How information geometry is encoded in bulk geometry

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Introduction

Emergent geometry (space-time)

Emergent geometry seems to play a crucial role in constructing quantum theory of gravity:

large-N reduction

matrix models for noncritical strings, M-theory and superstrings

gauge/gravity correspondence (AdS/CFT correspondence)

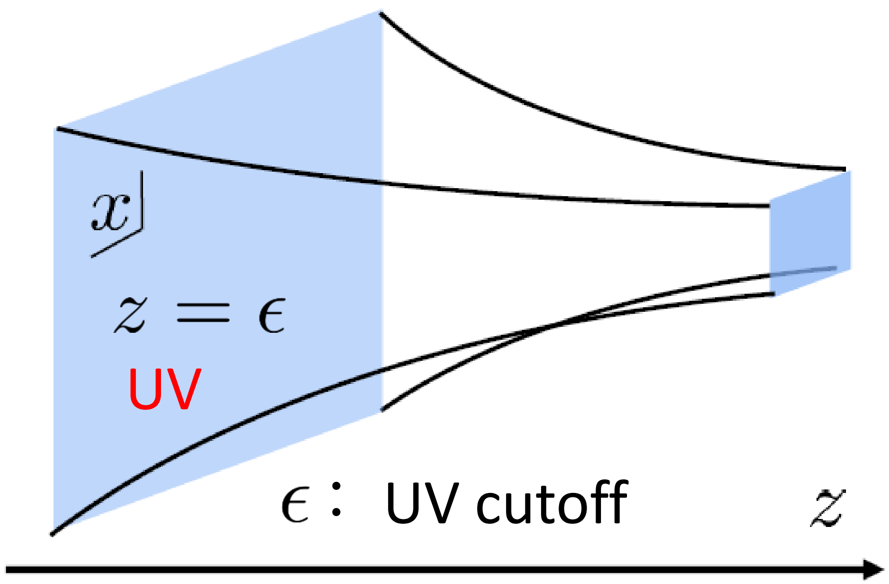
⋮

⋮

AdS/CFT correspondence

d-dim. CFT ⟺ (quantum) gravity (string theory) on AdS_{d+1}

$\mathrm{SO}(d,2)$



$z \gg 1$

IR

$$ds^2 = \frac{dz^2 + (dx^\mu)^2}{z^2}$$

$$x^\mu \to \rho x^\mu$$

$$z \to \rho z$$

scale of renormalization group

Susskind-Witten(1998)

de Boer-Verlinde-Verlinde (2000)

Emergence of bulk geometry

This observation motivates us to study the following problem:

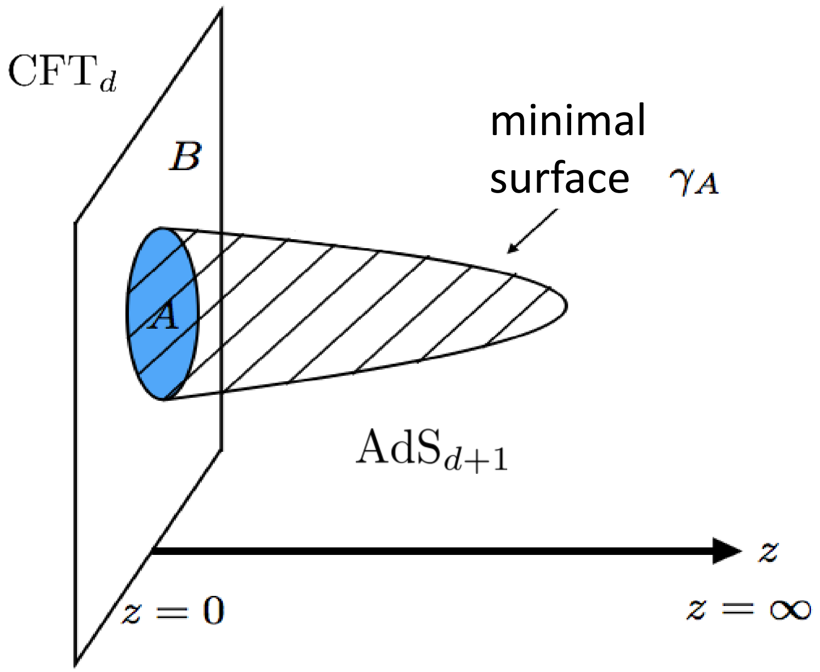
Can we reconstruct full bulk geometry directly from field theory ?

Entanglement entropy and geometry

Ryu-Takayanagi formula (2006)

$$S_A = \frac{\mathrm{Min}(\mathrm{area}(\gamma_A))}{4G_N}$$

a relation between
quantum information in field theory
and
bulk geometry



What we will do here

- consider information metric in quantum information theory other than entanglement entropy
- relate information metric to a more local quantity in bulk
- obtain dynamics of gravity (deviation from AdS) from information metric

cf.)Miyaji-Numasama-Shiba-Takayanagi- Watanabe('15), Bak('15)
Aoki-Yokoyama('17), Aoki-Yokoyama-Yoshida('19), …………

Plan of the present talk

✓ 1. Introduction

2. Information metric in field theory

3. Information metric encoded in bulk geometry

4. Conclusion and outlook

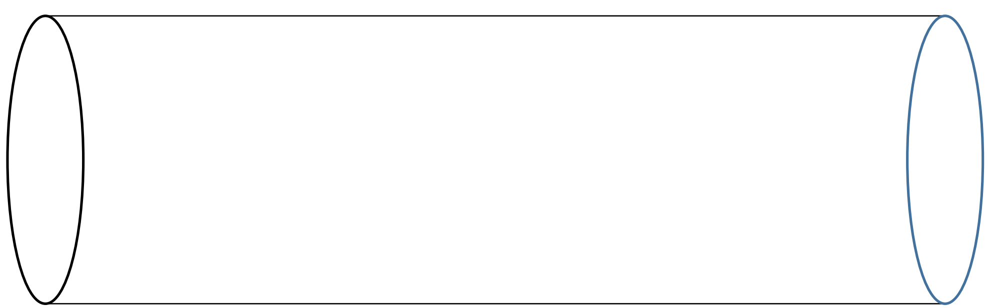
Information metric
in field theory

Wave function for ground state

$$\langle\tilde{\psi}|\Omega\rangle \sim \lim_{\Delta\tau\to\infty}\langle\tilde{\psi}|e^{-\Delta\tau H}|\Psi\rangle$$
$$= \frac{1}{Z^{1/2}}\int_{\psi(0,\vec{x})=\tilde{\psi}(\vec{x})}\mathcal{D}\psi\,\exp\left[-\int_{-\infty}^{0}d\tau\int d^{d-1}x\;\mathcal{L}\right]$$

$$\langle\Omega|\Omega\rangle = 1$$

$\tilde{\psi}(\vec{x}) = \psi(0,\vec{x})$



$\tau = 0$ $\qquad\qquad$ $\tau = -\infty$

Inner product between ground states

$|\Omega_1\rangle$: ground state of theory 1

$|\Omega_2\rangle$: ground state of theory 2

$$\langle\Omega_2|\Omega_1\rangle = \frac{1}{(Z_1 Z_2)^{1/2}} \int \mathcal{D}\psi \exp\left[-\int d^{d-1}x \left(\int_{-\infty}^{0} d\tau\ \mathcal{L}_1 + \int_0^{\infty} d\tau\ \mathcal{L}_2\right)\right]$$

$$\mathcal{L}_2 = \mathcal{L}_1 + \delta\mathcal{L}$$

$$\langle\Omega_2|\Omega_1\rangle = \frac{\left\langle \exp\left[-\int_0^{\infty} d\tau \int d^{d-1}x\ \delta\mathcal{L}\right]\right\rangle_1}{\left\langle \exp\left[-\int_{-\infty}^{\infty} d\tau \int d^{d-1}x\ \delta\mathcal{L}\right]\right\rangle_1^{1/2}}$$

$$\langle\mathcal{O}\rangle_1 = \frac{1}{Z_1}\int \mathcal{D}\varphi\ \mathcal{O}\ e^{-S_1} = \langle\Omega_1|\mathcal{O}|\Omega_1\rangle$$

Quantum information metric

perturbative expansion

$$\langle\Omega_2|\Omega_1\rangle = 1 - \frac{1}{2}\int_0^\infty d\tau \int_{-\infty}^0 d\tau' \int d^{d-1}x \int d^{d-1}x' \left(\langle\delta\mathcal{L}(\tau,\vec{x})\delta\mathcal{L}(\tau',\vec{x}')\rangle - \langle\delta\mathcal{L}(\tau,\vec{x})\rangle\langle\delta\mathcal{L}(\tau',\vec{x}')\rangle\right) + \mathcal{O}(\delta\mathcal{L}^3)$$

quantum information metric （Fischer's metric）

$$\mathcal{L}_2 = \mathcal{L}_1 + \delta\mathcal{L} = \mathcal{L}_1 + \phi(\vec{x})\mathcal{O}(\tau,\vec{x})$$

source indep. of time

T : volume of time direction

measuring distance between ground states of two theories

$$G = \frac{1}{T}(1 - \langle\Omega_2|\Omega_1\rangle) = \int d^{d-1}x \int d^{d-1}x'\ G_{\vec{x},\vec{x}'}\ \phi(\vec{x})\phi(\vec{x}')$$

$$G_{\vec{x},\vec{x}'} = \frac{1}{2T}\int_0^\infty d\tau \int_{-\infty}^0 d\tau' \left(\langle\mathcal{O}(\tau,\vec{x})\mathcal{O}(\tau',\vec{x}')\rangle - \langle\mathcal{O}(\tau,\vec{x})\rangle\langle\mathcal{O}(\tau',\vec{x}')\rangle\right)$$

Perturbation from CFT

$$\mathcal{L} = \mathcal{L}_{CFT} + \overbrace{\phi(\vec{x})\ \mathcal{O}(\tau,\vec{x})}^{\delta\mathcal{L}}$$

theory 2   theory 1   primary field with conformal dimension Δ

$$\langle \mathcal{O}(\tau,\vec{x})\mathcal{O}(\tau',\vec{x}')\rangle_{CFT} = \frac{c_{2pt}}{(\epsilon^2 + (\tau-\tau')^2 + (\vec{x}-\vec{x}')^2)^{\Delta}} \qquad \langle \mathcal{O}(\tau,\vec{x})\rangle_{CFT} = 0$$

ϵ : UV cutoff

quantum information metric

$$G = \frac{1}{T}(1 - \langle \Omega | \Omega_{CFT}\rangle)$$

$$= \frac{1}{2T}\int_0^{\infty} d\tau \int_{-\infty}^{0} d\tau' \int d^{d-1}x \int d^{d-1}x' \left(\langle \mathcal{O}(\tau,\vec{x})\mathcal{O}(\tau',\vec{x}')\rangle_{CFT}\, \phi(\vec{x})\phi(\vec{x}')\right)$$

$$= \frac{1}{8}\int_{-\infty}^{\infty} ds \int d^{d-1}x \int d^{d-1}x' \frac{c_{2pt}\phi(\vec{x})\phi(\vec{x}')}{(\epsilon^2 + s^2 + (\vec{x}-\vec{x}')^2)^{\Delta}}$$

Information metric encoded in bulk geometry

GKP-Witten relation

Gubser-Klebanov-Polyakov, Witten (1998)

$$\left\langle \exp\left[-\int d^d x\ \phi(x)\mathcal{O}(x)\right]\right\rangle = \int_{\Phi(\epsilon,x)=\epsilon^{d-\Delta}\phi(x)} \mathcal{D}g\mathcal{D}\Phi\ e^{-S}$$

$z=\epsilon$: boundary

large N
large 't Hooft coupling $\longrightarrow$ classical gravity

$$\exp(-S_{on-shell})$$

$$S = S_G + S_\Phi$$

$(\tau,\vec{x}) = x^\mu \qquad (z, x^\mu) = x^M$

$$S_G = \frac{1}{16\pi G_N}\left[\int d^{d+1}x\sqrt{G}\,(-R[G]+2\Lambda) - \int_{boundary} d^d x\sqrt{\gamma}K\right]$$

$$S_\Phi = \frac{1}{2}\int d^{d+1}x\sqrt{G}\,(G^{MN}\partial_M\Phi\partial_N\Phi + m^2\Phi^2)$$

$$\Lambda = -\frac{(d-1)(d-2)}{2} \qquad m^2 = \Delta(\Delta-d)$$

EOM for matter field

AdS space-time

$$ds^2 = G_{MN} dx^M dx^N = \frac{dz^2 + (dx^\mu)^2}{z^2}$$

EOM for matter field

$$-\frac{1}{\sqrt{G}} \partial_M (\sqrt{G} G^{MN} \partial_N \Phi) + m^2 \Phi = 0$$

solution

$$\Phi(z,x) = \int d^d x' K(z, x - x') \phi(x')$$

$$\to z^{d-\Delta} \phi(x) = z^{d-\Delta} \phi(\vec{x})$$

z small

$$K(z,x) = \frac{C(\Delta) z^\Delta}{(z^2 + x^2)^\Delta}$$

bulk-to-boundary propagator

On-shell action

$$S_{on-shell} = \frac{1}{2}\int d^{d+1}x\sqrt{G}\left(G^{MN}\partial_M\Phi\partial_N\Phi + m^2\Phi^2\right)$$

$$= \frac{1}{2}\int d^{d+1}x\partial_M\left(\sqrt{G}G^{MN}\Phi\partial_N\Phi\right)$$

$$-\frac{1}{2}\int d^{d+1}x\sqrt{G}\Phi\left\{\frac{1}{\sqrt{G}}\partial_M\left(\sqrt{G}G^{MN}\partial_N\Phi\right) - m^2\Phi\right\}$$

=0 thanks to EOM

$$= \frac{1}{2}\int_{z=\epsilon} d^dx\ \epsilon^{-d+1}\ \Phi\partial_z\Phi$$

$$= \underbrace{\Delta C(\Delta)}_{=c_{2pt}}\int_0^\infty d\tau\int_{-\infty}^{\infty} ds\int d^{d-1}x d^{d-1}x'\frac{\phi(\vec{x})\phi(\vec{x}')}{(\epsilon^2+s^2+(\vec{x}-\vec{x}')^2)^\Delta}$$

$$= 4TG$$

Back reaction

$$ds^2 = G_{MN}dx^M dx^N = \frac{1}{z^2}\left(dz^2 + g_{\mu\nu}(z,x)dx^\mu dx^\nu\right)$$

$$g_{\mu\nu}(z,x) = \delta_{\mu\nu} + h_{\mu\nu}(z,x)$$

back reaction
deviation from AdS

$\mathrm{Tr}A = \delta^{\mu\nu}A_{\mu\nu}$

$'$ = z-derivative

Einstein eq.

zz

$$\mathrm{Tr}g^{-1}g'' - \frac{1}{z}\mathrm{Tr}g^{-1}g' - \frac{1}{2}\mathrm{Tr}g^{-1}g'g^{-1}g' = -16\pi G_N\left(\partial_z\Phi\partial_z\Phi + \frac{m^2}{d-1}\frac{1}{z^2}\Phi^2\right)$$

$z\mu$

$$\nabla_\mu \mathrm{Tr}g^{-1}g' - \nabla^\lambda g'_{\lambda\mu} = -16\pi G_N \partial_z\Phi\partial_\mu\Phi$$

$\mu\nu$

$$2R_{\mu\nu}(g) + g''_{\mu\nu} - g'_{\mu\lambda}g^{\lambda\sigma}g'_{\sigma\nu} + \frac{1}{2}\mathrm{Tr}\left(g^{-1}g'\right)g'_{\mu\nu} - (d-1)\frac{1}{z}g'_{\mu\nu}$$
$$-\frac{1}{z}\mathrm{Tr}\left(g^{-1}g'\right)g_{\mu\nu} = -16\pi G_N\left(\partial_\mu\Phi\partial_\nu\Phi + \frac{m^2}{d-1}\frac{1}{z^2}g_{\mu\nu}\Phi^2\right)$$

EOM for Φ

$$\mathrm{tr}h' = \delta^{\mu\nu}h'_{\mu\nu} - h'_{00} = \delta^{ij}h'_{ij} = -8\pi G_N \Phi\partial_z\Phi$$

+ surface terms

Information metric encoded in bulk geometry

Integrating both sides over d-dim. space-time, we obtain

$$S_{on-shell} = \frac{1}{2}\int_{z=\epsilon} d^d x \ \epsilon^{-d+1} \ \Phi \partial_z \Phi = -\frac{1}{16\pi G_N}\int_{z=\epsilon} d^d x \ \epsilon^{-d+1} \ \mathrm{tr} h'$$

On the other hand, we have shown

$$= \Delta C(\Delta) \int_0^\infty d\tau \int_{-\infty}^{\infty} ds \int d^{d-1}x d^{d-1}x' \frac{\phi(\vec{x})\phi(\vec{x}')}{(\epsilon^2 + s^2 + (\vec{x} - \vec{x}')^2)^\Delta}$$

$$= 4TG$$

By dividing both by T, we obtain a formula

Encoding quantum information metric in bulk geometry

$$G = -\frac{1}{64\pi G_N}\int_{z=\epsilon} d^{d-1}x \ \epsilon^{-d+1} \ \mathrm{tr} h'$$

This holds for vector (and tensor) fields : universal relation

Interpretation of the bulk quantity

hypersurface with $z = \epsilon = \text{const.}$ and $\tau = \text{const.}$

induced metric

$$k_{ij} = \frac{\delta_{ij} + h_{ij}}{\epsilon^2} \quad \Longleftarrow \quad ds^2 = \frac{dz^2 + (\delta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu}{z^2}$$

deviation of volume

$$\delta V = \int_{z=\epsilon} d^{d-1}x \; \epsilon^{-d+1}\sqrt{k} - \underbrace{\int_{z=\epsilon} d^{d-1}x \; \epsilon^{-d+1}}_{\text{AdS case}} = \frac{1}{2}\int_{z=\epsilon} d^{d-1}x \; \epsilon^{-d+1} \; \mathrm{tr}h$$

$$\left.\frac{d\delta V}{dz}\right|_{z=\epsilon} = \underbrace{\frac{1}{2}\int_{z=\epsilon} d^{d-1}x \; (-d+1)\epsilon^{-d} \; \mathrm{tr}h}_{\text{canonical scaling}} + \underbrace{\frac{1}{2}\int_{z=\epsilon} d^{d-1}x \; \epsilon^{-d+1} \; \mathrm{tr}h'}_{\text{nontrivial scaling}}$$

~ RHS of the formula

Conclusion and outlook

Conclusion

$$G = \frac{1}{T}(1 - \langle \Omega | \Omega_{CFT} \rangle) = -\frac{1}{64\pi G_N} \int_{z=\epsilon} d^{d-1}x \ \epsilon^{-d+1} \ \mathrm{tr} h'$$

- holds for scalar, vector (and tensor) fields: universal relation
- Information metric in field theory gives local information w.r.t. z-direction in bulk geometry
- relates information metric to deviation from AdS (back reaction)

Outlook

- generalize the formula to a covariant one
- case of general gauge/gravity correspondence
- find more local relation to reconstruct full bulk geometry
- effects of strings and quantum gravity