Dynamics of Revolving D-Branes at Short Distances

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Based on collaboration with
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Introduction

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$\Rightarrow$ **Perturbative calculation for slow revolving.** **[IOS 17,18]**

$$\mathcal{V}(r) = -\int_0^\infty \frac{ds}{2s}(8\pi^2\alpha' s)^{-\frac{1}{2}} e^{-\frac{2r^2}{\pi\alpha'}s} \eta(is)^{-24} \left(1 - \frac{1}{3}v^2\right)^{-\frac{1}{2}}$$

$$\times \left[1 - 2\pi v^2 \left(-\frac{4}{\pi^2} s \sum_{n=1}^{\infty} \frac{n^{-1}q^n}{1-q^n} + \epsilon_0 s - \frac{4}{\pi} s^2 \sum_{n=1}^{\infty} \frac{2q^n}{(1-q^n)^2}\right)\right] + \mathcal{O}(v^4).$$

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cf. [Douglas et al. 97]

This led us to an efficient method for off-shall calculations.

⇒ A good approximation for the desired potential.

Warm up: D3-D3 at anlge

1-loop effective potential is known: cf. [Polchinski]

$$V(R) = -\int_0^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-\frac{1}{2}} e^{-\frac{R^2}{2\pi\alpha'}t} \frac{i\prod_{a=1}^4 \vartheta_{11}(\frac{i}{\pi}\phi'_a t, it)}{\eta(it)^3 \prod_{a=1}^3 \vartheta_{11}(\frac{i}{\pi}\phi_a t, it)}.$$

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$\Rightarrow$ **No! All states contribute.** ($e^{-2\pi n t}$ **not small for small** t**.)**

How to know the shape of the graph?

Partial modular transformation [IKOS 19]

We calculate as follows:

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$$-\int_0^1 \frac{dt}{t}(8\pi^2\alpha' t)^{-\frac{1}{2}} e^{-\frac{R^2}{2\pi\alpha'}t} \frac{i\prod_{a=1}^4 \vartheta_{11}(\frac{i}{\pi}\phi'_a t, it)}{\eta(it)^3 \prod_{a=1}^3 \vartheta_{11}(\frac{i}{\pi}\phi_a t, it)}$$

(Divide integration region into two.)

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$$+\int_1^\infty \frac{ds}{s}(8\pi^2\alpha')^{-\frac{1}{2}} s^{-\frac{1}{2}} e^{-\frac{R^2}{2\pi\alpha'}s^{-1}} \frac{\prod_{a=1}^4 \vartheta_{11}(\frac{1}{\pi}\phi'_a, is)}{\eta(is)^3\prod_{a=1}^3 \vartheta_{11}(\frac{1}{\pi}\phi_a, is)}$$

(Modular transf. for the 2nd half.)

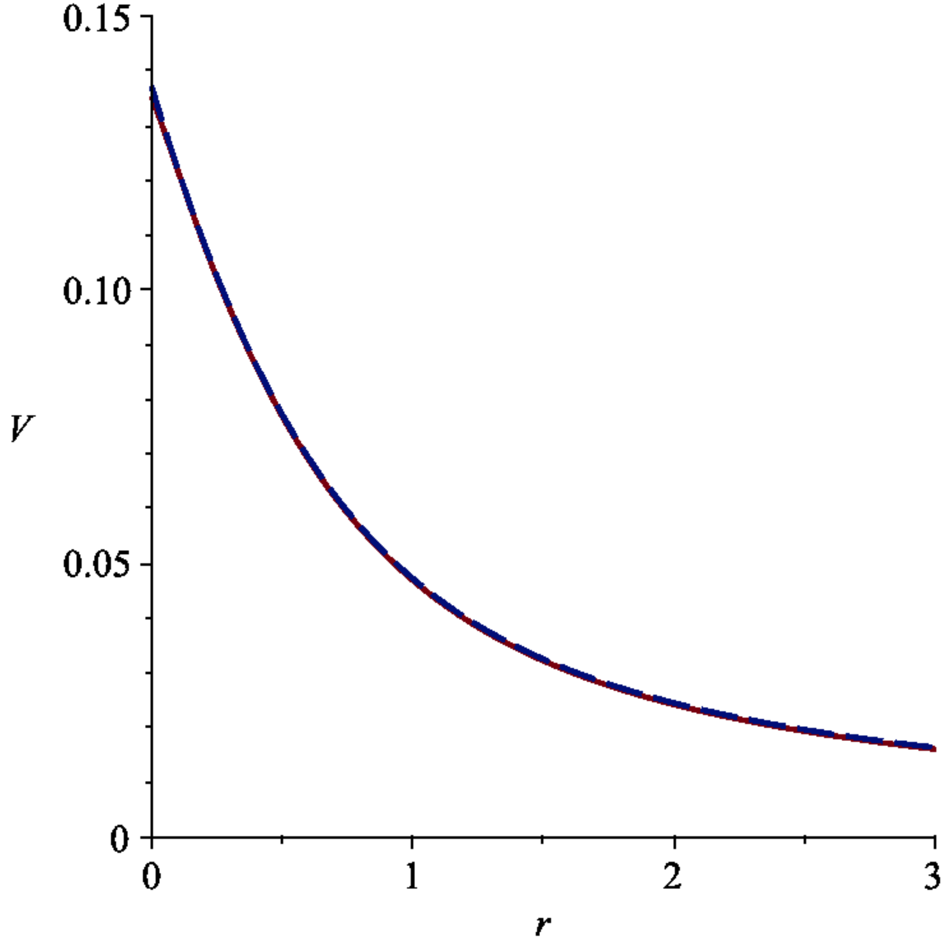
Partial modular transformation [IKOS 19]

We calculate as follows:

$$V(R) \sim \int_1^\infty \frac{dt}{t}(8\pi^2\alpha' t)^{-\frac{1}{2}} e^{-\frac{R^2}{2\pi\alpha'}t} \frac{2\sinh(\frac{3}{2}\phi t)\sinh^3(\frac{1}{2}\phi t)}{\sinh^3(\phi t)}$$
$$+ \int_1^\infty \frac{ds}{s}(8\pi^2\alpha')^{-\frac{1}{2}} s^{-\frac{1}{2}} e^{-\frac{R^2}{2\pi\alpha'}s^{-1}} \frac{2\sin(\frac{3}{2}\phi)\sin^3(\frac{1}{2}\phi)}{\sin^3\phi}$$

(Expand theta functions.)

This is more manageable.



Plot for exact and approximated potentials. Quite nice!

Reason for good approximation:

For $t \geq 1$**,**

$$e^{-2\pi t} \;\leq\; e^{-2\pi} \;=\; 0.001867. \qquad \textbf{Very small!}$$

$\Rightarrow$ **Higher-order terms are highly suppressed.**

(Degeneracies grow only $e^{c\sqrt{n}}$**.)**

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Note: Gaps in the mass spectrum is assumed to be $\mathcal{O}(1/\sqrt{\alpha'})$.

What we have obtained is

$$V(R) \sim \int_{\mathbf{1}}^{\infty} \frac{dt}{t} \text{ (light open string states)} + \int_{\mathbf{1}}^{\infty} ds \text{ (massless closed states)}.$$

Note: No double-counting.

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$\Rightarrow$ **Off-shell calculations are possible!**

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- **SYM: 1-loop effective action around**

$$X^8 = r\cos\omega t \cdot \sigma_3, \qquad X^9 = r\sin\omega t \cdot \sigma_3$$

in $\mathcal{N} = 4$ SU(2) **SYM**$_4$**.**

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in $\mathcal{N}=4$ SU(2) $\mathbf{SYM}_4$**.**

- **SUGRA: tree amplitude for exchanging** $\Phi, g_{\mu\nu}, C_3$ **with interaction vertices given by DBI+CS action.**

$$X^8 = \pm r\cos\omega t, \qquad X^9 = \pm r\sin\omega t$$

are embedding functions of revolving D3-branes.

SYM result: for small ω,

$$
\begin{aligned}
V_o(2r) \;=\; & -\frac{\omega^2 r^2}{\pi^2}\left[e^{-4r^2/m_{\rm str}^2} - \left(\frac{4r^2}{m_{\rm str}^2}\right) E_1(4r^2/m_{\rm str}^2)\right] \\
& -\omega^4\left[\left(\frac{1}{16\pi^2} + \frac{7r^2}{12\pi^2 m_{\rm str}^2} + \frac{10r^4}{3\pi^2 m_{\rm str}^4}\right) e^{-4r^2/m_{\rm str}^2} \right.\\
& \left. -\left(\frac{6r^4}{\pi^2 m_{\rm str}^4} + \frac{40r^6}{3\pi^2 m_{\rm str}^6}\right) E_1(4r^2/m_{\rm str}^2)\right] + \mathcal{O}(\omega^6),
\end{aligned}
$$

where

$$
E_1(x) \;:=\; \int_1^\infty dt \frac{e^{-xt}}{t}.
$$

Note: **This is exponentially suppressed for large** r.

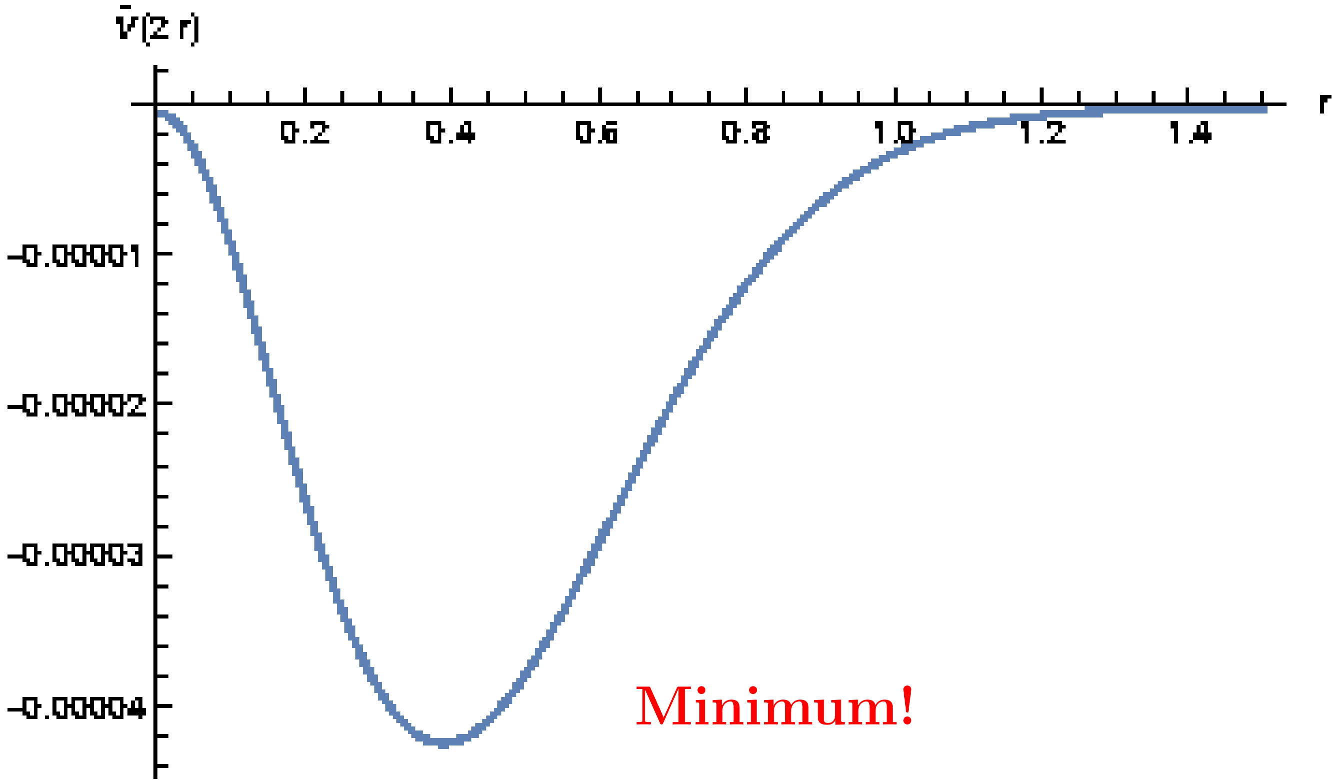
(No contribution to Newton potential.)

SUGRA result: for small ω,

$$V_c(2r) = -\frac{\omega^4}{16\pi^2}\left[1 - \left(1 + 4r^2/m_{\rm str}^2\right) e^{-4r^2/m_{\rm str}^2}\right] + \mathcal{O}(\omega^6)$$
$$\sim -\frac{v^4}{r^4}.$$

Note: For large r, the leading term $\propto v^4$.

$\Rightarrow$ It turned out to be quite general !



Plot for the approximated potential.

To shallow to form a bound state...

Note: **Bound states of D0-branes.**

- **Marginal bound states** [Yi 97][Sethi,Stern 98]

  The existence is well-understood.

  Corresponding to gravitons in BFSS.

- **Non-marginal bound states** [Danielsson et al. 96]

  Quantum-mechanical bound states with angular momentum.

  (A potential induced by integrating out fast modes.)

  $\Rightarrow$ **Possibility to investigate with our effective potential.**

Summary

- The partial modular transformation allows us to obtain good approximation to 1-loop potential.
- This reduces to calculations in SYM and SUGRA.
- Dynamical (non-marginal) bound states can be discussed.

Open issues

- Existence of D-brane bound states.
- Off-shell calculations in string theory.
- Higher loops. cf. [Douglas et al. 97]
- etc.