Does Boundary Distinguish Complexities?

Yoshiki Sato (佐藤 芳紀)

NCTS in Taiwan (← IPMU in Japan)

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In collaboration with Kento Watanabe (Univ. of Tokyo)

Introduction

"Complexity" plays an important role in AdS/CFT.

CV conjecture: [Susskind '14]

$$\text{complexity} = \frac{V}{G_N L} \qquad (V : \text{co-dim 1 maximum volume}, L : \text{length scale})$$

CA conjecture: [Brown-Roberts-Susskind-Swingle-Zhao '15]

$$\text{complexity} = \frac{I_{\mathrm{WDW}}}{\pi} \qquad (I_{\mathrm{WDW}} : \text{WDW action})$$

Both of conjectures provide similar

- divergence structures
- late time behaviours

$\Longrightarrow$ Basically, two proposals give us almost the same results.

Recently, Chapman *et. al.* argue that [Chapman-Ge-Policastro '18]

defect distinguishes action from volume!!

Gravity computation: AdS$_3$/DCFT$_2$ model [Azeyanagi-Karch-Takayanagi-Thompson '07]

$$\Delta C_{\rm V}^{\rm defect} := C_{\rm V}^{\rm DCFT} - C_{\rm V}^{\rm CFT} \neq 0\,, \qquad \Delta C_{\rm A}^{\rm defect} := C_{\rm A}^{\rm DCFT} - C_{\rm A}^{\rm CFT} = 0$$

CFT computation: Circuit complexities for specific DCFT models vanish,

candidate of definition

$$\Delta C_{\rm circuit}^{\rm defect} := C_{\rm circuit}^{\rm DCFT} - C_{\rm circuit}^{\rm CFT} = 0$$

Their argument: CA complexity is a good holographic dual

$\Longrightarrow$ Does this argument hold in other holographic models or in other definitions of complexity?

Plan of my talk

1. Introduction
2. Path-integral Optimization
3. AdS/BCFT model
   - CV conjecture
   - CA conjecture
4. Conclusion

- Ground state wave functional

$$\Psi^{\mathrm{BCFT}}_{\delta_{ab}}[\tilde{\varphi}(x)] = \int_{\mathcal{M}} \mathcal{D}\varphi \, \mathrm{e}^{-S_{\mathbf{BCFT}}[\varphi]} \prod_{x>0} \delta(\varphi(\epsilon, x) - \tilde{\varphi}(x))$$

- To estimate $\Psi^{\mathrm{BCFT}}_{\delta_{ab}}$ effectively, path integral is redundant since some high-energy degrees of freedom would be suppressed in the deep region of $\mathcal{M}$.

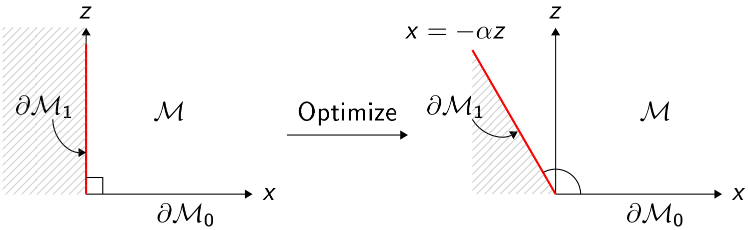
- To reduce such degrees of freedom, we deform the background metric with a boundary condition keeping the wave functional.

- In CFT_2, it can be realized by Weyl transformation,

$$\delta_{ab} \;\to\; \mathrm{e}^{2\phi}\delta_{ab} \quad \Rightarrow \quad \Psi^{\mathrm{BCFT}}_{\mathrm{e}^{2\phi}\delta_{ab}}[\tilde{\varphi}(x)] = \mathrm{e}^{S_{\mathrm{L}}[\phi]-S_{\mathrm{L}}[0]}\, \Psi^{\mathrm{BCFT}}_{\delta_{ab}}[\tilde{\varphi}(x)]$$

- The overall factor reflects how much redundant degrees of freedom (or lattice sites) can be reduced.

Optimized complexity: $C_{\mathrm{L}} = S_{\mathrm{L}}|_{\text{on-shell}}$ where S_{L} is Liouville action



- The path-integral optimization leads the time slice of the Poincaré AdS,

$$ds^2 = L^2 \frac{dz^2 + dx^2}{z^2}$$

with boundary $x = -\alpha z$ in the radial direction.

- This is the same geometry appearing in Takayanagi's AdS/BCFT model.

- Optimized complexity

$$\Delta C_{\mathrm{L}}^{\mathrm{bdy}} = C_{\mathrm{L}}^{\mathrm{BCFT}} - \frac{1}{2} C_{\mathrm{L}}^{\mathrm{CFT}} = \frac{c}{6\pi} \alpha \log\left(\frac{z_\infty}{\epsilon}\right)$$

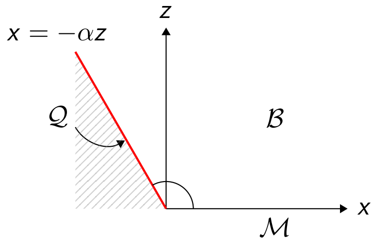
AdS/BCFT model [Takayanagi '11]

- Metric : $\mathrm{d}s^2 = G_{MN}\mathrm{d}X^M\mathrm{d}X^N = L^2\dfrac{\mathrm{d}z^2 + \eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu}{z^2}$
- Action :

$$8\pi G_{\mathrm{N}} I = \frac{1}{2}\int_{\mathcal{B}}\sqrt{-G}\left(R + \frac{d(d-1)}{L^2}\right) + \int_{\mathcal{Q}}\sqrt{-\hat{G}}\,(K - T) + \int_{\mathcal{M}}\sqrt{-\hat{G}}\,K$$

We introduce the boundary $\mathcal{Q}$ with a brane of tension $T = \frac{d-1}{L}\frac{\alpha}{\sqrt{1+\alpha^2}}$.

$\Longrightarrow$ the isometry of the metric is reduced from $SO(2,d)$ to $SO(1,d)$.



CV conjecture [Susskind '14]

$$C_{\mathrm{V}} = \frac{V}{G_{\mathrm{N}} L} \qquad (V : \text{co-dim maximal volume, } L : \text{length scale})$$

The the maximum volume, V, at $t = 0$ is just a $t = 0$ time-slice,

$$V = \int_{\epsilon}^{\infty} \mathrm{d}z \int_{-\alpha z}^{\infty} \mathrm{d}x_1 \int \prod_{i=2}^{d-1} \mathrm{d}x_i \frac{L^d}{z^d}$$
$$= \frac{1}{2} V_{d-1} L^d \int_{\epsilon}^{\infty} \frac{\mathrm{d}z}{z^d} + \alpha \frac{L^d V_{d-2}}{(d-2)\epsilon^{d-2}}$$

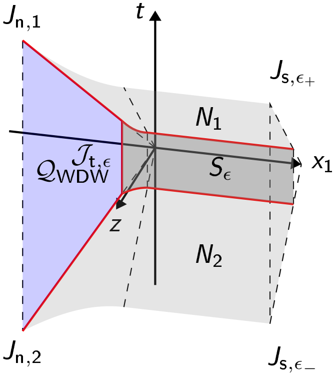
$$\Longrightarrow \Delta C_{\mathrm{V}}^{\mathrm{bdy}} = C_{\mathrm{V}}^{\mathrm{BCFT}} - \frac{1}{2} C_{\mathrm{V}}^{\mathrm{CFT}} = \alpha \frac{L^{d-1} V_{d-2}}{(d-2) G_{\mathrm{N}} \epsilon^{d-2}}$$

CA complexity

CA conjecture [Brown-Roberts-Susskind-Swingle-Zhao '15]

$$C_A = \frac{I_{\mathrm{WDW}}}{\pi}$$

$$\begin{aligned}
8\pi G_N I_{\mathrm{WDW}} = &\frac{1}{2}\int_{\mathcal{B}_{\mathbf{WDW}}} \sqrt{-G}\left(R + \frac{d(d-1)}{L^2}\right) \\
&+ \int_{\mathcal{Q}_{\mathbf{WDW}}} \sqrt{-\hat{G}}\,(K - T) + \sum_{i=\epsilon,\infty} \int_{S_i} \mathrm{d}^d X \sqrt{-\hat{G}}\, K \\
&+ \sum_{i=1}^{2} \epsilon_\kappa \left(\int_{N_i} \mathrm{d}\lambda \mathrm{d}\boldsymbol{x}\, \sqrt{\gamma}\kappa + \int_{N_i} \mathrm{d}\lambda \mathrm{d}\boldsymbol{x}\, \sqrt{\gamma}\Theta \log(\ell_{\mathrm{ct}}|\Theta|) \right) \\
&+ \sum_{J} \epsilon_a \int_J \mathrm{d}^{d-1} X \sqrt{h} a + \sum_{\mathcal{J}} \epsilon_\phi \int_{\mathcal{J}} \mathrm{d}^{d-1} X \sqrt{-h}\phi \,.
\end{aligned}$$



- $\underline{d > 2}$:

$$\Delta C_{\mathsf{A}}^{\mathsf{bdy}} = \frac{L^{d-1} V_{d-2}}{8\pi^2 G_{\mathsf{N}} \epsilon^{d-2}} (d-2) \left(\alpha \sqrt{1+\alpha^2} + \operatorname{arcsinh} \alpha \right) + \frac{L^{d-1} V_{d-2}}{4\pi^2 G_{\mathsf{N}} \epsilon^{d-2}} \log \left(\frac{\ell_{\mathsf{ct}}(d-2)}{L} \right) \operatorname{arcsinh} \alpha + \frac{L^{d-1} V_{d-2}}{4\pi^2 G_{\mathsf{N}} \epsilon^{d-2}} \left(\sqrt{1+\alpha^2} \arccos \left(\frac{\alpha}{\sqrt{1+\alpha^2}} \right) - \frac{\pi}{2} \right)$$

  Contributions from $z = \infty$ are ignored.

  For $d > 2$, the divergence structure is the same as CV.

- $\underline{d = 2}$:

$$\Delta C_{\mathsf{A}}^{\mathsf{bdy}} = \frac{L}{4\pi G_{\mathsf{N}}} \left(\sqrt{1+\alpha^2} - 1 \right)$$

  The boundary complexity does not diverge.

Conclusion

- We study the boundary complexity $\Delta C^{\mathrm{bdy}} = C^{\mathrm{BCFT}} - C^{\mathrm{CFT}}/2$.
- By applying the path-integral optimization to BCFT_2, we obtained

$$\Delta C^{\mathrm{bdy}}_{\mathrm{L}} = \frac{c}{6\pi}\alpha \log\left(\frac{z_\infty}{\epsilon}\right)$$

$\Rightarrow$ imply that vanishment of ΔC^{bdy} (or $\Delta C^{\mathrm{defect}}$) depends on the definition of the complexity or models in BCFT (or DCFT).

- In $\mathrm{AdS}_{d+1}/\mathrm{BCFT}_d$ model, we showed

$$\Delta C^{\mathrm{bdy}}_{\mathrm{V}} \sim \Delta C^{\mathrm{bdy}}_{\mathrm{A}} \propto 1/\epsilon^{d-2}$$

- Especially, in $\mathrm{AdS}_3/\mathrm{BCFT}_2$ model,

$$\Delta C^{\mathrm{bdy}}_{\mathrm{V}} = \frac{\alpha L}{G_{\mathrm{N}}} \log\left(\frac{z_\infty}{\epsilon}\right), \qquad \Delta C^{\mathrm{bdy}}_{\mathrm{A}} = \frac{L}{4\pi G_{\mathrm{N}}}\left(\sqrt{1+\alpha^2}-1\right)$$

$\Longrightarrow$ Boundary (or defect) can not detect the definite difference of CV and CA except a special case.

Thank you for your attention!