Spin-incoherent one-dimensional spin-1 Bose gas

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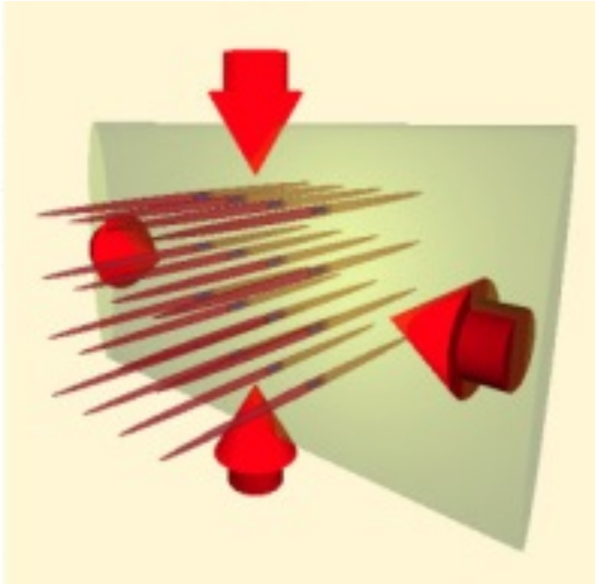
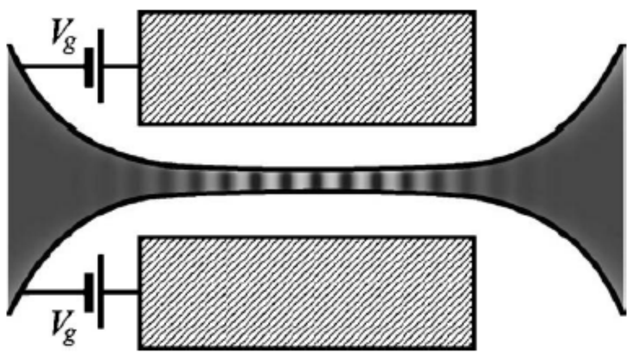
special thanks to Ming-Shien Chang



cold atoms



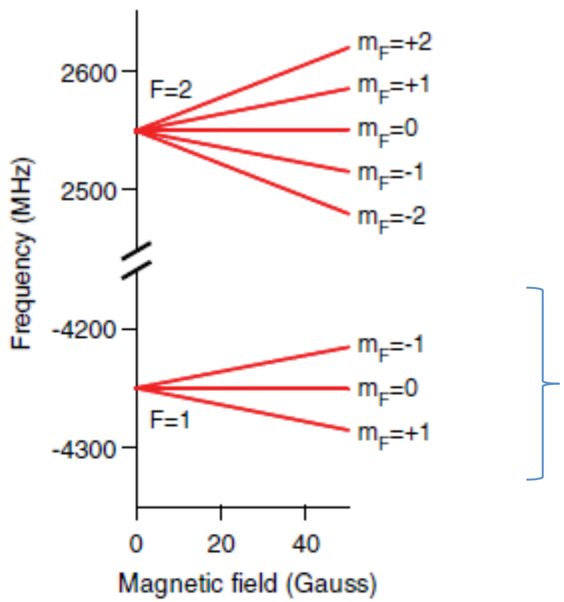
solid state

one-dimensional: occupation only in the lowest subband

Spin = hyperfine spin

Alkali atoms ^{87}Rb, ^{41}K, ^{7}Li, ^{23}Na S = ½, I=3/2, F = 1, 2

87Rb



“spin-1”

Dilute bose gas

interaction characterized by s-wave scattering lengths

two spin-1 particles: total F : 0 or 2 (Bose)

$$g_0 = \frac{4\pi a_0}{M} \qquad g_2 = \frac{4\pi a_2}{M}$$

^{7}Li, $F=1$	$a_0 = 23.9,\ a_2 = 6.8$	
^{23}Na, $F=1$	$a_0 = 50.0 \pm 1.6,\ a_2 = 55.0 \pm 1.7$	
^{41}K, $F=1$	$a_0 = 68.5 \pm 0.7,\ a_2 = 63.5 \pm 0.6$	a_B
^{87}Rb, $F=1$	$a_0 = 101.8 \pm 0.2,\ a_2 = 100.4 \pm 0.1$	

typically: $|a_0 - a_2| << a_{0,2}$

$$E_{spin} << E_{density/charge}$$

$$E_{spin} << T << E_{density}$$ "spin-incoherent"

$$H = \int [\frac{1}{2M} \nabla \psi_s^*(x) \nabla \psi_s(x) + \frac{1}{2} g_{1D} \psi_{s1}^*(x) \psi_{s2}^*(x) \psi_{s2}(x) \psi_{s1}(x)]$$ (SU(3))

$$g_{1D} \sim g / l_{\perp}^2 \qquad (a << l_{\perp})$$

One dimensional spinless Bose gas: (old problem) Lieb-Liniger

exotic properties

$$H = \int \left[\frac{1}{2M} \nabla \psi^*(x) \nabla \psi(x) + \frac{1}{2} g_{1D} \psi^*(x) \psi^*(x) \psi(x) \psi(x) \right]$$

$$g_{1D} \sim g / l_{\perp}^2 = -\frac{2}{M a_{1D}}$$

1D density = n

weak interaction: $n|a_{1D}| >> 1$ $\quad Mg_{1D} << n$

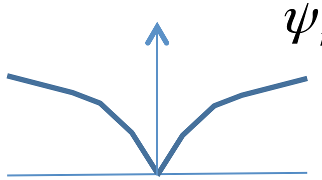
strong interaction: $n|a_{1D}| << 1$ $\quad Mg_{1D} >> n$

Tonk-Girardeau limit: $n|a_{1D}| \to 0$ $\quad g_{1D} \to \infty$

nodes / no exchange between Bosons

relative wavefunction:

$$\psi_{rel}(x, x') \propto |x - x'|$$



TG limit, wavefunction in (harmonic) trap:

one particle: $\varphi_n(x)$ $E = (n + \frac{1}{2})$

two: $[\varphi_{n1}(x_1)\varphi_{n2}(x_2) - \varphi_{n1}(x_2)\varphi_{n2}(x_1)]\,\mathrm{sgn}(x_2 - x_1)$ $E = (n_1 + n_2 + 1)$

three:

$$\begin{vmatrix} \varphi_{n1}(x_1) & \varphi_{n1}(x_2) & \varphi_{n1}(x_3) \\ \varphi_{n2}(x_1) & \varphi_{n2}(x_2) & \varphi_{n2}(x_3) \\ \varphi_{n3}(x_1) & \varphi_{n3}(x_2) & \varphi_{n3}(x_3) \end{vmatrix} \mathrm{sgn}(x_3 - x_2)\,\mathrm{sgn}(x_2 - x_1)\,\mathrm{sgn}(x_3 - x_1) \qquad E = (n_1 + n_2 + n_3 + \tfrac{3}{2})$$

....

n → k if uniform

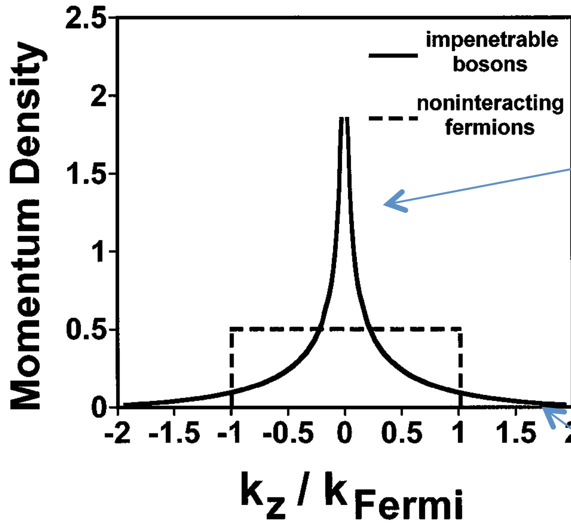
spinless Fermions: leave out sgn(…) density, potential energy same, but …

infinite:

Olshanii, PRL98

(power interaction dependent)

n(k)



$\propto k^{-1/2}$

$$\rho(x,x') \propto \frac{1}{|x-x'|^{1/2}}[1+...]$$

at long distances

$\propto k^{-4}$

Lenard, 60
Vaidya+Tracy, Jimbo ... 79-80

$$\rho(x,x') = \rho(x,x)[1+....+c_3|x-x'|^3+...]$$

Olshanii, Minguzzi, ...

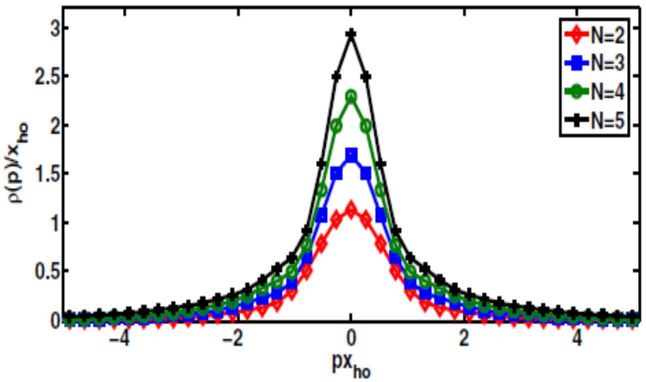
$$\psi_{rel}(x,x') \propto |x-x'|$$

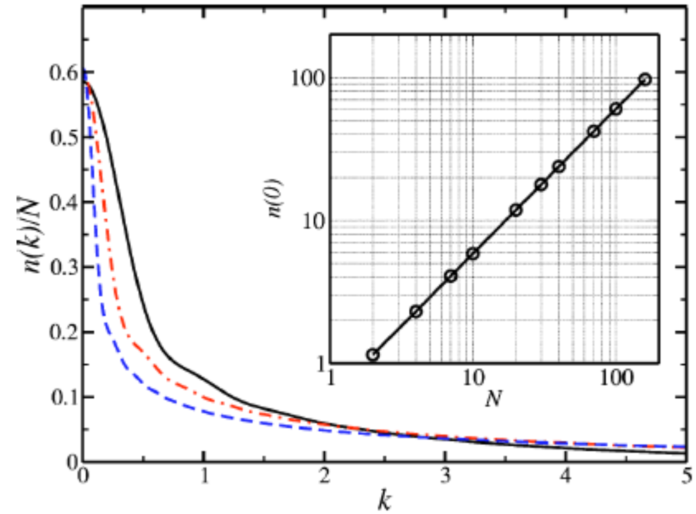
at short distances

(power indep of interaction; valid also in trap
coefficient depends on the many-body state and related to dE/da_{1D})

trap:

spinless Bosons:





Papenbrock PRA03

N=10,40,160

Put back spin, confine to TG limit

trivially all spin configurations degenerate

$$\begin{vmatrix} \varphi_{n1}(x_1) & \varphi_{n1}(x_2) \\ \varphi_{n2}(x_1) & \varphi_{n2}(x_2) \end{vmatrix}$$

wavefunctions:

two:

$x_1 < x_2$ $\quad \frac{1}{\sqrt{2}}[\varphi_{n1}(x_1)\varphi_{n2}(x_2) - \varphi_{n1}(x_2)\varphi_{n2}(x_1)]\,\mathrm{sgn}(x_2 - x_1) \quad |s_1 s_2\rangle$

$$\equiv \varphi^{sym}_{space,n1,n2}(x_1, x_2)$$

$\Psi(x_1 s'_1, x_2 s'_2) = \varphi^{sym}$ if $s'_1 = s_1 \quad s'_2 = s_2$

vanishes otherwise

wavefunction for $x_2 < x_1$ obtained by interchanging particles

$\Psi(x_1 s'_1, x_2 s'_2) = \Psi(x_2 s'_2, x_1 s'_1) = \varphi^{sym}$ if $s'_2 = s_1 \quad s'_1 = s_2$

vanishes otherwise

three particles

$$x_1 < x_2 < x_3 \qquad \varphi_{space}^{sym}(x_1, x_2, x_3) \qquad |\, s_1 s_2 s_3 \rangle$$

$$|||$$

$$\frac{1}{\sqrt{3!}}\begin{vmatrix} \varphi_{n1}(x_1) & \varphi_{n1}(x_2) & \varphi_{n1}(x_3) \\ \varphi_{n2}(x_1) & \varphi_{n2}(x_2) & \varphi_{n2}(x_3) \\ \varphi_{n3}(x_1) & \varphi_{n3}(x_2) & \varphi_{n3}(x_3) \end{vmatrix} \operatorname{sgn}(x_2 - x_1)\operatorname{sgn}(x_3 - x_1)\operatorname{sgn}(x_3 - x_2)$$

$$x_2 < x_1 < x_3 \qquad \Psi(x_1 s'_1, x_2 s'_2, x_3 s'_3) = \Psi(x_2 s'_2, x_1 s'_1, x_3 s'_3) = \varphi^{sym}$$

$$s'_2 = s_1 \qquad s'_1 = s_2 \qquad s'_3 = s_3$$

$$x_2 < x_3 < x_1 \qquad \Psi(x_1 s'_1, x_2 s'_2, x_3 s'_3) = \Psi(x_2 s'_2, x_1 s'_1, x_3 s'_3)$$

$$= \Psi(x_2 s'_2, x_3 s'_3, x_1 s'_1)$$

....

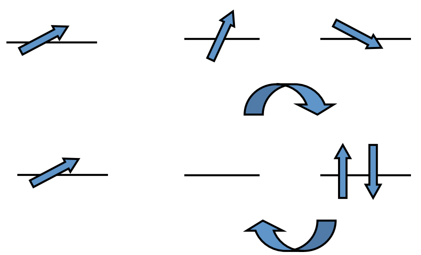
$$s'_2 = s_1 \qquad s'_3 = s_2 \qquad s'_1 = s_3$$

note: N=2, space symmetric (antisymmetric); spin symmetric (antisymmetric)
but typical state for N particles has mixed symmetry

large but finite g_{1D}

degenerate perturbation in $1/g_{1D}$

formally identical with derivation of spin model from large U Hubbard



Bosons: only symmetric combination can interchange

$$H_{\rm eff} = -\sum_l J_l \left(\frac{1}{g_0} P_0(l,l+1) + \frac{1}{g_2} P_2(l,l+1) \right)$$

$$J \propto \int d\mathbf{x} \left| \frac{\partial \phi_F}{\partial x_{ij}} \bigg|_{x_{ij}=0} \right|^2 \theta(\cdots < x_i = x_j < \cdots).$$

“quantum magnetism without lattice”

Deuretzbacher et al PRL 08, PRA 14
Volosniev et al, Nat. Comm. 14
Li Yang, Guan, Han Pu, PRA15
Lijun Yang and Cui PRA16
…

requires very low temperatures
$T < J/g$

below: consider $J/g < T$ → “spin incoherent regime”

(automatic for TG)

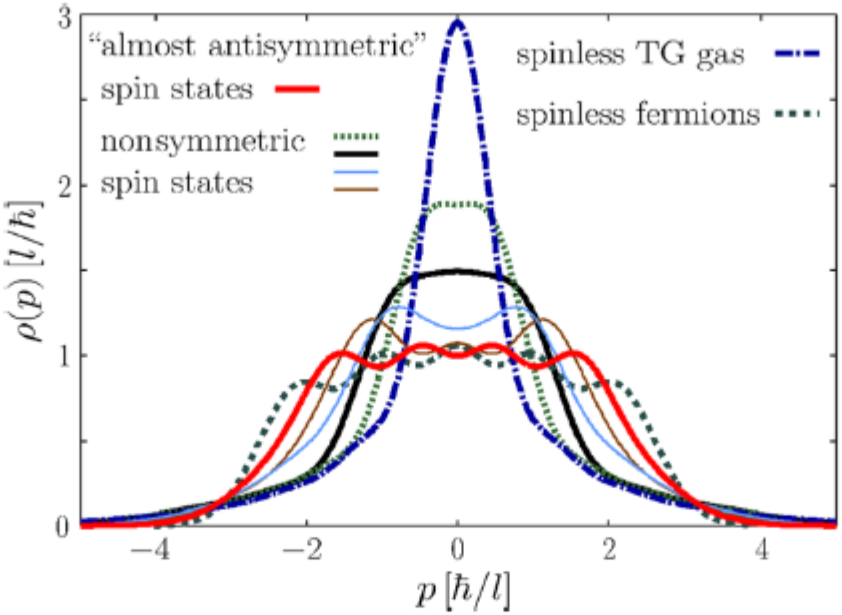
quantum wires Cheianov PRL04, Fiete + Balents, PRL04, Fiete, RMP07, $r_s >> 1$

t-J / Hubbard K Penc + … PRL06, PRB07
Feiguin +Fiete, PRB10

uniform two-component gas Cheianov + … PRA05

Deuretzbacher et al, PRL08

N=5



Three:

$x_1 < x_2 < x_3$ $\qquad \varphi^{sym}(x_1, x_2, x_3)$ $\qquad |s_1 s_2 s_3\rangle = |\chi\rangle$

$x_2 < x_1 < x_3$ $\qquad$ ...

...

average over all $|\chi\rangle$

$$\rho(x, x') = \sum_s \rho_s(x, x') = N\Sigma_{s,s2,s3....} \int_{x2,x3...} \psi^*_{s,s2,s3}(x, x_2, x_3, \ldots)\psi_{s,s2,s3}(x', x_2, x_3, \ldots)$$

spin overlaps

$$\begin{aligned}
x < x_2 < x_3, &\quad x' < x_2 < x_3, \quad \langle\chi|E|\chi\rangle = 1,\\
&\quad x_2 < x' < x_3, \quad \langle\chi|P_{12}|\chi\rangle,\\
&\quad x_2 < x_3 < x', \quad \langle\chi|P_{23}P_{12}|\chi\rangle,\\
x_2 < x < x_3, &\quad x_2 < x' < x_3, \quad \langle\chi|E|\chi\rangle = 1,\\
&\quad x_2 < x_3 < x', \quad \langle\chi|P_{12}^{-1}P_{23}P_{12}|\chi\rangle\\
x_2 < x_3 < x, &\quad x_2 < x_3 < x', \quad \langle\chi|E|\chi\rangle = 1,
\end{aligned}$$

Trace over all possible choices of $|\chi\rangle$

N=3:

$$\rho(x<x') = 3\times 2\times\left\{\int_{x<x'<x_2<x_3} 1 + \int_{x<x_2<x'<x_3} \frac{\mathrm{Tr}_\chi(P_{12})}{\mathrm{Tr}_\chi(E)} + \int_{x<x_2<x_3<x'} \frac{\mathrm{Tr}_\chi(P_{123})}{\mathrm{Tr}_\chi(E)} + \int_{x_2<x<x'<x_3} 1 + \int_{x_2<x<x_3<x'} \frac{\mathrm{Tr}_\chi(P_{13})}{\mathrm{Tr}_\chi(E)} + \int_{x_2<x_3<x<x'} 1\right\}$$

$$\times\, \psi_{\vec{n}}^{sym*}(x,x_2,x_3)\psi_{\vec{n}}^{sym}(x',x_2,x_3)dx_2dx_3,$$

same

$$\rho(x<x') = N!\left\{\int_{x<x'<x_2\cdots<x_N} 1 + \int_{x<x_2<x'\cdots<x_N} \frac{w_{2N}}{w_N} + \int_{x<x_2<x_3<x'\cdots<x_N} \frac{w_{3N}}{w_N} + \cdots + \int_{x_2<x<x'\cdots<x_N} 1 + \int_{x_2<x<x_3<x'\cdots<x_N} \frac{w_{2N}}{w_N} + \cdots + \int_{x_2<x_3\cdots<x_N<x<x'} 1\right\}\psi_{\vec{n}}^{sym*}(x,\bar{x})\psi_{\vec{n}}^{sym}(x',\bar{x})d\bar{x}$$

$$w_{jN} \equiv \mathrm{Tr}_\chi(P_{12\ldots j})$$

$$w_N \equiv \mathrm{Tr}_\chi(E)$$

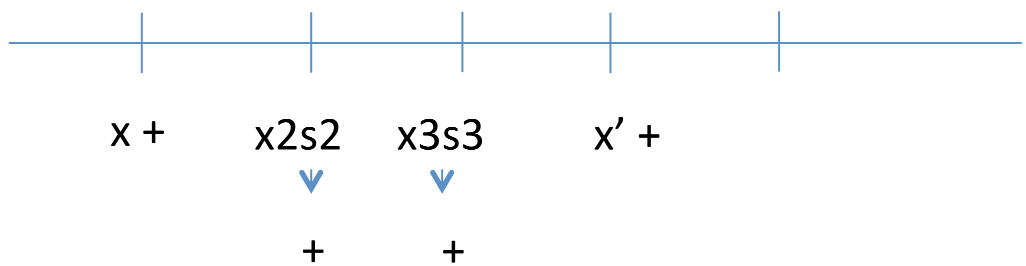
$w_{jN}/w_N \to 1$ if spinless

$\rho(x, x')$

$$\rho(x < x') = N!\Big\{ \int_{x<x'<x_2\cdots<x_N} 1 + \int_{x<x_2<x'\cdots<x_N} \frac{w_{2N}}{w_N} + \int_{x<x_2<x_3<x'\cdots<x_N} \frac{w_{3N}}{w_N} + \cdots$$
$$+ \int_{x_2<x<x'\cdots<x_N} 1 + \int_{x_2<x<x_3<x'\cdots<x_N} \frac{w_{2N}}{w_N} + \cdots + \int_{x_2<x_3\cdots<x_N<x<x'} 1 \Big\} \psi_{\vec{n}}^{sym*}(x,\bar{x})\psi_{\vec{n}}^{sym}(x',\bar{x})d\bar{x}$$

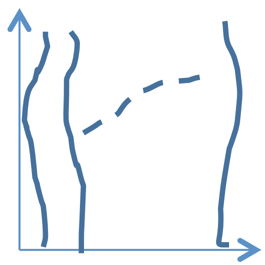
for not too small x, x' : possible contributions:

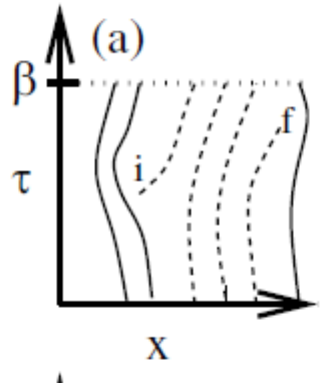
small
spatial integrals



finite contribution: need, e.g. $|\underbrace{+++++++}_{j} \ldots\ldots\rangle$, or $|\underbrace{0000000}_{j} \ldots\rangle$, ...

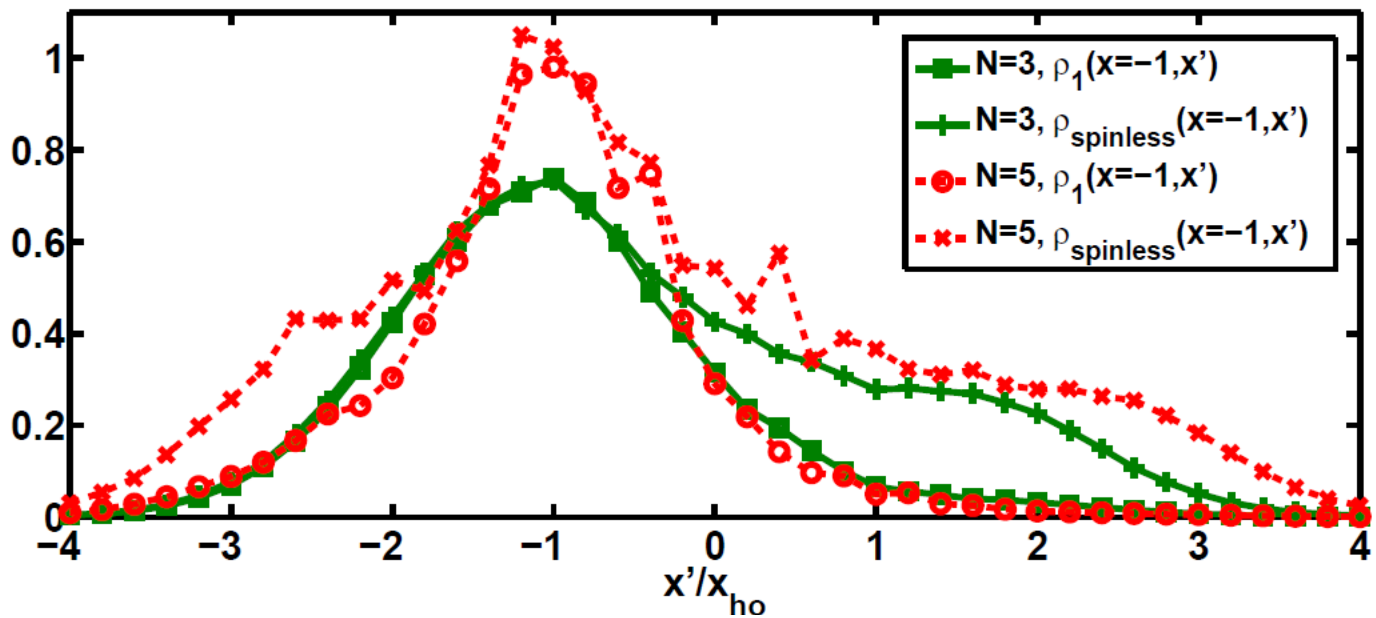
reduced by $Tr[P_{1\ldots j}]/Tr[E] = w_{jN}/w_N$ compared with spinless case c.f. Feynmann path integral picture (Fiete and Balents, PRL 04)





must be the same spin

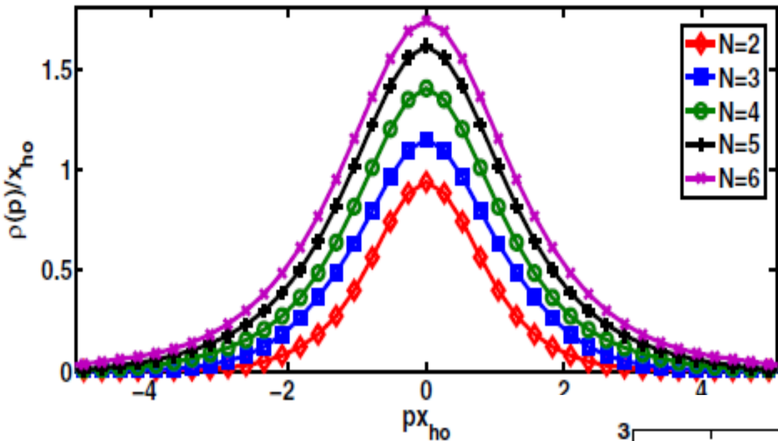
→ exponential decaying factor for propagators / single particle density matrix



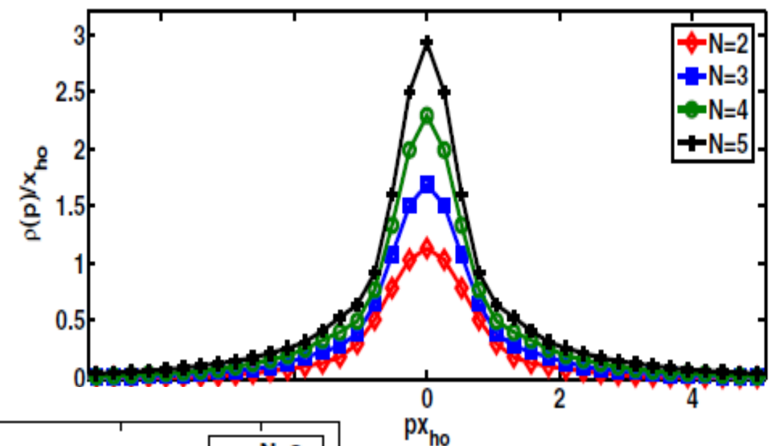
N=3, ρ1(x=−1,x')
N=3, ρspinless(x=−1,x')
N=5, ρ1(x=−1,x')
N=5, ρspinless(x=−1,x')
1
0.8
0.6
0.4
0.2
0
−4
−3
−2
−1
0
1
2
3
4
x'/xho

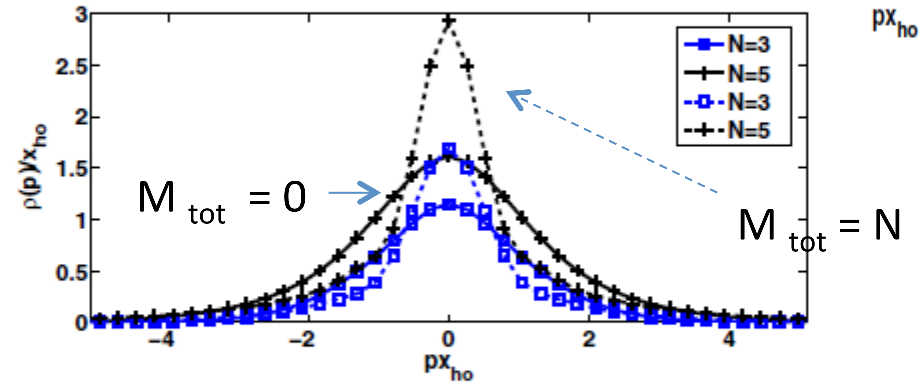
Spin 1

$M_{tot} = 0$ ground state manifold



Spinless, or $M_{tot} = N$



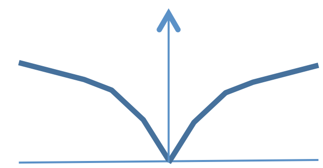


KE = PE = 0.5 E_{tot} virial theorem $\int dp p^2 \rho(p)$ same

spin incoherent Bose has broader momentum distribution (but see below !)

spinless:

$$\psi_{rel}(x,x') \propto |x - x'|$$



$$p \rightarrow \infty \qquad \rho(p) \propto 1/p^4$$

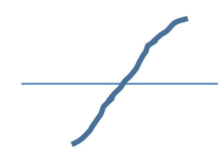
$$\rho(p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' e^{ip(x-x')} \rho(x,x')$$

$$\rho(x < x') = N! \Bigg\{ \int_{x<x'<x_2\cdots<x_N} 1 + \int_{x<x_2<x'\cdots<x_N} \frac{w_{2N}}{w_N} + \int_{x<x_2<x_3<x'\cdots<x_N} \frac{w_{3N}}{w_N} + \cdots$$

$$+ \int_{x_2<x<x'\cdots<x_N} 1 + \int_{x_2<x<x_3<x'\cdots<x_N} \frac{w_{2N}}{w_N} + \cdots + \int_{x_2<x_3\cdots<x_N<x<x'} 1 \Bigg\} \psi_{\vec{n}}^{sym*}(x,\bar{x}) \psi_{\vec{n}}^{sym}(x',\bar{x}) d\bar{x}$$

If replace w_{jN} / w_{N} by (-1)$^{j-1}$ → spinless fermions

$$\rho(p) \rightarrow e^{-p^2} \qquad \text{no } 1/p4$$



If replace w_{jN} / w_{N} by +1 → spinless hard core Bosons

Large p: x, x' close to each other

1/p4 singularity x, x' close to x2 (or x3, ….)

e.g. $\int_{x<x_2<x'}$ large p $\int_{-\infty}^{0} d\bar{y} \int_{0}^{\infty} d\bar{y}' e^{ip(\bar{y}-\bar{y}')} \bar{y}\bar{y}' = \frac{-1}{p^4}$

$\bar{y} = x - x_2 \qquad \bar{y}' = x' - x_2$

$$\rho(x < x') = N!\left\{ \int_{x<x'<x_2\cdots<x_N} 1 + \int_{x<x_2<x'\cdots<x_N} \frac{w_{2N}}{w_N} + \int_{x<x_2<x_3<x'\cdots<x_N} \frac{w_{3N}}{w_N} + \cdots \right.$$
$$\left. + \int_{x_2<x<x'\cdots<x_N} 1 + \int_{x_2<x<x_3<x'\cdots<x_N} \frac{w_{2N}}{w_N} + \cdots + \int_{x_2<x_3\cdots<x_N<x<x'} 1 \right\} \psi_{\vec{n}}^{sym*}(x,\bar{x})\psi_{\vec{n}}^{sym}(x',\bar{x})d\bar{x}$$

$w_{2N} / w_N < 1$ spin incoherent TG: `kink' weaker than spinless TG

$\rho(p) \propto 1/p^4$ with a smaller coefficient

$$\rho(x<x') = N!\Big\{ \int_{x<x'<x_2\cdots<x_N} 1 + \int_{x<x_2<x'\cdots<x_N} \frac{w_{2N}}{w_N} + \int_{x<x_2<x_3<x'\cdots<x_N} \frac{w_{3N}}{w_N} + \cdots$$
$$+ \int_{x_2<x<x'\cdots<x_N} 1 + \int_{x_2<x<x_3<x'\cdots<x_N} \frac{w_{2N}}{w_N} + \cdots + \int_{x_2<x_3\cdots<x_N<x<x'} 1 \Big\} \psi_{\vec{n}}^{sym*}(x,\bar{x}) \psi_{\vec{n}}^{sym}(x',\bar{x}) d\bar{x}$$

$$\rho(p) \underset{p\to\infty}{=} \frac{2[1+w_{2N}/w_N]}{2\pi p^4} \sum_{j=2,3,\ldots,N} \int_{x_2<x_3\cdots<x_N} d\bar{x} \begin{vmatrix} \phi'_{n_1}(x_j) & \phi'_{n_2}(x_j) & \cdots \\ \phi_{n_1}(x_j) & \phi_{n_2}(x_j) & \cdots \\ \vdots & \cdots & \vdots \\ \phi_{n_1}(x_N) & \cdots & \phi_{n_N}(x_N) \end{vmatrix}^2$$

↓

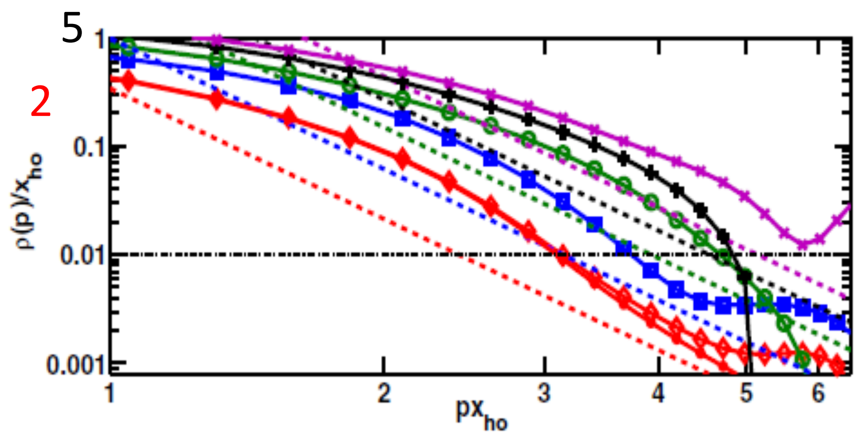
$$\frac{1}{(N-1)!} \frac{2[1+w_{2N}/w_N]}{2\pi p^4} \sum_{j=2,3,\ldots,N} \int_{-\infty}^{\infty} d\bar{x}$$

$$\rho(p) \underset{p\to\infty}{=} \frac{2\left[1+\frac{w_{2N}}{w_N}\right]}{2\pi p^4} \sum_{(n_i,n_j)} \int_{-\infty}^{\infty} dx \begin{vmatrix} \phi'_{n_i}(x) & \phi'_{n_j}(x) \\ \phi_{n_i}(x) & \phi_{n_j}(x) \end{vmatrix}^2$$

$w_{2N} / w_N \to 1$ spinless

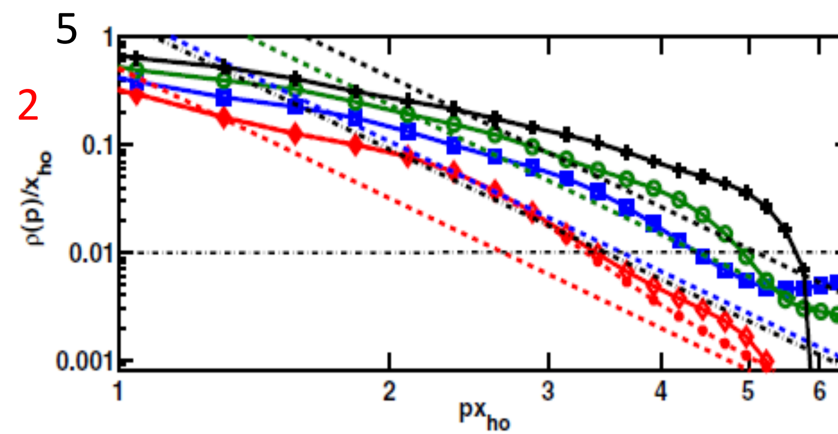
spin incoherent Bose has smaller momentum tail !

(spin part, hence also space, not nec symmetric) spinful



$0.34/p^4$, $0.98/p^4$ $2.38/p^4$, $4.27/p^4$,

spinless



$0.51/p^4$, $1.715/p^4$ $3.77/p^4$, and $6.8/p^4$

Summary:

1D strongly interacting Bose gas with spin

spin energy << T << "space"

"spin-incoherent"
(probably most common experimental regime)

properties very different from spinless / spin coherent

(exponentially) decaying spatial correlation functions

momentum distribution

broader and lower peak near k → 0 (incoherent,
smaller large k tail spin degree of freedom)

treated only TG limit (with a very inefficient scheme):

general case DMRG? (Feiguin+Fiete, PRB10, t-J)
QMC?

wait for experiments!