Interaction-induced Symmetry Protected Topological Phase in Harper-Hofstadter models

arXiv:1609.03760

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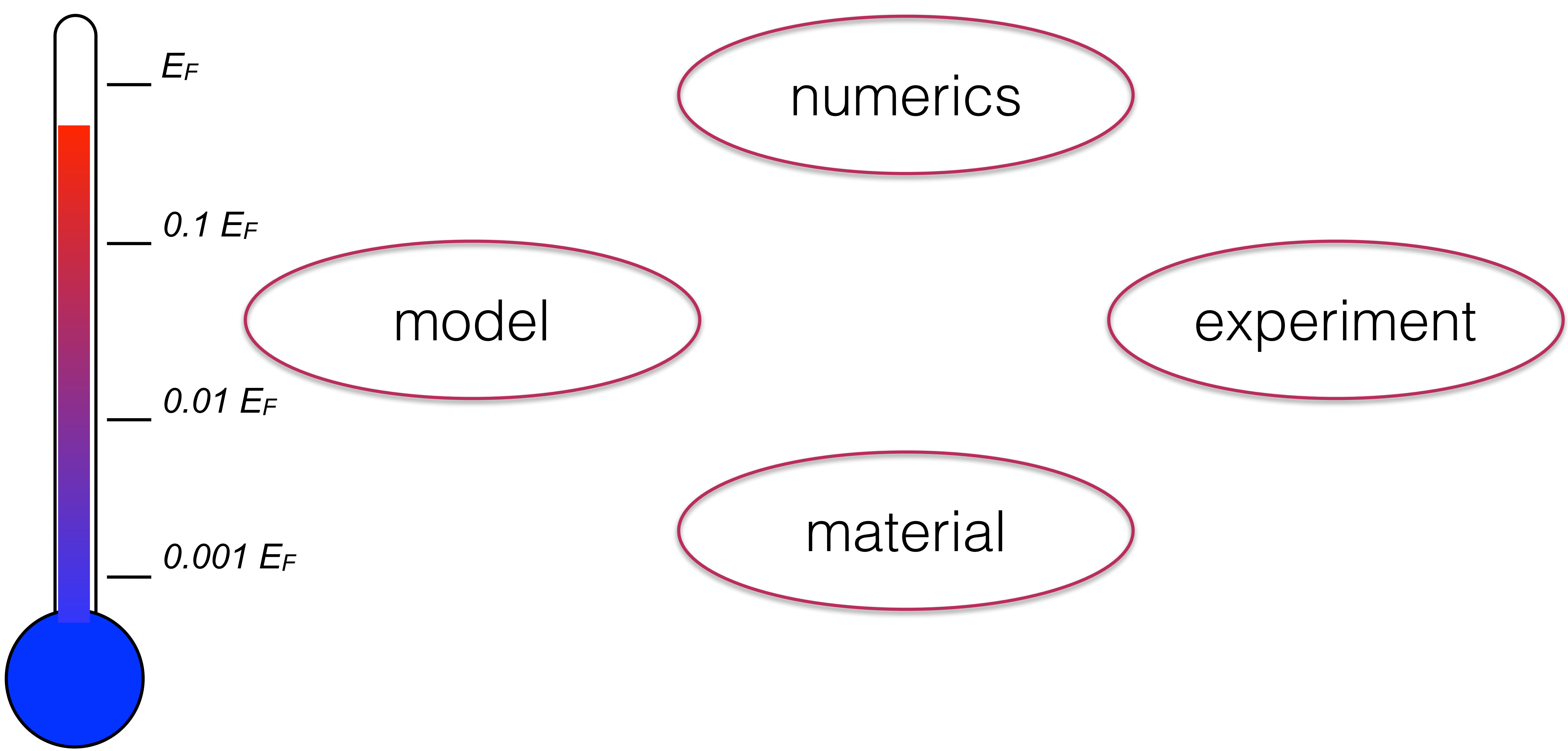
Algorithmic developments

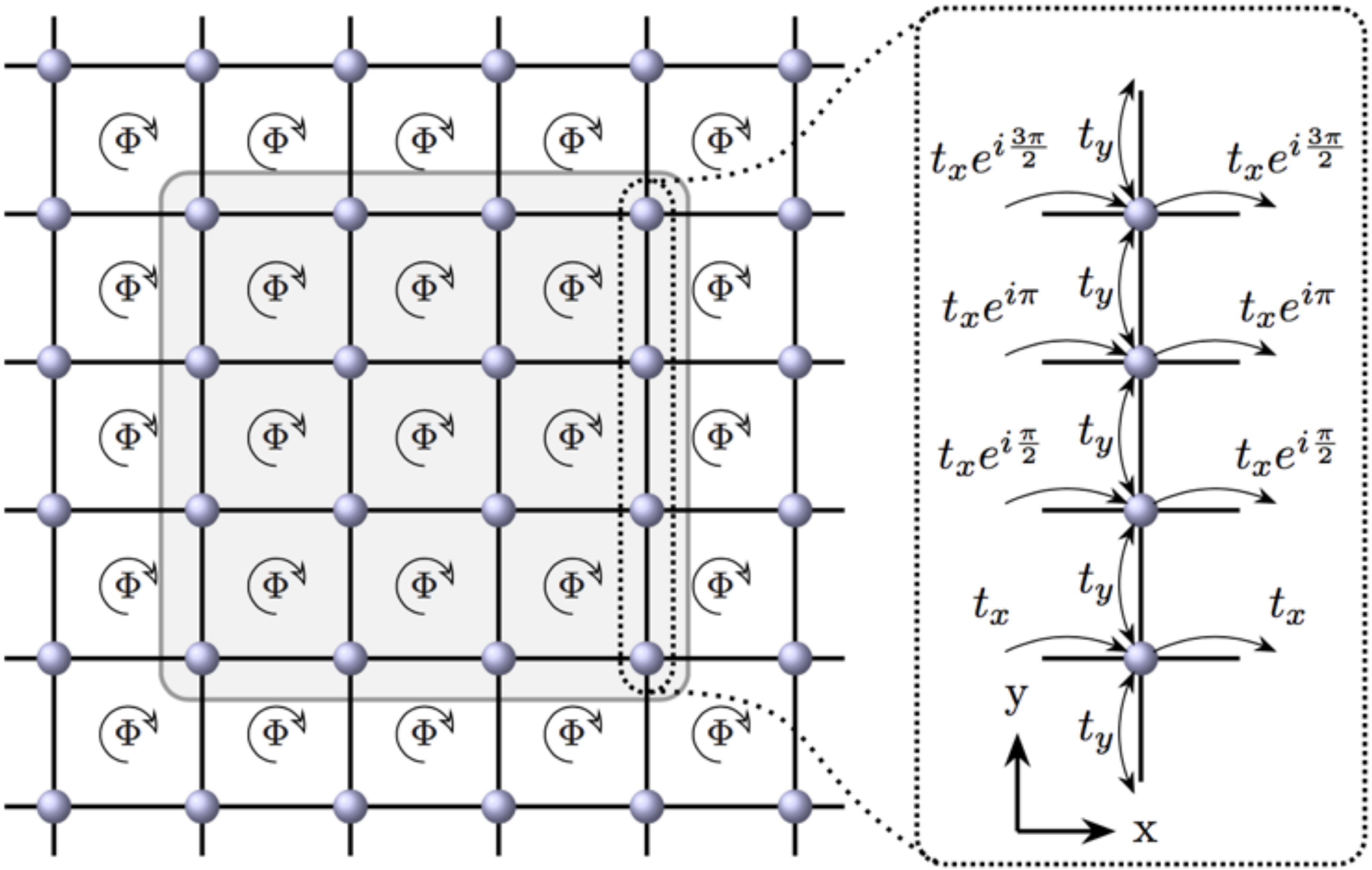
diagrammatic Monte Carlo, (bosonic) cluster methods, …

the quest for novel numerical methods going beyond the state of the art:



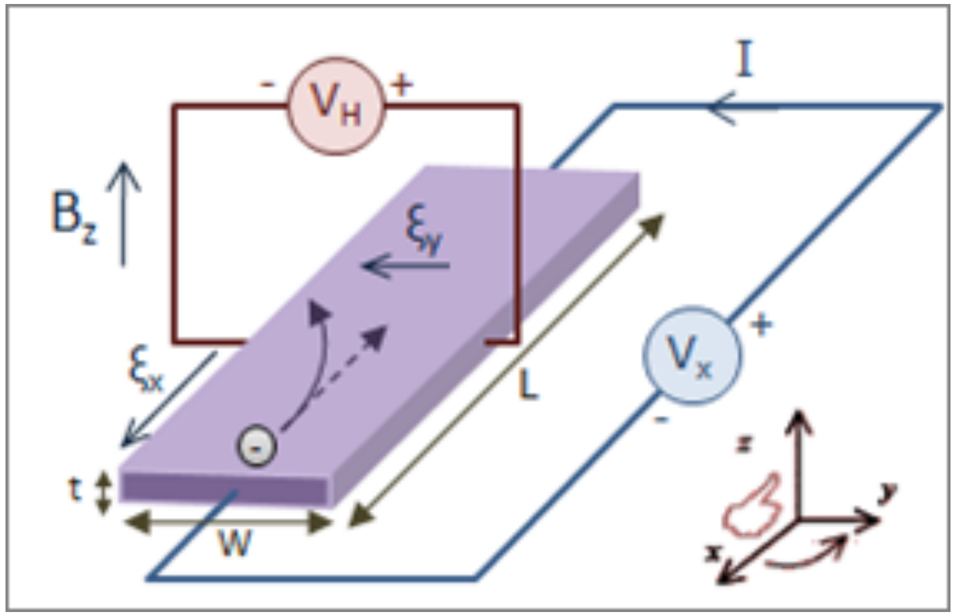
Harper-Hofstadter model

hardcore bosons



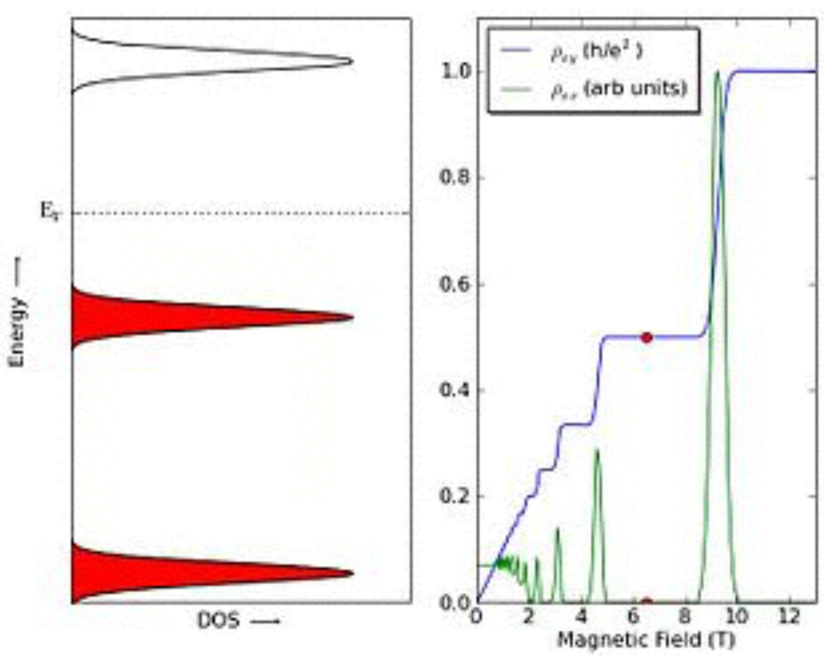
$$H_\Phi = -\sum_{x,y} \left(t_x e^{iy\Phi} b^\dagger_{x+1,y} b_{x,y} + t_y b^\dagger_{x,y+1} b_{x,y} \right) + h.c.$$

Hall effect



$$R_H = \frac{E_y}{j_x B_z} = -\frac{1}{ne}$$

Integer Quantum Hall effect



Landau levels

topological invariant: first Chern number (TKNN)

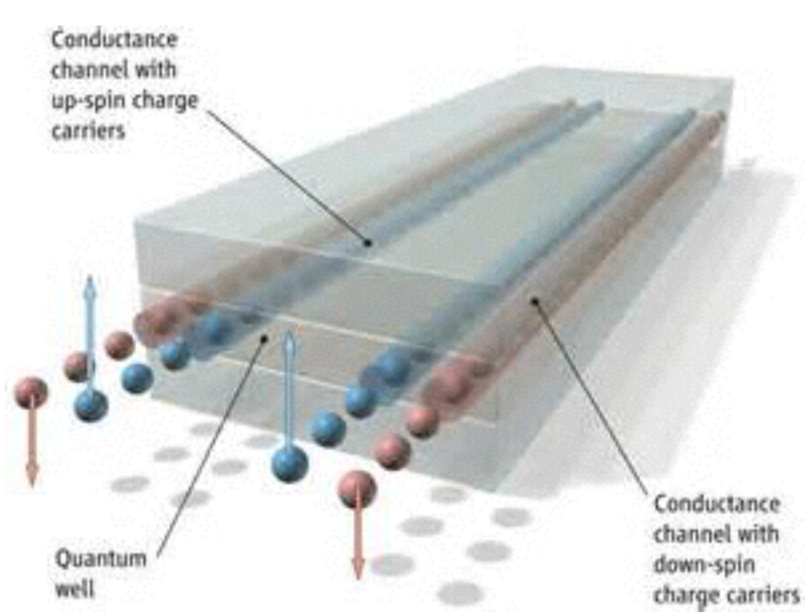
$$\sigma_{xy} = ne^2/h$$

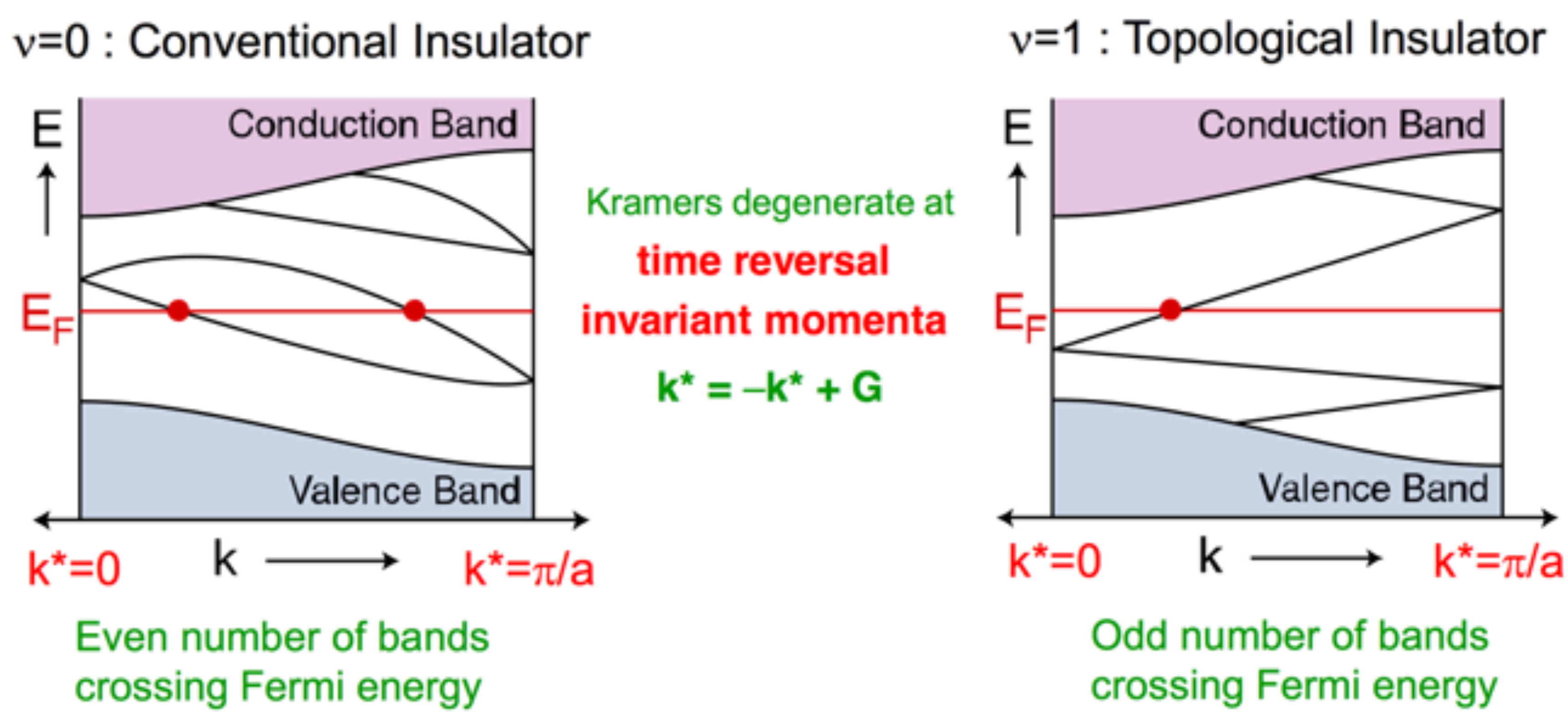
$$n = \frac{1}{2\pi i}\int_{\mathrm{BZ}} d^2k \, \langle \nabla_k u(k)| \times |\nabla_k u(k)\rangle$$

spin-Quantum Hall Effect

Z_2 topological invariant protected by time reversal symmetry (TKNN integer is 0)

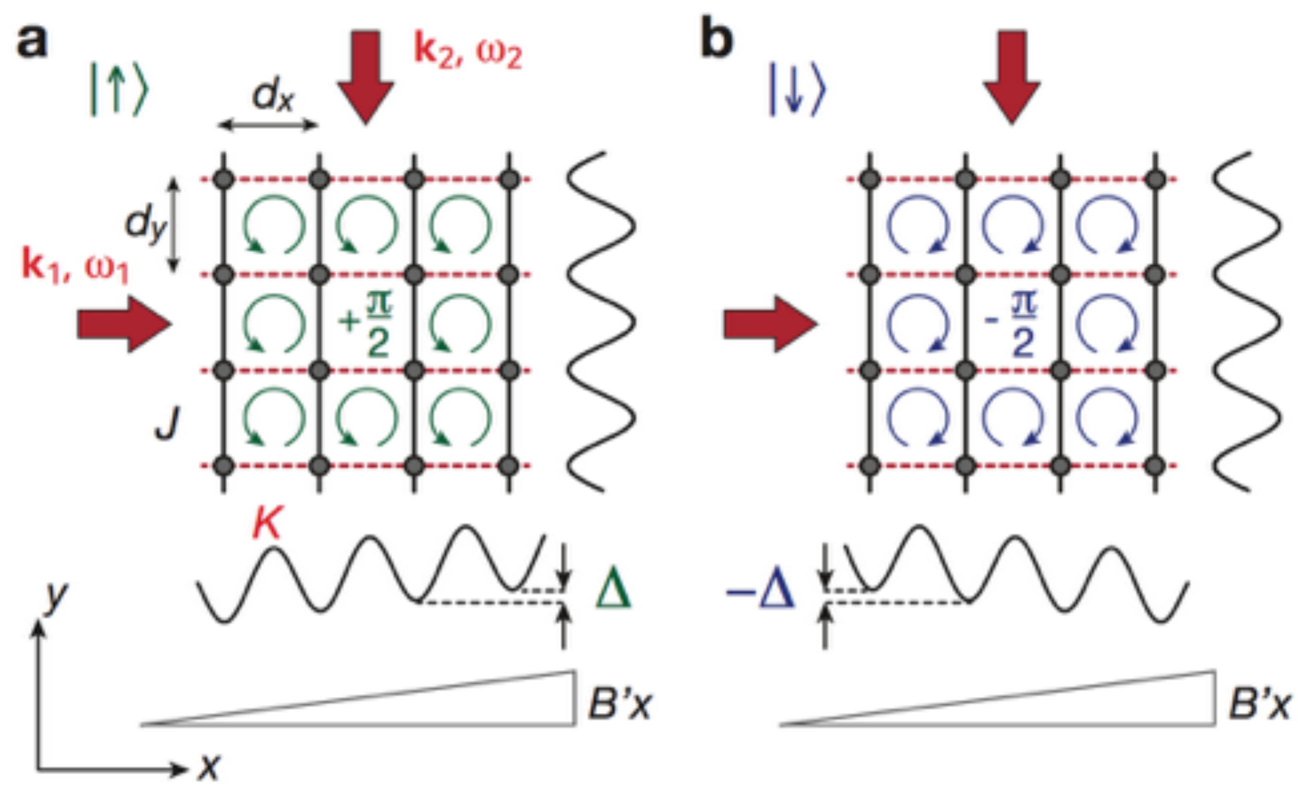
eg : spin-orbit coupling in graphene (but too weak),
CdTe/HgTe/CdTe structures (band inversion as function of thickness)





(CL Kane)

cold atom experiments

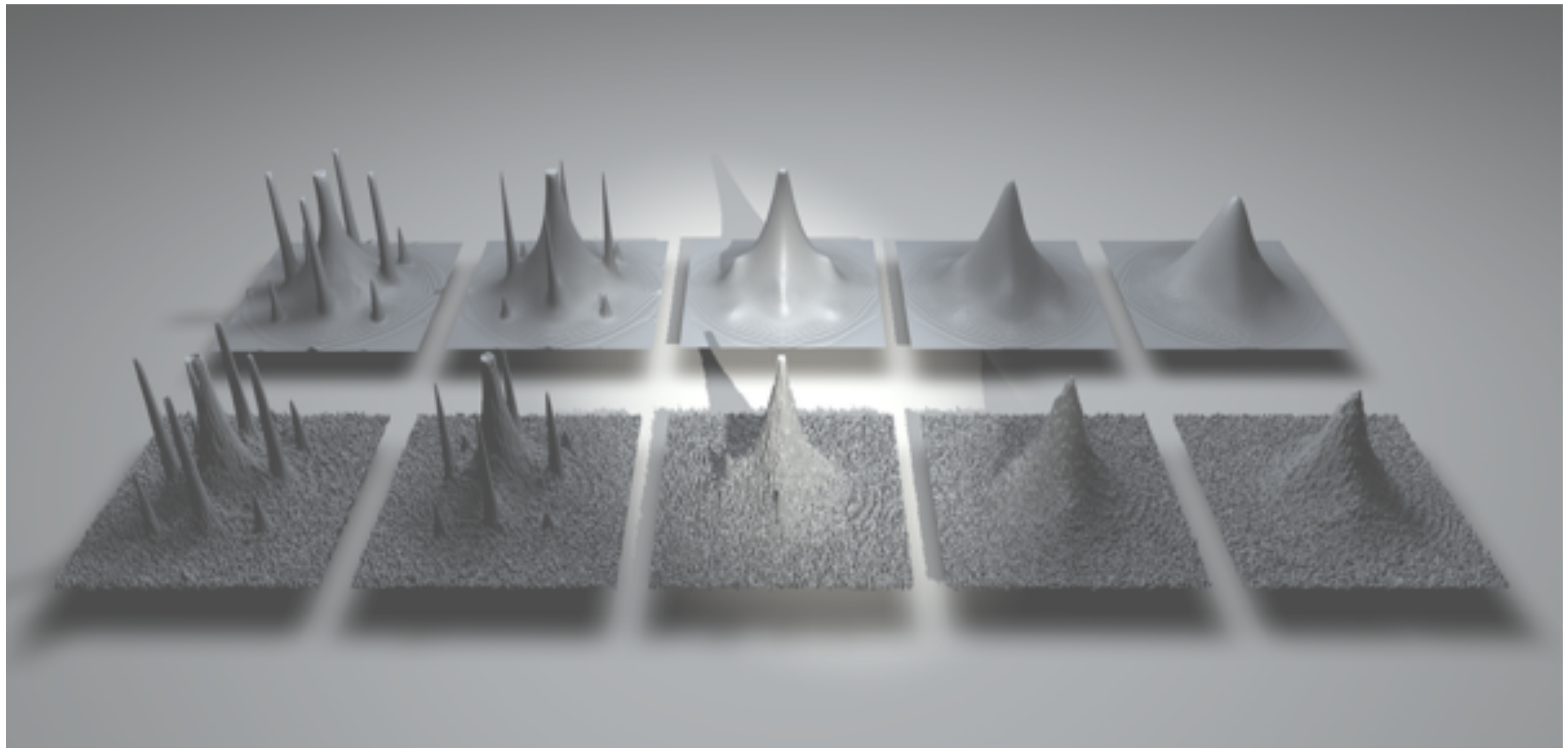


- very strong effective magnetic fields
- all optical setup with bosonic atoms
- Chern number has been measured (also for hexagonal lattice with fermions (ETHZ))
- add interactions?

M. Aidelsburger et al, Phys. Rev. Lett. 111, 185301 (2013)

M. Aidelsburger et al, Nature Physics 11, 162–166 (2015), AOP 3171 (2014)

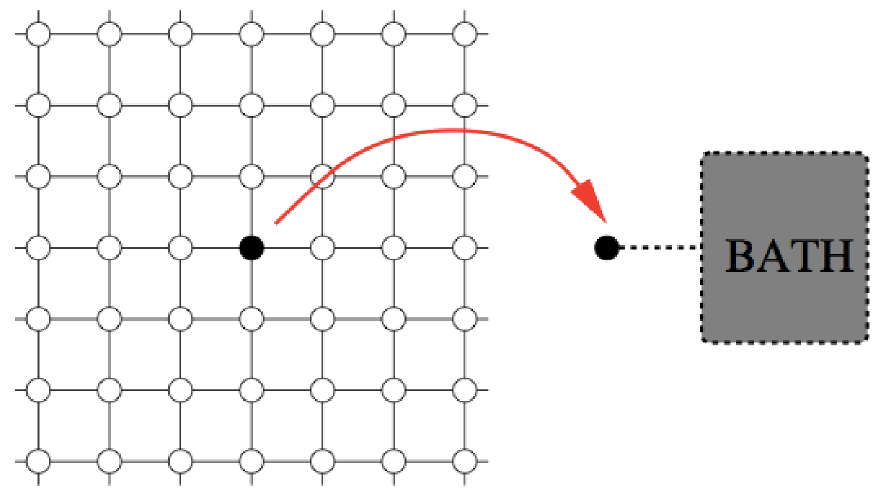
how to study?



The usual path integral Monte Carlo simulations do not work because of the infamous sign problem…

intermezzo: develop an approximate method and benchmark it

mean-field theory



classical Ising (ferromagnet J > 0):

$$H = -J \sum_{\langle i,j \rangle} S_i S_j + h \sum_i S_i$$

we are interested in the magnetization on every site:

$$m_i = \langle S_i \rangle$$

Weiss field:

$$H_{\mathrm{eff}} = -\sum_i h_i^{\mathrm{eff}} S_i \qquad \beta h_i^{\mathrm{eff}} = \tanh^{-1} m_i$$

approximation:

$$h_i^{\mathrm{eff}} \approx h + \sum_j J m_j = h + zJm$$

selfconsistency equation :

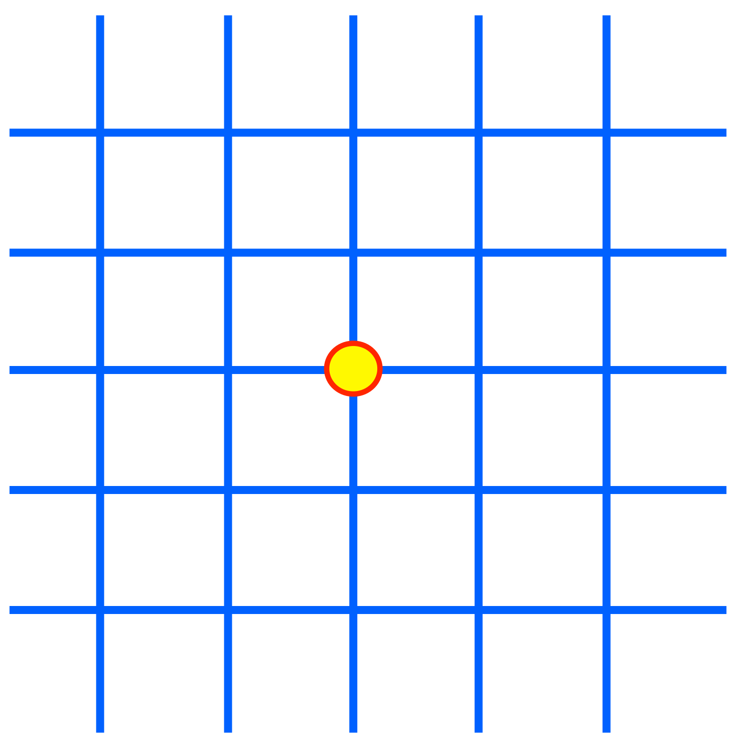
$$m = \tanh(\beta h + z\beta J m)$$

We want to develop the *dynamical* mean-field solution for the 3d Bose-Hubbard model

~2010

$$H = -t \sum_{\langle i,j \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) - \mu \sum_i n_i$$

write down single-site action :



$$S_{\mathrm{imp}} = \int_0^\beta d\tau b^\dagger(\tau) \left[\partial_\tau - \mu\right] b(\tau) + \frac{U}{2} \int_0^\beta d\tau n(\tau) \left[n(\tau) - 1\right]$$

we lose the momentum dependence for the self-energy

works fine for normal phase and Mott phase.

we want:
include the physics of the weakly interacting Bose gas non-perturbatively,
which is the limit t >>U

we want: correction on top of mean-field let's add a symmetry breaking field :

$$-zt\phi \int_0^\beta d\tau[b(\tau) + b^\dagger(\tau)]$$

this is the same as in static mean-field which can produce a condensate

Bogoliubov prescription : $$b(\tau) = \langle b \rangle + \delta b(\tau)$$

imag time dynamics can be added in the two-particle channel.
The second source field can only couple to the normal bosons, otherwise *double counting* will occur (Nambu notation):

for infinite coordination number, this term is *zero*

$$-\frac{1}{2}\int_0^\beta d\tau d\tau' \delta\mathbf{b}^\dagger(\tau)\mathbf{\Delta}(\tau - \tau')\delta\mathbf{b}(\tau')$$

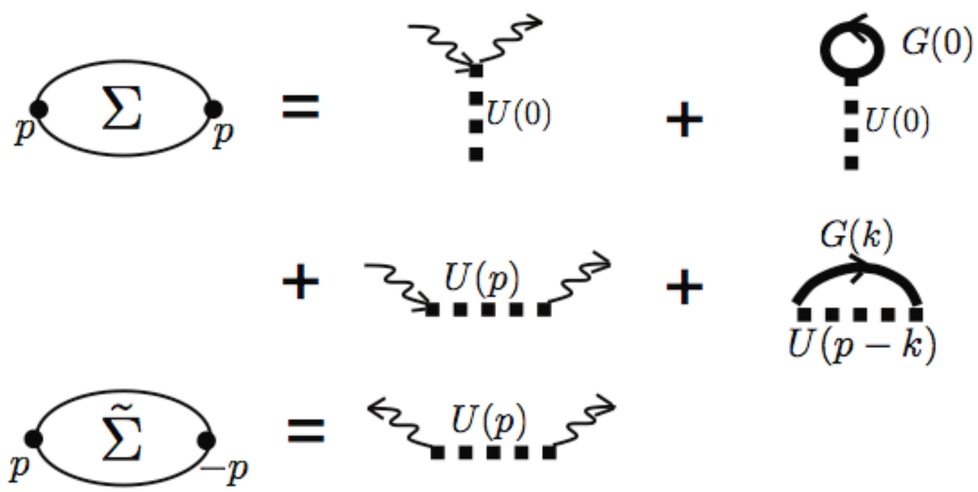
which contains normal and anomalous propagators.

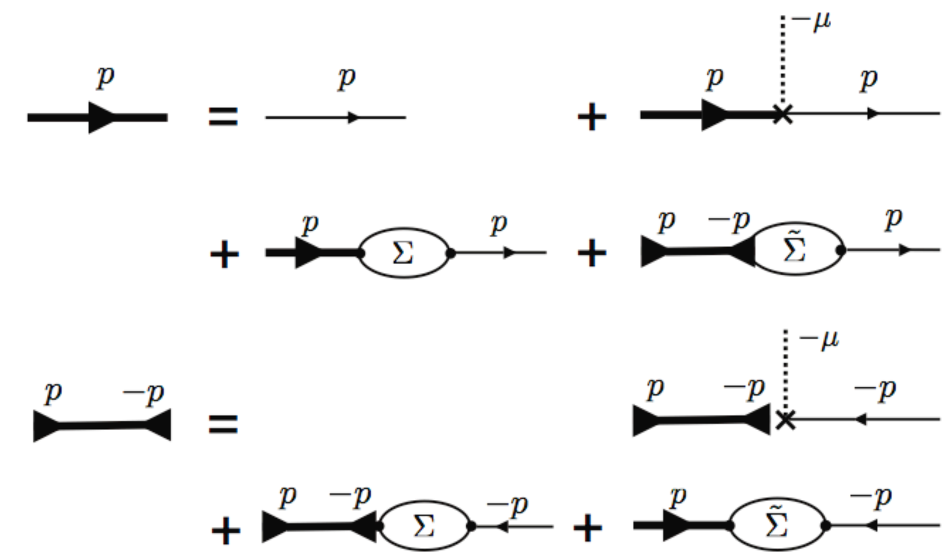
see J.W. Negele and H. Orland, Quantum Many-Particle Systems (Addison-Wesley Publishing Company 1988) ISBN 0-201-12593-5 for how to treat broken symmetry

Final step : re-express δb in terms of full b

weakly-interacting Bose gas

<u>*Why BDMFT should be good*:</u> look at self-energies of weakly interacting Bose gas (Beliaev)





$$\boxed{\begin{aligned} \Sigma(P) &= -2G(r=0,\tau=-0)U + 2n_0U = 2nU \\ \tilde{\Sigma}(P) &= n_0U\,. \end{aligned}}$$

momentum independent to leading order

B. Capogrosso-Sansone, S. Giorgini, S. Pilati, L. Pollet, N. V. Prokof'ev, B. V. Svistunov, and M. Troyer, New J. Phys. **12**, 043010 (2010)

$$G(P) = -\frac{\mathrm{i}\xi + \epsilon(k) + |\tilde{\mu}|}{\xi^2 + E^2(k)}$$

$$F(P) = \frac{|\tilde{\mu}|}{\xi^2 + E^2(k)}\,,$$

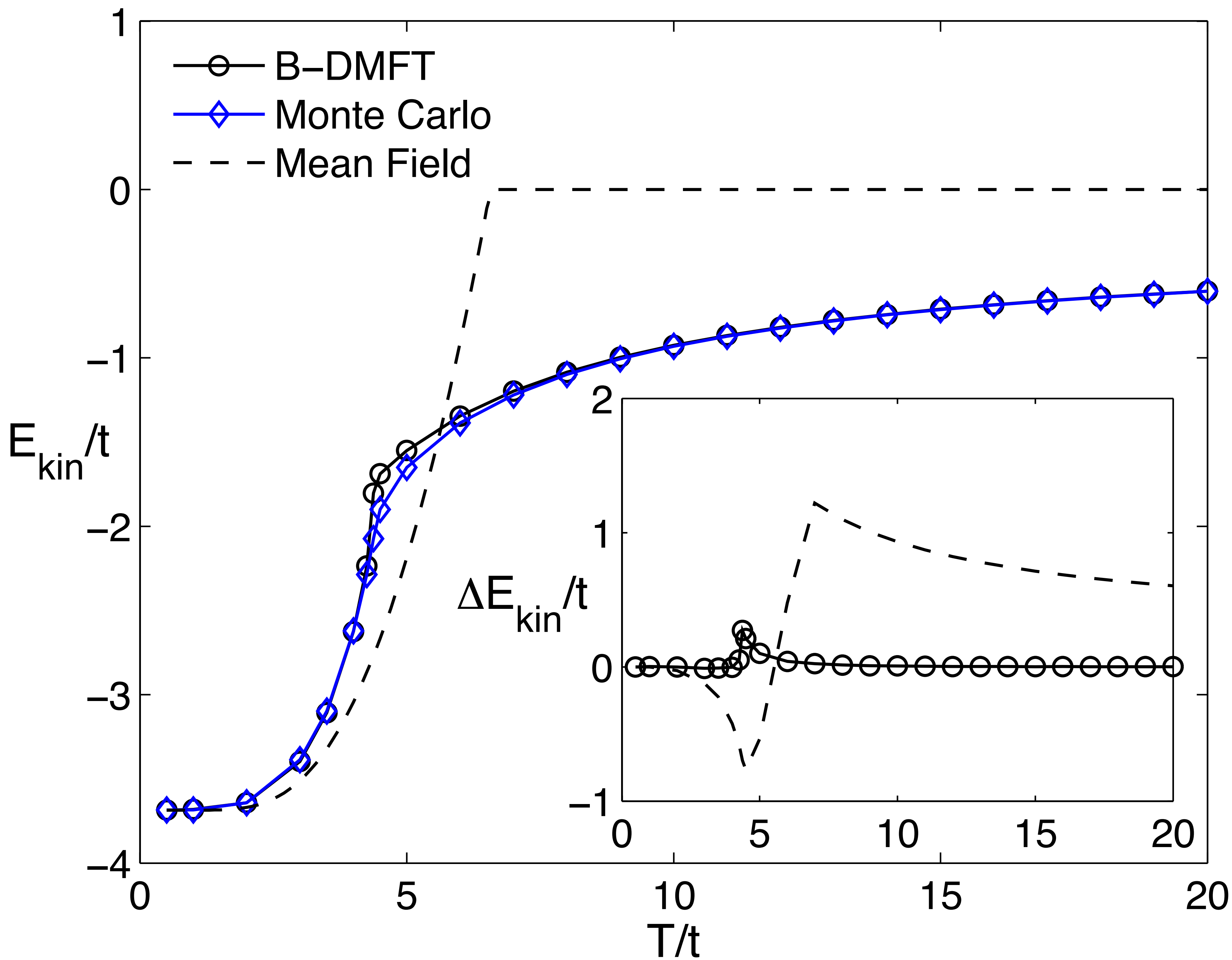
$$E^2(k) = \epsilon(k)[\epsilon(k) + 2|\tilde{\mu}|]$$

$$\tilde{\mu} = \mu - 2nU$$

similar in magnitude at low temperature, but opposite in sign

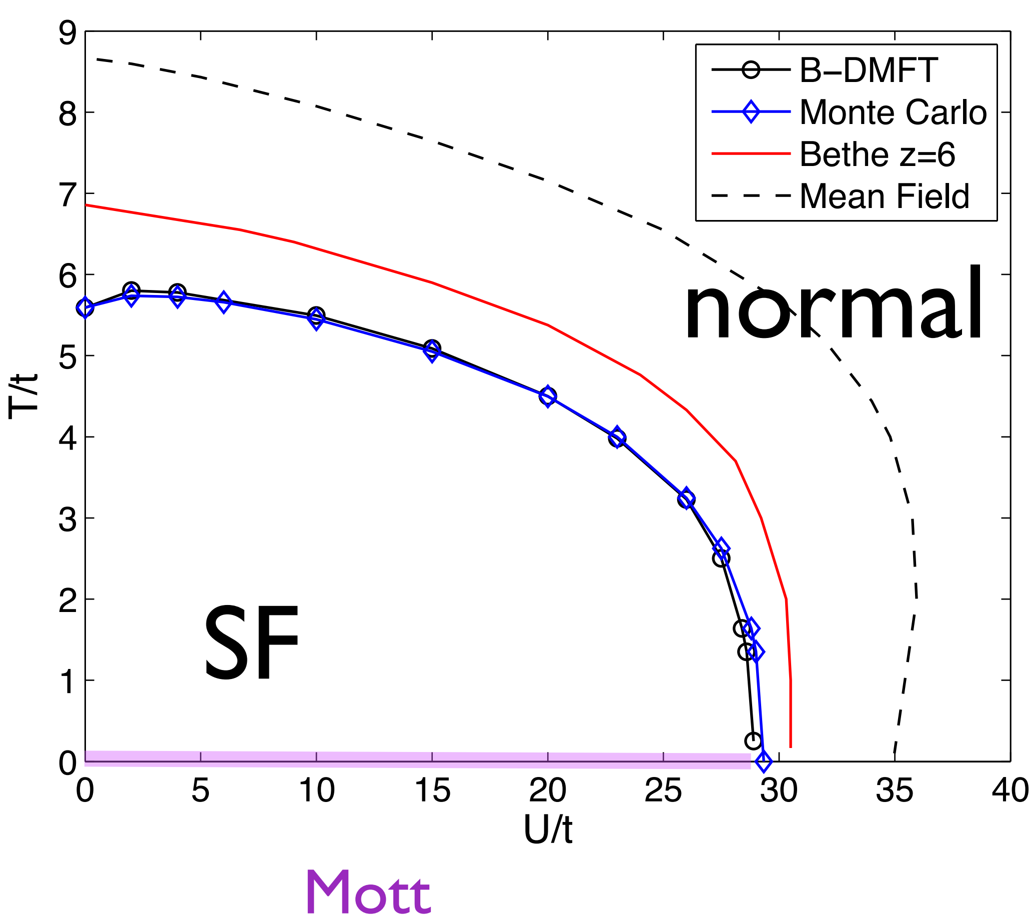
Hohenberg P C and Martin P C 1965 Ann. Phys. 34 291
Hugenholtz N M and Pines D S 1959 Phys. Rev. 116 489
Nepomnyashchii A A and Nepomnyashchii Yu A 1978 Zh. Eksp. Teor. Fiz. 75 976 [1978 Sov. Phys. JETP 48 493]
Nepomnyashchii Yu A 1983 Zh. Eksp. Teor. Fiz. 85 1244 [1983 Sov. Phys. JETP 58 722]

comparison in 3 dimensions

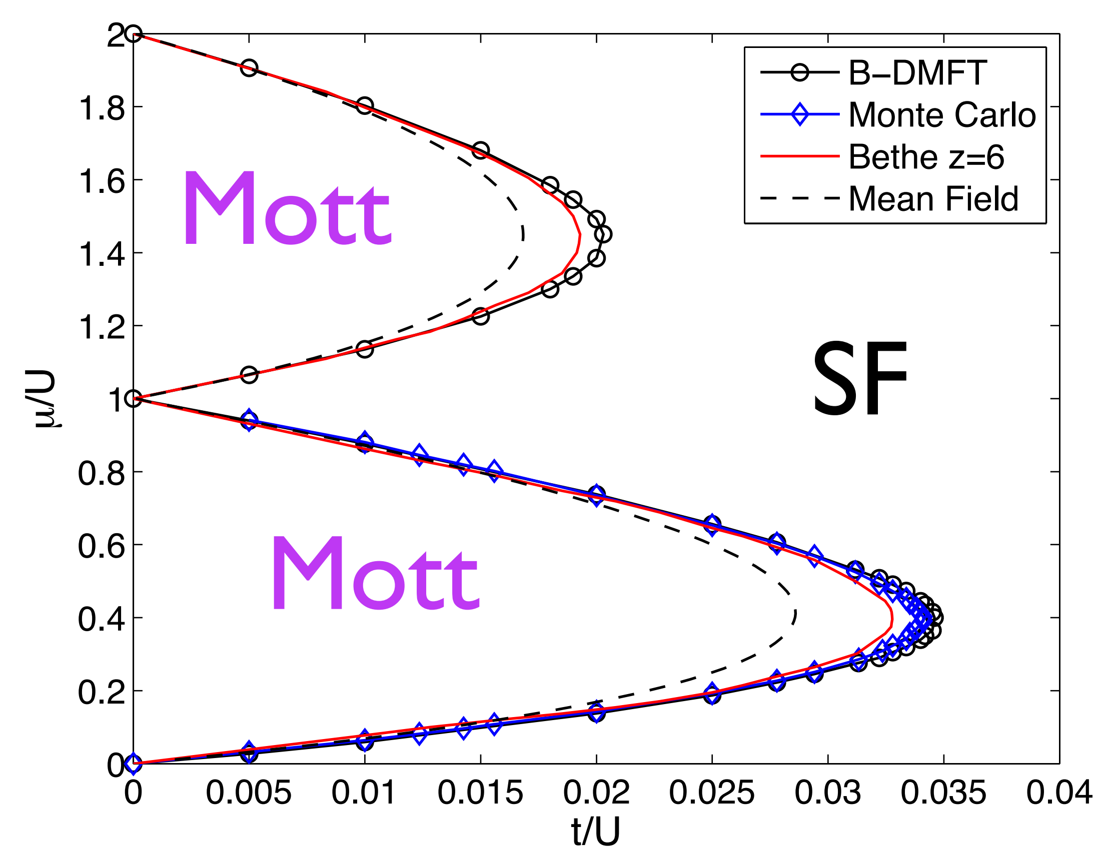


phase diagram in 3 dimensions

finite temperature, unit density

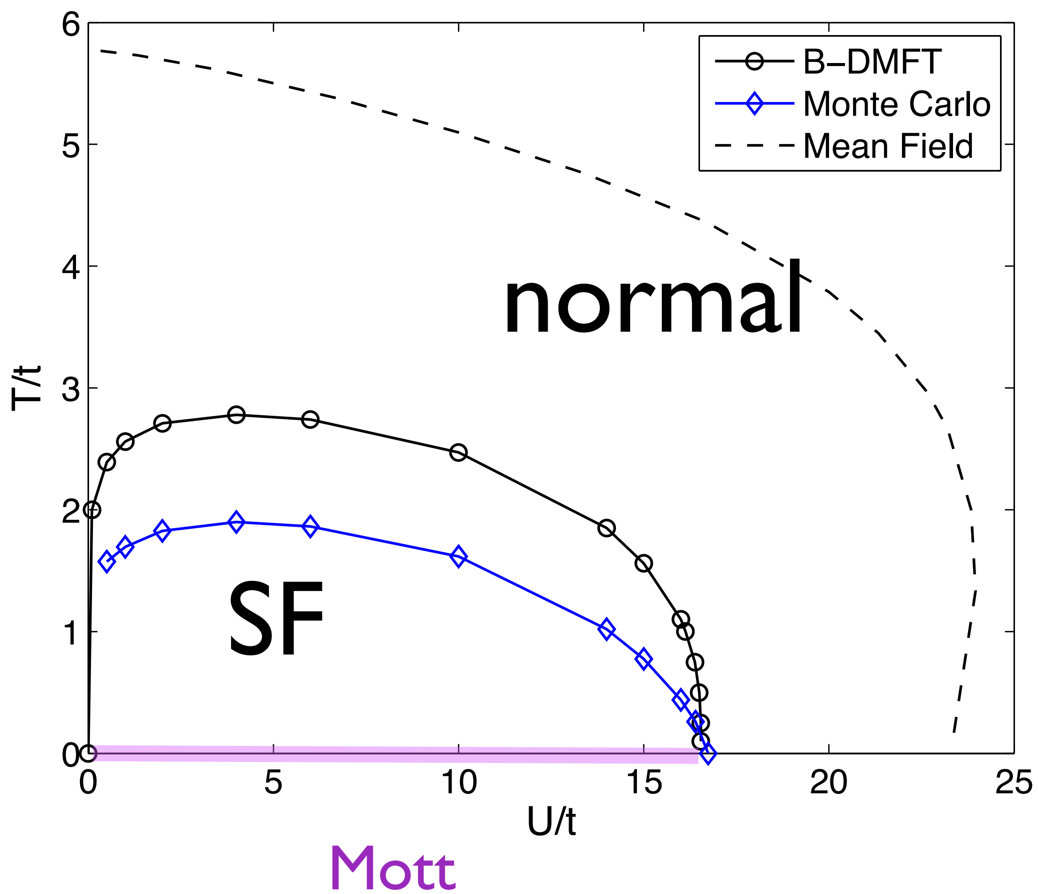


ground state

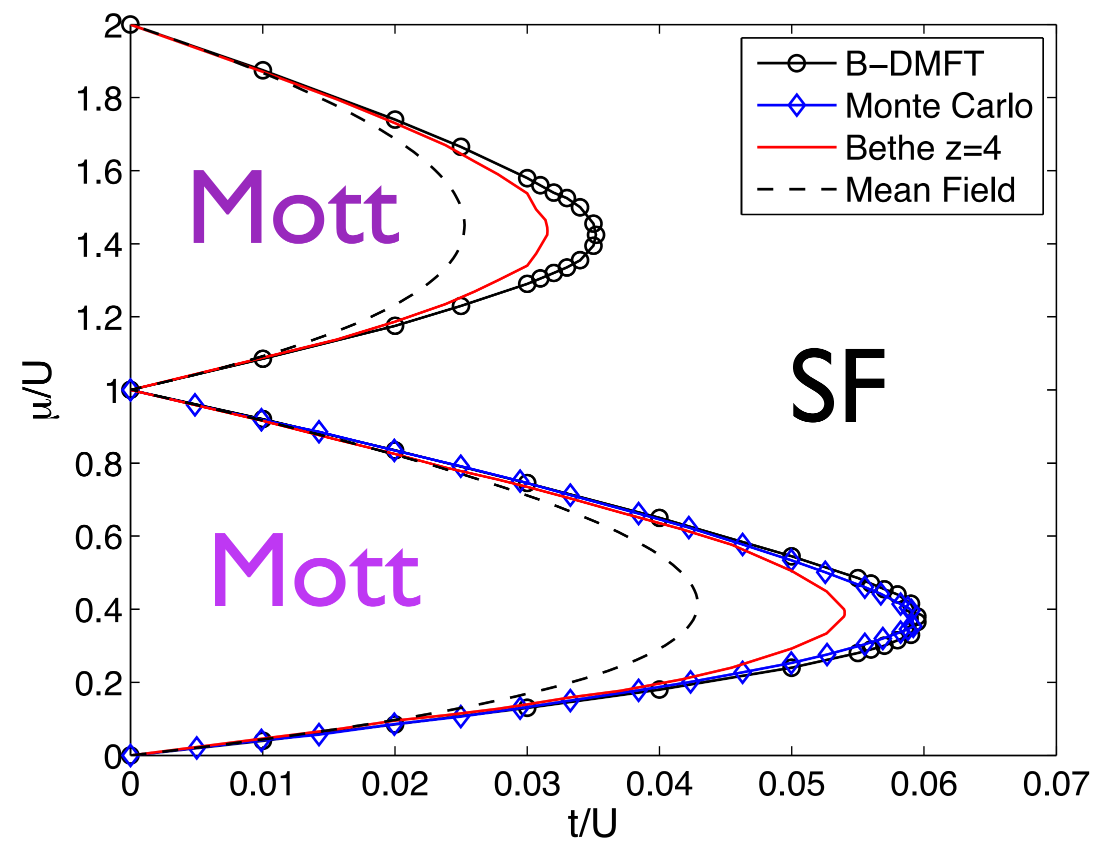


results in two dimensions

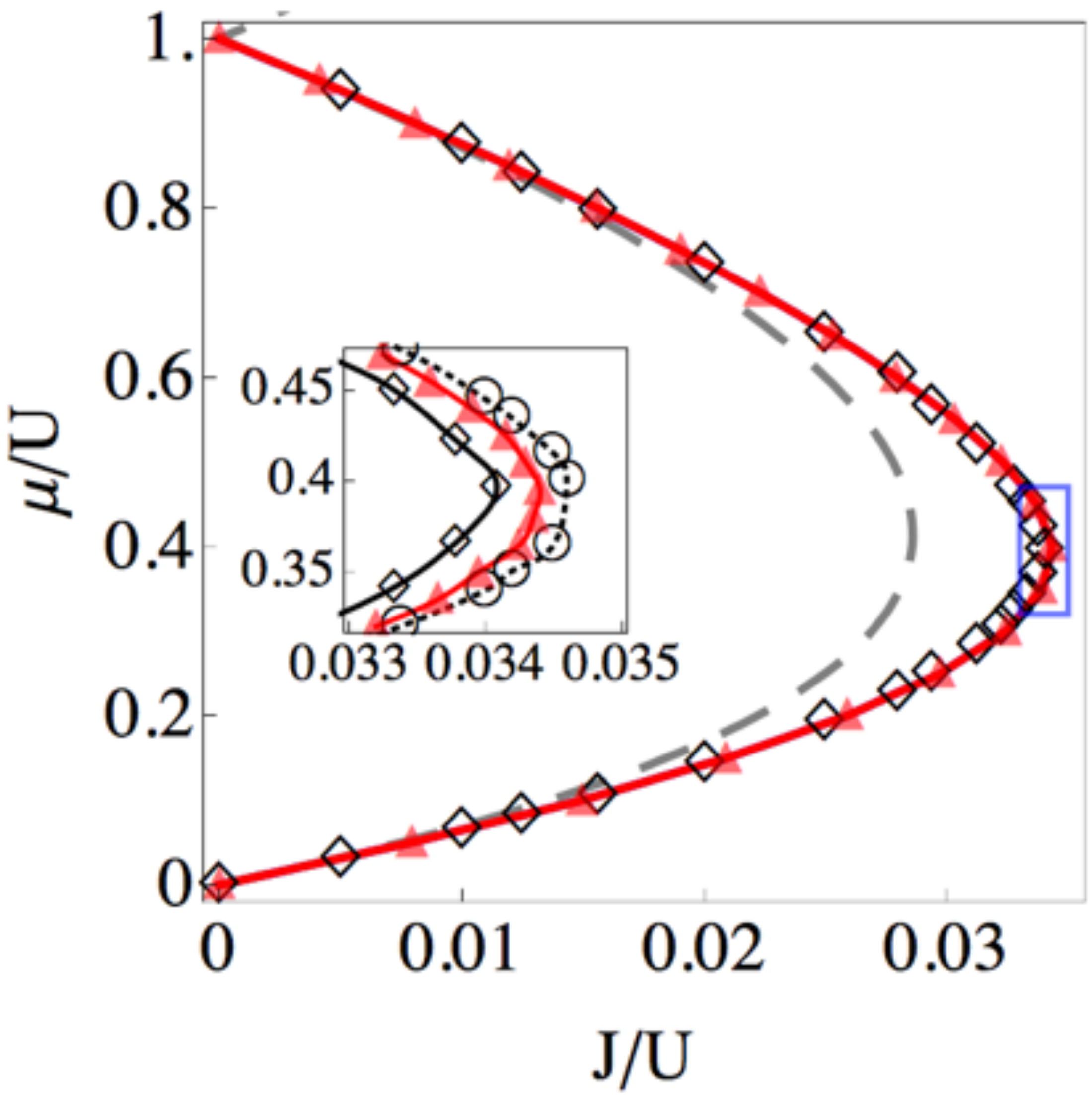
finite temperature, unit density

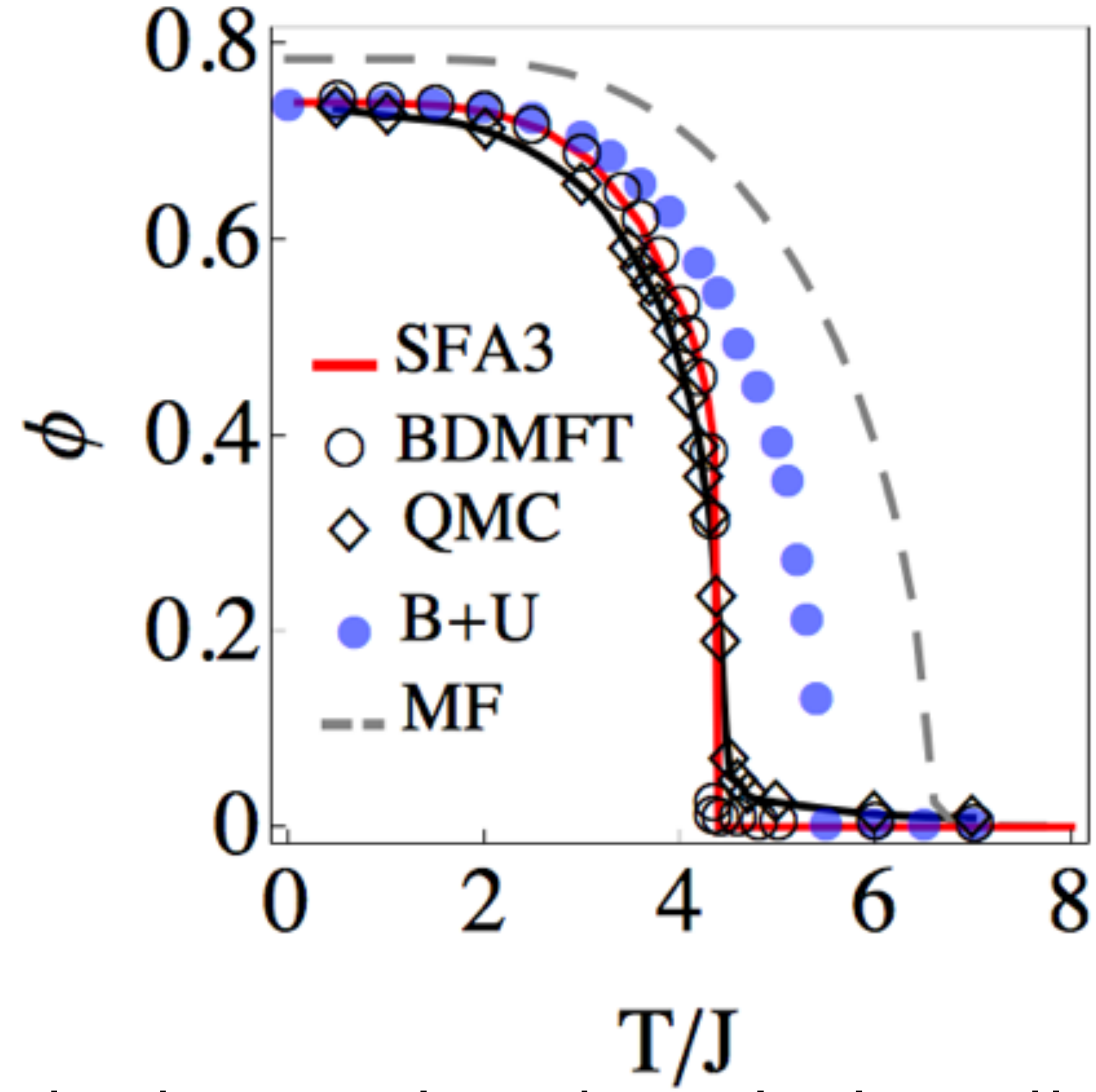


ground state



Bosonic self-energy functional theory





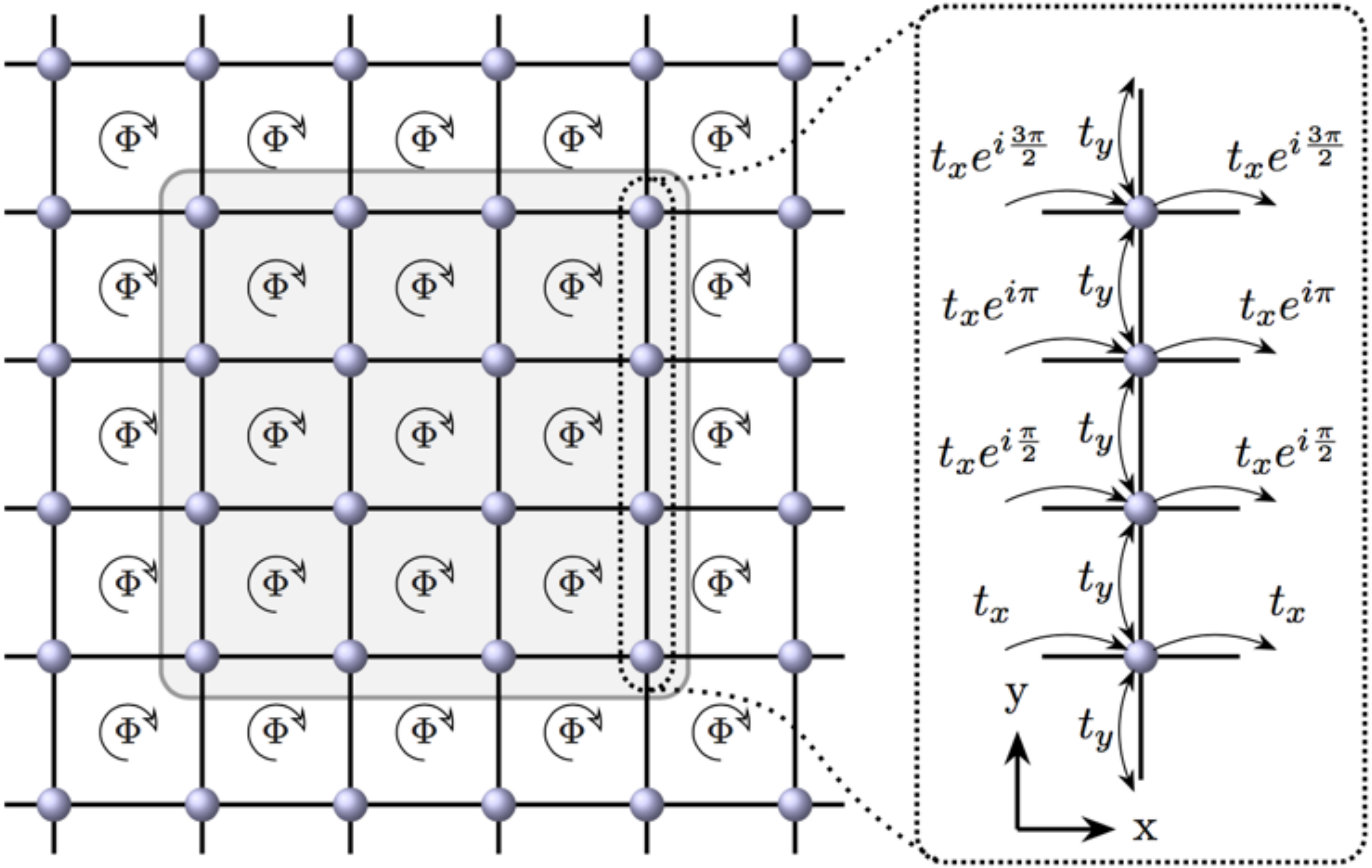
only 3 parameters needed, determined variationally
accuracy better than 1% for Bose-Hubbard model in every parameter regime

D. Hügel and L. Pollet, PRB 2015

D. Hügel, P. Werner, L. Pollet, and H. Strand, arXiv/1603.01401

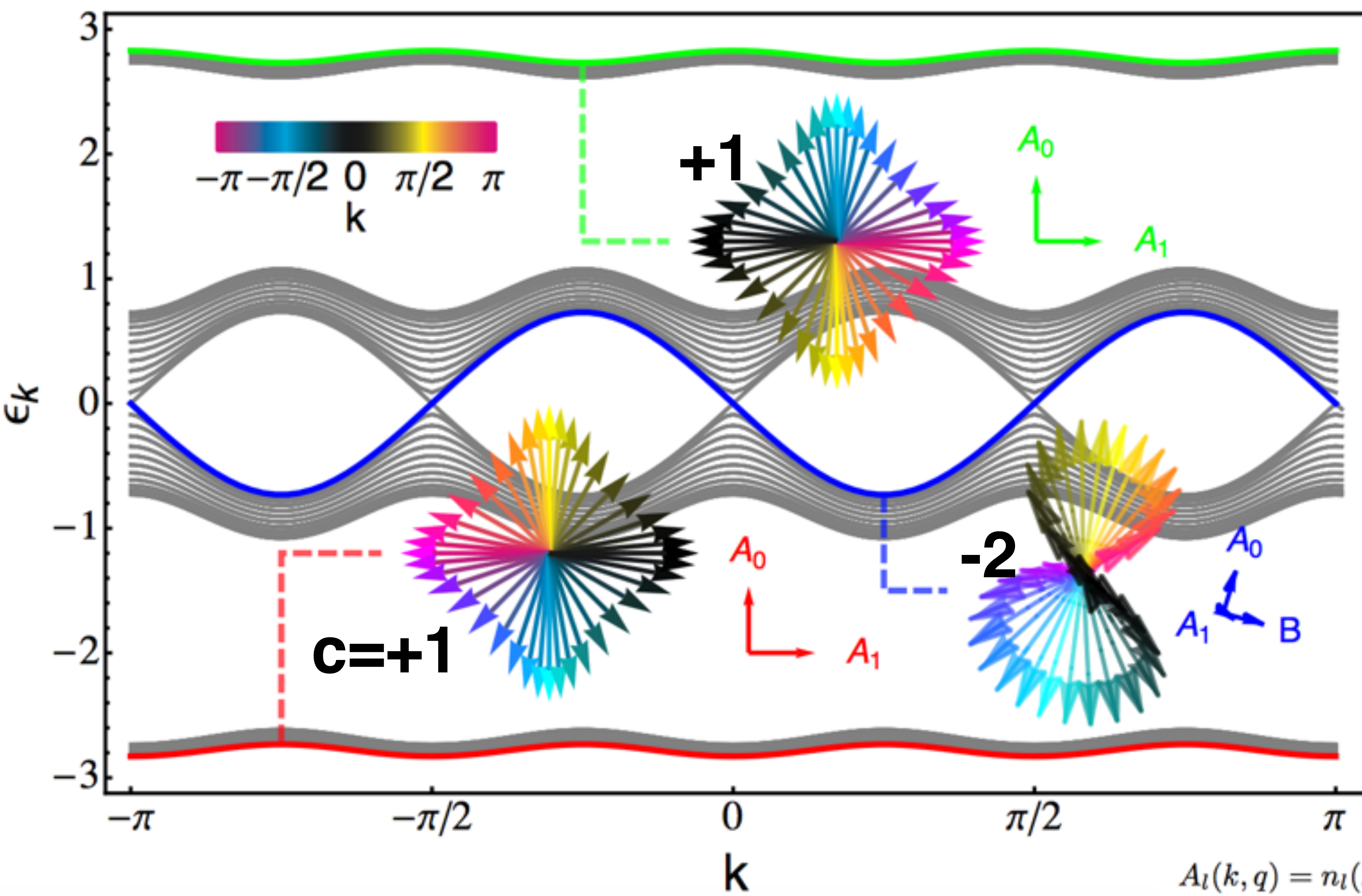
Harper-Hofstadter model

hardcore bosons



$$H_\Phi = -\sum_{x,y}\left(t_x e^{iy\Phi} b^\dagger_{x+1,y} b_{x,y} + t_y b^\dagger_{x,y+1} b_{x,y}\right) + h.c.$$

Chern numbers for non-interacting problem



diagonalize single-particle H:

$$H_\Phi = \int dkdq \left(\vec{v}_{k,q} \cdot \vec{h}_{k,q}\right)$$

$$\vec{v}_{k,q} = \begin{pmatrix} -2t_x \cos(k) \\ -2t_x \cos\left(k - \frac{\pi}{2}\right) \\ -2t_y \cos(q) \end{pmatrix}$$

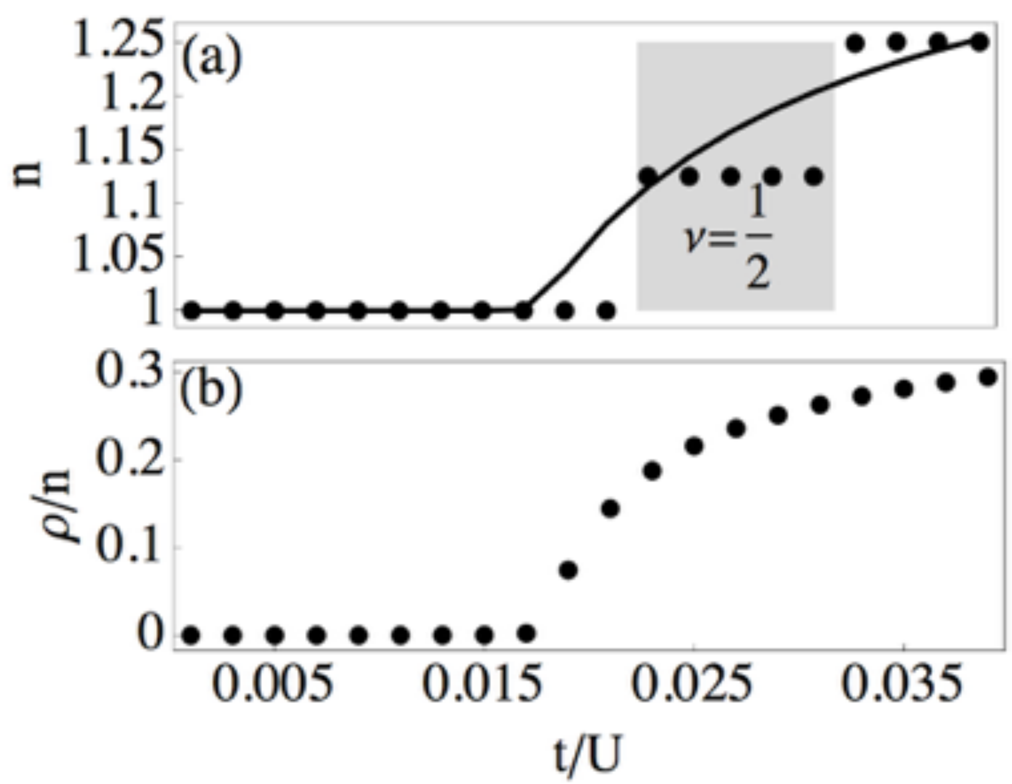
$$\vec{h}_{k,q} = \begin{pmatrix} A_0(k,q) \\ A_1(k,q) \\ B(k,q) \end{pmatrix}$$

$$A_l(k,q) = n_l(k,q) - n_{l+N_\phi/2}(k,q)\,,$$

$$B(k,q) = \frac{e^{-iq}}{2\cos(q)} \sum_l b^\dagger_{l+1}(k,q) b_l(k,q) + h.c.$$

Competing ground states of strongly correlated bosons in the Harper-Hofstadter-Mott model

Stefan S. Natu,[1,*] Erich J. Mueller,[2] and S. Das Sarma[1]



- real-space cluster mean-field
- studied several fluxes
- did not look at hopping anisotropy
- found metastable (f)QH phases, almost degenerate with the superfluid
- many density-wave instabilities

Phys. Rev. A **93**, 063610 (2016)

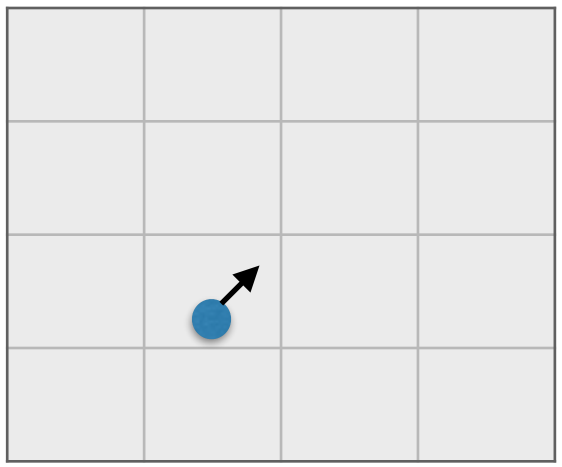
our method

- cluster mean-field but in momentum space
- hence: simpler than BDMFT or SFT (only Φ, no pair terms)
- impurity problem (4x4) solved with Lanczos
- no ‘connected’ Green function of non-condensed particles of original model

reciprocal cluster mean-field method

do NOT break translational invariance by working with clusters in momentum space instead of real space (but simpler than selfenergy functional theory):

divide the Brillouin zone into patches:



$$(K,Q),\ (\tilde{k},\tilde{q})$$
$$(X,Y),\ (\tilde{x},\tilde{y})$$

coarse grain the dispersion:

$$\bar{\epsilon}_{K,Q} = \frac{N_c M_c}{NM} \sum_{\tilde{k},\tilde{q}} \epsilon_{K+\tilde{k},Q+\tilde{q}}$$

this breaks up Hamiltonian:

$$H = H_c^{\mathrm{intra}} + \Delta H_c^{\mathrm{inter}}$$

care is needed in case of symmetry breaking:

$$\phi_{K,Q}(\tilde{x},\tilde{y}) = \phi_{K,Q}$$

cluster-Hamiltonian can be written as

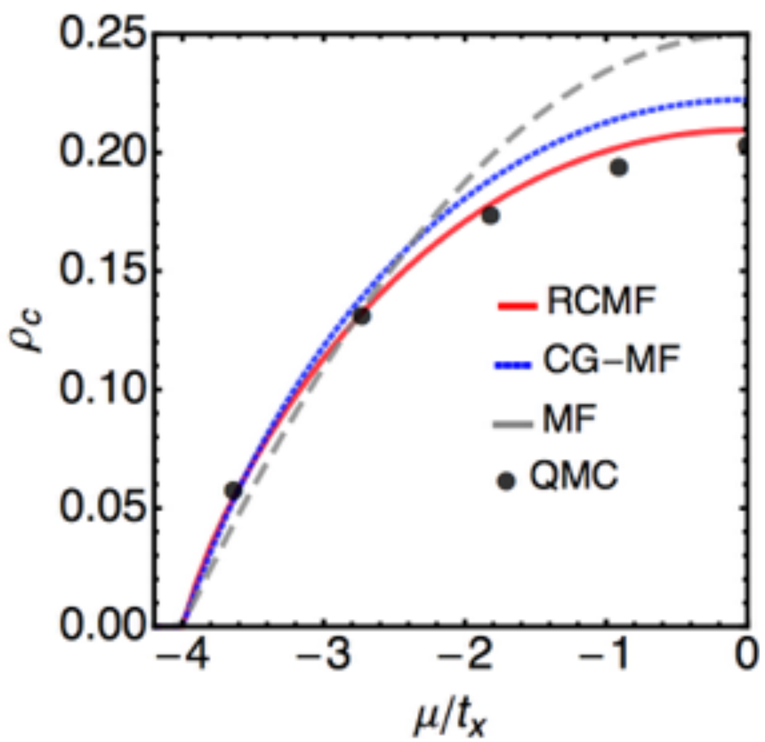
$$\begin{aligned} H'_{\mathrm{eff}} = & \sum_{X',Y'} \sum_{X,Y} \bar{t}_{(X',Y'),(X,Y)} b^{\dagger}_{X',Y'} b_{X,Y} \\ & - \mu \sum_{X,Y} n_{X,Y} + \frac{U}{2} \sum_{X,Y} n_{X,Y}(n_{X,Y}-1) \\ & + \sum_{X,Y} \left(b^{\dagger}_{X,Y} F_{X,Y} + F^{*}_{X,Y} b_{X,Y} \right), \end{aligned}$$

$$F_{X,Y} = \sum_{X',Y'} \delta t_{(X,Y),(X',Y')} \phi_{X',Y'}, \qquad \phi_{X,Y} = \langle b_{X,Y} \rangle$$

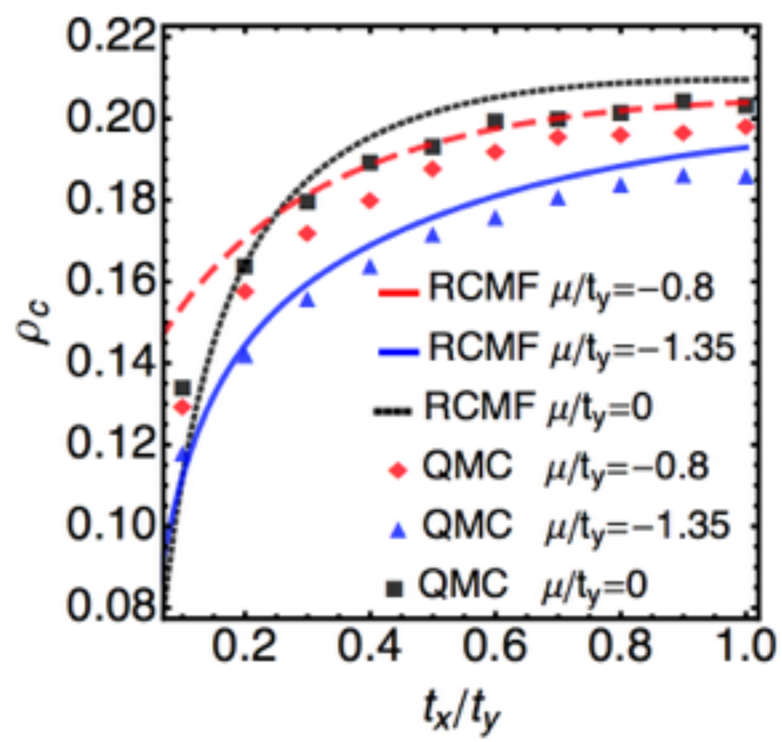
$$\bar{t}_{(X',Y'),(X,Y)} = \frac{1}{N_c M_c} \sum_{K,Q} e^{i(K(X'-X)+Q(Y'-Y))} \bar{\epsilon}_{K,Q}$$

$$\delta t_{(X',Y'),(X,Y)} = t_{(X',Y'),(X,Y)} - \bar{t}_{(X',Y'),(X,Y)}$$

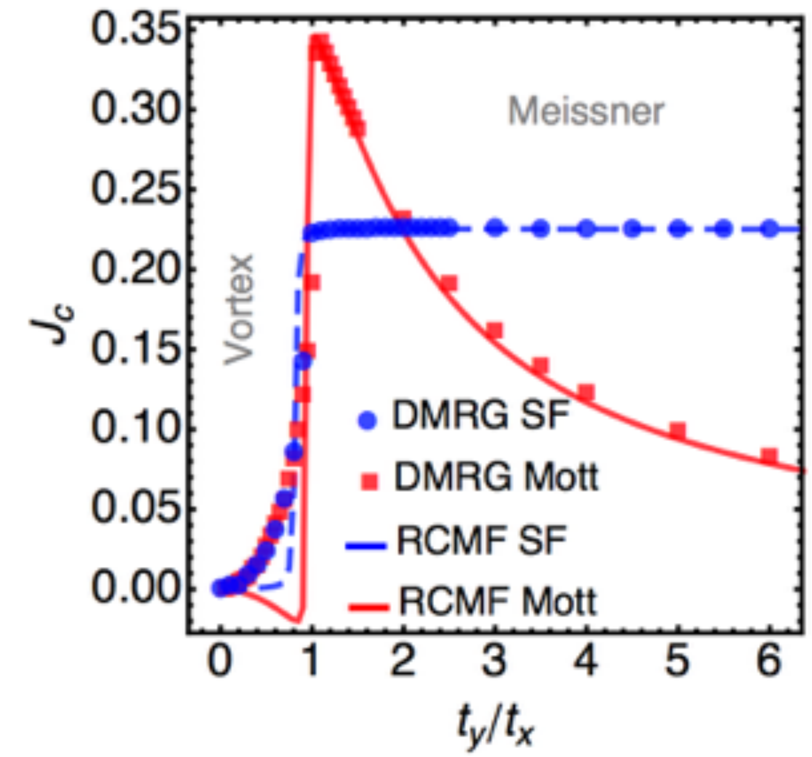
benchmarking



2d Bose Hubbard model, no anisotropy, no flux (MF: mean-field; CG cluster Gutzwiller)

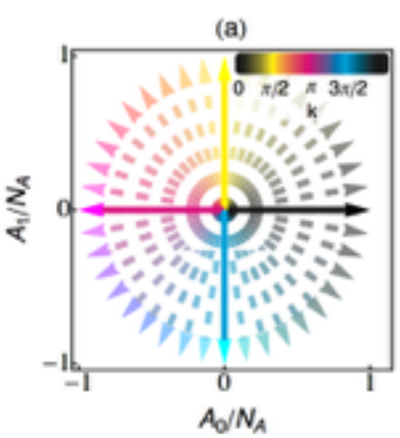


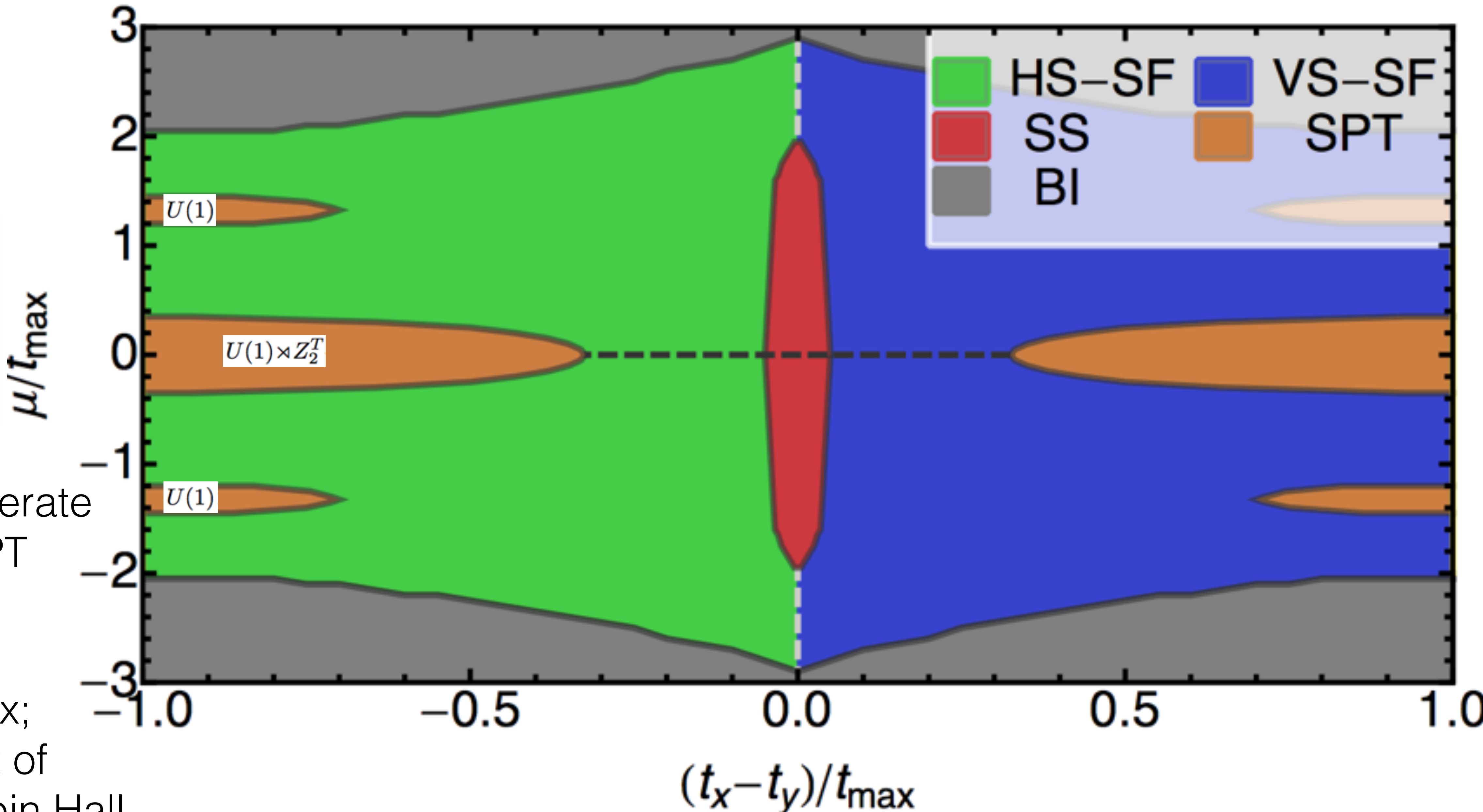
2d Bose Hubbard model, no flux; black = half filling



chiral ladder system

ground state phase diagram





non-degenerate gapped SPT phase $U(1)\rtimes Z_2^T$

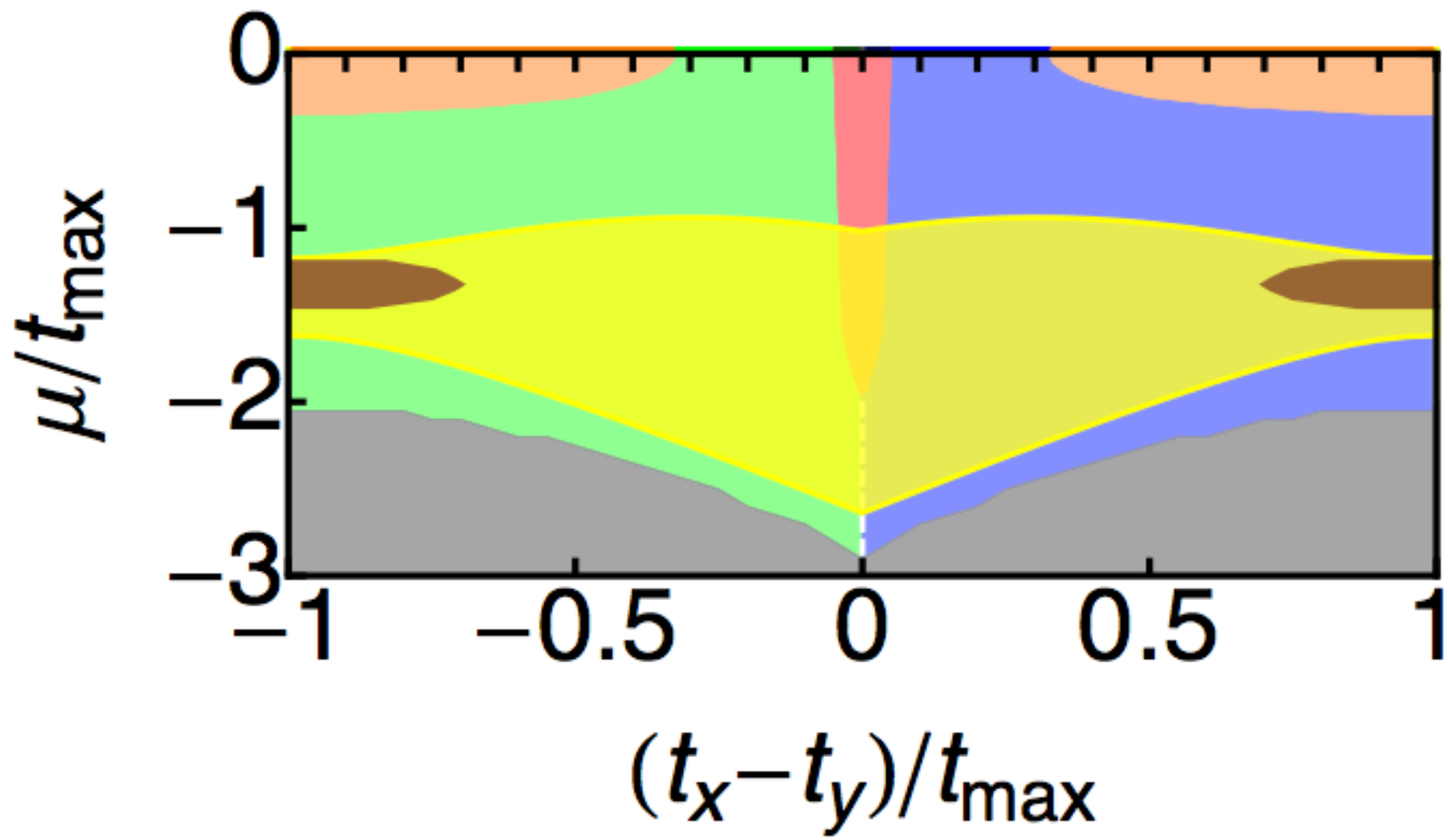
has Z_2 index; reminiscent of quantum spin Hall

symmetry operation: $\mathcal{T}U_\theta\mathcal{T}U_\theta = e$

FQHE: found, but all unstable against superfluidity

https://arxiv.org/abs/1205.3156

compare with free fermions:



SPT at filling 1 strongly reduced
SPT at filling 2 absent for free fermions

Chern numbers for interacting problem

twisted boundary conditions:

$$C = \frac{1}{2\pi}\int_0^{2\pi} d\theta_x \int_0^{2\pi} d\theta_y \left(\partial_{\theta_x}\mathcal{A}_y - \partial_{\theta_y}\mathcal{A}_x\right)$$

$$\mathcal{A}_j(\theta_x,\theta_y) \;=\; i\langle\Psi\left(\theta_x,\theta_y\right)|\partial_{\theta_j}|\Psi\left(\theta_x,\theta_y\right)\rangle$$

$$T_{x/y}\Psi(\theta_x,\theta_y) = e^{i\theta_{x/y}}\Psi(\theta_x,\theta_y)$$

$$t_y \to t_y e^{i\theta_y/L_y}$$

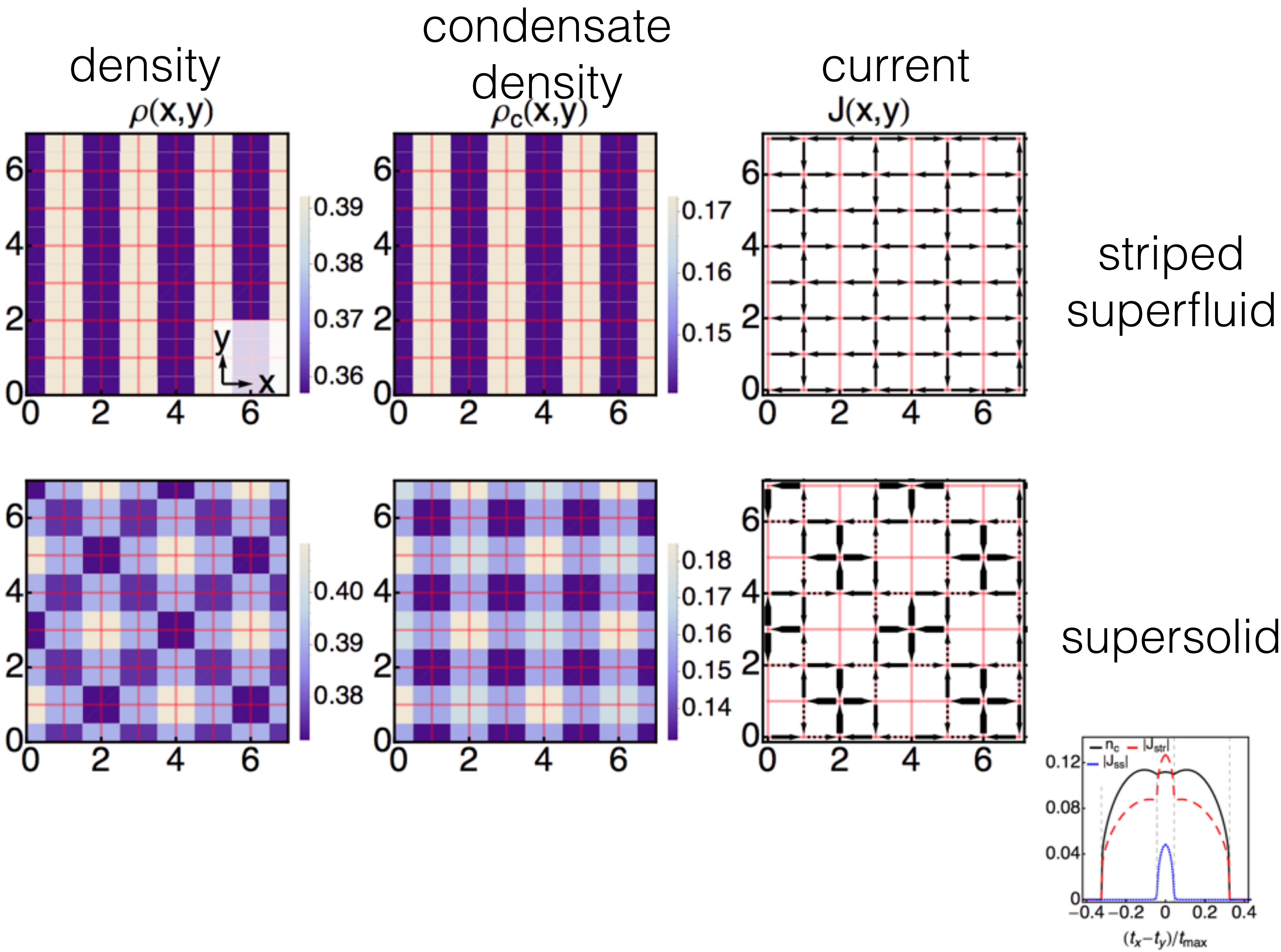
in the thermodynamic limit reciprocal space is continuous, and the phase twist infinitesimal

$$\vec{v}_{k,q} \to \vec{v}_{k,q}(\theta_x,\theta_y) = \begin{pmatrix} -2t_x\cos\left(k-\theta_x/L_x\right) \\ -2t_x\cos\left(k-\theta_x/L_x-\frac{\pi}{2}\right) \\ -2t_y\cos\left(q-\theta_y/L_y\right) \end{pmatrix}$$

this is just a momentum shift for every momentum

$$\left\langle\Psi(\theta_x,\theta_y)\left|\vec{h}_{k,q}\right|\Psi(\theta_x,\theta_y)\right\rangle =$$
$$\left\langle\Psi(0,0)\left|\vec{h}_{k+\theta_x/L_x,q+\theta_y/L_y}\right|\Psi(0,0)\right\rangle$$

we hence look at the winding of h projected on the space of hard-core bosons


density
condensate density
current
ρ(x,y)
ρc(x,y)
J(x,y)
striped superfluid
supersolid
y
x
0.39
0.38
0.37
0.36
0.17
0.16
0.15
0.40
0.39
0.38
0.18
0.17
0.16
0.15
0.14
nc
|Jstr|
|Jss|
(tx−ty)/tmax

coincidence?

project interacting problem onto non-interacting bands:

$$n = 1/4 \ (\nu = 1)$$

occupation numbers:

$$\nu_0 = 1, \nu_1 = 0, \nu_2 = 0$$

observe:

$$c_0 \nu_0 = -1$$

$$n = 1/2 \ (\nu = 2)$$

occupation numbers:

$$\nu_0 = 1.45, \nu_1 = 0.25, \nu_2 = 0.05$$

observe:

$$\nu_0 c_0 + \nu_1 c_1 + \nu_2 c_2 \approx -1.45 + 0.5 - 0.05 = -1$$

(the result of this procedure is 0 for the trivial band insulators)

(not the same as the approach by T. Neupert et al)

Summary

- Bosonic dynamical mean-field theory, bosonic self-energy functional theory
- cluster extensions
- SPT phases in interacting Harper-Hofstadter models; one purely due to interactions and of quantum spin-Hall like nature
- perhaps the easiest around to check experimentally
- checks: extend to selfenergy functional methods, other fluxes, seeing topological phase transition?
- many interesting extensions possible (disorder, dynamics)