On N=1 Superconformal Currents

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Preamble

Mostly based on:

- Two papers by Callan, Coleman and Jackiw (CCJ):
  - A new improved energy-momentum tensor, Ann. Phys. 59 (1970) 42 (CCJ)
  - Why dilatation generators do not generate dilatations?, Ann. Phys. 67 (1971) 552 (Coleman, Jackiw)

  CCJ tensor, existence condition, show its existence for renormalisable *lagrangians* and for *one real* scalar, anomalous dimensions.

- Wess, Zumino, Supergauge transformations in four dimensions, Nucl. Phys 70 (1974) 39:
  $N = 1$ *superconformal* (and then super-Poincaré) theories (linear susy).

- Ferrara, Zumino, Transformation properties of the supercurrent, Nucl. Phys. B87 (1975) 207:
  Supercurrent superfield, and its superfield conservation equation.

- But also many more . . . (incl. Arnold, Hartong, JPD 1208.1648)

Preliminaries

Contents:

- The $\mathcal{N} = 0$ case (scalar theories): Poincaré, dilatation, conformal

  Condition for scale $\Longrightarrow$ conformal (scalar theories, lagrangian level)

  A bosonic R–symmetry

- The $\mathcal{N} = 1$ case: global superconformal symmetry

  Current structure, supercurrents Q_μ and S_μ, improvements

  Obtaining $\partial^\mu S_\mu = \gamma^\mu Q_\mu$

- Currents in supermultiplets:

  “Supercurrent structures", superfield improvements

  The case of scale-invariant non-conformal theories

  Bosonic to superconformal $U(1)_R$ symmetry

Of course, $D = 4$ only

Preliminaries

- Currents are important and ambiguous quantities. They can be "improved"

$$j_\mu \to j_\mu + \partial^\nu \sigma_{\mu\nu} = \widetilde{j}_\mu \qquad \sigma_{\mu\nu} = -\sigma_{\nu\mu} \qquad Q = \int d^3x\, j_0 = \widetilde{Q}$$

  Energy-momentum tensor :

$$T_{\mu\nu} \to T_{\mu\nu} + \partial^\rho \chi_{\mu\rho\nu} = \widetilde{T}_{\mu\nu} \qquad \chi_{\mu\rho\nu} = -\chi_{\rho\mu\nu} \qquad P_\nu = \widetilde{P}_\nu$$

  Same physics.
  Any preferred choice, for a given transformation of fields/coordinates ??
  Physics will not choose . . . (Other requirements may suggest . . .)

- The interplay $T_{\mu\nu}$ — dilatation current j^D_μ is (should be) well-known.
- Under superconformal transformations:
  Who is in supermultiplets ? Current and source superfields ?
  Superfield current improvements, conservation equation ?
- *(Old) literature often superficial or imprecise*

Poincaré and scale / dilatations

- A closed algebra, $[D, P_\mu] = -P_\mu$, $[D, M_{\mu\nu}] = 0$.
- On coordinates: $\delta x^\mu = c^\mu - \lambda_D x^\mu$
- Noether variation (on-shell variation): $(c^\nu - \lambda_D x^\nu)T_{\mu\nu}$
  Conserved energy-momentum tensor defined up to $\delta T_{\mu\nu} = \langle V\rangle \eta_{\mu\nu}$
  And $\langle V\rangle$ is unobservable (but useful ...).
- Suggests for curents: $j^D_\mu = x^\nu T_{\mu\nu}$ $\quad \partial^\mu j^D_\mu = T^\mu{}_\mu$
- But there are supplementary terms: scale variations of fields
- Generators: $[D, P_\mu] = P_\mu$ $\quad P_\mu = \partial_\mu,$ $\quad D = x^\mu\partial_\mu + \mathcal{D}$
  $\mathcal{D}$: matrix of scale dimensions w_i of the fields
- Then: $j^D_\mu = \mathcal{V}_\mu + x^\nu T_{\mu\nu}$ $\quad \partial^\mu j^D_\mu = \partial^\mu \mathcal{V}_\mu + T^\mu{}_\mu$

  With Noether energy-momentum tensor

$$\mathcal{V}_\mu = \sum_i w_i \frac{\partial \mathcal{L}}{\partial^\mu \phi_i}\phi_i \qquad \mathcal{V}_\mu\text{: “virial current"}$$

Poincaré and scale / dilatations

- If $T_{\mu\nu}$ can be improved to $\Theta_{\mu\nu}$ such that $\widetilde{j}^{D}_{\mu} = x^{\nu}\Theta_{\mu\nu}$ one has found the CCJ energy-momentum tensor.
- Then, if $\partial^{\mu} j^{D}_{\mu} = \Theta^{\mu}{}_{\mu} = 0$ (scale invariance), the theory is conformal.
- For theories with scalar fields, the CCJ tensor is not the Belinfante tensor (symmetric, derivable from a formally introduced space-time metric)
- CCJ showed: for all renormalizable lagrangians and single scalar two-derivative theories, $\Theta_{\mu\nu}$ exists.
- But starting with two real scalars, many scale-invariant lagrangians are not conformal invariant (usually non-unitary repres., live in broken phase)
- A simple toy-model: $\mathcal{L} = (z + \overline{z})(\partial_{\mu} z)(\partial^{\mu}\overline{z})$

  Scale-invariant, $w = 2/3$, not conformal, kählerian, derives from

  $$K = \frac{1}{2}\left(z^2\overline{z} + \overline{z}^2 z\right) = \frac{1}{6}\left(z + \overline{z}\right)^3 + \text{Kähler transformation}$$

  Can be supersymmetrized . . . Look at superconformal currents ?

Conformal transformations

Poincaré $+$ *scale* $+$ *conformal boosts* K^a

On coordinates

$$\delta x^\mu = -\xi^\mu \quad = \quad c^\mu + \omega^{\mu\nu} x_\nu - \lambda_D x^\mu + a_\nu (2x^\mu x^\nu - \eta^{\mu\nu} x^2)$$

$$= \quad [c^\nu + \omega^{\nu\rho} x_\rho - (\lambda_D - 2ax) x^\nu - a^\nu x^2] \partial_\nu x^\mu$$

$\lambda_D - 2ax$: Conformal factor

Expect for conformal currents

$$K^\rho_\mu = 2x^\rho j^D_\mu - x^2 T_\mu{}^\rho = 2x^\rho \mathcal{V}_\mu + (2x^\rho x^\nu - x^2 \eta^{\rho\nu}) T_{\mu\nu}$$

(for the symmetric Belinfante $T_{\mu\nu}$ and the associated virial)

$$\partial^\mu K^\rho_\mu = 2\mathcal{V}^\rho + 2\, x^\rho (\partial_\mu \mathcal{V}^\mu + T^\mu{}_\mu) + 2\, x_\nu (T^{\rho\nu} - T^{\nu\rho})$$

If $\mathcal{V}_\mu = 0$, $T_{\mu\nu} = \Theta_{\mu\nu}$: $\quad \Theta^\mu{}_\mu = 0$ implies scale and conformal invariance.

- Usually: fields transformations for Lorentz and dilatations, not for translations or conformal boosts.

Scale $\Longrightarrow$ conformal: a bosonic R–symmetry

- CCJ also found: if the virial current is $\boxed{\mathcal{V}_\mu = \partial^\nu \sigma_{\mu\nu}}$

  an improvement to $\Theta_{\mu\nu}$ with related $\mathcal{V}_\mu = 0$ exists.

  And then:

  Scale invariance $\quad 0 = \Theta^\mu{}_\mu = \partial^\mu \mathcal{V}_\mu + T^\mu{}_\mu \quad$ implies

  $$\partial^\mu \overline{K}^\rho_\mu = \partial^\mu (K^\rho_\mu - 2\,\sigma^\rho{}_\mu) = 0 \quad : \quad \text{conformal invariance}$$

- Two-derivative scalar theories: $\quad \sigma_{\mu\nu} = \eta_{\mu\nu} f(\phi) \qquad \mathcal{V}_\mu = \partial_\mu f(\phi)$

  $\rightarrow \mathcal{V}_\mu$ needs to be “pure gauge", $\quad \partial_{[\mu} \mathcal{V}_{\nu]} = 0$

- Not verified for complex z with Kähler potential $K = \frac{1}{2}(z^2\overline{z} + \overline{z}^2 z)$.

- Or for instance by any $K(z+\overline{z})$ for which $\partial_{[\mu}\mathcal{V}_{\nu]} \sim K'''(z-\overline{z})$

Scale $\Longrightarrow$ conformal: a bosonic R–symmetry

For a kählerian model (with supersymmetry to follow), a curious characterization.

$$\mathcal{L} = K_{m\overline{n}}(z,\overline{z})\,(\partial_\mu z^m)(\partial^\mu \overline{z}^{\overline{n}}) \qquad\qquad K_{m\overline{n}} = \frac{\partial^2 K}{\partial z^m \partial \overline{z}^{\overline{n}}}$$

the virial current is (with assigned scale dimensions w_n)

$$\mathcal{V}_\mu = \sum_{m,\overline{n}} K_{m\overline{n}}[w_m z^m(\partial_\mu \overline{z}^{\overline{n}}) + w_n \overline{z}^{\overline{n}}(\partial_\mu z^m)] \qquad\qquad \text{(Belinfante)}$$

The CCJ tensor exists if $\boxed{\sum_m K_{mn\overline{p}} w_m z^m = \sum_{\overline{m}} K_{n\overline{pm}} w_m \overline{z}^{\overline{m}}}$

$\Rightarrow$ The Kähler potential (only, a potential may break it) must have a $U(1)$ symmetry with charges w_m: $\boxed{w_m K_m z^m = w_m K_{\overline{m}} \overline{z}^{\overline{m}}}$

"Bosonic R–symmetry"

Superconformal transformations

- All multiplets of (global) $N = 1$ Poincaré supersymmetry are multiplets of the $N = 1$ superconformal algebra
- Needed: assign scale dimensions w and $U(1)_R$ charges q to the multiplets (with constraints on q like $w = q$ for chiral multiplets, as in the earlier discussion ...).
- New in the algebra: $\mathcal{Q}$ and $\mathcal{S}$ supersymmetries

$$[\delta_\mathcal{Q}, \delta_\mathcal{Q}] = \frac{1}{2}\overline{\epsilon}_2\gamma^a\epsilon_1\, P_a, \qquad [\delta_\mathcal{S}, \delta_\mathcal{S}] = \frac{1}{2}\overline{\eta}_2\gamma^a\eta_1\, K_a,$$

$$[\delta_\mathcal{Q}, \delta_\mathcal{S}] = -\frac{1}{2}\overline{\eta}\epsilon\, D + \frac{i}{2}\,\overline{\eta}\gamma_5\epsilon\, T + \frac{1}{8}\,\overline{\eta}[\gamma^a, \gamma^b]\epsilon\, M_{ab}.$$

- In variations, replace the parameter ϵ of $\mathcal{Q}$ supersymmetry:

$$\epsilon \quad , \quad \overline{\epsilon} \qquad \longrightarrow \qquad \alpha = \epsilon + x_a\gamma^a\eta \quad , \quad \overline{\alpha} = \overline{\epsilon} - \overline{\eta}\gamma^a x_a$$

η: the constant spinor parameter of $\mathcal{S}$ supersymmetry (WZ, 74)
Some explicit $\mathcal{S}$ supersymmetry variations are needed.

Superconformal transformations

- Conformal: $\delta x^\mu = -\xi^\mu$ $\quad\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \frac{1}{2}\eta_{\mu\nu}\partial_\rho \xi^\rho$
- Conformal Killing spinor equation: $(\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu)\alpha = \frac{1}{2}\eta_{\mu\nu}\gamma^\rho \partial_\rho \alpha$

  Solution: $\alpha = \epsilon + x^\mu \gamma_\mu \eta$

  $\xi^\mu \sim \overline{\alpha}_2 \gamma^\mu \alpha_1 = -\overline{\alpha}_1 \gamma^\mu \alpha_2$: a conformal Killing vector

  The superconformal algebra $[\delta_1, \delta_2]$ is easily verified.
- But since fields are assigned w and q weights, some η-explicit terms are needed in variations: $D \to w$, $\quad T \to q$ in

$$[\delta_{\mathcal{Q}}, \delta_{\mathcal{S}}] = -\frac{1}{2}\overline{\eta}\epsilon\, D + \frac{i}{2}\,\overline{\eta}\gamma_5\epsilon\, T + \frac{1}{8}\,\overline{\eta}[\gamma^a, \gamma^b]\epsilon\, M_{ab}.$$

  See the case of chiral multiplets, later on.

Superconformal currents

Current structure:

- Bosonic: all conformal currents and the chiral $U(1)_R$ current
- Fermionic: supercurrents $Q_{\mu\alpha}$ and $S_{\mu\alpha}$ for $\mathcal{Q}$ and $\mathcal{S}$ supersymmetries.

Since $\mathcal{Q}$ variation parameter is now in $\alpha = \epsilon - x^\mu\gamma_\mu\eta$, expect

$$S_\mu = x^\rho\gamma_\rho Q_\mu, \qquad \partial^\mu S_\mu = \gamma^\mu S_\mu$$

in a super-Poincaré theory ($\partial^\mu Q_\mu = 0$)

- But since there are explicit variations in η,

$$\boxed{S_\mu = x^\rho\gamma_\rho Q_\mu + \Delta S_\mu} \qquad [S_\mu] = [\Delta S_\mu] = 5/2 \quad [Q_\mu] = 7/2$$

Analogy with the virial current in the dilatation current.

Can one improve Q_μ to eliminate Δ_μ ? Yes (no condition)

Supercurrents: arbitrary Kähler potential

Generic lagrangian, one chiral superfield Φ, extension to many is easy:

$$\mathcal{L} = \mathcal{L}_K + \mathcal{L}_W$$

$$\mathcal{L}_K = \int d^2\theta d^2\overline{\theta}\, K(\Phi, \overline{\Phi}) \qquad \mathcal{L}_W = \int d^2\theta\, W(\Phi) + \int d^2\overline{\theta}\, \overline{W}(\overline{\Phi})$$

Apply superconformal variations $\qquad (\alpha = \epsilon + x^\mu \gamma_\mu \eta \quad \overline{\alpha} = \overline{\epsilon} - x^\mu \overline{\eta} \gamma_\mu)$

$$\delta\, z = \frac{1}{\sqrt{2}} \overline{\alpha} \psi_L \qquad \delta\, \psi_L = \frac{1}{\sqrt{2}} (f \alpha_L + \gamma^\mu \partial_\mu z\, \alpha_R) + \sqrt{2}\, wz\, \eta_L$$

$$\delta\, f = \frac{1}{\sqrt{2}} \overline{\alpha} \gamma^\mu \partial_\mu \psi_L - \sqrt{2}(w-1) \overline{\eta} \psi_L$$

- (scale dimension , $U(1)_R$ charge) are:

  (w, w) for z, $\qquad (w + \frac{1}{2}, w - \frac{3}{2})$ for ψ_L, $\qquad (w_1, w - 3)$ for f

Supercurrents: arbitrary Kähler potential

Superconformal variation of K term:

$$
\begin{aligned}
\delta\,\mathcal{L}_K \quad = \quad & -\frac{1}{2\sqrt{2}}\,\partial_\mu\Big[K_{z\bar{z}}\Big(f\,\overline{\psi}\gamma_\mu\alpha_L + \partial_\nu z\,\overline{\psi}\gamma^\nu\gamma^\mu\alpha_R + \text{h.c.}\Big) \\
& \qquad -\frac{1}{2}\left[K_{zz\bar{z}}\,(\overline{\psi}\psi_L)(\overline{\psi}\gamma^\mu\alpha_L) + K_{z\bar{z}\bar{z}}\,(\overline{\psi}\psi_R)(\overline{\psi}\gamma^\mu\alpha_R)\right] \\
& \qquad +2w\,K_{z\bar{z}}\,(z\,\overline{\eta}\gamma^\mu\psi_R + \bar{z}\,\overline{\eta}\gamma^\mu\psi_L)\Big] \\
& +\sqrt{2}\,[wzK_{zz\bar{z}} + (w-1)K_{z\bar{z}}]\Big(\overline{\eta}\gamma^\mu\psi_R\,\partial_\mu z - \overline{f}\,\overline{\eta}\psi_L\Big) \\
& +\sqrt{2}\,[w\bar{z}K_{z\bar{z}\bar{z}} + (w-1)K_{z\bar{z}}]\Big(\overline{\eta}\gamma^\mu\psi_L\,\partial_\mu\bar{z} - f\,\overline{\eta}\psi_R\Big) \\
& +\frac{1}{\sqrt{2}}\,[(w-1)K_{zz\bar{z}} + w\bar{z}K_{zz\bar{z}\bar{z}}](\overline{\psi}\psi_L)(\overline{\eta}\psi_R) \\
& +\frac{1}{\sqrt{2}}\,[(w-1)K_{z\bar{z}\bar{z}} + wzK_{zz\bar{z}\bar{z}}](\overline{\psi}\psi_R)(\overline{\eta}\psi_L)
\end{aligned}
$$

Some results for arbitrary Kähler potential

Superconformal variation of W term:

$$\delta\,\mathcal{L}_W = \frac{1}{\sqrt{2}}\partial_\mu\Big[W_z\,\overline{\psi}\gamma^\mu\alpha_R + \overline{W}_{\overline{z}}\,\overline{\psi}\gamma^\mu\alpha_L\Big] + \sqrt{2}\Big[(w-3)W_z + wzW_{zz}\Big]\overline{\eta}\psi_L + \mathrm{h.c.}$$

All α terms assemble in a derivative (super-Poincaré invariance).

All η terms cancel and the theory is superconformal if three conditions are verified:

$wz\,W_z = 3\,W$	W has scale dimension three
$wz\,K_z + w\overline{z}\,K_{\overline{z}} = 2\,K$	K has scale dimension two
$wz\,K_z - w\overline{z}\,K_{\overline{z}} = 0$	K is R–symmetric

Supercurrents

The supercurrents are (using the algebraic field equation of f):

$$\begin{aligned}\mathcal{Q}_\mu &= -\frac{1}{\sqrt{2}} K_{z\bar{z}}\Big(\partial_\nu z\,\gamma^\nu\gamma_\mu\psi_R + \partial_\nu\bar{z}\,\gamma^\nu\gamma_\mu\psi_L - \gamma_\mu\psi_L\,\overline{f} - \gamma_\mu\psi_R\,f\Big) \\ &\quad -\frac{1}{2\sqrt{2}}\Big(K_{z\bar{z}\bar{z}}\gamma_\mu\psi_L(\overline{\psi}\psi_R) + K_{zz\bar{z}}\gamma_\mu\psi_R(\overline{\psi}\psi_L)\Big) \\ \mathcal{S}_\mu &= x^\nu\gamma_\nu\,\mathcal{Q}_\mu - \sqrt{2}\,w\,K_{z\bar{z}}\,\gamma_\mu\,(z\,\psi_R + \bar{z}\,\psi_L) \;=\; x^\nu\gamma_\nu\,\mathcal{Q}_\mu + \Delta S_\mu\end{aligned}$$

Can we improve the supercurrent $\widehat{\mathcal{Q}}_\mu$ to eliminate $\Delta\mathcal{S}_\mu$?

Improvement of $\mathcal{Q}_\mu$: $\mathcal{Q}_\mu \longrightarrow \mathcal{Q}_\mu + \partial^\nu[\gamma_\mu,\gamma_\nu]\Upsilon \equiv \widetilde{\mathcal{Q}}_\mu$

for some spinor (off-shell) Υ. Then:

$$\begin{aligned}x^\rho\gamma_\rho\mathcal{Q}_\mu &= x^\rho\gamma_\rho\widetilde{\mathcal{Q}}_\mu - x^\rho\gamma_\rho\partial^\nu[\gamma_\mu,\gamma_\nu]\Upsilon \\ &= x^\rho\gamma_\rho\widetilde{\mathcal{Q}}_\mu - 6\,\gamma_\mu\Upsilon - \partial^\nu(x^\rho\gamma_\rho[\gamma_\mu,\gamma_\nu]\Upsilon)\end{aligned} \tag{1}$$

The last contribution is an improvement term: omitted.

Supercurrents

Improved supercurrent $\widetilde{\mathcal{S}}_\mu$:

$$\widetilde{\mathcal{S}}_\mu = x^\rho \gamma_\rho \widetilde{\mathcal{Q}}_\mu - 6\,\gamma_\mu \Upsilon - \sqrt{2}\, w\, K_{z\bar{z}}\, \gamma_\mu\, (z\,\psi_R + \bar{z}\,\psi_L). \tag{2}$$

Taking: $$\boxed{\Upsilon = -\frac{\sqrt{2}}{6}\, w\, K_{z\bar{z}}(z\psi_R + \bar{z}\psi_L)}$$

leads to the expected $\widetilde{\mathcal{S}}_\mu = x^\rho \gamma_\rho \widetilde{\mathcal{Q}}_\mu$ $\qquad \partial^\mu \widetilde{\mathcal{S}}_\mu = \gamma^\mu \widetilde{\mathcal{Q}}_\mu$

- Hence: no obstruction/condition to obtain the structure $\mathcal{S}_\mu = x^\rho \gamma_\rho \widetilde{\mathcal{Q}}_\mu$
- Notice that

$$\boxed{\overline{\alpha}\Upsilon = -\frac{\sqrt{2}}{6}\, w\, K_{z\bar{z}}(z\overline{\alpha}\psi_R + \overline{z\alpha}\psi_L) = -\frac{1}{3}\, w\, K_{z\bar{z}}\delta(z\bar{z})}$$

- If the Kähler potential is R–symmetric: $\quad zK_z = \bar{z}K_{\bar{z}}$, then

$$\boxed{\overline{\alpha}\Upsilon = -\frac{1}{6}\, w\, \delta[zK_z + \bar{z}K_{\bar{z}}]}$$

Supercurrents, results

- If the Kähler potential is R–symmetric:

Use the superfield $\mathcal{G} = -\frac{1}{6}\, w\,[\Phi K_\Phi + \overline{\Phi} K_{\overline{\Phi}}]$ to simultaneously improve:

$$\text{Natural structure} \quad \left[\begin{array}{ccc} U(1)_R,\, w=0 & \longrightarrow & U(1)_R,\, w \\ \text{Belinfante } T_{\mu\nu} & \longrightarrow & \text{CCJ } \Theta_{\mu\nu} \\ Q_\mu,\, S_\mu & \longrightarrow & \widetilde{Q}_\mu,\, \widetilde{S}_\mu \end{array}\right] \quad \text{CCJ structure}$$

All currents in supermultiplets:

- Natural (Belinfante) structure:

  $j_\mu^{R,w=0}$, Q_μ, $T_{\mu\nu}$ are in $J_{\alpha\dot{\alpha}}$ (supercurrent superfield)

  $\mathcal{V}_\mu$, ΔS_μ (and the improvement from $T_{\mu\nu}$ to $\Theta_{\mu\nu}$) are in $\mathcal{G}$.

- CCJ structure:

  $j_\mu^{R,w}$, $\widetilde{Q}_\mu$, $\Theta_{\mu\nu}$ are in $\widetilde{J}_{\alpha\dot{\alpha}}$

Supercurrents, results

- If K is not R–symmetric and the CCJ tensor does not exist:

  The natural structure certainly exists (in a supercurrent superfield)

  Improved supercurrents not in a supermultiplet.

- If K is scale invariant: $\mathcal{G} = -\frac{1}{3} K$:

  The improvement leads to the Ferrara-Zumino structure.

(W is not at all involved)

Supercurrent superfields

Supercurrent structure: $J_{\alpha\dot{\alpha}}$ (supercurrent superfield),

$X, \chi_\alpha = -\frac{1}{4}\overline{DD}D_\alpha U$ (sources, anomalies) and

$$\boxed{\overline{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = D_\alpha X + \chi_\alpha \qquad \text{On-shell}}$$

$J_{\alpha\dot{\alpha}}$ includes: $U(1)_R$ current (lowest), Q_μ $(\theta, \overline{\theta})$, $T_{\mu\nu}$ $(\theta\sigma^\mu\overline{\theta})$ ...

$D_\alpha X + \chi_\alpha$ induces: $\partial^\mu j^R_\mu$, $\gamma^\mu S_\mu$, $T^\mu{}_\mu$

Superconformal if $X = \chi_\alpha = 0$ (or a superfield improvement)

Sources are such that $\partial^\mu T_{\mu\nu} = \partial^\mu Q_\mu = 0$ (super-Poincaré)

[Ferrara, Zumino, 1974]

Standard superfield improvement formula

$$\boxed{\overline{D}^{\dot{\alpha}}\Big(J_{\alpha\dot{\alpha}} + 2[D_\alpha, \overline{D}_{\dot{\alpha}}]\mathcal{G}\Big) = D_\alpha\Big(X + \overline{DD}\mathcal{G}\Big) + \Big(\chi_\alpha + 3\overline{DD}D_\alpha\mathcal{G}\Big)}$$

An identity with $\mathcal{G}$ real and arbitrary. But: *what are the new currents ?*

Supercurrent superfields

[Arnold, Hartong, JPD]

The superfield improvement improves Q_α and $T_{\mu\nu}$.
But it changes the charges of $U(1)_R$ to *another* $U(1)_R$.

- Natural structure: $U(1)_R$: $q = 0$ on all superfields, always a symmetry of K, anomaly $\propto \frac{3}{2}\left[C(G) - T/R\right]$, j_μ^R is fermionic.

$$\boxed{J_{\alpha\dot{\alpha}} = -2(\overline{D}_{\dot{\alpha}}\overline{\Phi})K_{\Phi\overline{\Phi}}(D_\alpha\Phi) \qquad X = 4W \qquad \chi_\alpha = \overline{D}\overline{D}D_\alpha K}$$

Belinfante $T_{\mu\nu}$, $T^\mu{}_\mu \neq 0$ (even if scale invariant for some w)

- Ferrara-Zumino structure with $\chi_\alpha = 0$: use $\mathcal{G} = -K/3$:

$$J_{\alpha\dot{\alpha}} = -2(\overline{D}_{\dot{\alpha}}\overline{\Phi})K_{\Phi\overline{\Phi}}(D_\alpha\Phi) - \frac{2}{3}[D_\alpha, \overline{D}_{\dot{\alpha}}]K \quad X = 4W - \frac{1}{3}\overline{D}\overline{D}K$$

$U(1)_R$ without physical relevance in general
$X = 0$ in the superconformal case, with $\Phi\overline{D}\overline{D}K_\Phi = \overline{D}\overline{D}K = 4\Phi W_\Phi$

Supercurrent superfields

- Improvement to the CCJ tensor: as already mentioned use

$$\boxed{\mathcal{G} = -\frac{w}{6}(\Phi K_\Phi + \overline{\Phi} K_{\overline{\Phi}})}$$

The $U(1)_R$ with charge $q = w$ is a symmetry of K (does not fix w in general, and scale does not need to be a symmetry).

The particular role of $U(1)_R$ in supersymmetric (algebra-related), but also in scalar theories