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Gravitational perturbations as $T\overline{T}$-deformations in the JT gravity

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Based on Ishii-Okumura-Sakamoto-KY, 1906.03865

An overview of this talk

1) $T\bar{T}$-deformation of 2D QFT

A composite operator composed of energy-momentum tensor

2) Gravitational perturbation in the flat-space JT gravity
can be regarded as $T\bar{T}$-deformation of 2D QFT (JT = Jackiw-Teitelboim)

[Dubovsky-Gorbenko-Mirbabayi, 1706.06604] (flat)

Our work:

We have discussed a generalization to curved spaces.

[Ishii-Okumura-Sakamoto-KY, 1906.03865] (AdS,dS) [Ishii-Okumura-Sakamoto-KY, in progress]

Plan of this talk

1. Some relevant aspects of $T\overline{T}$-deformation of 2D QFT (10 slides)

2. Gravitational perturbation as $T\overline{T}$-deformation (10 slides)

   [Dubovsky-Gorbenko-Mirbabayi, 1706.06604]

   [Ishii-Okumura-Sakamoto-KY, 1906.03865]

   [Ishii-Okumura-Sakamoto-KY, in progress]

3. Summary and outlook

1. Some relevant aspects of $T\overline{T}$-deformation of 2D QFT

For a nice review, see Y. Jiang, 1904.13376.

Setup - notation and terminology

We consider 2D QFT with local energy-momentum tensor $T_{\mu\nu}$.

One may consider a determinant operator defined as

$$\mathrm{T}\overline{\mathrm{T}} \equiv -\frac{1}{4}\det T_{\mu\nu} = T\overline{T} - \Theta^2$$

where

$$T(z,\bar{z}) \equiv T_{zz}(z,\bar{z}), \quad \overline{T}(z,\bar{z}) \equiv T_{\bar{z}\bar{z}}(z,\bar{z}), \quad \Theta(z,\bar{z}) \equiv T_{z\bar{z}}(z,\bar{z})$$

NOTE: For general 2D QFT, T and $\overline{T}$ are not chiral, and Θ does not vanish.

FACT:

This operator is well-defined as a composite operator in two dimensions and called $T\overline{T}$-operator. [Sasha Zamolodchikov, hep-th/0401146]

Factorization of expectation value

$$\langle T\bar{T}\rangle = \langle T\rangle\langle\bar{T}\rangle - \langle\Theta\rangle^2$$

[Sasha Zamolodchikov, hep-th/0401146]

This factorization is valid for stationary states under the following assumptions:

1. Local translational and rotational invariance (L)

   The existence of local $T_{\mu\nu}$ and $T_{\mu\nu} = T_{\nu\mu}$

2. Global translational invariance (G)

   $\langle\mathcal{O}_i(z)\rangle$ does not depend on z (for any local field $\mathcal{O}_i(z)$) .

3. Infinite separations (G) There should exist at least one direction,

   such that for any $\mathcal{O}_i$ and $\mathcal{O}_j$, $\lim_{t\to\infty}\langle\mathcal{O}_i(z+et)\mathcal{O}_j(z')\rangle = \langle\mathcal{O}_i\rangle\langle\mathcal{O}_j\rangle$

   Note: Assumps. 2 & 3 $\Rightarrow$ 2D space is infinite plane or infinitely long cylinder

4. CFT limit at short distances (L) To make definition of $T\bar{T}$-op. unambiguous.

Application of the factorization

Let us consider a 2D QFT on an infinitely long cylinder.

$(\tau : \text{time}, \; x : \text{compactified with a period } L)$

Note: $\langle T \rangle$, $\langle \overline{T} \rangle$ vanish on infinite plane.

The factorization enables us to compute the expectation value of $T\overline{T}$-op.

With an arbitrary non-degenerate eigenstate of the energy $|n\rangle$, such that

$$H|n\rangle = E_n|n\rangle$$

one obtains that

[Sasha Zamolodchikov, hep-th/0401146]

$$\begin{aligned} \langle n|\mathrm{T}\overline{\mathrm{T}}|n\rangle &= \langle n|T|n\rangle\langle n|\overline{T}|n\rangle - \langle n|\Theta|n\rangle^2 \\ &= -\frac{1}{4}\left(\langle n|T_{\tau\tau}|n\rangle\langle n|T_{xx}|n\rangle - \langle n|T_{\tau x}|n\rangle^2\right) \end{aligned} \quad .$$

With the physical meaning of the stress tensor,

$$\langle T_{\tau\tau}\rangle = \frac{E_n}{L}, \qquad \langle T_{xx}\rangle = \frac{\partial E_n}{\partial L}, \qquad \langle T_{\tau x}\rangle = \frac{iP_n}{L} \quad .$$

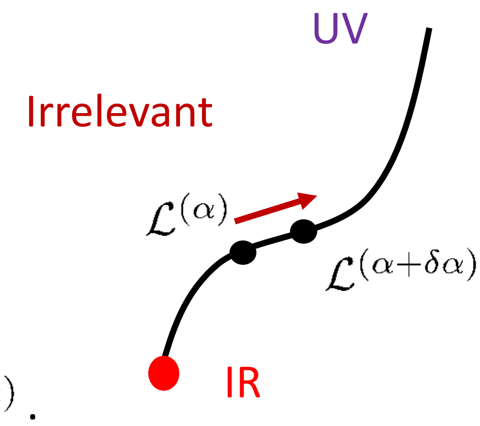
(energy density) (pressure) (momentum density)

What is $T\overline{T}$-deformation?

Assume the set of 2D QFTs described by Lagrangian.

Consider a trajectory in the theory space parametrized by α

and denote the Lagrangian at each point of the trajectory by $\mathcal{L}^{(\alpha)}$.



For $T\overline{T}$-deformed CFT

A flow for theories on the trajectory may be triggered by the $T\overline{T}$-operator as

$$\begin{aligned} \mathcal{L}^{(\alpha+\delta\alpha)} &= \mathcal{L}^{(\alpha)} + \frac{\delta\alpha}{4} \det T^{(\alpha)}_{\mu\nu} \\ &\equiv \mathcal{L}^{(\alpha)} - \delta\alpha \, \mathrm{T\overline{T}}^{(\alpha)} \qquad (T\overline{T}\text{-flow}) \end{aligned}$$

Irrelevant perturbation

$\delta\alpha$: coupling constant with dimension (length)^2

NOTE 1: This $T\overline{T}$- flow is not the usual RG-flow! c.f., $\mathcal{L}^{(\alpha)} = \mathcal{L}^{(0)} - \alpha \, \mathrm{T\overline{T}}^{(0)}$

$T\overline{T}$- flow gives rise to a one-parameter family of 2D QFTs.

NOTE 2: The undeformed theory $\mathcal{L}^{(0)}$ may be a general 2D QFT.

In fact, the $T\overline{T}$-deformation is governed by a simple differential equation

$T\overline{T}$-flow equation

$$\frac{\mathrm{d}S^{(\alpha)}}{\mathrm{d}\alpha} = -\int \mathrm{d}^2x \, (\mathrm{T}\overline{\mathrm{T}})^{(\alpha)}$$

[Smirnov -Zamolodchikov, 1608.05499]

[Cavaglia-Negro-Szecsenyi -Tateo, 1608.05534]

This flow equation was proposed as an operator equation at quantum level.
But at least so far, classical aspects of this equation has been well studied.

In the following, we will see two classical aspects of $T\overline{T}$-deformation:

1. $T\overline{T}$-deformed Lagrangian
2. $T\overline{T}$-deformed spectrum

Here we will not discuss quantum aspects of the $T\overline{T}$-flow equation.
But for some argument, see a nice paper [Rosenhaus-Smolkin, 1909.02640]
and hear Haruna-kun's talk

1) $T\overline{T}$-deformed Lagrangian

EX a free massless scalar $\Rightarrow$ Nambu-Goto action (with static gauge)

The ansatz: $\mathcal{L}^{(\alpha)} = \frac{1}{\alpha} F(\alpha\, \partial\phi\bar{\partial}\phi)$ $\quad (F(0) = 0)$

Then $\quad T = \frac{\partial \mathcal{L}^{(\alpha)}}{\partial(\bar{\partial}\phi)} \partial\phi = F'(x)(\partial\phi)^2, \quad \overline{T} = \frac{\partial \mathcal{L}^{(\alpha)}}{\partial(\partial\phi)} \bar{\partial}\phi = F'(x)(\bar{\partial}\phi)^2, \quad x \equiv \alpha\, \partial\phi\bar{\partial}\phi$

$$\Theta = \frac{x\, F'(x)}{\alpha} - \frac{F(x)}{\alpha} \qquad \partial_\alpha \mathcal{L}^{(\alpha)} = -\frac{F(x)}{\alpha^2} + \frac{x\, F'(x)}{\alpha^2}$$

By putting them into the flow equation, we obtain a differential eq.,

$$F^2 - 2xF'F - xF' + F = 0 \quad \Rightarrow \quad F(x) = -\frac{1}{2} + \frac{1}{2}\sqrt{1 + 4\mathrm{e}^C x}$$

(C: integration const.)

Thus the Nambu-Goto action is obtained

$$\mathcal{L}^{(\alpha)} = -\frac{1}{2\alpha} + \frac{1}{2\alpha}\sqrt{1 + 4\alpha\, \partial\phi\bar{\partial}\phi} \qquad (C\text{=0})$$

Now α is α' !

It would be instructive to see the expanded form of the Nambu-Goto action:

$$\mathcal{L}^{(\alpha)} = \mathcal{L}^{(0)} - \alpha T^{(0)}\overline{T}^{(0)} + 2\alpha^2 (T^{(0)}\overline{T}^{(0)})^{3/2} - 5\alpha^3 (T^{(0)}\overline{T}^{(0)})^2 + \cdots$$

NG action

free massless scalar

This is nothing but an RG flow-type interpretation of $T\overline{T}$-flow,

but a perturbation with an infinite number of irrelevant operators.

c.f., An analogous perturbative expansion is the sinh Gordon model

$$\mathcal{L}^{(b)} = \partial\phi\bar{\partial}\phi + \frac{m^2}{b^2}\left(\cosh(b\phi) - 1\right)$$

(relevant perturbation!)

$$\mathcal{L}^{(b)} = \mathcal{L}^{(0)} + \frac{1}{2}m^2\phi^2 + \frac{m^2 b^2}{4!}\phi^4 + \frac{m^2 b^4}{6!}\phi^6 + \cdots$$

NOTE: The sinh Gordon model is integrable, and the NG action is also integrable.

2) $T\overline{T}$-deformed spectrum

Let us evaluate the expectation value of the flow eq. with excited states $|n\rangle$

$$\frac{\partial E_n}{\partial \alpha} = -L\langle n|\mathrm{T}\overline{\mathrm{T}}|n\rangle \quad \Longrightarrow \quad 4\frac{\partial E_n}{\partial \alpha} = E_n \frac{\partial E_n}{\partial L} + \frac{P_n^2}{L}$$

A forced inviscid Burgers eq.

$$\left((\because) \quad \frac{\partial}{\partial \alpha}\langle n|H|n\rangle = \int \mathrm{d}x\, \langle n|\mathrm{T}\overline{\mathrm{T}}|n\rangle = L\langle n|\mathrm{T}\overline{\mathrm{T}}|n\rangle \right)$$

By solving the Burgers eq., the spectrum of the $T\overline{T}$-deformed system is computed exactly.

- One has to know the original spectrum E_n $\Longrightarrow$ CFT$_2$, Integrable QFT$_2$ (IQFT$_2$).

- Even if the original spectrum is unknown, the deformation effect itself can be examined.

$\Longrightarrow$ ``Integrable'' deformation

In integrable QFTs, this is really an integrable deformation in the usual sense.

EX $T\overline{T}$-deformation of CFT on a cylinder

The energy and momentum of a primary state in CFT on a cylinder with period *L*,

$$E_n(L) = \frac{2\pi}{L}\left(\Delta_n + \overline{\Delta}_n - \frac{c}{12}\right), \qquad P_n(L) = \frac{2\pi}{L}(\Delta_n - \overline{\Delta}_n)$$

are deformed as follows:

$$E_n(L,\alpha) = \frac{2\pi}{L}\left(\frac{1}{b}\right)\left[-1 + \sqrt{1 + 2bM_n + b^2 J_n^2}\right]$$

$$\left[\begin{array}{l} M_n \equiv \Delta_n + \overline{\Delta}_n - \frac{c}{12}, \\ J_n \equiv \Delta_n - \overline{\Delta}_n, \quad b \equiv \frac{\pi\alpha}{L^2} \end{array}\right.$$

The associated entropy can also be computed by using the Cardy formula.

Holographic duals for $T\overline{T}$-deformed CFT_2 on a cylinder

These results were employed to figure out the gravity duals:

Positive sign: RG flow: LST to AdS (single trace) [Giveon-Itzhaki-Kutasov, 1701.05576]

Negative sign: cut-off AdS (double trace) [McGough-Mezei-Verlinde, 1611.03470]

Why is $T\overline{T}$-deformation so interesting?

At least so far, we have presented classical aspects of $T\overline{T}$-flow.

The resulting deformed spectrum would be quite analogous to the Landau level, where the quantum energy level is deformed by turning on a classical magnetic field.

In the case of $T\overline{T}$-deformation, classical gravity is turned on, instead of magnetic field.

In fact, by performing a field-dependent coordinate transformation, $T\overline{T}$-deformation can be undone while the base space is deformed with a classical metric perturbation.

(quite analogous to the Seiberg-Witten map)

[Cardy, 1801.06895]

[Dubovsky-Gorbenko-Mirbabayi, 1706.06604]

[Conti-Negro-Tateo, 1809.09593]

Ambitious motive

If the $T\overline{T}$-flow eq. can be understood at quantum level, a clue to quantum gravity may be captured.

2. Gravitational perturbation as $T\overline{T}$-deformation

[Dubovsky-Gorbenko-Mirbabayi,1706.06604]

[Ishii-Okumura-Sakamoto-KY, 1906.03865]

[Ishii-Okumura-Sakamoto-KY, in progress]

2D dilaton gravity system with matter field

Total classical action

$$S[g_{\mu\nu}, \phi, \psi] = S_{\rm dg}[g_{\mu\nu}, \phi] + S_{\rm m}[\psi, g_{\mu\nu}, \phi]$$

$g_{\mu\nu}$: 2D metric, ϕ : dilaton, ψ : matter field

2D dilaton gravity action (or JT gravity)

[Jackiw, NPB252 (1985) 343] [Teitelboim, PLB126 (1983) 41]

$$S_{\rm dg}[g_{\mu\nu}, \phi] = \frac{1}{16\pi G_N} \int \mathrm{d}^2x \sqrt{-g}\, [\, \phi\, R - \underline{U(\phi)}\,]$$

Dilaton Potential

(The dilaton kinematic term has been removed by a Weyl transformation)

Matter action

$$S_{\rm m}[\psi, g_{\mu\nu}, \phi] = \int \mathrm{d}^2x \sqrt{-g}\, F(\phi)\, \mathcal{L}_m[g_{\mu\nu}, \psi]$$

Dilaton Coupling to matter

In the following, we take $F(\phi) = 1$ and $\kappa \equiv 8\pi G_N$.

Classical equations of motion

$$R - U'(\phi) = 0 \ , \qquad \frac{1}{2} g_{\mu\nu} U(\phi) - \left(\nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \nabla^2 \phi\right) = \kappa \, T_{\mu\nu}$$

(dilaton) (metric)

In the following, we will consider a gravitational perturbation around a classical solution

$$g_{\mu\nu} = \overline{g}_{\mu\nu} + h_{\mu\nu} \ , \qquad \phi = \overline{\phi} + \sigma \ , \qquad \psi = 0 + \psi$$

(The perturbation parameter is the Newton const.)

Then the classical action can be expanded as

e.o.m.

$$\begin{aligned} S[g_{\mu\nu}, \phi, \psi] &= S^{(0)} + S^{(1)} + S^{(2)} + \cdots \\ &= \underline{S^{(0)}_{\rm dg}[\overline{g}_{\mu\nu}, \overline{\phi}]} + \cancel{S^{(1)}_{\rm dg}[\overline{g}_{\mu\nu}, \overline{\phi}; h_{\mu\nu}, \sigma]} + S^{(2)}_{\rm dg}[\overline{g}_{\mu\nu}, \overline{\phi}; h_{\mu\nu}, \sigma] \\ &\quad + S^{(1)}_{\rm m}[\overline{g}_{\mu\nu}; \psi] + S^{(2)}_{\rm m}[\overline{g}_{\mu\nu}; \psi, h_{\mu\nu}] + \text{higher-order terms} \end{aligned}$$

classical energy

The second order part can be evaluated as

$$
\begin{aligned}
S_{\rm dg}^{(2)} \;&=\; \frac{1}{2\kappa}\int \mathrm{d}^2x\,\sqrt{-\overline{g}}\,\Bigg(\bigg[\overline{\nabla}^\mu\overline{\nabla}^\nu h_{\mu\nu} - \overline{\nabla}^2 h - \frac{1}{2}h\,U'(\overline{\phi}) - \frac{1}{2}U''(\overline{\phi})\sigma\bigg]\,\sigma \\
&\quad -\frac{1}{8}\overline{\nabla}^2\overline{\phi}\,h_{\mu\nu}h^{\mu\nu} - \overline{\nabla}^\rho\overline{\phi}\left[-\frac{1}{2}h_{\rho\sigma}\overline{\nabla}_\mu h^{\mu\sigma} + \frac{1}{4}h\overline{\nabla}^\sigma h_{\sigma\rho} + \frac{3}{4}h_{\rho\mu}\overline{\nabla}^\mu h\right]
\end{aligned}
$$

$$
S_{\rm m}^{(2)} = \delta g^{\mu\nu}\,\frac{\delta S_{\rm m}}{\delta g^{\mu\nu}}\bigg|_{g_{\mu\nu}=\overline{g}_{\mu\nu}} = \frac{1}{2}\int \mathrm{d}^2x\,\sqrt{-\overline{g}}\,h^{\mu\nu}\underline{t_{\mu\nu}}
$$

EM tensor with $S_{\rm m}^{(1)}$

In summary, the total action is expressed as (without the classical energy)

$$
S[g_{\mu\nu},\phi,\psi] \;=\; \underline{S_{\rm m}^{(1)}[\overline{g}_{\mu\nu};\psi]} + \boxed{\frac{1}{2}\int \mathrm{d}^2x\,\sqrt{-\overline{g}}\,h^{\mu\nu}t_{\mu\nu} + S_{\rm dg}^{(2)}[\overline{g}_{\mu\nu},\overline{\phi};h_{\mu\nu},\sigma] + \text{higher-order terms}}
$$

The matter action
on the undeformed b.g.

This part can be seen as a deformation of $S_{\rm m}^{(1)}$.

The deformation part can be rewritten furthermore by supposing

$$\boxed{h_{\mu\nu} = -2\kappa(t_{\mu\nu} - \bar{g}_{\mu\nu} t^{\rho}_{\rho})\, k}$$

A covariant generalization of the ansatz employed in [Dubovsky-Gorbenko-Mirbabayi,1706.06604]

Then the quadratic action is given by

$$\boxed{S^{(2)} = \int \mathrm{d}^2 x \sqrt{-\bar{g}} \left[\frac{1}{4\kappa} U''\left(\bar{\phi}\right) \sigma^2 - \kappa \left(k - \frac{k^2}{4} U\left(\bar{\phi}\right) \right) \left(t_{\mu\nu} t^{\mu\nu} - t^2 \right) \right]}$$

In the flat-space JT with $U(\phi) = \Lambda$, this part can be seen as $T\overline{T}$-deformation.

(const.) [Dubovsky-Gorbenko-Mirbabayi,1706.06604]

It is still necessary to solve the e.o.m. for σ .

The existence of a solution: [Dubovsky-Gorbenko-Mirbabayi,1706.06604]

An explicit non-local solution: [Ishii-Okumura-Sakamoto-KY, 1906.03865]

Explicit expression of σ

A byproduct of [Ishii-Okumura-Sakamoto-KY, 1906.03865]

The dilaton fluctuation σ is composed of the local and non-local part:

$$\sigma(x^+, x^-) = \sigma_{\rm L}(x^+, x^-) + \sigma_{\rm NL}(x^+, x^-)\,.$$

The local part:

$$\sigma_0(x^+, x^-) = a_1 + a_2\, x^+ + a_3\, x^-\,, \qquad a_i\ (i = 1, 2, 3)\text{: arbitrary real consts.,}$$

The non-local part:

$$\begin{aligned}
\sigma_{\rm NL}(x^+, x^-) \;\; = \;\; & \frac{1}{2}\kappa\Bigg[k\Lambda\int_0^{x^+}\mathrm{d}s\, s\, t_{++}(s, x^-) + k\Lambda\int_0^{x^-}\mathrm{d}s\, s\, t_{--}(x^+, s) \\
& -2\,(k\Lambda - 1)\int_0^{x^+}\mathrm{d}s\int_0^{x^-}\mathrm{d}s'\, t_{+-}(s, s') \\
& +(k\Lambda - 2)\left(\int_{u_1^+}^{x^+}\mathrm{d}s\int_{u_2^+}^{s}\mathrm{d}s'\, t_{++}(s', 0) + \int_{u_1^-}^{x^-}\mathrm{d}s\int_{u_2^-}^{s}\mathrm{d}s'\, t_{--}(0, s')\right)\Bigg]
\end{aligned}$$

$u^{\pm}_{1,2}$: arbitrary constants

FACT: In the case of the flat-space JT gravity, the metric perturbation can be seen as $T\overline{T}$-deformation. In other words, any $T\overline{T}$-deformation can also be seen as a gravitational perturbation.

$$\boxed{T\overline{T}\text{-deformation} \quad \Longleftrightarrow \quad \text{metric perturbation}}$$

Question: How about in more general JT gravity?

In general, the metric perturbation cannot be seen as $T\overline{T}$-deformation.

Strategy here:

1) Look for appropriate potentials

2) Solve the equations of motion for σ, while the metric ansatz is the same.

In the following, let us consider a famous example,

The Almheiri-Polchinski (AP) model [Almheiri-Polchinski, 1402.6334]

The dilaton potential: $U(\phi) = \Lambda - \frac{2}{L^2}\phi$ + conformal matter i.e., $t^\mu_\mu = 0$

$$\Rightarrow \quad U''(\phi) = 0 \ , \qquad L^2 \ : \ \text{curvature radius}$$

NOTE: The cosmological constant is contained in addition to the flat-space JT gravity.

The most general vacuum solution [Almheiri-Polchinski, 1402.6334]

$$\mathrm{d}^2 s = \bar{g}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -2\mathrm{e}^{2\bar{\omega}}\mathrm{d}x^+\mathrm{d}x^- \qquad \text{(conformal gauge)}$$

$$\mathrm{e}^{2\bar{\omega}} = \frac{2\,L^2}{(x^+ - x^-)^2}\,, \qquad \bar{\phi} = \frac{L^2}{2}\left(\Lambda + \frac{a + b\,(x^+ + x^-) + c\,x^+x^-}{x^+ - x^-}\right)$$

where *a, b, c* are arbitrary constants.

The remaining task is to derive a consistent dilaton fluctuation σ .

$$\boxed{\sigma(x^+,x^-) \equiv \frac{M(x^+,x^-)}{x^+ - x^-}}$$

The general solution *M*

[Ishii-Okumura-Sakamoto-KY, 1906.03865]

$$M(x^+,x^-) = \underbrace{I_0(x^+,x^-)}_{\text{Local part}} + \underbrace{I^+(x^+,x^-) - I^-(x^+,x^-)}_{\text{Non-local part}}$$

$$I_0(x^+,x^-) \equiv A + B\,(x^+ + x^-) + C\,x^+x^- \,, \qquad A,\ B,\ C\text{: arbitrary real consts.,}$$

$$I^\pm(x^+,x^-) \equiv 8\pi G_N \int_{u^\pm}^{x^\pm} \mathrm{d}s\,(s - x^+)(s - x^-)\,\mathcal{T}_\pm(s)$$

Here

$$\mathcal{T}_\pm(x^\pm) \equiv (1 \mp b\,k \mp c\,k\,x^\pm)\,t_{\pm\pm} \mp \frac{k}{4}\,(a + 2b\,x^\pm + c\,(x^\pm)^2)\,\partial_\pm t_{\pm\pm}$$

where *a, b, c* are the constants contained in the background dilation.

Some remarks:

The dilaton fluctuation σ satisfies the following simple differential equation.

$$\partial_+ \partial_- \sigma + \frac{2\,\sigma}{(x^+ - x^-)^2} = 0$$

This is really a familiar equation in the study of AdS_2, and it is solved by employing hypergeometric functions or Gegenbauer polynomials. These are local solutions.

But here we have found out a new non-local solution to this simple differential equation.

This solution may play an important role in the context of (N)AdS2/(N)CFT1.

Note here that we have assumed the conformal matter, so $T\overline{T}$-flow type interpretation is not applicable. In addition, the coefficients of $T\overline{T}$-part contain the background dilaton and hence depend on the space-time coordinates.

Towards removing the conformal matter condition

[Ishii-Okumura-Sakamoto-KY, in progress]

A possible resolution

Consider a deformation depending on dilaton ϕ

Let us discuss the $T\overline{T}$-deformed massless scalar theory

$$S_{\mathrm{m}}^{(\alpha)} = \int \mathrm{d}^2x\,\sqrt{-g}\,\frac{1}{2\alpha(\phi)}\left(\sqrt{1-2\alpha(\phi)\,g^{\mu\nu}\nabla_\mu\psi\nabla_\nu\psi}-1\right)$$

Then, under the condition $U''(\phi)=0$, the e.o.m. of the metric fluctuation can be solved without using the conformal matter condition.

For example, for $U(\phi)=\Lambda-\frac{2}{L^2}\phi$, the deformation may be measured by

$$\boxed{\alpha(\phi)=\kappa\exp\left(\frac{2k}{L}\phi\right)}$$

and the conformal matter condition can be removed.

3. Summary and Outlook

Summary

Relation between gravitational perturbation and $T\overline{T}$-deformation of 2D QFT

In the flat-space JT gravity, this is well established. [Dubovsky-Gorbenko-Mirbabayi, 1706.06604]

In this talk, we have tried to consider a generalization to curved spaces.

In the AP model with conformal matter, gravitational perturbation can be seen as $T\overline{T}$-like deformation (with the RG-type interpretation). [Ishii-Okumura-Sakamoto-KY, 1906.03865]

Anyway, a linear perturbation in the AP model has been solved exactly.
In particular, the dilaton fluctuation takes an impressive form.

Outlook

Interpretation of the background dilaton dependence? [Ishii-Okumura-Sakamoto-KY, in progress]

$T\overline{T}$-deformation of 2D QFT on a curved space? $T\overline{T}$-flow equation on a curved space?

Outlook 2

Other directions to study $T\overline{T}$-deformation apart from the JT gravity.

- $T\overline{T}$-deformation in general dims.

  Some proposals

  $$\left(\det T\right)^{1/(D-1)} \quad , \quad T^{ij}T_{ij} - \frac{1}{D-1}T_i^i T_j^j$$

  Problem in regularization

  [Cardy, 1801.06895]
  [Bonelli-Doround-Zhu, 1804.10967]

  [M. Taylor, 1805.10287]
  [T. Hartman-Kruthoff-Shaghoulian-Tajdini, 1807.11401]

- $J\overline{T}$ -deformation of 2D CFT and holography

  [Guica, 1710.08415] [Bzowksi-Guica, 1803.09753]
  [Apolo-Song, 1806.10127, 1907.03745]
  [Chakraborty-Giveon-Kutasov, 1806.09667, 1905.00051]
  [Aharony-Datta-Giveon-Jiang-Kutasov, 1808.08978] and more.

- $T\overline{T}$-deformation of non-Lorentz invariant theory [Cardy, 1809.07849]

  (with non-symmetric energy-momentum tensor, $T_{\mu\nu} \neq T_{\nu\mu}$)

- $T\overline{T}$-deformation with SUSY

  [Baggio-Sfondrini-Tartaglino Mazzucchelli-Walsh, 1811.00533]
  [Jiang-Sfondrini-Tartaglino Mazzucchelli, 1904.04760]
  [Chang-Ferko-Sethi-Sfondrini-Tartaglino Mazzucchelli, 1906.00467]

- $T\overline{T}$-deformation of dS/dS [Gorbenko-Silverstein-Torroba, 1811.07965]

- Relation between $T\overline{T}$-deformation for single trace ($\alpha > 0$) and Yang-Baxter deformation

[Araujo-O Colgain-Sakatani-Sheikh Jabbari-Yavartanoo, 1811.03050] [Borsato-Wulff, 1812.07287]

- Entanglement Entropy in the cut-off AdS case

[Chakraborty-Giveon-Itzhaki-Kutasov, 1805.06286] [Donnelly-Shyam, 1806.07444]
[Chen-Chen-Hao, 1807.08293] [Park, 1812.00545] [Sun-Sun, 1901.08796] [Caputa-Datta-Shyam, 1902.10893]
[Banerjee-Bhattacharyya-Chakraborty, 1904.00716] [Ota, 1904.06930] [Jeong-Kim-Nishida, 1906.03894]
[Murdia-Nomura-Rath-Salzetta, 1907.12603] [Chen-Chen-Zhang, 1907.12110] [He-Shu, 1907.12603]
and more

The Ryu-Takayanagi formula works well even for $T\overline{T}$-deformed case.

Thank you!

Backup

A comment on $T\overline{T}$-deformation of 2D IQFT

Indeed, the $T\overline{T}$-deformation is really integrable deformation of relativistic IQFT_2 .

In relativistic IQFT_2 , it is well known that the *N*-body S-matrix is factorized to the product of the 2-body S-matrices,

$$S(p_1, p_2, \ldots, p_N) = \prod_{i<j} S(p_i, p_j)$$ ➡ Quantum Integrability

Then the 2-body S-matrix can be determined from the assumptions, Lorentz symmetry, unitarity, crossing symmetry, Yang-Baxter eq. (S-matrix bootstrap)

$$S(\theta) \sim S(\theta) f(\theta)$$ up to the CDD factor $f(\theta)$

[Castilejo-Dalitz-Dyson, Phys. Rev. 101 (1956) 453]

<u>FACT</u>

The $T\overline{T}$-deformation deforms only the CDD factor. [Mussardo-Simon, hep-th/9903072]

➡ The quantum integrability is preserved (integrable deformation).

Why is the signature so significant?

You may wonder why the signature of the coupling should be significant.

Consider a Φ^4 theory in four dimensions, for example.

Then deform this system by adding a term to the original action,

$$-\alpha \int \mathrm{d}^4 x \, \Phi^6 \qquad .$$

If $\alpha > 0$, then the potential is still bounded and the vacuum is stable.

But, if $\alpha < 0$, then the potential is not bounded any more
and the vacuum becomes unstable.

Thus, the signature of irrelevant perturbation is significant to physics.

Two types of $T\overline{T}$-like operators

[Giveon-Itzhaki-Kutasov, 1701.05576]

According to the product structure, one may consider two types of $T\overline{T}$-like operators:

1) Double Trace

$$\left(\sum_{i=1}^{p} T_i\right)\left(\sum_{i=1}^{p} \overline{T}_i\right)$$

This corresponds to the usual $T\overline{T}$-deformation.

This typically leads to a non-local deformation of the string world-sheet.

2) Single Trace

$$\sum_{i=1}^{p} T_i \overline{T}_i$$

This leads to a current-current deformation of the string world-sheet.

$J\bar{J}$-deformation of the world-sheet (well known)

A relevant perspective of this duality

At large *p*, the boundary CFT has the form of a symmetric product,

$$\mathcal{M}^p/S_p \quad , \qquad S_p \text{ : symmetric group}$$

[Argurio-Giveon-Shomer, hep-th/0009242]

[Giveon-Kutasov, 1510.08872]

Each $\mathcal{M}$ is a CFT with central charge $\boxed{c_{\mathcal{M}} = 6k}$.

Thus, the total central charge is $\boxed{c_{\text{tot}} = p\, c_{\mathcal{M}} = 6\, p\, k}$.

Roughly speaking, $\mathcal{M}$ can be regarded as the CFT associated with a single F-string. The above structure relies on the fact at large *p* the interaction between the *p* strings in the background goes to zero,

$$\boxed{g_s^2 \sim 1/p}$$