Worldvolume Effective Action of Winding Corrected Five-branes in Double Field Theory

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30 Oct. 2019

East Asia String Workshop @ NCTS (NTHU)

JHEP07(2018)001, **JHEP12(2018)095** (joint work with Tetsuji Kimura and Shin Sasaki)
and **work in progress** (with Shin Sasaki)

Spacetime Geometry

We are interested in Spacetime Geometry considering string effects

What is the difference between Einstein and string theories?

Einstein gravity (Supergravity)

- probed by point particles
- described by a Riemannian manifold

String Theory

- probed by strings
- described by new stringy geometry

What are the characteristic quantities in stringy geometry?

T-duality

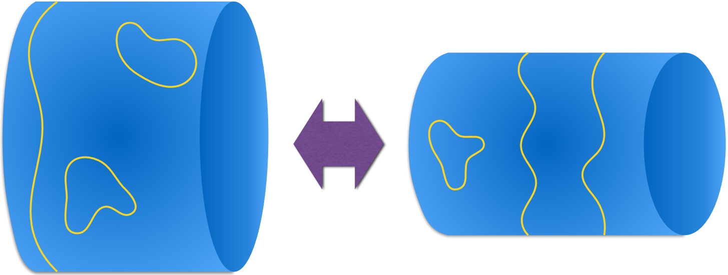
Strings on compact spaces

- can have momentum along compact directions $\rightarrow$ KK mode n
- can wind compact directions (unlike point particles) $\rightarrow$ winding mode w

String theory is invariant under exchange of KK-mode n and winding mode w

$$E^2 = \left(\frac{n}{R}\right)^2 + \left(\frac{R}{\alpha'} w\right)^2 \qquad \mathsf{T}: \begin{pmatrix} n \\ R \end{pmatrix} \leftrightarrow \begin{pmatrix} w \\ \alpha'/R \end{pmatrix}$$

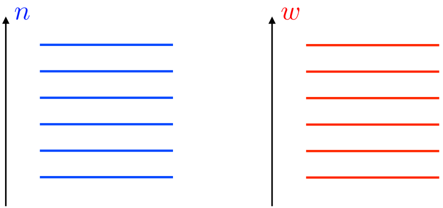
This invariance is known as T-duality.



A Puzzle for 5-branes

Analysis by [Gregory-Harvey-Moore '97]

Smeared NS5 on $S^1 \longleftrightarrow$ Taub-NUT space



tower of KK-modes   tower of winding modes

- Smeared NS5-brane geometry receives KK-mode corrections

$\rightarrow$ Does the Taub-NUT geometry receive string winding corrections?

— yes.

Dual coordinate dependence

Smeared NS5 becomes NS5-brane geometry by KK-mode corrections [Tong '02]

- NS5-brane geometry:

$$g_{\mu\nu} = g_{\mu\nu}(r, y)$$

- y: Fourier dual of KK momentum

Taub-NUT space is modified by string winding corrections [Harvey-Jensen '05]

- Modified Taub-NUT space:

$$g_{\mu\nu} = g_{\mu\nu}(r, \tilde{y})$$

- $\tilde{y}$: Fourier dual of winding momentum

— has winding coordinate dependence

The corrections break isometries of the geometries
$\rightarrow$ this means "Non-isometric T-duality" [Dabholkar-Hull '05]

NS5-brane $\longleftrightarrow$ Modified Taub-NUT

Winding corrections to geometries

T-duality relation of background geometries extends as follows

[de Boer-Shigemori '10, '13]

$$\text{NS5} \longleftrightarrow \text{KK5} \longleftrightarrow 5^2_2$$

5^2_2-brane geometry is modified by winding corrections [Kimura-Sasaki '13]

- Modified 5^2_2 geometry:

$$g_{\mu\nu} = g_{\mu\nu}(\rho, \tilde{y})$$

- $\tilde{y}$: Fourier dual of winding momentum

"Non-isometric T-duality" relation also extends as

$$\text{NS5-brane} \longleftrightarrow \text{Modified Taub-NUT} \longleftrightarrow \text{Modified } 5^2_2\text{-brane}$$

Winding corrected geometries are not solutions to type II supergravity
$\rightarrow$ what kind of solutions are they?

— solutions to double field theory

Double Field Theory (DFT)

A manifestly T-duality invariant formulation of supergravity theory

[Hull-Zwiebach '09]

Double Field Theory

DFT is a gravity theory defined on the doubled space $\mathcal{M}^{2D} = M^D \times \tilde{M}^D$

- $\mathcal{M}^{2D}$ is parametrized by the generalized coordinates $X^M = (\tilde{x}_\mu, x^\mu)$

momentum p $\xleftrightarrow{\text{T-dual}}$ winding momentum w

$\updownarrow$ Fourier conj. | $\updownarrow$ Fourier conj.

ordinary coord. x | winding coord. $\tilde{x}$

M^D | $\tilde{M}^D$

Dynamical fields in DFT:

"generalized metric" $\mathcal{H}^{MN}(X)$ and "DFT dilaton" $d(X)$

- $\mathcal{H}^{MN}$ is parametrized by metric $g_{\mu\nu}$ (on M^D) and NSNS B-field $B_{\mu\nu}$:

$$\mathcal{H}^{MN} = \begin{pmatrix} g_{\mu\nu} - B_{\mu\rho} g^{\rho\sigma} B_{\sigma\nu} & B_{\mu\rho} g^{\rho\nu} \\ -g^{\mu\rho} B_{\rho\nu} & g^{\mu\nu} \end{pmatrix}$$

- d is rescaled the dilaton ϕ: $\quad e^{-2d} = \sqrt{-g} e^{-2\phi}$

DFT action

- DFT action: represented by Einstein-Hilbert form [Hohm-Hull-Zwiebach '10]

$$S_{\mathrm{DFT}} = \int \mathrm{d}^{2D}X \; e^{-2d}\mathcal{R}(\mathcal{H}, d)$$

- T-duality is a manifest symmetry of the theory

- DFT never requires isometries
  $\rightarrow$ inherits "Non-isometric T-duality" [Dabholkar-Hull '05]

- The equation of motion obtained from S_{DFT} is the generalized Einstein eq.

$$\mathcal{R}_{MN} - \frac{1}{2}\mathcal{H}_{MN}\mathcal{R} = 0$$

- DFT is formulated in the same form as Einstein gravity theory,
  except that the space is doubled to respect T-duality.

Strong Constraint

- DFT has extra degrees of freedom since the base space is doubled. It is necessary to reduce.
- The constraint that makes DFT be a physical theory is called the strong constraint
$$\eta^{MN}\partial_M * \partial_N * = 0$$
- The strong constraint originally came from the level matching condition of closed strings
- By choosing one of the solution to the strong constraint $\partial_M = (0, \partial_\mu)$, DFT reduces to the NSNS sector of type II supergravity

$$S_{\mathrm{DFT}} \xrightarrow{\tilde{\partial}=0} S_{\mathrm{sugra}} = \int \mathrm{d}^D x \, \sqrt{-g} e^{-2\phi} \left[R + 4(\partial\phi)^2 - \frac{1}{12} H^2 \right]$$

DFT $\supset$ Supergravity

Classical solutions of DFT

These known supergravity solutions are classical solutions of DFT:

[Berkeley-Berman-Rudolph '14]
[Berman-Rudolph '15]
[Bakhmatov-Kleinschmidt-Musaev '16]
[and more ...]

- NS5-brane
- Taub-NUT space
- F-string
- pp-wave
- 5_2^2-brane

and so on

Non-supergravity DFT solution

We are interested in the **non-supergravity solutions peculiar to DFT**

- DFT inherits "Non-isometric T-duality"
- T-duality transformation **without isometries** is possible

- We find new solutions in DFT
  - Modified Taub-NUT space
  - Modified 5^2_2-brane
  - Locally non-geometric R-brane

  and so on

New solutions

Modified Taub-NUT space

$$
\begin{aligned}
ds^2 &= \eta_{mn} dx^m dx^n + H^{-1}(dy^9 + b_{i9} dy^i)^2 + H \delta_{ij} dy^i dy^j , \\
B &= b_{ij}\; dy^i \wedge dy^j , \\
e^{2\phi} &= \text{const.}, \qquad\qquad 3\partial_{[a} b_{bc]} = \varepsilon_{abcd} \partial^d H \\
& \qquad (i,j = 6,7,8; \quad a,b,c,d = 6,7,8,9; \quad m,n = 0,\ldots,5)
\end{aligned}
$$

Harmonic function of this solution:

$$
H(y^i, \tilde{y}_9) = c + \frac{Q}{|y^i|} \left[1 + \sum_{n \neq 0} \exp \left(in \frac{\tilde{y}_9}{\tilde{R}_9} - |y^i| \left| \frac{n}{\tilde{R}_9} \right| \right) \right]
$$

— has winding coordinate dependence

→ this exhibits **exponential behavior**

New solutions

Modified 5^2_2-brane [Kimura-Sasaki-K.S. '18]

$$
\begin{aligned}
ds^2 &= \eta_{mn}\, dx^m dx^n + H \delta_{\alpha\beta}\, dy^\alpha dy^\beta \\
&\qquad + \frac{H}{H^2 + b_{89}^2}\left[(dy^9 + b_{\alpha 9} dy^\alpha)^2 + (dy^8 + b_{\alpha 8} dy^\alpha)^2\right], \\
B &= b_{\alpha\beta}\, dy^\alpha \wedge dy^\beta - \frac{b_{89}}{H^2 + b_{89}^2}\Big[(dy^8 + b_{\alpha 8} dy^\alpha) \wedge (dy^9 + b_{\beta 9} dy^\beta)\Big], \\
e^{2\phi} &= \frac{H}{H^2 + b_{89}^2}, \qquad (\alpha, \beta = 6, 7)
\end{aligned}
$$

Harmonic function of this solution:

$$
\begin{aligned}
H &\simeq \frac{Q}{2\pi \tilde{R}_8 \tilde{R}_9} \log \frac{\mu}{|y^\alpha|} \\
&\quad + \frac{Q}{2\pi \tilde{R}_8 \tilde{R}_9} \sum_{n,m \neq (0,0)} \exp\left[-|y^\alpha| \sqrt{\left(\frac{m}{\tilde{R}_8}\right)^2 + \left(\frac{n}{\tilde{R}_9}\right)^2} + i \left(m \frac{\tilde{y}_8}{\tilde{R}_8} + n \frac{\tilde{y}_9}{\tilde{R}_9} \right) \right]
\end{aligned}
$$

— has winding coordinate dependence

New solutions

Locally non-geometric R-brane [Kimura-Sasaki-K.S. '18]

$$
\begin{aligned}
ds^2 &= \eta_{mn}\, dx^m dx^n + H(dy^6)^2 \\
&\quad + K_2^{-1}\left[H^2(dy_{\hat{k}} + b_{6\hat{k}}dy^6)^2 + \left(\frac{1}{2}\varepsilon^{\hat{\imath}\hat{\jmath}\hat{k}} b_{\hat{\imath}\hat{\jmath}}(dy_{\hat{k}} + b_{6\hat{k}}dy^6)\right)^2\right], \\
B &= -\frac{H b_{\hat{\imath}\hat{\jmath}}}{K_2}\,(dy_{\hat{\imath}} + b_{6\hat{\imath}}dy^6)\wedge dy_{\hat{\jmath}}, \qquad (\hat{\imath},\hat{\jmath},\hat{k} = 7,8,9) \\
e^{2\phi} &= H K_2^{-1}, \qquad K_2 \equiv H(H^2 + b_{89}^2 + b_{79}^2 + b_{78}^2).
\end{aligned}
$$

Harmonic function of this solution:

$$
H = \text{const.} - c|y^6| + c \sum_{l,m,n\neq(0,0,0)} \frac{1}{s}\exp\left[-|y^6|s + i\left(l\frac{\tilde{y}_7}{\tilde{R}_7} + m\frac{\tilde{y}_8}{\tilde{R}_8} + n\frac{\tilde{y}_9}{\tilde{R}_9}\right)\right]
$$

$$
c = \frac{Q}{4\pi\tilde{R}_7\tilde{R}_8\tilde{R}_9}, \quad s = \sqrt{\left(\frac{l}{\tilde{R}_7}\right)^2 + \left(\frac{m}{\tilde{R}_8}\right)^2 + \left(\frac{n}{\tilde{R}_9}\right)^2}
$$

R-brane has **No corresponding solution in supergravity**

R-brane is mysterious object

New solutions

We obtain various five-brane solutions in DFT:

[Kimura-Sasaki-K.S. '18]

	5_2	5_2^1	5_2^2	5_2^3	5_2^4
codim 4	NS5	KK5 $+ w^1$	$5_2^2 + w^2$	$5_2^3 + w^3$	$5_2^4 + w^4$
codim 3	sNS5	KK5	$5_2^2 + w^1$	$5_2^3 + w^2$	$5_2^4 + w^3$
codim 2	dsNS5	sKK5	5_2^2	$5_2^3 + w^1$	$5_2^4 + w^2$
codim 1	tsNS5	dsKK5	s5_2^2	5_2^3	5_2^4
codim 0	qsNS5	tsKK5	ds5_2^2	s5_2^3	5_2^4

- black: the supergravity solutions
- blue: the previously known solutions
- red: newly obtained solutions by our calculation

DFT solutions naturally have the winding coordinate dependence

Winding coordinate dependence shows **exponential behavior**

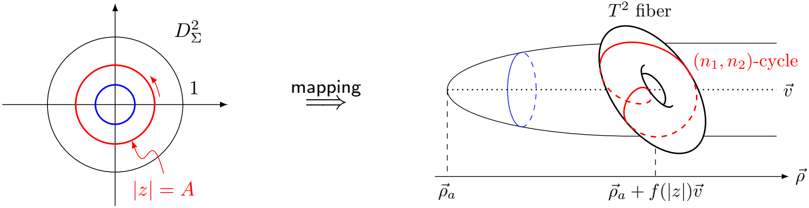
The exponential behavior suggests:

the winding corrections are interpreted as the instanton corrections to the spacetime

$$e^{-S_{\text{inst.}}}$$

Winding dependence as worldsheet instanton effects

- We give an interpretation of the winding corrections to geometries in DFT as the string worldsheet instanton effects [Kimura-Sasaki-K.S. '18]
- In particular, we discuss that the worldsheet instanton in 5^2_2-brane geometry is given by a generalization of the disk instanton



- The disk instanton becomes the worldsheet instanton when it takes the appropriate limit
- The discussion of worldsheet instanton corrections to R-brane geometry is completely parallel

Five-brane worldvolume effective theory in DFT

- In the above discussion, we obtained how the winding dependence is given
- However, we did not discuss physical properties of locally non-geometric solutions (locally non-geometric means that a solution depends on the winding coordinate)
- In particular, we would like to understand about the energy gained by the winding coordinate dependence

- We expect that winding depended five-branes have fluctuation along the winding space directions
- We focus on the brane fluctuation effective action in DFT

DFT five-brane fluctuation effective action

We decompose the generalized metric as follows

$$
\begin{aligned}
ds^2_{\text{DFT}} &= ds^2_{\text{doubled worldvolume}} + ds^2_{\text{doubled transverse space}} \\
&= \mathcal{H}_{\hat{M}\hat{N}}(X^{\hat{M}})dX^{\hat{M}}dX^{\hat{N}} + \mathcal{H}_{\check{M}\check{N}}(Y^{\check{M}}, t^A(X^{\hat{M}}))dY^{\check{M}}dY^{\check{N}}
\end{aligned}
$$

- $X^{\hat{M}}$ ($\hat{M} = 1, \ldots, 12$): doubled worldvolume coordinates
- $Y^{\check{M}}$ ($\check{M} = 1, \ldots, 8$): doubled transverse coordinates
- $\mathcal{H}_{\check{M}\check{N}}$ in transverse dir. depends on $X^{\hat{M}}$ through parameters t^A

$\rightarrow$ parameters $t^A = (t^a, \tilde{t}_a)$ represent **scalar fields on the worldvolume**

Under this decomposition of $\mathcal{H}_{MN}$, DFT action becomes

$$
S = \int d^{12}X \int d^8Y \; e^{-2d}\left[\frac{1}{8}\mathcal{H}^{\hat{M}\hat{N}}(\partial_{\hat{M}}t^A)\left(\frac{\partial}{\partial t^A}\mathcal{H}^{\check{K}\check{L}}\right)(\partial_{\hat{N}}t^B)\left(\frac{\partial}{\partial t^B}\mathcal{H}_{\check{K}\check{L}}\right) + \hat{\mathcal{R}}(X) + \check{\mathcal{R}}(t(X), Y)\right]
$$

$\hat{\mathcal{R}}(X), \check{\mathcal{R}}(t(X), Y)$ are potential term $\rightarrow$ now, we focus on the kinetic term of t^A

DFT five-brane fluctuation effective action

We consider the DFT five-brane solution
(it contains NS5-, KK5-, 5^2_2-, R- and 5^4_2-brane simultaneously)

$$\mathcal{H}_{\check{M}\check{N}} = \begin{pmatrix} H(\delta_{ab} - H^{-2} b_{ac} b^c{}_b) & H^{-1} b_a{}^b \\ -H^{-1} b^a{}_b & H^{-1} \delta^{ab} \end{pmatrix}, \qquad \mathcal{H}^{\hat{M}\hat{N}} = \begin{pmatrix} \eta^{mn} & 0 \\ 0 & \eta_{mn} \end{pmatrix}$$

$$H(y,t) = c + \frac{Q}{(y^a - t^a)^2}, \qquad (a = \{6,7,8,9\})$$

$$e^{-2d} = H e^{-2\phi_0}, \qquad \phi_0 = \text{const.}$$

We obtain **the doubled action of scalar fields on 5-branes in DFT**

$$S = \int d^6x d^6\tilde{x}\; e^{-2\phi_0} \left[\frac{1}{2} \eta^{mn} (\partial_m t^a)(\partial_n t^b) + \frac{1}{2} \eta_{mn} (\tilde{\partial}^m t^a)(\tilde{\partial}^n t^b) + (\text{potential}) \right]$$

This action contains [work in progress]

- NS5-brane: $(t^6, t^7, t^8, t^9) = (X^6, X^7, X^8, X^9)$
- KK5-brane: $(t^6, t^7, t^8, t^9) = (X^6, X^7, X^8, \tilde{X}_9)$
- 5^2_2-brane: $(t^6, t^7, t^8, t^9) = (X^6, X^7, \tilde{X}_8, \tilde{X}_9)$
- R-brane: $(t^6, t^7, t^8, t^9) = (X^6, \tilde{X}_7, \tilde{X}_8, \tilde{X}_9)$
- 5^4_2-brane: $(t^6, t^7, t^8, t^9) = (\tilde{X}_6, \tilde{X}_7, \tilde{X}_8, \tilde{X}_9)$

DFT five-brane worldvolume effective action

The above discussion agrees with the discussion of five-brane DBI actions in DFT
[Blair-Musaev '17]

Winding depended five-branes are locally non-geometric and mysterious objects
$\rightarrow$ it may be able to explore these physical properties by using w.v. action

However, the above discussion is only for the kinetic term of scalar fields
$\rightarrow$ It is necessary to discuss the potential and the gauge field term

Therefore, the worldvolume theory of winding depended five-brane in DFT is under construction and is incomplete

In supergravity, winding direction is isometry, but it breaks in this case
$\rightarrow$ we expect that scalar fields corresponding winding coordinate are interpreted as Goldstone modes

Summary

- There is many mysterious branes in string theory. The winding coordinate dependence is necessary to understand these branes.
- DFT is useful tool to examine the winding coordinate dependence.

- We obtained the DFT solutions that naturally has the winding coordinate dependence.
- We interpreted the winding coordinate dependence of DFT solutions as the string worldsheet instanton corrections to supergravity solutions.
- We discussed the action of scalar fields on the five-branes in DFT.

Thank you for your attention

extra materials

Non-geometric Flux

[Shelton-Taylor-Wecht '05]

Non-geometric flux

$$H_{abc} \xleftrightarrow{T_c} f_{ab}{}^{c} \xleftrightarrow{T_b} Q_a{}^{bc} \xleftrightarrow{T_a} R^{abc}$$

- H-flux source: NS5-brane
- geometric f-flux: associated with KK-monopole
- globally non-geometric Q-flux: Q-brane ($= 5^2_2$-brane)
- locally non-geometric R-flux: R-brane (realized as 5^3_2-brane)

R-brane is the mysterious object