



Randomness and chaos in qubit models

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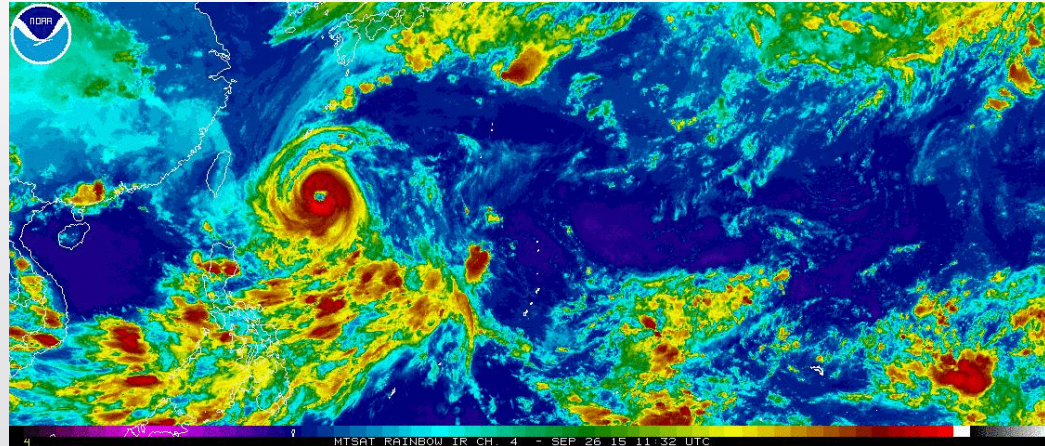
with Chen-Te Ma (SCNU), Jeff Murugan (Cape Town U.) and Masaki Tezuka (Kyoto U)

Phys. Lett. B795 (2019) 230-235

Classical chaos

- Non-linear system
 - Double rod pendulum
 - Weather
- Sensitivity to initial condition

$$\frac{\delta x(t)}{\delta x(0)} \sim e^{\lambda t}$$





Quantum chaos

- Quantum $\rightarrow$ Classical chaos
- Out of time-ordered correlator (OTOC)
- Random matrix theory
 - Spectral form factor
 - Level spacing distribution

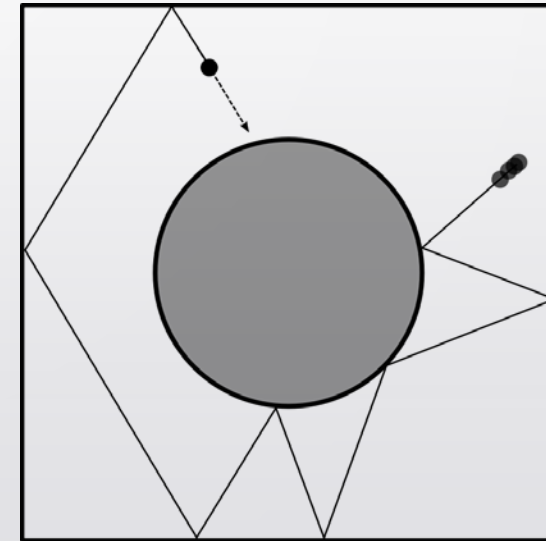


Diagnostic tool

- Quantum → Probabilistic
 - Statistic properties of the system
- Random matrix theory (RMT)
 - Spectra of heavy nuclei
 - Wigner 1955

Classical chaos & RMT

- Sinai billiard
 - Chaotic
- Quantum Sinai billiard
 - Spectrum
 - RMT



K. Hashimoto, K. Murata, R. Yoshii JHEP 1710 (2017) 138

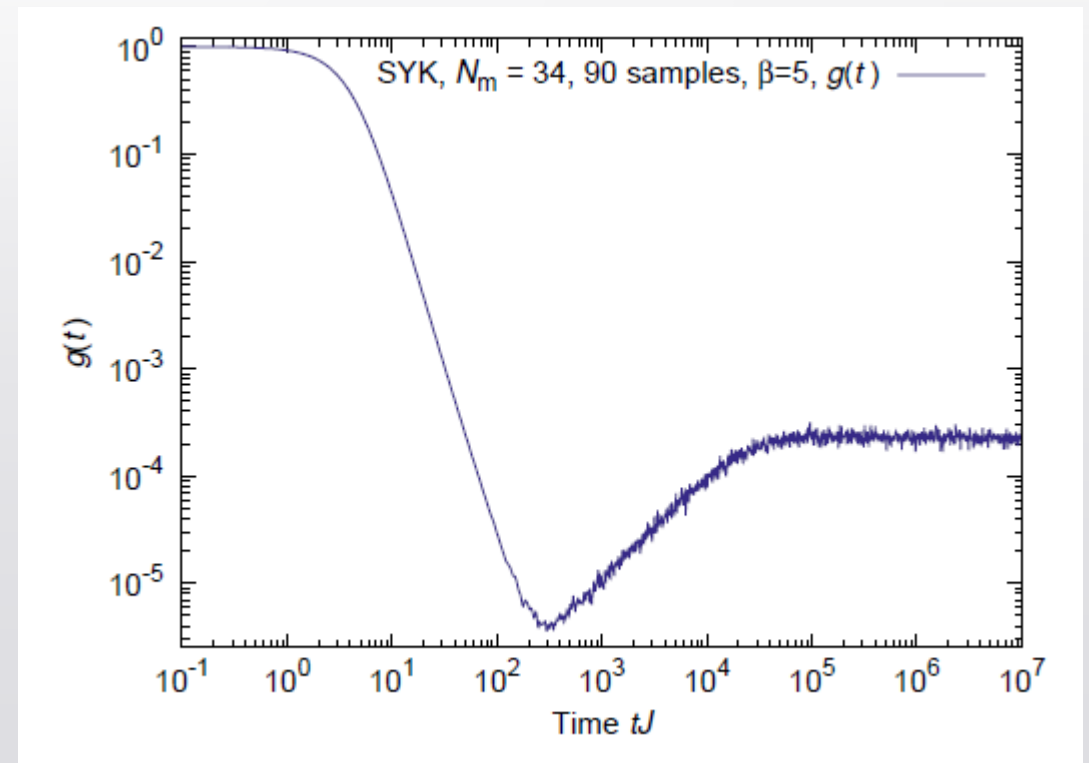
https://en.wikipedia.org/wiki/Dynamical_billiards

Diagnostic - RMT

- Spectral form factor (SFF)
 - Correlation between eigenvalues

$$g(t) = \frac{|Z(\beta, t)|^2}{|Z(\beta, 0)|^2}$$

$$Z(\beta, t) = \text{Tr}(e^{(-\beta + it)H})$$



Sachedev-Ye-Kitaev model

- Hamiltonian

$$H = i^{q/2} \sum_{1 \leq i_1 < \dots < i_q \leq N} J_{i_1 \dots i_q} \psi_{i_1} \dots \psi_{i_q}$$

- Majorana fermions $\{\psi_i, \psi_j\} = \delta_{ij}$
- Random coupling
 - Gaussian variance $(q-1)! \frac{J^2}{N^{q-1}}$
- Most studied case
 - $q = 4$ large N (chaotic)

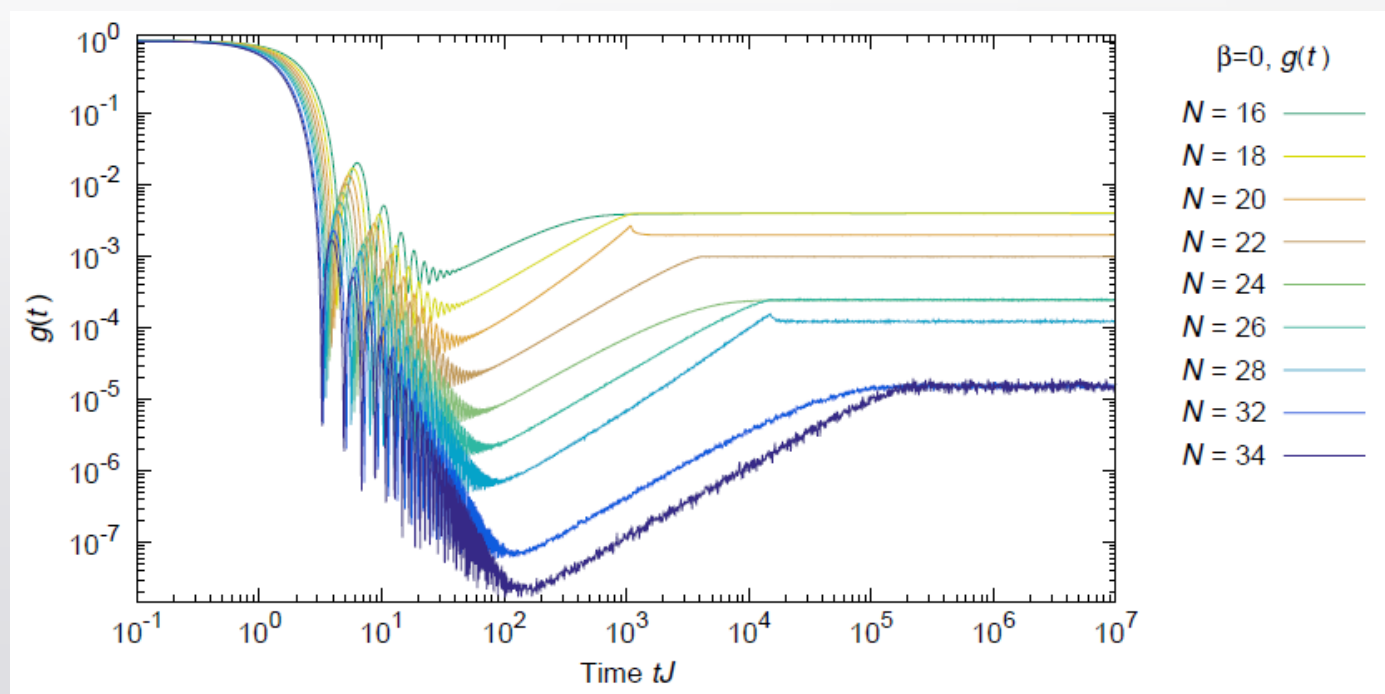
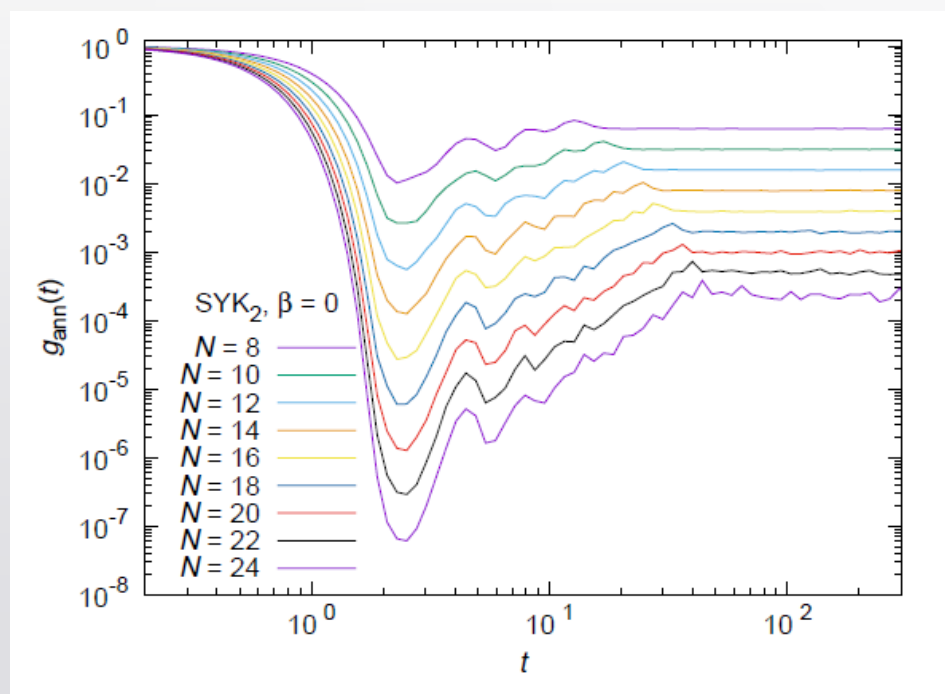
Model

- $q = 2$ Sachdev-Ye-Kitaev model (SYK)
 - Majorana fermions
 - Random coupling
 - Gaussian
 - Integrable

$$\mathcal{P} = \exp \left(- \sum_{j_1, j_2=1}^N \mathcal{J}_{j_1 j_2}^2 \frac{N}{4J^2} \right)$$

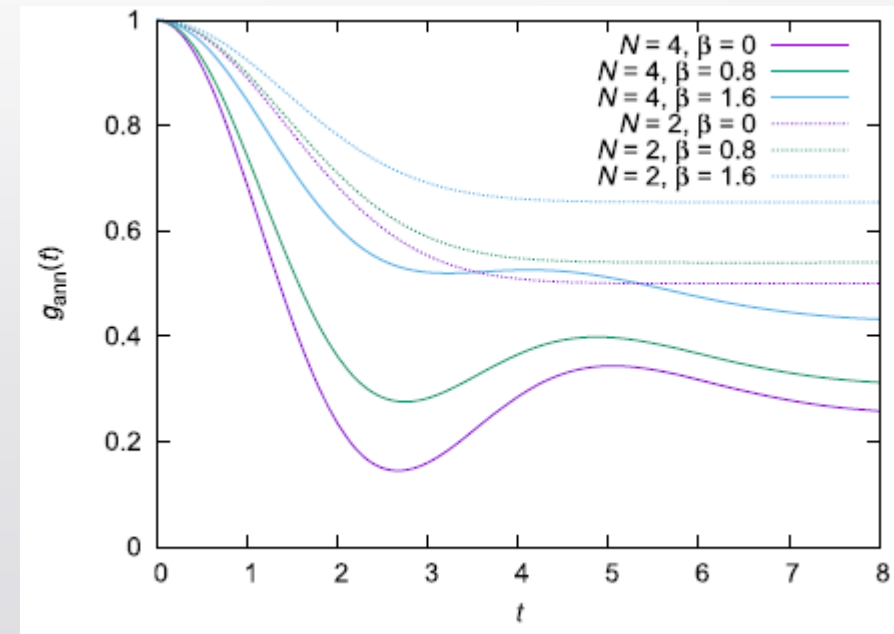
$$H_{\text{SYK2}} \equiv \frac{i}{2} \sum_{j_1, j_2=1}^N \mathcal{J}_{j_1 j_2} \psi_{j_1} \psi_{j_2},$$

SYK₂ - Spectral form factor



Small N SFF

- Only $N = 2$
 - No dip-ramp-plateau
- $N = 2$ solvable analytically
- Eigenvectors
 - Independent of coupling
 - Origin of dip-ramp-plateau?





Comparison and summary

- Same behaviour as the SYK4 model (chaotic) for $N > 2$
 - Transverse Ising model gives consistent result
 - General integrable spin system?
- Dip-ramp-plateau
 - Generate from large set of eigenstates
 - $N = 2$ eigenvectors independent of couplings
 - Not sufficient condition to diagnose chaos