On detecting the open string pair production

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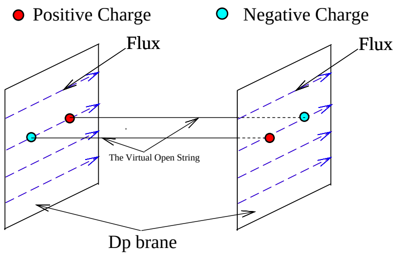
Outline

- A brief introduction/motivation
- The D3/D3 system
- The D3/D1 system
- Discussion & Implication

The open string pair production



Stringy computations show indeed a non-vanishing pair production rate for this setup. However, this rate is usually vanishing small for any realistic electric fields and so has no any practical use.

This rate can be greatly enhanced if we add in addition a magnetic flux in a particular manner on each Dp.

The pair production rate

For this purpose, consider the electric/magnetic tensor $\hat{F}^1$ on one Dp brane and the $\hat{F}^2$ on the other Dp brane, respectively, as

$$\hat{F}^a = \begin{pmatrix} 0 & -\hat{f}_a & 0 & 0 & 0 & \dots \\ \hat{f}_a & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & -\hat{g}_a & 0 & \dots \\ 0 & 0 & \hat{g}_a & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}_{(p+1)\times(p+1)}, \qquad (1.1)$$

where $\hat{f}_a$ denotes the dimensionless electric field ($|\hat{f}_a| < 1$) while g_a the dimensionless magnetic one ($|\hat{g}_a| < \infty$) with $a = 1, 2$, and $6 \geq p \geq 3$. Note $\hat{F} = 2\pi\alpha' F$.

The open string pair production rate

The pair production rate can be computed to be Lu'17

$$\mathcal{W}^{(1)} = \frac{8\,|\hat f_1 - \hat f_2||\hat g_1 - \hat g_2|}{(8\pi^2\alpha')^{\frac{p+1}{2}}} \nu_0^{\frac{p-3}{2}} e^{-\frac{y^2}{2\pi\nu_0\alpha'}} \frac{\left[\cosh\frac{\pi\nu_0'}{\nu_0} + 1\right]^2}{\sinh\frac{\pi\nu_0'}{\nu_0}} Z_1(\nu_0, \nu_0'), \tag{1.2}$$

where

$$Z_1(\nu_0, \nu_0') = \prod_{n=1}^{\infty} \frac{\left[1 + 2e^{-\frac{2n\pi}{\nu_0}}\cosh\frac{\pi\nu_0'}{\nu_0} + e^{-\frac{4n\pi}{\nu_0}}\right]^4}{\left[1 - e^{-\frac{2n\pi}{\nu_0}}\right]^6 \left[1 - e^{-\frac{2\pi}{\nu_0}(n-\nu_0')}\right]\left[1 - e^{-\frac{2\pi}{\nu_0}(n+\nu_0')}\right]}. \tag{1.3}$$

In the above, the parameters $\nu_0 \in [0, \infty)$ and $\nu_0' \in [0, 1)$ are

$$\tanh\pi\nu_0 = \frac{|\hat f_1 - \hat f_2|}{1 - \hat f_1 \hat f_2}, \quad \tan\pi\nu_0' = \frac{|\hat g_1 - \hat g_2|}{1 + \hat g_1 \hat g_2}. \tag{1.4}$$

The pair production rate

In practice, $\nu_0 \ll 1$ & $\nu_0' \ll 1$ ($Z_1(\nu_0, \nu_0') \approx 1$). Let us explore the posibility for a large rate (note $p \geq 3$) .

The rate (1.2) becomes

$$\mathcal{W}^{(1)} \approx \frac{8\pi^2 \nu_0 \nu_0'}{(8\pi^2\alpha')^{\frac{1+p}{2}}} \nu_0^{\frac{p-3}{2}} e^{-\frac{y^2}{2\pi\alpha'\nu_0}} \frac{\left[\cosh \frac{\pi\nu_0'}{\nu_0} + 1\right]^2}{\sinh \frac{\pi\nu_0'}{\nu_0}}. \qquad (1.5)$$

- It is clear the $p = 3$ gives the largest rate (Lu'19) and the rate, say, for $p = 4$, is smaller by a factor of $(\nu_0/4\pi)^{1/2}$ and so on.
- Adding more magnetic flux doesn't help (Jia & Lu'19).

Let us estimate this factor to see how large it is (set $\hat{f}_2 = \hat{g}_2 = 0$ for simplicity).

The pair production rate

Note $M_s = 1/\sqrt{\alpha'} \sim$ a few TeV upto $10^{16} \sim 10^{17}$ GeV (Berenstein'14),

The current lab. limit $eE \sim 10^{-8} m_e^2 = 2.5 \times 10^{-21} \mathrm{TeV}^2$,

$\hat{f}_1 = 2\pi\alpha' eE = 2\pi\, m_e^2/M_s^2 \leq\sim 10^{-21} \ll 1$

$$\nu_0 = \frac{|\hat{f}_1|}{\pi} = 2\frac{m_e^2}{M_s^2} \leq\sim 10^{-21} \to \left(\frac{\nu_0}{4\pi}\right)^{1/2} \sim 10^{-11} \ll 1 \qquad (1.6)$$

In other words, the dimensionless rate for any other $p > 3$ brane is at least smaller than that of the D3-brane by a factor of 10^{-11}!

So the pair production for D3 is the only hope for detection!

The pair production rate

In terms of the lab. field E and B via

$$\hat{f}_1 = 2\pi\alpha' eE \ll 1, \qquad \hat{g}_1 = 2\pi\alpha' eB \ll 1, \tag{2.1}$$

the pair production rate(1.5) for D3 brane is now

$$\mathcal{W}^{(1)} = \frac{2(eE)(eB)}{(2\pi)^2} \frac{\left[\cosh\frac{\pi B}{E} + 1\right]^2}{\sinh\frac{\pi B}{E}} e^{-\frac{\pi m^2}{eE}}, \tag{2.2}$$

where we have introduced a mass scale

$$m = T_f y = \frac{y}{2\pi\alpha'}. \tag{2.3}$$

Keep in mind, we need to have a nearby D3 brane for this rate!

The pair production rate

Let us try to understand (2.2) a bit more.

In the absence of both E and B, the mass spectrum for the open string connecting the two D3 is

$$\alpha' M^2 = -\alpha' p^2 = \begin{cases} \frac{y^2}{4\pi^2\alpha'} + N_{\rm R} & ({\rm R-sector}), \\ \frac{y^2}{4\pi^2\alpha'} + N_{\rm NS} - \frac{1}{2} & ({\rm NS-sector}), \end{cases} \tag{2.4}$$

where $p = (k, 0)$ with k the momentum along the brane worldvolume directions, $N_{\rm R}$ and $N_{\rm NS}$ are the standard number operators in the R-sector and NS-sector, respectively, as

$$\begin{aligned} N_{\rm R} &= \sum_{n=1}^{\infty}(\alpha_{-n}\cdot\alpha_n + n d_{-n}\cdot d_n), \\ N_{\rm NS} &= \sum_{n=1}^{\infty}\alpha_{-n}\cdot\alpha_n + \sum_{r=1/2}^{\infty} r d_{-r}\cdot d_r. \end{aligned} \tag{2.5}$$

The pair production rate

The R-sector gives fermions with $N_{\rm R} \geq 0$ while the NS-sector gives bosons with $N_{\rm NS} \geq 1/2$. The $N_{\rm R} = 0$, $N_{\rm NS} = 1/2$ give the usual massless $4(8_{\rm F} + 8_{\rm B})$ degrees of freedom (The 4D N = 4 U(2) SYM) when $y = 0$.

Among these, $2(8_{\rm F} + 8_{\rm B})$ become massive ones, all with mass $T_f y = y/(2\pi\alpha')$ due to unbroken SUSY, when $y \neq 0$. This just reflects the spontaneously symmetry breaking $U(2) \to U(1) \times U(1)$ when $y = 0 \to y \neq 0$. The two broken generators give 16 pairs of charged/anti-charged DOF with respect to the brane observer.

The pair production rate (2.2) is obtained in the weak field limit $\pi\nu_0 = \hat{f}_1 \ll 1, \pi\nu'_0 = \hat{g}_1 \ll 1$ and all massive other than the lowest 16 charged/anti-charged pair dof are dropped since $Z_1 \approx 1$. In other words, only these 16 pair dof actually contribute to this rate.

The pair production rate

We now compare the open string pair production rate (2.2) with QED charged scalar, spinor and W-boson pair production rate with the same E and B. The present rate is

$$\mathcal{W}^{(1)} = \frac{2(eE)(eB)}{(2\pi)^2}\frac{\left[\cosh\frac{\pi B}{E}+1\right]^2}{\sinh\frac{\pi B}{E}}\, e^{-\frac{\pi m^2}{eE}}, \qquad (2.6)$$

while for the QED scalar Nikishov'70

$$\mathcal{W}_{\rm QEDscalar} = \frac{(eE)(eB)}{2(2\pi)^2}\operatorname{csch}\left(\frac{\pi B}{E}\right) e^{-\frac{\pi m^2}{eE}}, \qquad (2.7)$$

for spinor

$$\mathcal{W}_{\rm QEDspinor} = \frac{(eE)(eB)}{(2\pi)^2}\coth\left(\frac{\pi B}{E}\right) e^{-\frac{\pi m_e^2}{eE}}, \qquad (2.8)$$

and for W-boson Kruglov'01,

$$\mathcal{W}_{\rm W-boson} = \frac{(eE)(eB)}{2(2\pi)^2}\frac{2\cosh\frac{2\pi B}{E}+1}{\sinh\frac{\pi B}{E}}\, e^{-\frac{\pi m_W^2}{eE}}. \qquad (2.9)$$

The pair production rate

Identifying the mass, when $B = 0$, we have

$$\begin{aligned} \mathcal{W}^{(1)} &= 16\,\mathcal{W}_{\rm QEDscalar} = 8\,\mathcal{W}_{\rm QEDspinor} \\ &= \frac{16}{3}\,\mathcal{W}_{\rm W-boson} = \frac{8(eE)^2}{(2\pi)^2}\,e^{-\frac{\pi m^2}{eE}}, \end{aligned} \tag{2.10}$$

While for large B/E (or $B \neq 0, E \sim 0$),

$$\mathcal{W}^{(1)} \approx \frac{(eE)(eB)}{(2\pi)^2}\,e^{-\frac{\pi(m^2-eB)}{eE}}, \quad \mathcal{W}_{\rm Wboson} \approx \frac{(eE)(eB)}{(2\pi)^2}\,e^{-\frac{\pi(m_W^2-eB)}{eE}}$$

$$\mathcal{W}_{\rm Scalar} \approx \frac{(eE)(eB)}{(2\pi)^2}\,e^{-\frac{\pi(m^2+eB)}{eE}}, \quad \mathcal{W}_{\rm Spinor} \approx \frac{(eE)(eB)}{(2\pi)^2}\,e^{-\frac{\pi m_e^2}{eE}}, \tag{2.11}$$

The pair production rate

For this case, the pre-factor is the same for all cases but the exponential suppressing factor is different. How to understand this?

It is well-known that an electrically charged particle with mass m_S and spin S in a weak magnetic field B background has energy

$$E_S^2 = (2N+1)eB - g_S eB \cdot S + m_S^2, \qquad (2.12)$$

with g_S the gyromagnetic ratio ($g_S = 2$) and N the Landau level. So for the lowest Landau level ($N = 0$), the lowest energy

$$E_{S=0}^2 = eB + m_0^2, \quad E_{S=1/2}^2 = m_{1/2}^2, \quad E_{S=1}^2 = -eB + m_1^2. \qquad (2.13)$$

which come from $B \cdot S = 0, B/2, B$ for $S = 0, 1/2, 1$, respectively, counting only one DOF for each spin.

So the large B/E pair production rate (2.11) can all be written

$$\mathcal{W}_S \approx \frac{(eE)(eB)}{(2\pi)^2} e^{-\frac{\pi E_S^2}{eE}}. \qquad (2.14)$$

The pair production rate

This completely explains the rate behavior for the respective $S = 0, 1/2, 1$ for large B/E.

In particular, the behavior of our rate $\mathcal{W}^{(1)}$ for the applied general weak fields can also be easily explained due to the following.

The two broken gauge generators, each giving the 4D N $= 4$ massive SYM (5 massive scalars, 4 massive fermions and 1 massive vector) but charged oppositely under the U(1), all have the same mass $m = y/(2\pi\alpha')$.

Once turning on magnetic field, different spin polarization gives different mass splitting, indicating the underlying SUSY breaking. It is clear that the strength of this SUSY breaking depends on that of the applied magnetic field.

Discussion of the rate

Given the above, we expect to have,

$$\mathcal{W}^{(1)} = 5W_{\rm QED-scalar} + 4W_{\rm QED-spinor} + W_{\rm w-boson}, \quad (2.15)$$

if we take all the modes having the same mass and with the same $E\&B$.

One can check this holds indeed true and this relation explains all the previous results for $B = 0$ and large B/E, respectively.

It is also very satisfied to have this since they are computed completely differently, one in string theory and the other in QFT.

Discussion of the rate

We now move to discuss the rate (1.5) which is valid for $\nu_0 \ll 1$ and finite $\nu_0' \in (0,1)$. This is our focus in what follows.

For this, consider the presence of magnetic flux B $(\tan \pi\nu_0' = 2\pi\alpha' eB)$. The lowest mass in the R-sector is still given by the mass $y/(2\pi\alpha')$ as before.

However, the spectrum in the NS-sector is now,

$$\alpha' E_{\rm NS}^2 = (2N+1)\frac{\nu_0'}{2} - \nu_0'\, S + \alpha' M_{\rm NS}^2, \qquad (2.16)$$

where $b_0^+ b_0 = N$ defines the Landau level, the mass $M_{\rm NS}$, the spin operator S in the 23-direction and the number operator $N_{\rm NS}$ are

$$\alpha' M_{\rm NS}^2 = \frac{y^2}{4\pi^2\alpha'} + N_{\rm NS} - \frac{1}{2}, \qquad (2.17)$$

and

Discussion of the rate

$$
\begin{aligned}
S &= \sum_{n=1}^{\infty}(a_n^+ a_n - b_n^+ b_n) + \sum_{r=1/2}^{\infty}(d_r^+ d_r - \tilde{d}_r^+ \tilde{d}_r), \\
N_{\rm NS} &= \sum_{n=1}^{\infty} n(a_n^+ a_n + b_n^+ b_n) + \sum_{r=1/2}^{\infty} r(d_r^+ d_r + \tilde{d}_r^+ \tilde{d}_r) + N_{\rm NS}^{\perp}.
\end{aligned} \tag{2.18}
$$

So the lowest mass is given by the GSO projected state $d_{1/2}^+|0\rangle_{\rm NS}$ with now the spin $S = 1$ and the effective mass

$$\alpha' E_{\rm NS}^2 = -\nu_0'/2 + y^2/(4\pi^2\alpha'), \tag{2.19}$$

with a tachyonic shift $-\nu_0'/2$ and reducing to the energy of charged massive vector mode in the weak magnetic limit as

$$E_{\rm NS}^2 = -eB + m^2, \tag{2.20}$$

with $m = y/(2\pi\alpha')$.

Discussion of the rate

Can the stringy rate be useful for actual detection in practice?

$$\mathcal{W}^{(1)} = \frac{2(eE)(eB)}{(2\pi)^2} \frac{\left[\cosh \frac{\pi B}{E} + 1\right]^2}{\sinh \frac{\pi B}{E}} e^{-\frac{\pi m^2}{eE}}. \tag{2.21}$$

Since the modes contributing to the above rate all have the same mass m, due to unbroken SUSY, we expect $m > \mathrm{TeV}$. A detection of the pair production requires either $eE \sim m^2 > \mathrm{TeV}^2$ if $B/E \sim \mathcal{O}(1)$ or if $B/E \gg 1$,

$$\mathcal{W}^{(1)} \sim \frac{2(eE)(eB)}{(2\pi)^2} e^{-\frac{\pi(m^2 - eB)}{eE}}, \tag{2.22}$$

$eB \sim m^2$, which is impossible in practice in either case.

(The current lab limit $eE \sim eB \sim 10^{-8} m_e^2 \sim 10^{-21} \mathrm{TeV}^2$)

A possibility of detection

For a potential detection, we need to have

- a non-supersymmetric system of D-branes even in the absence of worldvolume fluxes,
- the corresponding mass scale is comparable to the current lab electric field.

One such system is the D3/D1 (Lu&Xu' 09, Jia et al'19), which has no SUSY and the D1 appears effectively as a stringy scale magnetic such that it can gives a small effective mass scale.

A possibility of detection

The corresponding pair production rate can be computed to be (Lu'19),

$$W^{(1)} = \frac{eE}{2\pi} e^{-\frac{\pi(m^2 - \nu_0'/(2\alpha'))}{eE}}, \tag{3.1}$$

where on D3 $\hat{f} = 2\pi\alpha' eE \ll 1$ and $\hat{g} = 2\pi\alpha' eB \ll 1$ but D1 doesn't carry any electric flux. Note

$$m = \frac{y}{2\pi\alpha'}, \qquad \tan \pi\nu_0' = \frac{1}{\hat{g}} \to \nu_0' \sim 1/2. \tag{3.2}$$

A possibility of detection

Now among the 16 + 16 modes contributing to this rate, the lowest energy modes are the charged/anti-charged massive vector ones with its effective $m_{\rm eff}^2 = m^2 - 1/(4\alpha')$. Only for this charged pair, we have a possibility for detection.

Concretely, if $m \approx 1/(2\sqrt{\alpha'})$, or $y \sim \pi\sqrt{\alpha'}$, giving a small effective $m_{\rm eff}$, we need then only a small $eE \sim m_{\rm eff}^2$ to detect the pair production.

Further, with a small tunable $\hat{g} = 2\pi\alpha' eB \ll 1$, we can check the rate behavior against E and B and this can be used to check the stringy computations.

Implication

Implications: If such a detection, for example as an electric current due to the pair production, is indeed possible,

- it first implies the existence of extra dimensions.
- secondly, it gives a new way to verify the underlying string theory without the need to compactify 10 D to 4D so long the pair production is concerned.

THANK YOU!

Welcome

To the 5th East Asia Joint Workshop on Fields & Strings

XiAn, China