Light Dark Matter Scattering in Gravitational Wave Detectors

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NCTS

Energy Frontier in Particle Physics:
LHC and Future Colliders
5-6 October 2020

Based on arXiv:2007.07908
in collaboration with Chun-Hao Lee and Martin Spinrath

Outline

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The GW Interferometer

Toy Model for DM Hit

KAGRA

DM Signal at KAGRA

Summary

Interferometer Used by GW Experiments

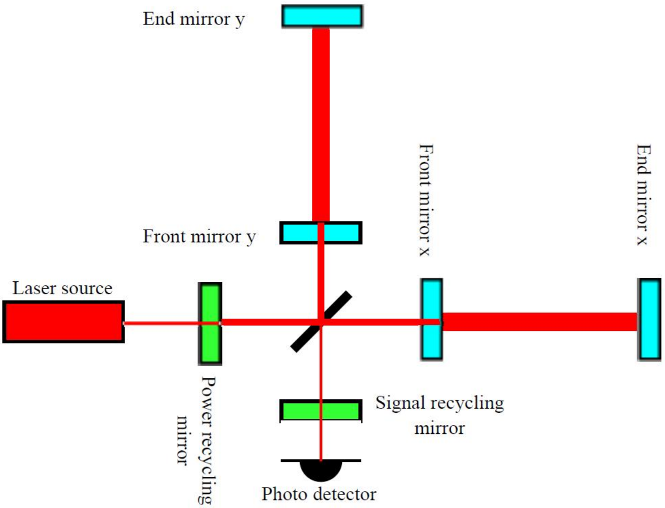

Figure: The interferometer with the arm length (distance between front mirror and end mirror) equals to L. [**Taken from KAGRA PhD Thesis: D.Chen, 2016**]

Estimated Sensitivity

- The change in the interference of the light is proportional to

$$\Delta L = \Delta L_x - \Delta L_y$$

- A gravitational wave will induce a strain, h_{GW}

$$h_{\text{GW}} \sim \frac{\Delta L}{L} \leq 10^{-20}$$

- What about strain induced by DM ?
- Is it visible compared to the noises? What noises?

Strain Amplitude Budgets

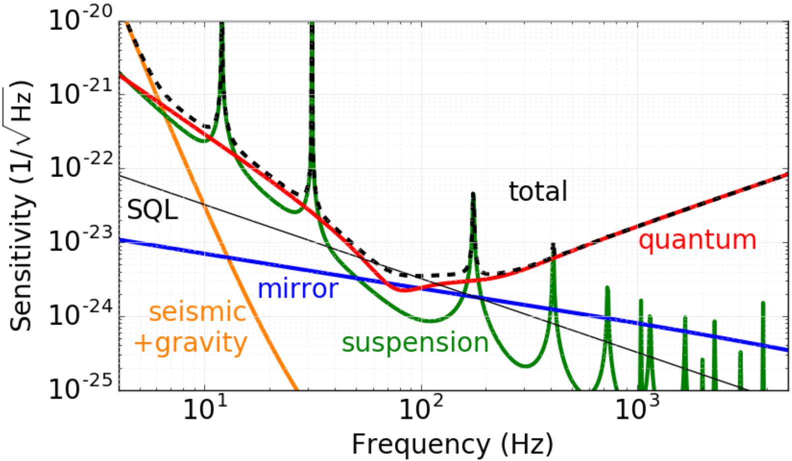

Figure: Example of the strain amplitude of the noises.
[**https://gwcenter.icrr.u-tokyo.ac.jp/en/researcher/parameter**]

Interferometer Isolation

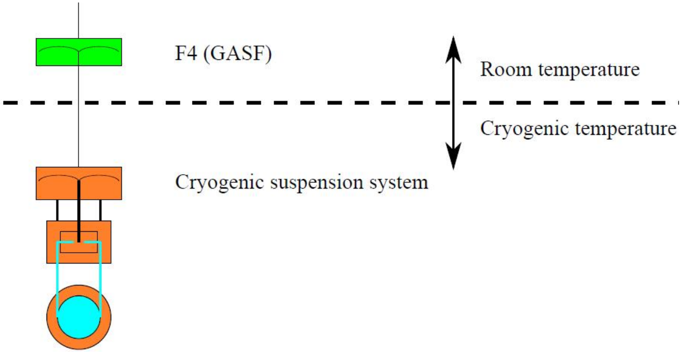

Figure: Schematic of the interferometer isolation. [**Taken from KAGRA PhD Thesis: D.Chen, 2016**]

Outline

Toy Model to Estimate DM Induced Strain

- Simple pendulum describing the TM and its suspension system

$$m\ddot{x}_c + k_c(1 + \mathrm{i}\phi)x_c = \frac{F_{\mathrm{ext},c}}{L}$$

- We model the total displacement [Moore, Cole, Berry '14]

$$x_{\mathrm{tot},c}(t) = x_{\mathrm{th},c}(t) + x_{\mathrm{qu},c}(t) + x_{\mathrm{DM},c}(t)$$

- The strain can be obtained from one-sided power spectral density (PSD)

$$h_i(\omega) = \sqrt{S_i(\omega)}$$

Suspension Thermal Noise

- According to fluctuation-dissipation theorem, the thermal noise PSD is given by [Callen, Welton '51]

$$S_{\rm th}(\omega) = \frac{4k_B T}{L^2\omega^2}\Re[Y(\omega)]$$

- The admittance for $F_{\rm ext} \sim exp(\mathrm{i}\omega t)$ in this case

$$Y(\omega) \equiv \frac{\dot{x}_{{\rm th},c}}{F_{\rm ext}/(mL)} = \mathrm{i}\omega D_c^{-1}(\omega) = \frac{\mathrm{i}\omega}{(-m\omega^2 + k_c(1+\mathrm{i}\phi))}$$

- $D_c(\omega)$ is the Fourier transform of the differential operator.

Suspension Thermal Noise

- The Thermal noise PSD of the Toy model ($\omega_c^2 \equiv k_c/m$)

$$S_{\mathrm{th},c}(\omega) = \frac{4k_B T}{L^2} \frac{\phi\,\omega_c^2/(m\omega)}{(\omega^2-\omega_c^2)^2+\omega_c^4\phi^2}$$

- The associate Thermal noise strain

$$h_{\mathrm{th}}(\omega) = \sqrt{S_{\mathrm{th}}(\omega)}$$

Quantum Noise

- For quantum noise, we use the standard quantum limit (SQL)

$$S_{\mathrm{qu}}(\omega) = \frac{8\hbar}{m\omega^2 L^2}$$

- The corresponding strain

$$h_{\mathrm{qu}}(\omega) = \sqrt{S_{\mathrm{qu}}(\omega)}$$

- The total noise reads

$$h_{\mathrm{n}}(\omega) = \sqrt{h^2_{\mathrm{th}}(\omega) + h^2_{\mathrm{qu}}(\omega)}$$

DM Signal

- Recall the simple pendulum motion

$$m\,\ddot{x}_{\rm DM} + k_c(1+{\rm i}\phi)\,x_{\rm DM} = \frac{F_{\rm ext}}{L}$$

- Using Fourier expansion

$$g(t) = \int_{-\infty}^{\infty} d\omega\,\tilde{g}(\omega)e^{{\rm i}\omega t}$$

- We assume single DM hit at t = 0 [Lee, Nugroho, MS '20; Tsuchida et al.'19]

$$F_{\rm ext}(t) = q_R\,\delta(t)$$

- The displacement induced by DM in frequency domain reads

$$\tilde{x}_{\rm DM}(\omega) = \frac{q_R}{m\,L}\frac{1}{(-\omega^2+\omega_c^2(1+{\rm i}\phi))}$$

- The DM induced strain

$$h_{\rm DM}(\omega) = \sqrt{\frac{2\,\omega}{\pi}}|\tilde{x}_{DM}(\omega)|$$

The Strain for the Toy Model

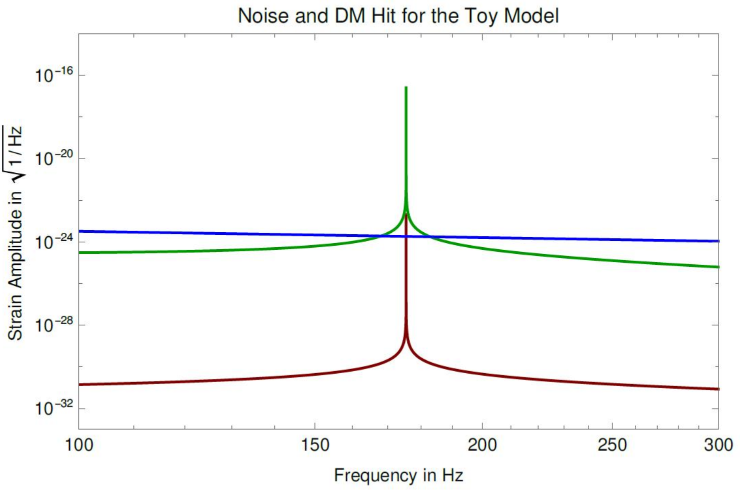

Figure: **Thermal noise**, **SQL**, **DM Signal**.

Signal to Noise Ratio (SNR)

- The optimal SNR [Moore, Cole, Berry '14]

$$\varrho^2 = \int_{f_{\rm min}}^{f_{\rm max}} df \, \frac{4\,|\tilde{x}_{\rm DM}(2\pi f)|^2}{S_n(2\pi f)}$$

- For: $f_0 = 175.4\,\text{Hz}$, $\phi = 6.32 \times 10^{-12}$, $m = 22.8\,\text{kg}$, $T = 19\,\text{K}$, $L = 3\,\text{km}$, $m_{\rm DM} = 1\text{GeV}/c^2$, and $|\vec{v}_{DM}| = 220\,\text{km/s}$
- Around the peak at full width half maximum (FWHM)

$$\varrho_{\rm th}^2 = \frac{1}{2\pi}\frac{q_R^2}{m\,k_B\,T} = \frac{1}{2\pi}\frac{E_R}{E_{\rm th}} = 4.09 \times 10^{-24}$$

- $E_R = 3.37 \times 10^{-45}\,\text{J}$ and $E_{\rm th} = 1.31 \times 10^{-23}\,\text{J}$
- Lighter mirror and colder for better SNR.

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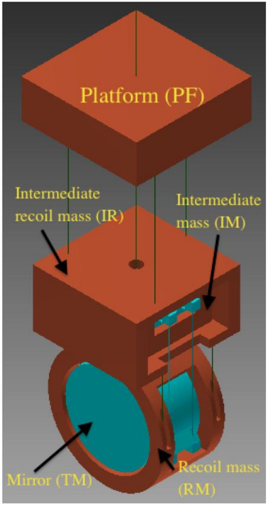

- KAGRA is GW detector currently running in Japan
- Equipped with cryogenic system with mirror temperature around 20 K
- The suspension system is modelled as triple pendulum consist of: IM, Blade Springs, and TM

Figure: [**KAGRA PhD Thesis: D.Chen, 2016**]

KAGRA Suspension Thermal Noise

- Vertical suspension thermal noise [KAGRA Document, JGW-T1707038v9]

$$\left(M\frac{d^2}{dt^2} + K_v\right)\vec{x}_v(t) = \frac{\vec{F}_{\mathrm{ext},v}(t)}{L}$$

$$M = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \quad K_v = \begin{pmatrix} K_{1v} + K_{2v} & -K_{2v} & 0 \\ -K_{2v} & K_{2v} + K_{3v} & -K_{3v} \\ 0 & -K_{3v} & K_{3v} \end{pmatrix}$$

$$\vec{x}_v(t) = \begin{pmatrix} x_{1v}(t) \\ x_{2v}(t) \\ x_{3v}(t) \end{pmatrix} \text{ and } \vec{F}_{\mathrm{ext},v}(t) = \begin{pmatrix} F_{\mathrm{ext},1v}(t) \\ F_{\mathrm{ext},2v}(t) \\ F_{\mathrm{ext},3v}(t) \end{pmatrix}.$$

$$K_{iv} \equiv k_{iv}(1 + \mathrm{i}\,\phi_{iv})$$

- The index i = (1, 2, 3) stands for (IM, BS, and TM)

KAGRA Suspension Thermal Noise

- The equation in frequency domain

$$D_v(\omega)\vec{\tilde{x}}_v \equiv \left(-\omega^2 M + K_v\right)\vec{\tilde{x}}_v = \frac{\vec{\tilde{F}}_{\mathrm{th},v}}{L}$$

- The vertical admittance reads

$$Y_v(\omega) \equiv \mathrm{i}\omega D_v^{-1}(\omega)$$

- Thermal noise PSD of the vertical thermal noise

$$S_{\mathrm{th},v}(\omega) = \frac{4\,k_B\,T\,\Re(Y_v(\omega))_{33}}{L^2\omega^2}$$

- The strain amplitude of vertical suspension thermal noise

$$h_{\mathrm{th},v}(\omega) = \mathrm{VHC}\sqrt{4|S_{\mathrm{th},v}(\omega)|}$$

- VHC due to the tilt of the baseline. Its value is $\frac{1}{200}$

Other Noise

- Horizontal thermal noise proceed in similar manner as the verctical one
- Mirror thermal noise and quantum noise
- Sum over the total noise

$$h_{\text{tot}}(\omega) = \sqrt{h^2_{\text{th},v} + h^2_{\text{th},h} + h^2_{\text{qu}}(\omega) + h^2_{\text{mir}}(\omega)}$$

KAGRA Noise

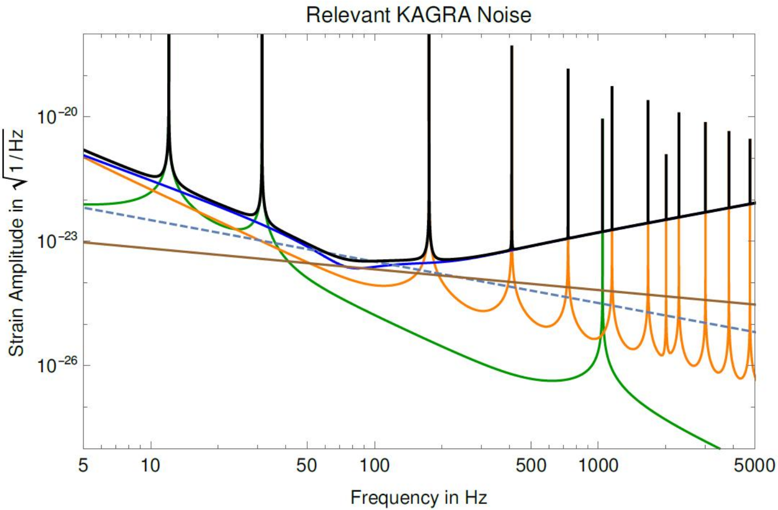

Figure: **total noise**: **vert.susp.**, **horz.susp.**, **mirror noise**, **quantum noise (solid blue)**, and **SQL (dashed blue)** [Lee, Nugroho, MS '20; JGW-T1707038v9]

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DM Signal at KAGRA

- DM hit the j-th component from vertical direction [Lee, Nugroho, MS '20]

$$F_{\mathrm{DM},jv}(t) = q_{R,jv}\delta(t)$$

- The Fourier transform of the displacement of the i-th component

$$\tilde{x}_{iv}(\omega) = \sum_{j=1}^{3} (D_v^{-1}(\omega))_{ij} \frac{\tilde{F}_{j,v}(\omega)}{L}$$

- Since we probe the third component, we have

$$|\tilde{x}_{\mathrm{DM},v}(\omega)|^2 \equiv |\tilde{x}_{3v}(\omega)|^2 = \left| \sum_{j=1}^{3} (D_v^{-1}(\omega))_{3j} \frac{\tilde{F}_{j,v}(\omega)}{L} \right|^2$$

- The strain is given by

$$h_{\mathrm{DM},v}(\omega) = \mathrm{VHC}\sqrt{\frac{2\,\omega}{\pi}|\tilde{x}_{\mathrm{DM},v}(\omega)|^2}$$

DM Signal at KAGRA

- The DM induced strain in horizontal direction

$$h_{\mathrm{DM},v}(\omega) = \sqrt{\frac{2\omega}{\pi}|\tilde{x}_{\mathrm{DM},h}(\omega)|^2}$$

- The total DM signal

$$|\tilde{x}_{\mathrm{DM}}(\omega)|^2 = \mathrm{VHC}^2|\tilde{x}_{\mathrm{DM},v}(\omega)|^2 + |\tilde{x}_{\mathrm{DM},h}(\omega)|^2$$

DM Signal at KAGRA

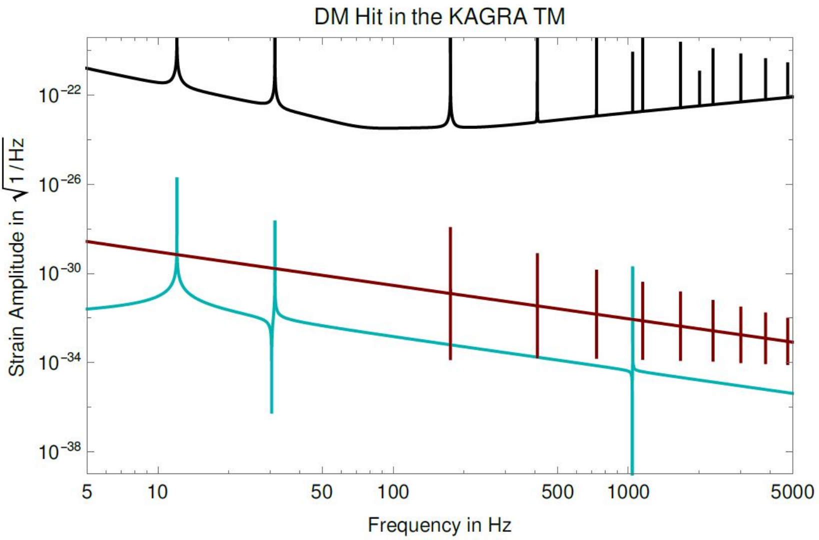
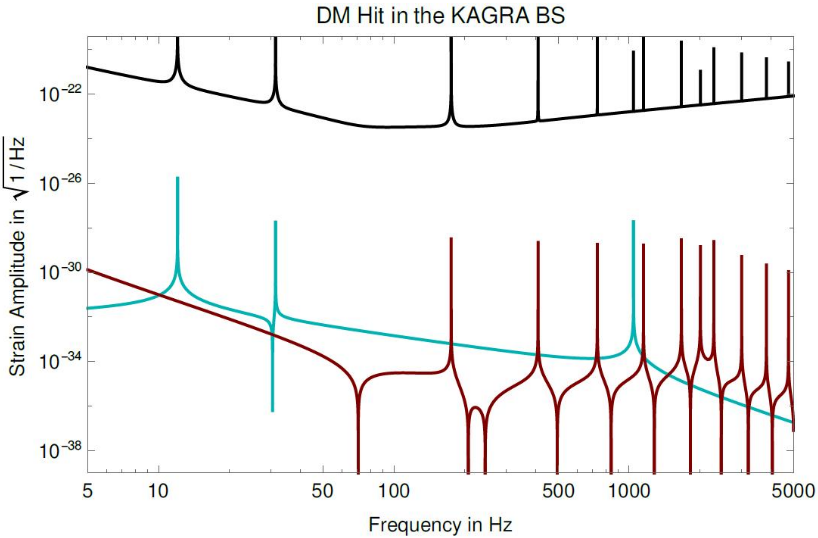


Figure: **total noise**, **DM vertical hit**, and **DM horizontal hit**

DM Signal at KAGRA



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DM Signal at KAGRA

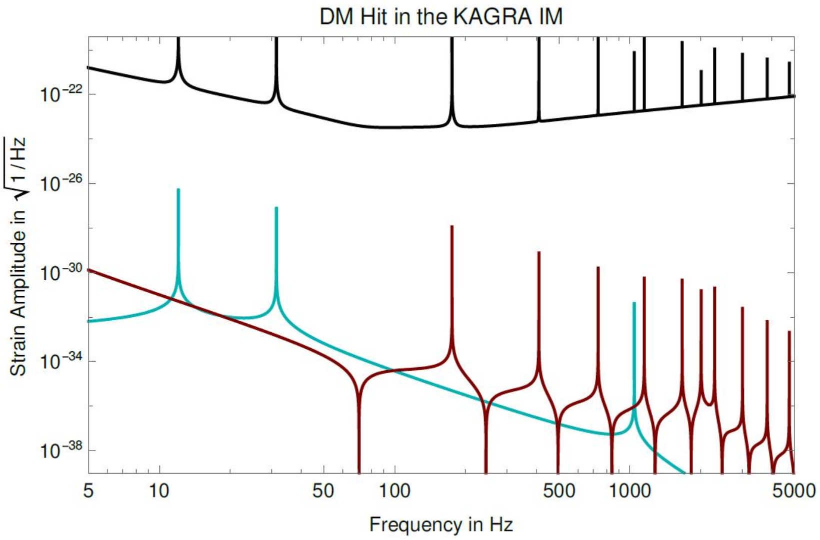

Figure: **total noise**, **DM vertical hit**, and **DM horizontal hit**

SNR

- For horizontal DM hit

$$\varrho_h^2 = \int_{f_{\min,h}}^{f_{\max,h}} df \frac{4|\tilde{x}_{\rm DM}(2\,\pi\, f)|^2}{S_{\rm tot}(2\,\pi\, f)}$$

- We integrate around the peak which is located at 175.4 Hz

$$f_{\min,h} = 170\text{Hz and } f_{\max,h} = 180\text{Hz}$$

- This gives the SNR

$$\varrho_h^2 = 6.94 \times 10^{-18}$$

- For vertical hit around 31.4 Hz

$$f_{\min,h} = 30.4\text{Hz and } f_{\max,h} = 32.4\text{Hz}$$

- The corresponding SNR

$$\varrho_v^2 = 1.89 \times 10^{-21}$$

Space-Based Experiment

- LISA path finder sensitivity within 0.1 mHz $\leq f \leq$ 30 Hz [LISA Pathfinder '16]

$$\sqrt{S_{\Delta g}} \leq 3\sqrt{2}\ \mathrm{fm\,s^{-2}}/\sqrt{\mathrm{Hz}} \times \sqrt{1 + (f/8\,\mathrm{mHz})^4}$$

- DM induced strain

$$\sqrt{S_{\Delta g,\mathrm{DM}}} \sim 4.1 \times 10^{-7} \sqrt{\frac{f}{\mathrm{Hz}}}\,\mathrm{fm\,s^{-2}}/\sqrt{\mathrm{Hz}}$$

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Summary

- The current interferometer used in GW experiments are very sensitive.
- DM can excite mechanical resonance as suspension thermal noise does.
- For DM detection, lighter and colder mirror are needed.
- LPF is quite sensitive, but not sensitive enough to detect DM.