Chapter 2

Basic Emission mechanisms

§1 Inverse-Compton Scatterings

§2 Radiation from Relativistic Charges

§3 Synchrotron Radiation

§4 Synchro-curvature Radiation

§5 Electron-positron Pair Production

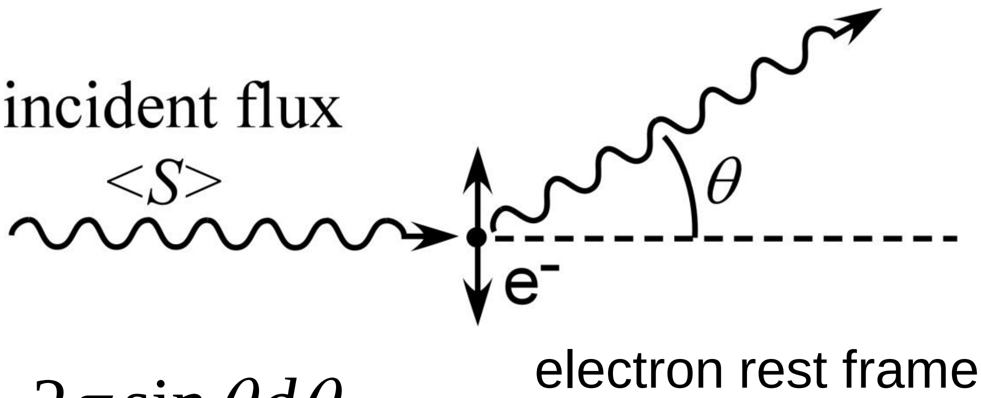
§1 Inverse-Compton Scatterings

From Rybicki & Lightman (1979),
"Radiative Processes in Astrophysics"

§1 Inverse-Compton Scatterings

Emission processes can be interpreted as the Compton scatterings of real or virtual photons. Thus, let us begin by considering its classical limit, Thomson scatterings,

A free e^- oscillates in the $\boldsymbol{E}$ field to radiate (by dipole formula)



$$\frac{dP}{d\Omega} = \langle S \rangle \frac{d\sigma}{d\Omega}. \qquad d\Omega = 2\pi \sin\theta d\theta$$

For unpolarized radiation field, the differential cross becomes

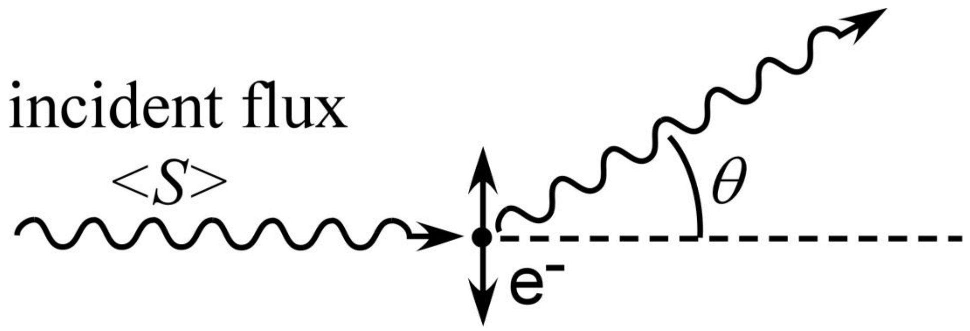
$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}\left(1+\cos^2\theta\right), \text{ where } r_0 \equiv \frac{e^2}{m_e c^2}$$

§1 Inverse-Compton Scatterings

The total cross section becomes the Thomson cross section,

$$\sigma_{\mathrm{T}} = \int \frac{d\sigma}{d\Omega} d\Omega = 2\pi r_0^2 \int_{-1}^{1} (1-\mu^2) d\mu = \frac{8\pi}{3} r_0^2$$

Note that the scattering can also occur if the incident photon is a virtual photon (§ 3).



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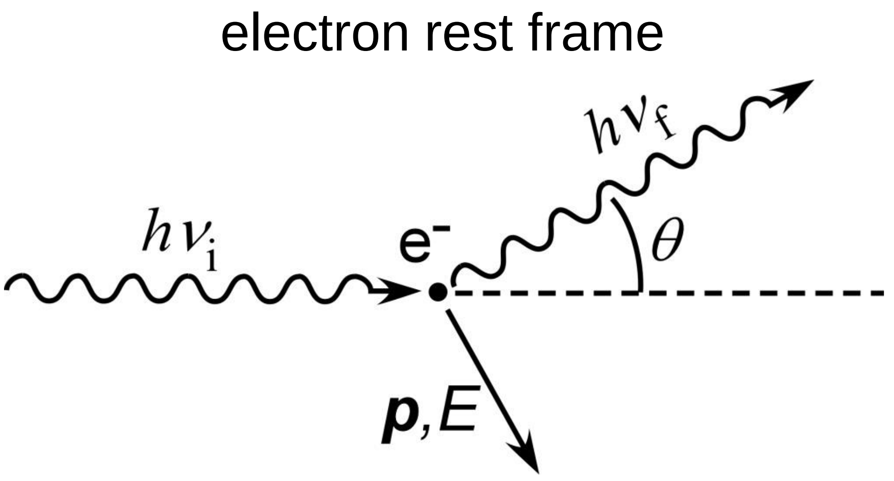
§1 Inverse-Compton Scatterings



If incident photon energy becomes comparable or greater than $m_e c^2$, quantum effects appear in two ways:

- recoil of e^-,
- reduction of cross section.

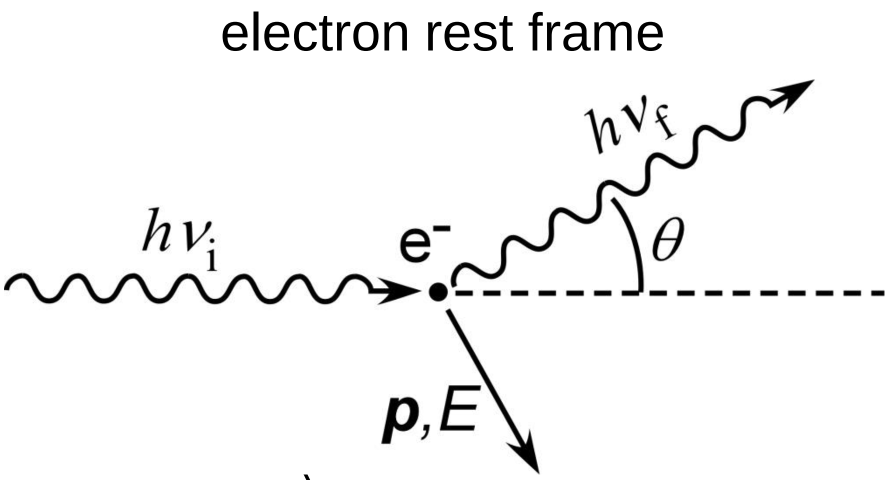
Consider the scattering of a photon off an electron in the electron rest frame.

Energy and momentum conservation gives

$$h\nu_f = \frac{h\nu_i}{1 + \frac{h\nu_i}{m_e c^2}(1 - \cos\theta)}$$

§1 Inverse-Compton Scatterings

QED gives the differential cross section for unpolarized radiation, the Klein-Nishina formula,



$$\frac{d\sigma}{d\Omega}=\frac{3}{16\pi}\sigma_T\left(\frac{\nu_f}{\nu_i}\right)^2\left(\frac{\nu_i}{\nu_f}+\frac{\nu_f}{\nu_i}-\sin^2\theta\right),$$

where

$$\sigma_T\equiv\frac{8\pi}{3}r_0^2=6.652462\times10^{-25}\ \mathrm{cm}^2$$

Integrating $d\sigma/d\Omega$ over the solid angle, we obtain the total cross section

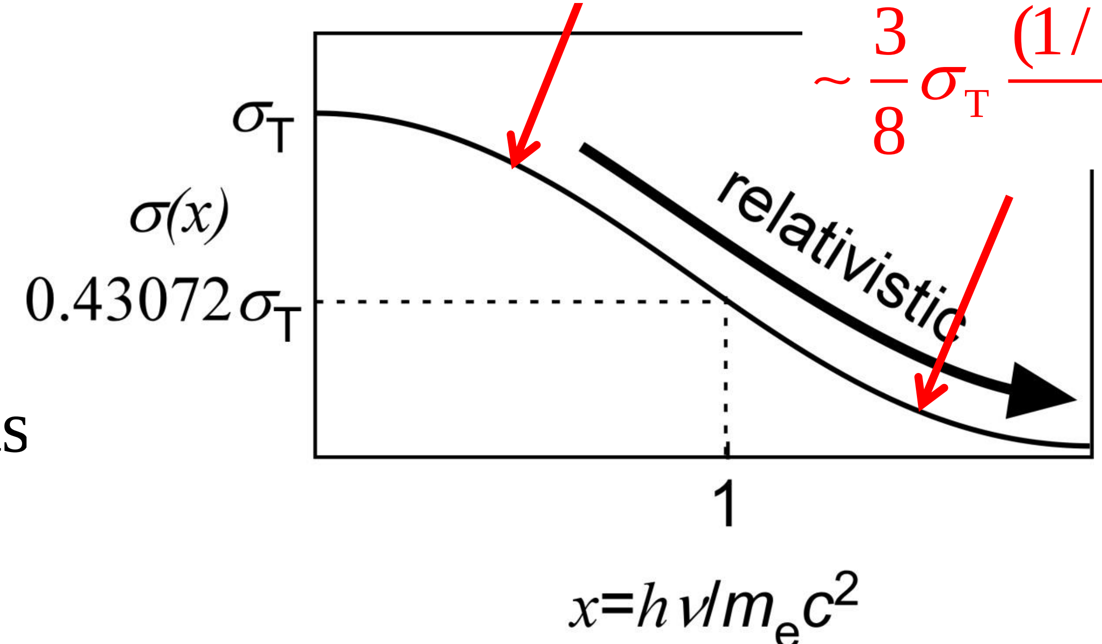
$$\sigma = \sigma_T \cdot \frac{3}{4}\left\{\frac{1+x}{x^3}\left[\frac{2x(1+x)}{1+2x} - \ln(1+2x)\right] + \frac{\ln(1+2x)}{2x} - \frac{1+3x}{(1+2x)^2}\right\}$$

where $x \equiv h\nu / m_e c^2$.

$$\sim \sigma_T\left(1 - 2x + \frac{26}{5}x^2\right)$$

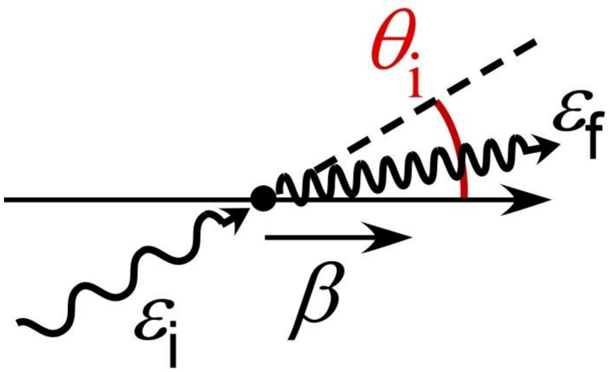
$$\sim \frac{3}{8}\sigma_T \frac{(1/2) + \ln 2x}{x}$$

Cross section reduces due to quantum effects

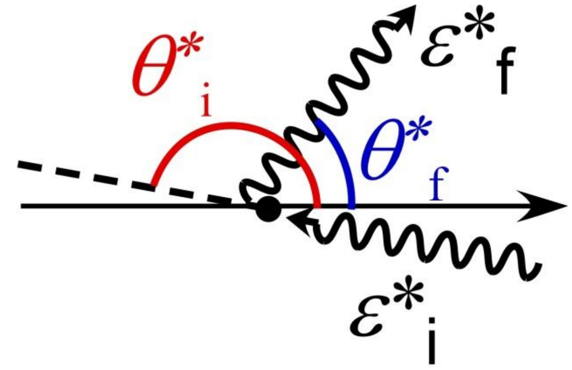


§1 *Inverse-Compton Scatterings*

Consider in the observer's frame. The Doppler shift formula gives



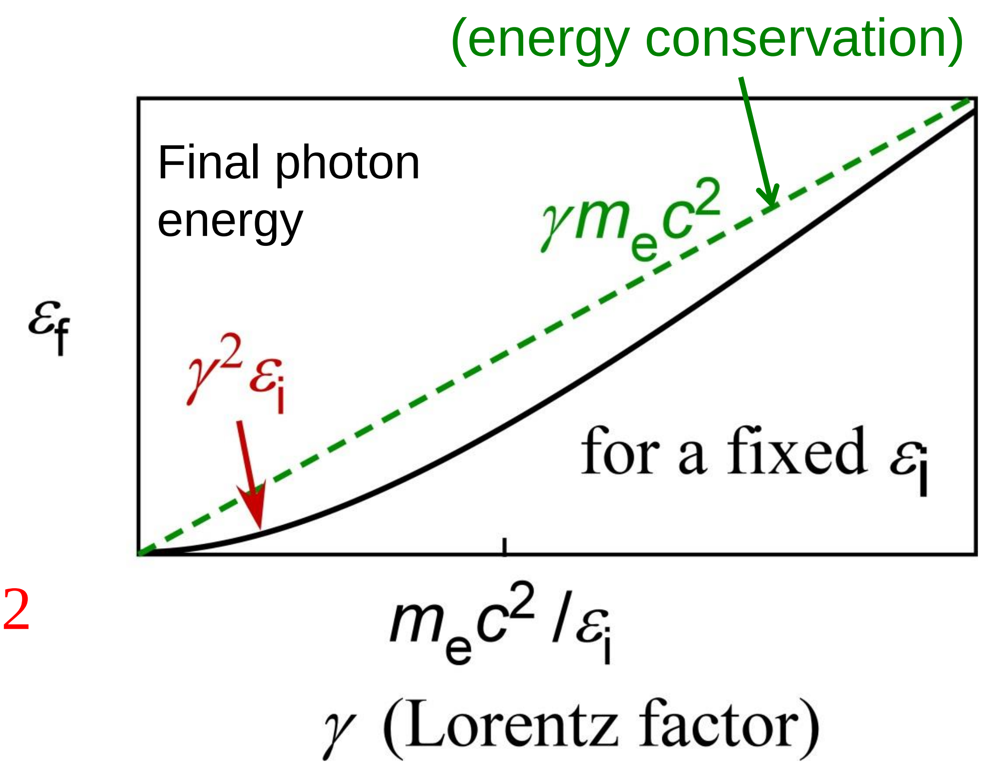
observer's frame *K*



e⁻ rest frame *K'*

$$\varepsilon^*_i = \varepsilon_i \gamma (1 - \beta \cos\theta_i)$$

$$\varepsilon_f = \varepsilon_f^* \gamma (1 + \beta \cos\theta_f^*).$$



If elastic ($\varepsilon^*_f \approx \varepsilon^*_i$) in the e^--rest frame, we obtain

$$\varepsilon_i : \varepsilon^*_i : \varepsilon_f \approx 1 : \gamma : \gamma^2$$

§1 Inverse-Compton Scatterings

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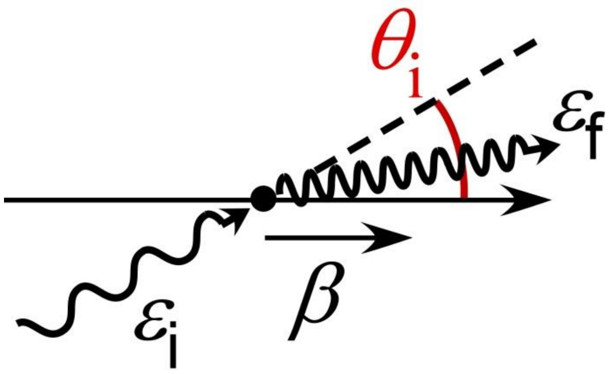

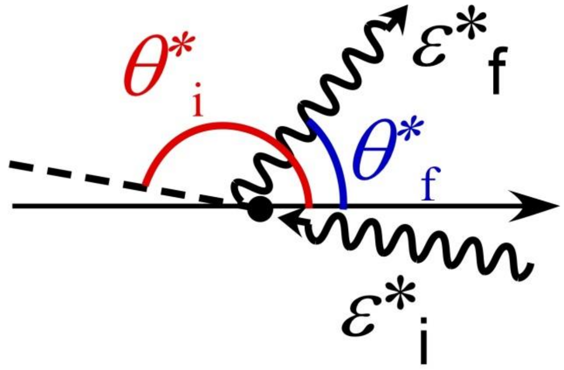

$$\varepsilon^*_{\rm i} = \varepsilon_{\rm i}\gamma(1-\beta\cos\theta_{\rm i})$$

$$\varepsilon_{\rm f} = \varepsilon_{\rm f}^*\gamma(1+\beta\cos\theta_{\rm f}^*).$$

If elastic ($\varepsilon^*_{\rm f} \approx \varepsilon^*_{\rm i}$) in the e^--rest frame, we obtain

$$\varepsilon_{\rm i} : \varepsilon^*_{\rm i} : \varepsilon_{\rm f} \approx 1 : \gamma : \gamma^2$$

For example, if $\varepsilon_{\rm i}$ =100 eV and γ=10^3,

$\varepsilon_{\rm i}^*$~$\varepsilon_{\rm f}^*$~0.1 MeV
$\varepsilon_{\rm f}$ ~100 MeV

§1 Inverse-Compton Scatterings

For isotropic distribution of photons, an e^- emits the inverse-Compton radiation at a rate [ergs s^{-1}],

$$P_{\mathrm{IC}} = \frac{4}{3}\sigma_{\mathrm{T}} c \gamma^2 U_{\mathrm{ph}} \ ,$$

where U_{ph} denotes the energy density of the photons.

cU_{ph} : incident photon flux [erg s^{-1} cm^{-2}]
$\sigma_{\mathrm{T}} cU_{\mathrm{ph}}$: collision rate [erg s^{-1}]
γ^2 : energy amplification factor by a single IC.

§1 Inverse-Compton Scatterings

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$$P_{\mathrm{IC}} = \frac{4}{3}\sigma_{\mathrm{T}} c \gamma^2 U_{\mathrm{ph}} \ ,$$

where U_{ph} denotes the energy density of the photons.

The factor 4/3 comes from the angle average of

$$\left\langle (1-\beta\cos\theta)^2 \right\rangle = 1 + \frac{\beta^2}{3} = \frac{4}{3} \,,$$

which comes from the Lorentz transformation,

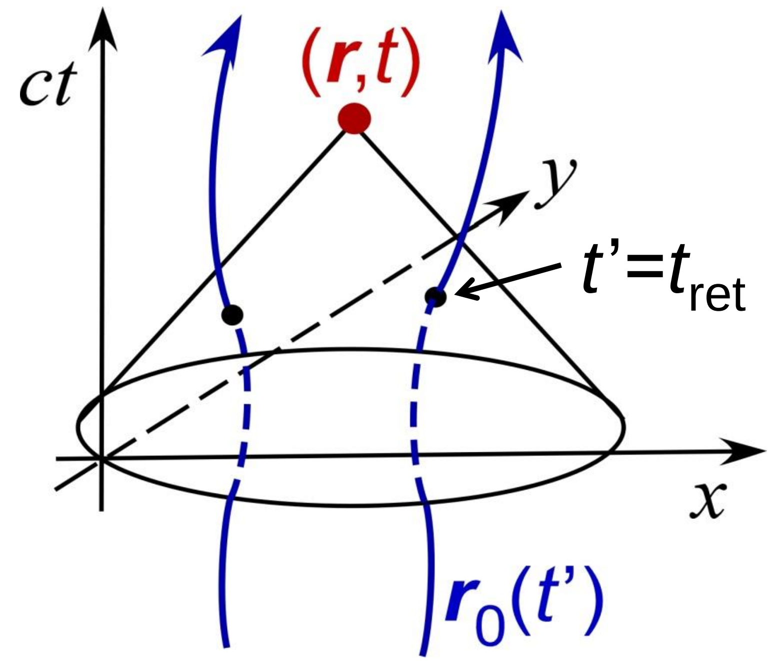
$$\varepsilon^*{}_{\mathrm{i}} = \varepsilon_{\mathrm{i}} \gamma (1-\beta\cos\theta_{\mathrm{i}}) \ .$$

§2 Radiation from Relativistic Charges

From Rybicki & Lightman (1979),
“Radiative Processes in Astrophysics”

§2 Radiation from Relativistic Charges

Consider a charge q moving along a world line, $\mathbf{r}'=\mathbf{r}_0(t')$. It produces an electro-magnetic field at point $(\mathbf{r},t)$, if it is causality connected to the particle's motion.



$$\mathbf{u}(t') \equiv d\mathbf{r}_0/dt'$$

§2 Radiation from Relativistic Charges

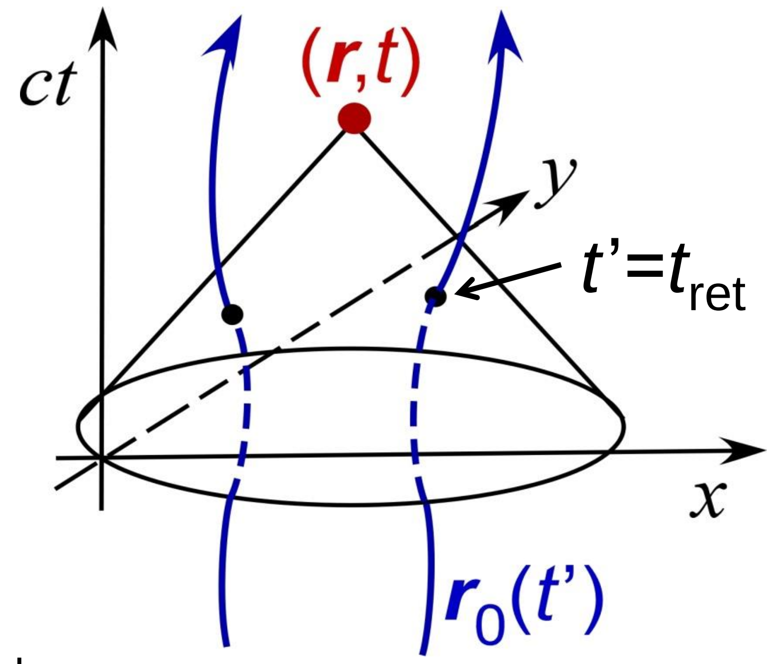
Maxwell eq. $\nabla^2\phi-(1/c^2)\partial^2\phi/\partial^2 t=-4\pi\rho$ gives the Lienard-Wiechart (scalar) potential,

$$\phi(\mathbf{r},t)=\int_{-\infty}^{t}dt'\iiint d^3r'\;4\pi\rho(\mathbf{r}',t')\frac{\delta\left(t-t'-\left|\mathbf{r}-\mathbf{r}'\right|/c\right)}{4\pi\left|\mathbf{r}-\mathbf{r}'\right|}$$

$$=\left[\frac{q}{\kappa R}\right]_{t'=t_{\mathrm{ret}}},$$

where [ ] denotes time t' is evaluated at retarded time, t_{ret},

$$t-t_{\mathrm{ret}}-\frac{\left|\mathbf{r}-\mathbf{r}_0(t_{\mathrm{ret}})\right|}{c}=0,$$



$$\mathbf{R}(t')=\mathbf{r}-\mathbf{r}_0(t'),\quad R(t')=\left|\mathbf{R}(t')\right|,$$

$$\mathbf{u}(t')\equiv d\mathbf{r}_0/dt'$$

$$\kappa(t')=1-\mathbf{n}(t')\bullet\mathbf{u}(t')/c\quad \mathbf{n}=\mathbf{R}/R,\quad \rho(\mathbf{r}',t')=q\delta(\mathbf{r}'-\mathbf{r}_0(t'))$$

§2 Radiation from Relativistic Charges

The vector potential, ***A*(*r*,*t*)**, is given in the same way,

$$\phi(\mathbf{r},t)=\left[\frac{q}{\kappa R}\right]_{t'=t_{\text{ret}}}, \quad \mathbf{A}(\mathbf{r},t)=\left[\frac{q\mathbf{u}}{c\kappa R}\right]_{t'=t_{\text{ret}}}.$$

Thus, a moving charge produces an EM field,

$$\mathbf{E}(\mathbf{r},t)=q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3 R^2}\right]_{t'=t_{\text{ret}}}+\frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R}\times\left\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right\}\right]_{t'=t_{\text{ret}}}$$

$$=\mathbf{E}_{\text{vel}}+\mathbf{E}_{\text{acc}}$$

$$\mathbf{B}(\mathbf{r},t)=\left[\mathbf{n}\times\mathbf{E}(\mathbf{r},t)\right]_{t'=t_{\text{ret}}}$$

§2 Radiation from Relativistic Charges

$$\mathbf{E}(\mathbf{r},t) = q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3 R^2}\right]_{t'=t_{\text{ret}}} + \frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R}\times\left\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right\}\right]_{t'=t_{\text{ret}}}$$

$$= \mathbf{E}_{\text{vel}} + \mathbf{E}_{\text{acc}}$$

Noting $\quad E_{\text{vel}} \propto 1/R^2, \quad E_{\text{rad}} \propto 1/R$

$$\frac{E_{\text{rad}}}{E_{\text{vel}}} \sim \frac{R\dot{u}}{c^2} \sim \frac{Ru\nu}{c^2} \sim \frac{u}{c}\frac{R}{\lambda} \qquad (\dot{u} \sim u\nu),$$

we find that the $\boldsymbol{E}_{\text{vel}}$ dominates in the near zone, $R<\lambda$,
while the $\boldsymbol{E}_{\text{acc}}$ dominates in the far zone, $R \gg \lambda$.

The velocity field does not carry energy to large distances,
while the radiation field does. $\qquad (\mathbf{S} = \mathbf{E}\times\mathbf{B}/4\pi)$

$$\mathbf{E}(\mathbf{r},t) = q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3 R^2}\right]_{t'=t_{\text{ret}}} + \frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R}\times\left\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right\}\right]_{t'=t_{\text{ret}}}$$

$$= \mathbf{E}_{\text{vel}} + \mathbf{E}_{\text{acc}}$$

Usually, we use $\mathbf{E}_{\text{acc}}$ to compute the emission spectrum, by Fourier analyzing the time-dependent **E** field.

However, we can derive the same results using $\mathbf{E}_{\text{vel}}$ by introducing the virtual quanta.

For example, the synchrotron radiation can be interpreted as the inverse Compton scatterings of virtual photons in an external **B** field.

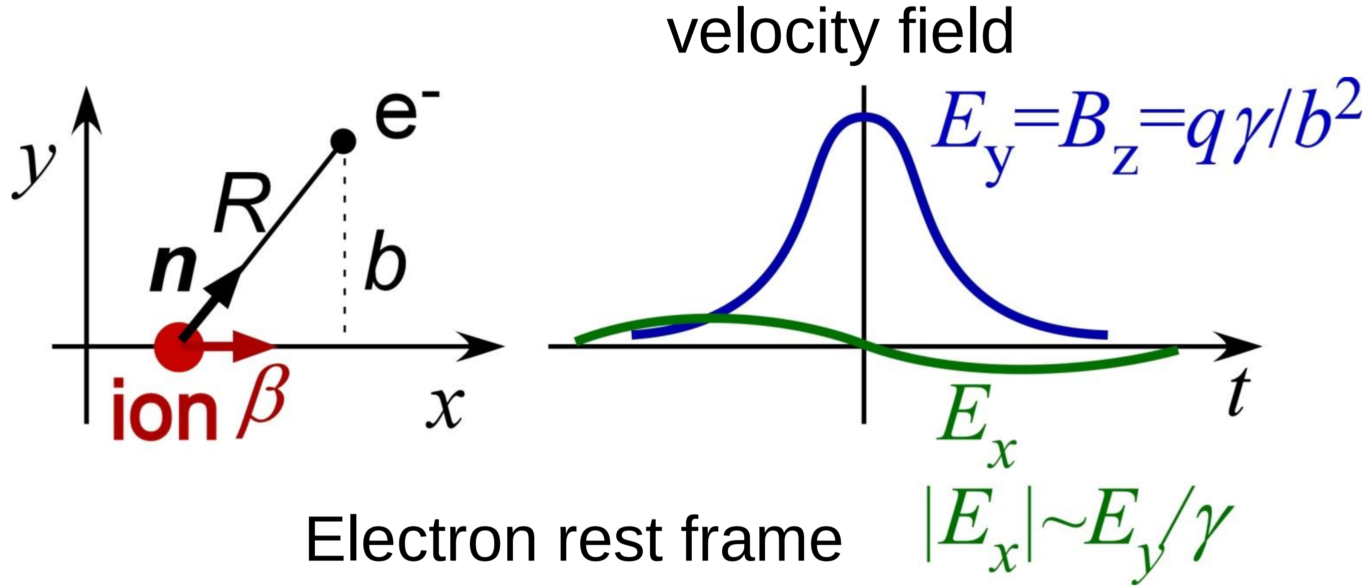
We thus consider such an explanation in what follows.

§2 Radiation from Relativistic Charges

$$\mathbf{E}(\mathbf{r},t) = q\left[\frac{(\mathbf{n}-\boldsymbol{\beta})(1-\beta^2)}{\kappa^3 R^2}\right]_{t'=t_{\text{ret}}} + \frac{q}{c}\left[\frac{\mathbf{n}}{\kappa^3 R}\times\left\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right\}\right]_{t'=t_{\text{ret}}}$$

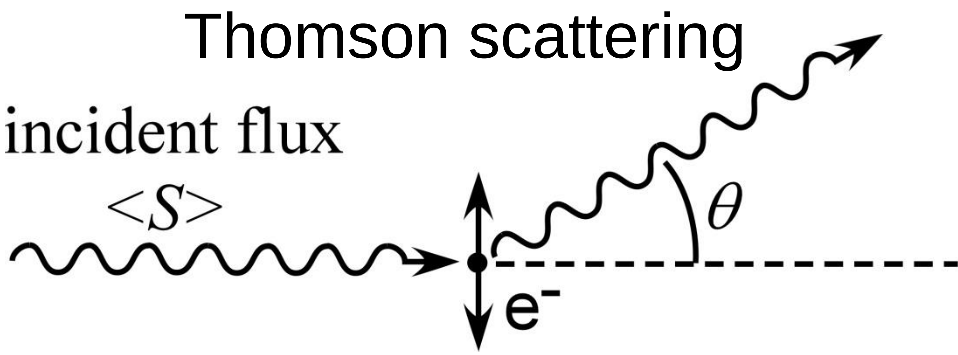
$$= \mathbf{E}_{\text{vel}} + \mathbf{E}_{\text{acc}}$$

Consider e^--ion Bremsstrahlung (free-free emission).
In the e^- rest frame, relativistic ion produces a pulse of $\mathbf{E}_{\text{vel}}$ in the near zone, $R<\lambda$,

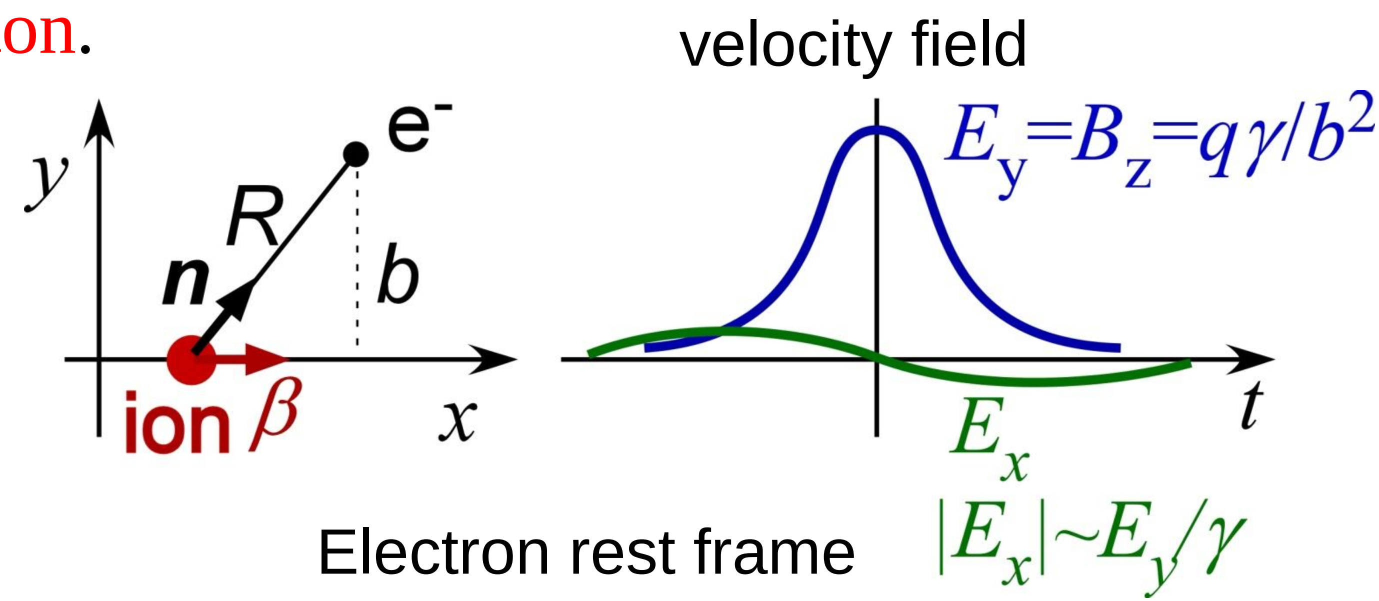


§2 Radiation from Relativistic Charges

Because of this pulsed $\mathbf{E}_{\mathrm{vel}}$, e^- oscillates to radiate by the dipole formula. That is, a virtual photon, which does not carry energy to infinity, is up-scattered to become a real photon.

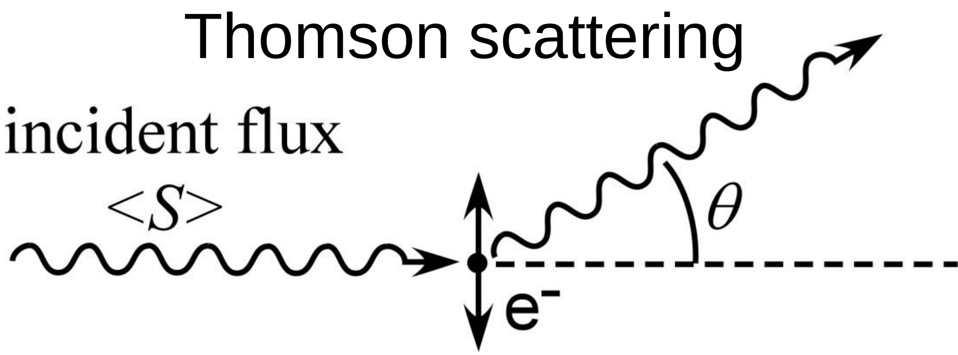


= Relativistic bremsstrahlung



Electron rest frame

§2 Radiation from Relativistic Charges



Because of this pulsed $\mathbf{E}_{\mathrm{vel}}$, e^- oscillates to radiate by the dipole formula.
That is, a virtual photon, which does not carry energy to infinity, is up-scattered to become a real photon.

= Relativistic bremsstrahlung

The same idea can be applied when a charge is moving in an external magnetic field.

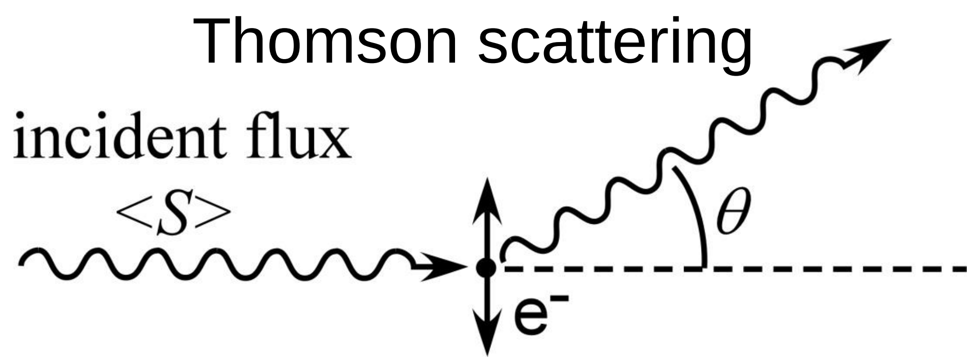
$\rightarrow$ Synchrotron radiation

§3 Synchrotron Radiation

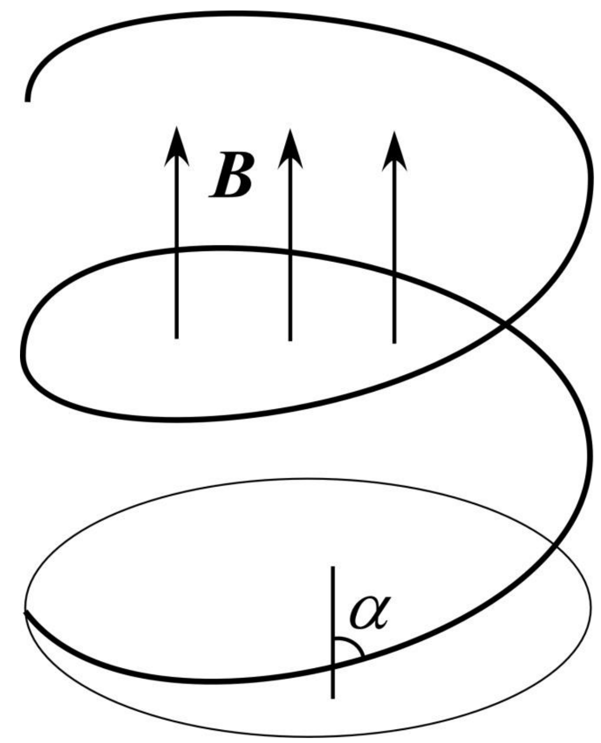
From Rybicki & Lightman (1979),
"Radiative Processes in Astrophysics"

§3 Synchrotron Radiation

If an e^- is moving in a **B** field, a time-varying $\mathbf{E}_{\mathrm{vel}}$ field arises in the e^--rest frame.



Then, the e^- oscillates to emit synchrotron photons.



The virtual photon has an energy,

$$\sim \hbar\omega_{\mathrm{cyclotron}} \sin\alpha = \hbar \frac{qB}{m_{\mathrm{e}}c} \sin\alpha$$

Thus, the up-scattered photon has an energy $\sim \gamma^2 \hbar\omega_{\mathrm{cyclotron}} \sin\alpha$.

§3 Synchrotron Radiation

A detailed argument shows an additional factor, 3/2, arises; thus, the synchrotron characteristic energy becomes

$$\hbar\omega_{\mathrm{c}} = \frac{3}{2}\gamma^2\hbar\omega_{\mathrm{cyclotron}}\sin\alpha = \frac{3}{2}\gamma^2\hbar\frac{qB}{m_{\mathrm{e}}c}\sin\alpha = \frac{3}{2}\hbar\gamma^3\frac{c}{r_{\mathrm{g}}},$$

where the gyro radius is defined by $r_{\mathrm{g}} \equiv \dfrac{\gamma m_{\mathrm{e}}c^2}{qB\sin\alpha}$.

Since $\hbar\omega_{\mathrm{cyclotron}} = 11.576765\left(\dfrac{B}{10^{12}\,\mathrm{G}}\right)$ keV , synchrotron photons have the typical energy,

$$\hbar\omega_{\mathrm{c}} = 17.365148\left(\frac{B}{10^{12}\,\mathrm{G}}\right)\gamma^2\sin\alpha \quad \mathrm{keV}$$

The synchrotron radiation power can be estimated by

$$P_{\mathrm{synch}} = N_{\mathrm{virtual}} \sigma_{\mathrm{T}} c \gamma^2 \hbar \omega_{\mathrm{cyclotron}}$$

$$\approx \sigma_{\mathrm{T}} c \gamma^2 U_{\mathrm{B}} \ ,$$

where $U_{\mathrm{B}} = B^2 / 8\pi \approx N_{\mathrm{virtual}} \hbar \omega_{\mathrm{cyclotron}}$.

For an isotropic distribution of α, a detailed argument shows

$$P_{\mathrm{synch}} = \frac{4}{3} \sigma_{\mathrm{T}} c \gamma^2 U_{\mathrm{B}} \ .$$

Reminding $P_{\mathrm{IC}} = \frac{4}{3} \sigma_{\mathrm{T}} c \gamma^2 U_{\mathrm{ph}}$, we find $\dfrac{P_{\mathrm{synch}}}{P_{\mathrm{IC}}} = \dfrac{U_{\mathrm{B}}}{U_{\mathrm{ph}}}$.

The trans-field motion is quantized with discrete energy values, called Landau levels,

$$E = \hbar\omega_{\text{cyclotron}} \left(n + \frac{-m + |m|}{2} + \frac{1}{2} \right),$$

$$n + \frac{-m + |m|}{2} = 0,\ 1,\ 2,\ 3,\ \ldots$$

n: radial quantum number
m: magnetic quantum number

The ground state has the “zero-point” energy, $\dfrac{\hbar\omega_{\text{cyclotron}}}{2}$.

After falling onto the ground state, an e^- no longer emits synchrotron photons.

§4 Synchro-curvature Radiation

§4 Synchro-curvature Radiation

The longitudinal motion (along ***B***) is not quantized, allowing continuous $P_{||}$.

After the particle falls onto the ground Landau state, only the longitudinal motion contributes to an emission.

If the macroscopic particle motion follows a curved trajectory with curvature radius R_c, it emits the pure curvature radiation with characteristic energy,

$$\boxed{\hbar\omega_{\mathrm{curv}} = \frac{3}{2}\hbar\gamma^3 \frac{c}{R_c}.} \qquad \text{cf.} \qquad \hbar\omega_c = \frac{3}{2}\hbar\gamma^3 \frac{c}{r_g}$$

(synchrotron case)

§4 Synchro-curvature Radiation

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cf. synchrotron case

characteristic energy: $\hbar\omega_{\mathrm{curv}} = \frac{3}{2}\hbar\gamma^3 \frac{c}{R_c}$ $\quad\quad$ $\hbar\omega_c = \frac{3}{2}\hbar\gamma^3 \frac{c}{r_g}$

radiation power: $P_{\mathrm{curv}} = \frac{3e^2}{2c^3}\gamma^4 \left(\frac{c^2}{R_c}\right)^2$ $\quad\quad$ $P_{\mathrm{synch}} = \frac{3e^2}{2c^3}\gamma^4 \left(\frac{c^2}{r_g}\right)^2$

§4 Synchro-curvature Radiation

If the particle has not fallen onto the ground Landau state, but moves along a curved trajectory with curvature radius R_c, it emits the synchro-curvature radiation.

(Cheng & Zhang 1996, ApJ 463, 271-283)

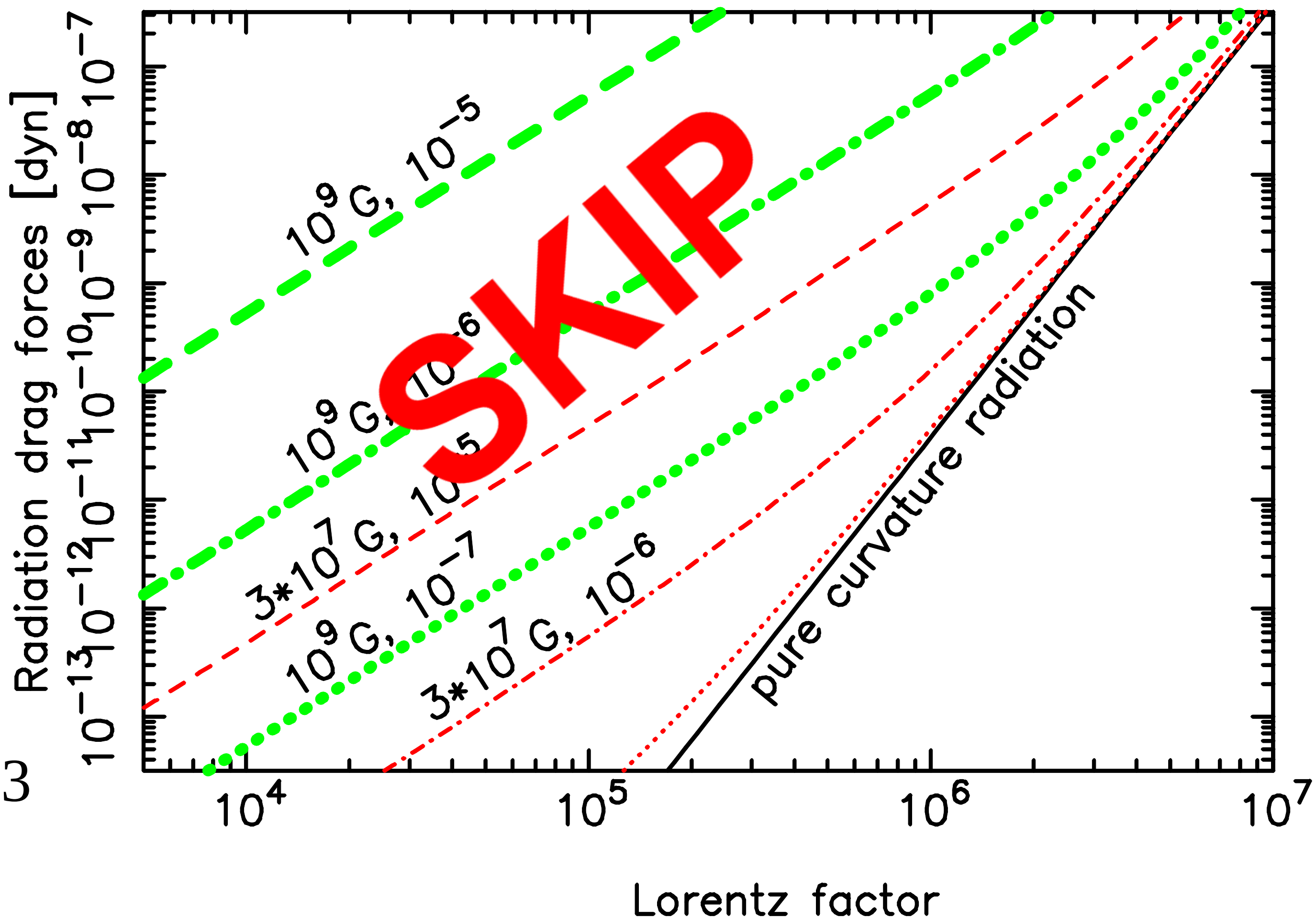
In $\alpha \to 0$, it reduces to the pure curvature process.
In $R_c \to \infty$, it reduces to the pure synchrotron process.

SKIP

General expression of SC radiation is complicated. Thus, we consider typical e^- motion in a pulsar magnetosphere and show how it deviates from the pure curvature process when both ***B*** and α are large.

§4 Synchro-curvature Radiation

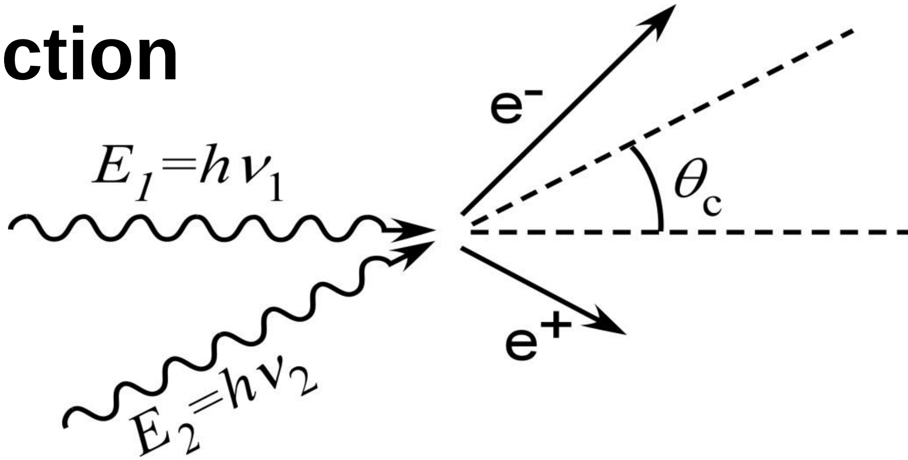
If $B > 10^7$G and $\alpha > 10^{-6}$, synchro-curvature process deviates from the pure curvature process



KH 2006,
ApJ 652, 1475-93

§5 Electron-positron Pair Production

Photon-photon pair production



If two photons collide with angle θ_c, e^--e^+ pair may be produced. The total cross section of $\gamma\gamma \rightarrow ee$ becomes

$$\sigma_p(u) = \frac{3}{16}\sigma_T(1-u^2)\left[(3-u^4)\ln\frac{1+u}{1-u} - 2u(1-u^2)\right],$$

where $u(E_1, E_2, \theta_c) \equiv \sqrt{1 - \frac{2}{1-\cos\theta_c}\frac{(m_e c^2)^2}{E_1 E_2}}$

Note that $\gamma\gamma \rightarrow ee$ takes place only when the threshold is satisfied,

$$E_1 E_2 > \frac{1-\cos\theta_c}{2}(m_e c^2)^2$$

Magnetic pair production

When a photon propagates in a ***B*** field, it may be absorbed to materialize as a pair.

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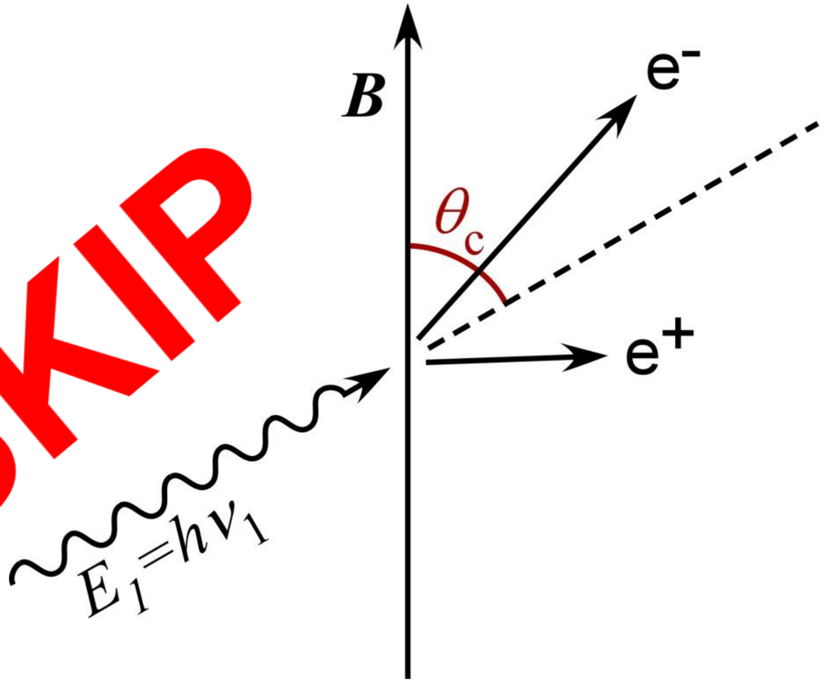

The photon mean-free path to $\gamma B \rightarrow ee$ becomes
(Erber 1966, Rev. mod. Phys. 38, 626)

$$\lambda_B = 600 \frac{c}{\frac{eB}{m_e c} \sin\theta_c} \exp\left[\frac{8}{3} \frac{B_{cr}}{B \sin\theta_c} \frac{m_e c^2}{E_1} \right] \text{cm} ,$$

where $$B_{cr} \equiv \frac{m_e^2 c^3}{e\hbar} = 4.413 \times 10^{13}\,\text{G}$$

END OF CHAPTER 2