mathbf{8}_V \otimes (\mathbf{1}, \mathbf{14})$ • **two** $\mathbf{8}_S \otimes (\mathbf{16}, \mathbf{1})$

⇒ **SO(16)×SO(14)×U(1)** ⟶ **SO(16)×SO(16)**

$\mathbf{8}_S \otimes (\mathbf{16}, \mathbf{14}) \longrightarrow \mathbf{8}_S \otimes (\mathbf{16}, \mathbf{16})$

$\longrightarrow \quad n_F^0 - n_B^0 = 64$

- **SUSY SO(32) —SO(16)×SO(16) interpolating model with WL**

Massless spectrum ∃**a few conditions under which the gauge group gets enhanced**

$$Z_{\rm int}^{(9)} = Z_{\rm boson}^{(7)} \Big\{ \bar{V}_8 \left(O_{16}^{(0,0)} O_{16} + S_{16}^{(0,0)} S_{16} \right) - \bar{S}_8 \left(V_{16}^{(0,0)} V_{16} + C_{16}^{(0,0)} C_{16} \right)$$
$$+ \bar{O}_8 \left(V_{16}^{(0,1/2)} C_{16} + C_{16}^{(0,1/2)} V_{16} \right) - \bar{C}_8 \left(O_{16}^{(0,1/2)} S_{16} + S_{16}^{(0,1/2)} O_{16} \right)$$
$$+ \bar{V}_8 \left(V_{16}^{(1/2,0)} V_{16} + C_{16}^{(1/2,0)} C_{16} \right) - \bar{S}_8 \left(O_{16}^{(1/2,0)} O_{16} + S_{16}^{(1/2,0)} S_{16} \right)$$
$$+ \bar{O}_8 \left(O_{16}^{(1/2,1/2)} S_{16} + S_{16}^{(1/2,1/2)} O_{16} \right) - \bar{C}_8 \left(V_{16}^{(1/2,1/2)} C_{16} + C_{16}^{(1/2,1/2)} V_{16} \right) \Big\}$$

condition ② $\sqrt{2}A + \sqrt{1+A^2}\, a n_2 = 0, \quad n_2 \in 2\boldsymbol{Z} + 1$

new massless state : • **two** $\mathbf{8}_V \otimes (\mathbf{16}, \mathbf{1})$ • **two** $\mathbf{8}_S \otimes (\mathbf{1}, \mathbf{14})$

⇒ { **SO(16)×SO(14)×U(1)** ⟶ **SO(18)×SO(14)**
$\mathbf{8}_S \otimes (\mathbf{16}, \mathbf{14}) \longrightarrow \mathbf{8}_S \otimes (\mathbf{18}, \mathbf{14})$ }

⟶ $$n_F^0 - n_B^0 = 0$$

V)

- have added the WL background to the radius R and constructed two parameter interpolating models
- have found the conditions for (R, A) under which the gauge group enhances
- $\exists$ an example under which the cosmological const. is exponentially suppressed simultaneously with the gauge group enhancement.
-

Conditions	$\tilde{\tau}_1 = n_1/\sqrt{2} \ \ (n_1 \in \boldsymbol{Z})$	$\tilde{\tau}_1 = n_2/\sqrt{2} \ \ (n_2 \in \boldsymbol{Z} + 1/2)$
Gauge group	$SO(16) \times SO(16)$	$SO(14) \times SO(18)$
$N_F - N_B$	positive	zero

$$\tilde{\tau}_1 = \frac{A}{\sqrt{1+A^2}} a^{-1}$$

Conditions	$\tilde{\tau}_1 = n_1/\sqrt{2} \ \ (n_1 \in 2\boldsymbol{Z})$	$\tilde{\tau}_1 = n_1/\sqrt{2} \ \ (n_1 \in 2\boldsymbol{Z} + 1)$	$\tilde{\tau}_1 = n_2/\sqrt{2} \ \ (n_2 \in \boldsymbol{Z} + 1/2)$
Gauge group	$SO(16) \times SO(16)$	$SO(16) \times E_8$	$SO(16) \times SO(14) \times U(1)$
$N_F - N_B$	positive	negative	negative

- gauge group enhancement $\approx$ extrema of $\mathcal{V}_D$

- two & more Wilson lines
- 4d
- higher loops