Impossible anomalies in CFT

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CFT saga

- Conformal symmetry is very powerful to understand critical phenomena (in any dimensions e.g. by conformal bootstrap)
- Conformal symmetry determines the form of correlation functions
- For example, we learn two and three-point functions are completely fixed
- The condition is same in d=2 and not stronger (because local part of Virasoro is always spontaneously broken so there is no further constraint)

Something you might not know

Myth: CFT two-point functions of primary operators are non-zero only when conformal dimensions are same

$$\langle O_1(x)O_2(y)\rangle = \frac{c_{12}}{(x-y)^{\Delta_1+\Delta_2}}\delta_{\Delta_1,\Delta_2}$$

Reality: they can be non-zero in contact terms

$$\langle O_1(x)O_2(y)\rangle = c_{12}\delta(x-y) \qquad \Delta_1+\Delta_2=d$$

More complicated with derivatives or in semi-local terms

$$\partial^2\delta(x-y)\frac{1}{(y-z)^{\Delta}}$$

Contact terms?

- Why do we (have to) care?
- One may eliminate them by local counter-terms (unless symmetry forbids them → unambiguous observables!)
- WT-identities and anomalies are all in this category (chiral anomaly, trace anomaly, shortening anomaly…) $\langle \partial^{\mu} J_{\mu}(x) O_1(x_1) \cdots \rangle = \delta(x - x_1) \langle \delta O_1(x_1) \cdots \rangle$
- Sometimes they are observables (Chern-Simons contact terms)
- Applications to de-Sitter cosmology(?) where the contact terms may dominate the amplitudes (in momentum space)

Conformal invariance of contact terms or semi-local terms

- A sample questions: When are these correlation functions conformal invariant?

$$\langle O_1(x)O_2(y)\rangle = c_{12}\delta(x-y)$$

$$\langle O_1(x)O_2(y)O_3(z)\rangle = \delta(x-y)\frac{1}{(y-z)^\Delta}$$

- Of course, we can check it by solving conformal WT eq, but…

- Better idea: use embedding space formalism (d-dim conf = d+2 dim Lorentz: SO(d+1,1))
- But delta function is non-trivial

$$\delta^{(d+2)}(X_1-X_2) \neq \delta^{(d)}(x_1-x_2)$$

- I developed embedding space delta functions

$$\delta_k^{(d)}(X,Y) = \int_{-\infty}^{\infty}\frac{ds}{s^{k+1}}\left(\int_{-\infty}^{\infty} d(R^2)\delta^{(D)}(X-sY)\right)|_{X^2=Y^2=0}\ .$$

$$R^2 = \eta_{MN}X^M X^N$$

Application to impossible anomaly

A debate on trace anomaly in d=4

$$T^\mu_\mu = c\mathrm{Weyl}^2 - a\mathrm{Euler} + bR^2 + d\Box R + e\mathrm{Pontryagin}$$

- a, c, d and e are all consistent (Wess-Zumino condition)
- b is not consistent
- d is trivial (i.e. can be removed by local counterterms)
- While we were chatting when we were students, Yuji Tachikawa suggested the possibility of e.

$$\mathrm{Pontryagin} = \epsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta\mu\nu} R^{\mu\nu}{}_{\gamma\delta}$$

- Pontryagin term breaks CP (P as well as T)
- When I wrote a paper in 2012, I concluded that there is no known example, but it could be non-perturbatively generated (“everything that can happen do happen”)

One loop Pontryagin trace anomaly?

- Long time ago, Christensen and Duff (1978) computed the Seeley De-Witt B coefficient for a Euclidean Weyl fermion (i.e. (1/2,0) of SO(4)) via heat kernel method

$$b_4(D^\dagger D) = \frac{1}{180}\left(\frac{11}{4}\mathrm{Euler} - \frac{9}{2}\mathrm{Weyl}^2 + \frac{15}{4}\mathrm{Pontryagin}\right)$$

- Does this mean the existence of Pontryagin trace anomaly for a Euclidean Weyl fermion?
- Bonora et al argued that the same value (in Minkowski space) can be obtained from the direct 1-loop computation of $\langle T^\mu_\mu(x) T_{\alpha\beta}(y) T_{\rho\sigma}(z)\rangle$

  1403.2606, 1503.03326, 1703.10473, 1807.01249

- Others say computation by Bonora et al is not correct

  Bastianelli Martelli Broccoli, Frob Zahn

▪ Bonora et al claims back that the regularization used by these people are incorrect (Weyl vs Majorana) 1909.11991

- Controversial?

"Impossible" anomaly

- The name may be misleading, but there are two distinct anomalies.
- Conventional anomaly: anomaly eq is semi-local (or contact term), but the parent correlation functions are non-local

$$\langle \partial^\mu J_\mu(x) J_\alpha(y) J_\beta(z) \rangle = \epsilon_{\alpha\beta\rho\sigma} \partial^\rho \partial^\sigma \delta(x-y)\delta(y-z)$$

$$\langle J_\mu(x) J_\alpha(y) J_\beta(z) \rangle = \text{non-local in } \ x, \ y, \ z$$

- Impossible anomaly: anomaly eq is semi-local but the parent correlation functions are not non-local (with conformal invariance)

$$\langle \partial^\mu J^B_\mu J_\nu J_\rho \rangle = \delta_{\nu\rho} \partial^\alpha \delta^4(x-y) \partial_\alpha \delta^4(x-z) - \partial_\nu \delta^4(x-y) \partial_\rho \delta^4(x-z)$$

$$\langle J^B_\mu J_\nu J_\rho \rangle = \delta(k+p+q) \frac{k_\mu}{k^2} (q_\mu p_\nu - pq\delta_{\mu\nu})$$

"Impossibility" of Pontryagin trace anomaly

- Consider three-point functions $\langle T_{\mu\nu}(x) T_{\alpha\beta}(y) T_{\rho\sigma}(z)\rangle$
- Pontryagin trace anomaly is related to

$$\langle T^{\mu}_{\mu}(x) T_{\sigma\rho}(y) T_{\alpha\beta}(z)\rangle = \epsilon_{\sigma\alpha\epsilon\kappa}((\partial_\beta\partial_\rho - \partial^2\delta_{\beta\rho})(\partial^\epsilon\delta(x-y)\partial^\kappa\delta(x-y))$$

and this contact term is conformal invariant

- By using various techniques (such as embedding space formalism), we can show there is no parity odd non-local terms in d=4 (Zhiboedov et al)

- The best we could do is semi-local terms

“Possibility” of Pontryagin trace anomaly

- It is NOT obvious if the anomaly coming from the entire contact terms are unphysical
- It is similar to the parity violating contact terms in EM tensor two-point functions of Maldacena and Pimentel (They hope they will find it in the sky)

$$\langle T_{\mu\nu}(x)T_{\alpha\beta}(y)\rangle = \epsilon_{\mu\alpha\sigma}(\partial_\nu\partial_\beta - \eta_{\nu\beta}\partial^2)\partial^\sigma\delta(x-y)$$

- At least the (dilaton) effective action exists

$$S = \int d^4x\sqrt{g}\left(\frac{1}{2}\phi\Delta_4\phi - Q\mathcal{Q}\phi - \phi\epsilon^{\alpha\beta\gamma\delta}R_{\alpha\beta\mu\nu}R^{\mu\nu}{}_{\gamma\delta}\right) ,$$

- The existence of the effective action really means that the Pontryagin trace anomaly is a consistent anomaly
- Euclidean Weyl fermion (i.e. (1/2,0) alone) does not have proper energy-momentum tensor, so the physical meaning of heat kernel is unclear…

Summary

- Contact terms in CFT may be physically important/interesting
- Embedding space formalism may be useful
- Application to de-Sitter cosmology?

- Pontryagin trace anomaly?
- SUSY generalization of Pontryagin anomaly is possible (with Nakagawa)
- The Pontryagin trace anomaly → central charge “c” is complexified.
- It also predicts the R-current trace anomaly (!?)

$$T^\mu_\mu = \tilde{e} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

SUSY Pontryagin trace anomaly

- Consider Super Weyl transformation with the super Weyl parameter given by a chiral superfield σ
- The super Weyl variation in the superpotential

$$\int d^4x d^2\theta c\sigma W_{\alpha\beta\gamma} W^{\alpha\beta\gamma}$$

- This c cane be a complex number (while a in Kahler potential must be real)
- Real parti → Usual Weyl^2 term in Weyl anomaly
- Imaginary part → Pontryagin trace anomaly

- Seems consistent but no known examples…

“Impossible” anomaly is not always impossible

- Consider d=2 CFT with the anomalous current conservation

$$\partial^\mu J_\mu = R$$

- This means that the current-energy-momentum two-point function has the divergence of

$$\langle \bar{\partial} J(x) T(y) \rangle = \partial^2 \delta(x-y)$$

- But the parent current-EM tensor two-point function cannot be conformal invariant (impossible anomaly)

$$\langle J(z_1) T(z_2) \rangle = \frac{1}{(z_1 - z_2)^3}$$

- But we do know it exists (i.e. ghost number current)
- A resolution here is ghost current is not primary