Chapter 4

The Black Hole Gap Model

§1 Formulation

§2 Soft photons emitted from hot accretion disk

§1 Formulation

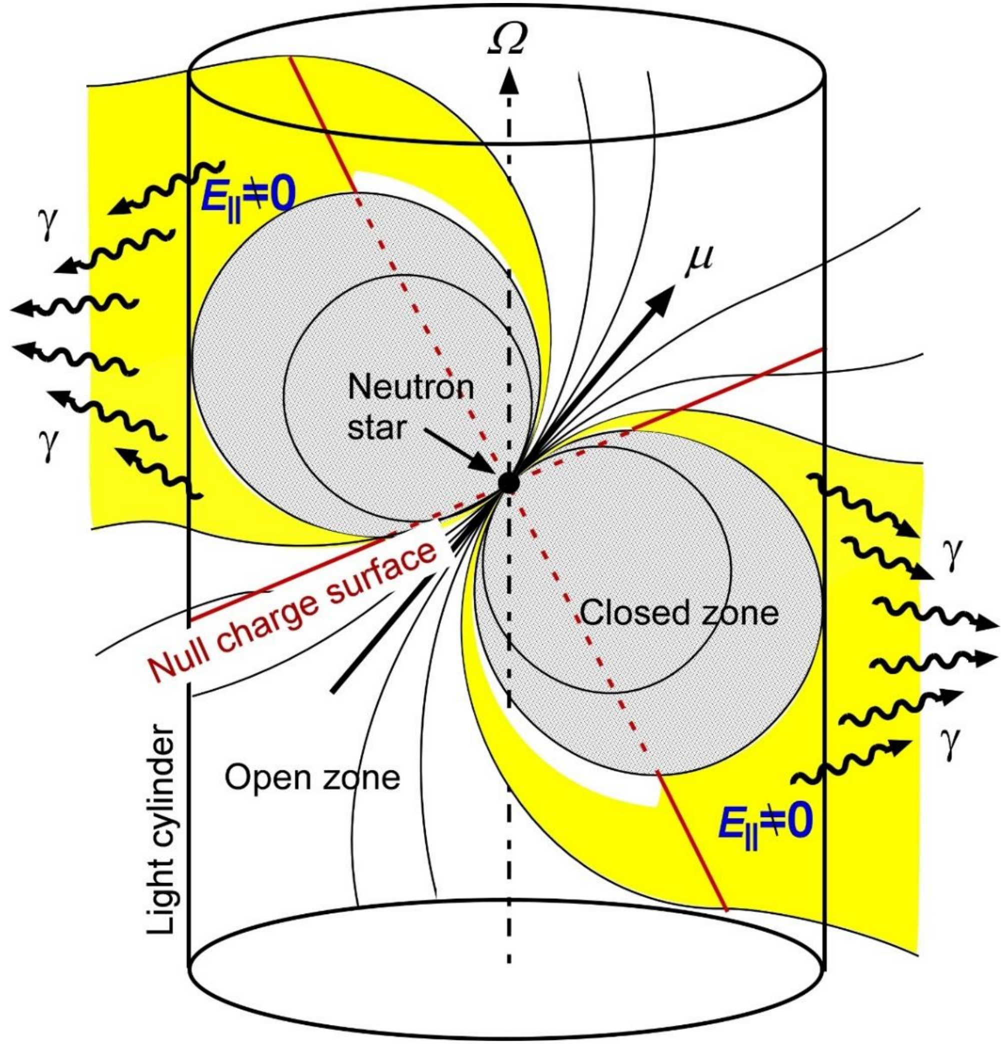
§ 1 Basic equations

Around a *pulsar*, $e^{\pm}$ accelerator (i.e., a *gap*) arises around the null-charge surface ($\rho_{GJ}=0$), which is formed at $r \gg r_{NS}$ due to the convex $\boldsymbol{B}$-field geometry.

(Cheng +, 1986, ApJ 300, 500)

Goldreich-Julian charge density:

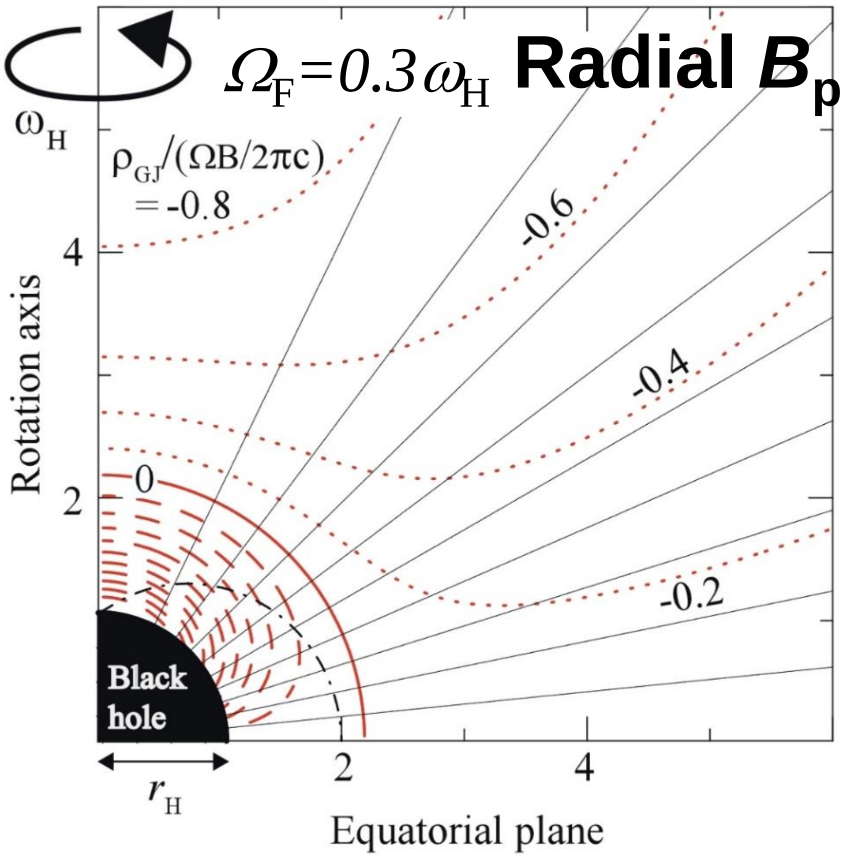
$$\rho_{GJ} \approx -\frac{\mathbf{\Omega} \cdot \mathbf{B}}{2\pi c}$$



Pulsar outer magnetospheric accelerato

§ 1 Basic equations

Around a BH, due to frame dragging, the GR Goldreich-Julian charge density vanishes near the $\Omega=\omega$ surface (and hence in the direct vicinity of the horizon).



The GR Goldreich-Julian charge density:

$$\rho_{GJ} \equiv -\frac{1}{4\pi}\nabla\cdot\left(\frac{\Omega-\omega}{2\pi\alpha c}\nabla\Psi\right)$$

Ω: angular freq. of $\boldsymbol{B}$

ω: angular freq. of frame dragging

α: redshift factor (lapse function)

Ψ: magnetic flux function

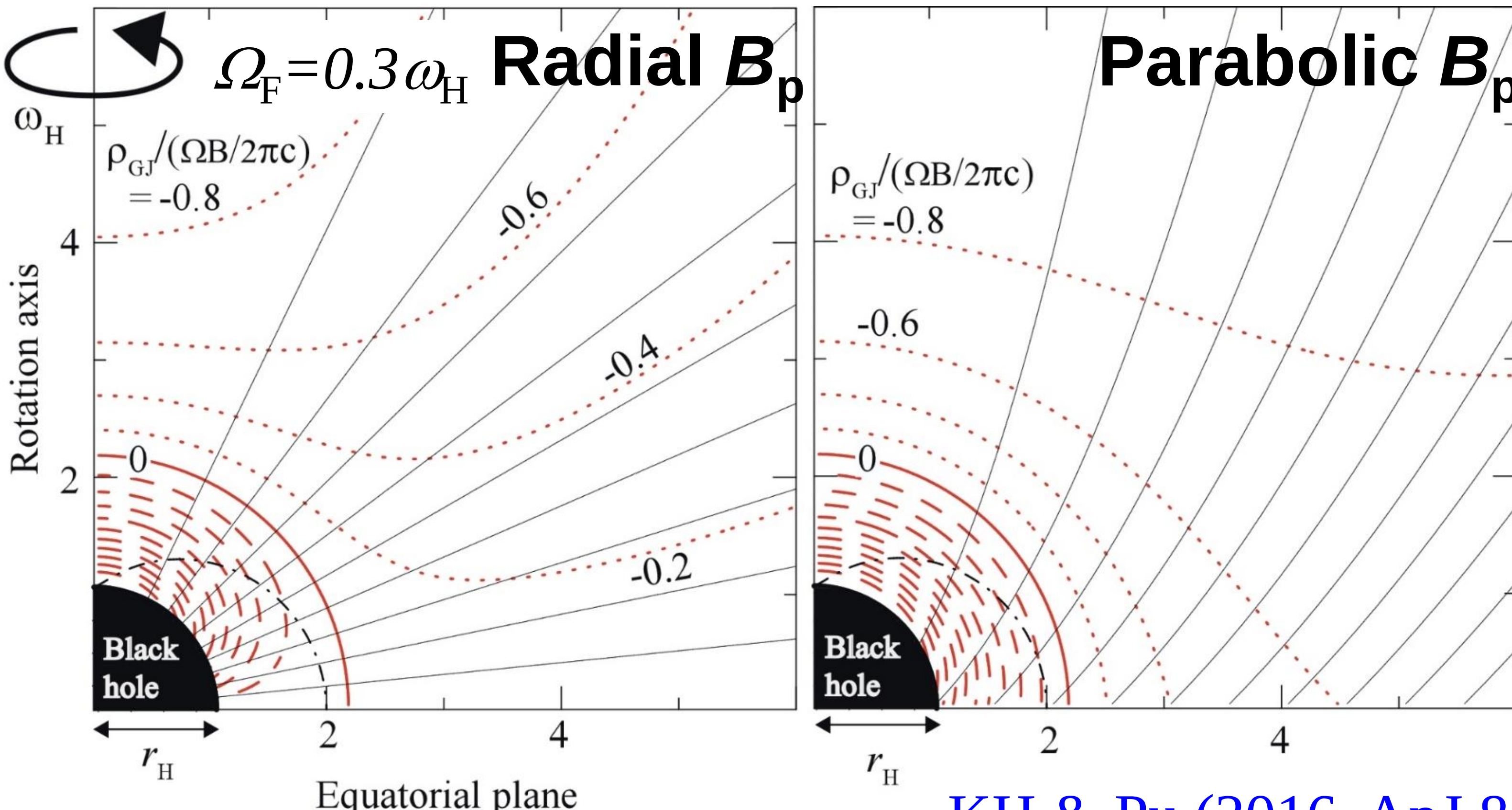
Beskin + 1992, Sov. Astron., 36, 642

KH & Pu (2016, ApJ 818, 50)

§ 1 Basic equations

$$\rho_{\mathrm{GJ}} \equiv -\frac{1}{4\pi}\nabla\bullet\left(\frac{\Omega-\omega}{2\pi\alpha c}\nabla\Psi\right)$$

$\rho_{\mathrm{GJ}}(r,\theta)$ little depends on the $\boldsymbol{B}$ geometry.



KH & Pu (2016, ApJ 818, 50)

§ 1 Basic equations

Since ρ_{GJ}, and hence the gap solution little depends on the ***B*** geometry, we assume a radial ***B*** on the poloidal (r, θ) plane.

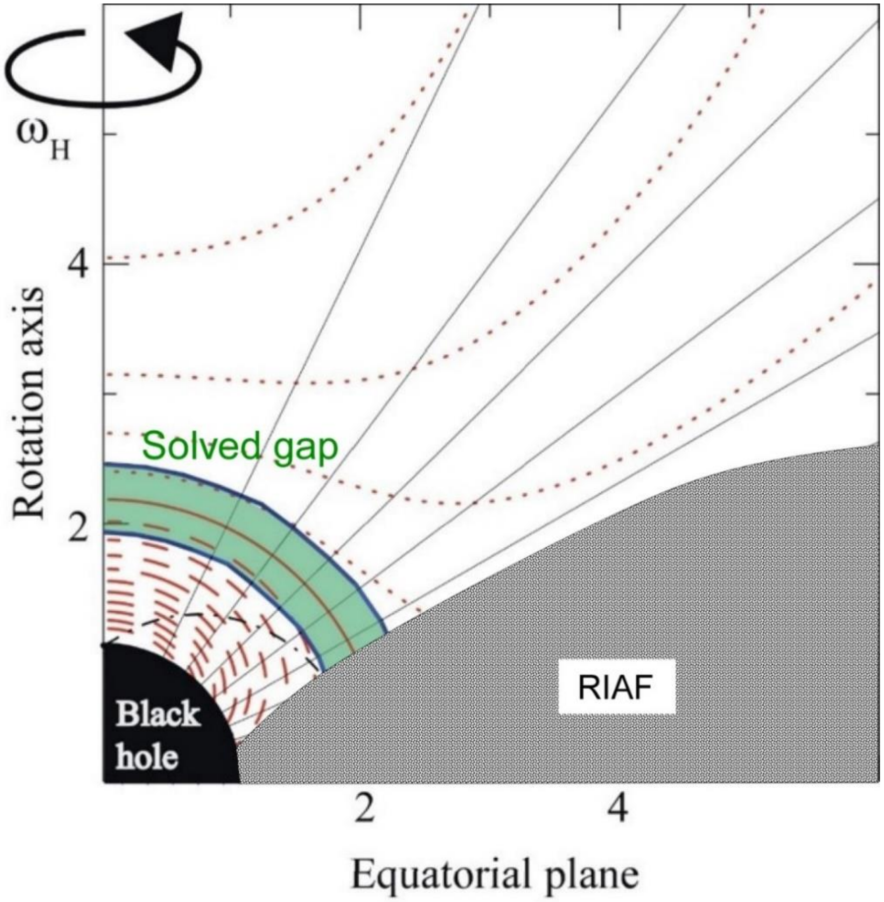


Fig. Side view of a BH gap.

KH & Pu (2016, ApJ 818, 50)

Physical processes

Since the null surface appears in a BH magnetosphere, the same method as the pulsar outer-gap model can be applied to BH magnetospheres.

$e^{\pm}$'s are accelerated by $E_{\parallel}$

↓

Relativistic e^+/e^- emit γ-rays via synchro-curvature, and IC processes

↓

γ-rays collide with soft photons/B to materialize as pairs in the accelerator

Basic equations

KH+ (‘16, ApJ 818, 50)

$E_{\|}$ is solved from the **Poisson eq.**

Near the horizon ($\Delta \equiv r^2 - 2Mr + a^2 \ll M^2$), it becomes

$$-\left(\frac{r^2+a^2}{\Delta}\right)^2 \frac{\partial^2\Phi}{\partial r_*^2} + \frac{2(r-r_g)(r^2+a^2)}{\Delta^2}\frac{\partial\Phi}{\partial r_*}$$

$$-\frac{\Sigma}{\Delta\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\sin\theta}{\Sigma}\frac{\partial\Phi}{\partial\theta}\right) = \left(\frac{\Sigma}{r^2+a^2}\right)^2 (n_+ - n_- - n_{\mathrm{GJ}})$$

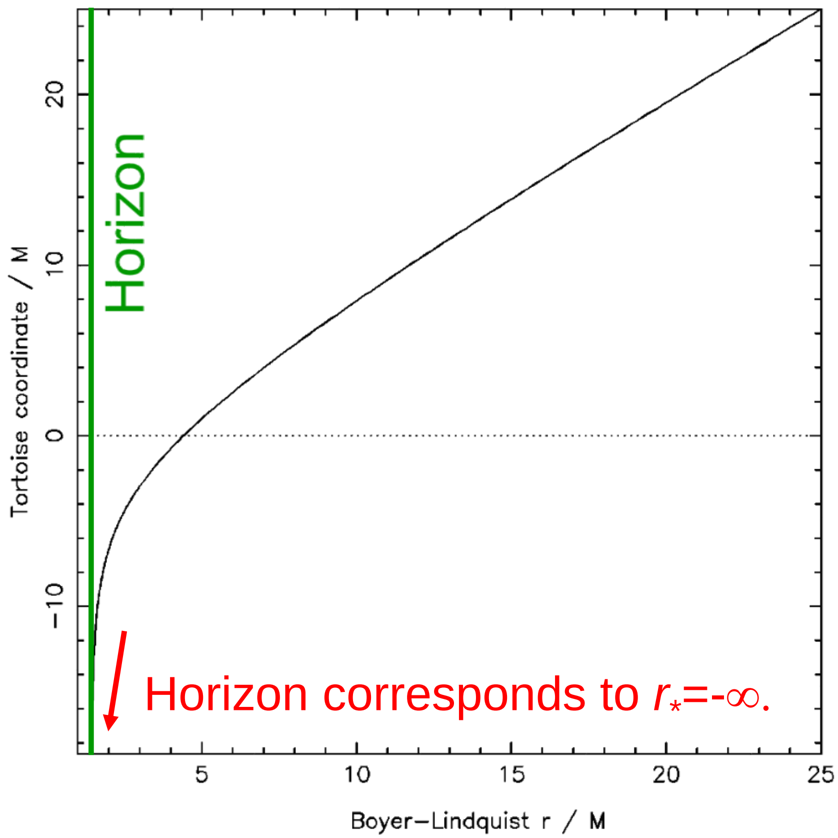
where $E_{\|} \equiv -(\mathbf{B}\bullet\nabla)\Phi / B$ and $\Sigma \equiv r^2 + a^2\cos^2\theta$.

The tortoise coordinate r_* is related to the polar r by

$$\frac{dr_*}{dr} = \frac{r^2+a^2}{\Delta}$$

Basic equations

KH+ ('16, ApJ 818, 50)



Tortoise coordinate vs. Boyer-Lindquist *r* coordinate

The tortoise coordinate r_* is related to the polar r by

$$\frac{dr_*}{dr} = \frac{r^2 + a^2}{\Delta}$$

Basic equations

Poisson eq.:

$$-\left(\frac{r^2+a^2}{\Delta}\right)^2\frac{\partial^2\Phi}{\partial r_*^{\,2}}+\frac{2(r-r_g)(r^2+a^2)}{\Delta^2}\frac{\partial\Phi}{\partial r_*}$$

$$-\frac{\Sigma}{\Delta\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\sin\theta}{\Sigma}\frac{\partial\Phi}{\partial\theta}\right)=\left(\frac{\Sigma}{r^2+a^2}\right)^2(n_+-n_--n_{\mathrm{GJ}})$$

***e*$^\pm$ Boltzmann eqs.:**

$$\frac{\partial N_\pm}{\partial t}+\vec{v}\cdot\nabla N_\pm+\left(e\vec{E}_\parallel+\frac{\vec{v}}{c}\times\vec{B}\right)\cdot\frac{\partial N_\pm}{\partial\vec{p}}$$

$$=S_{IC}+S_{SC}+\int\alpha_\nu d\nu\int\frac{I_\nu}{h\nu}d\omega$$

Radiative transfer eq.: $$\frac{dI_\nu}{dl}=-\alpha_\nu I_\nu+j_\nu$$

§2 Soft photons emitted from the hot accretion disk

We consider the mass accretion rate that is much less than the Eddington rate.

Accordingly, the equatorial plasma flow becomes an advection-dominated accretion flow (ADAF).

The soft photons emitted from ADAF materialize as seed pairs that are to be accelerated in the gap.

§ 2 ADAF soft photon field

KH+ ('17, ApJ **845**, 77)

Compute ADAF photon flux @ $(r,\theta;\theta_\gamma)$.

Ray-trace the ADAF-emitted photons in Kerr spacetime, and tabulate the specific intensity at each point in the static frame.

Then convert it to the value measured in the Zero-Angular-Momentum Observer (ZAMO), using the invariance of $I_\nu \nu^{-3}$.

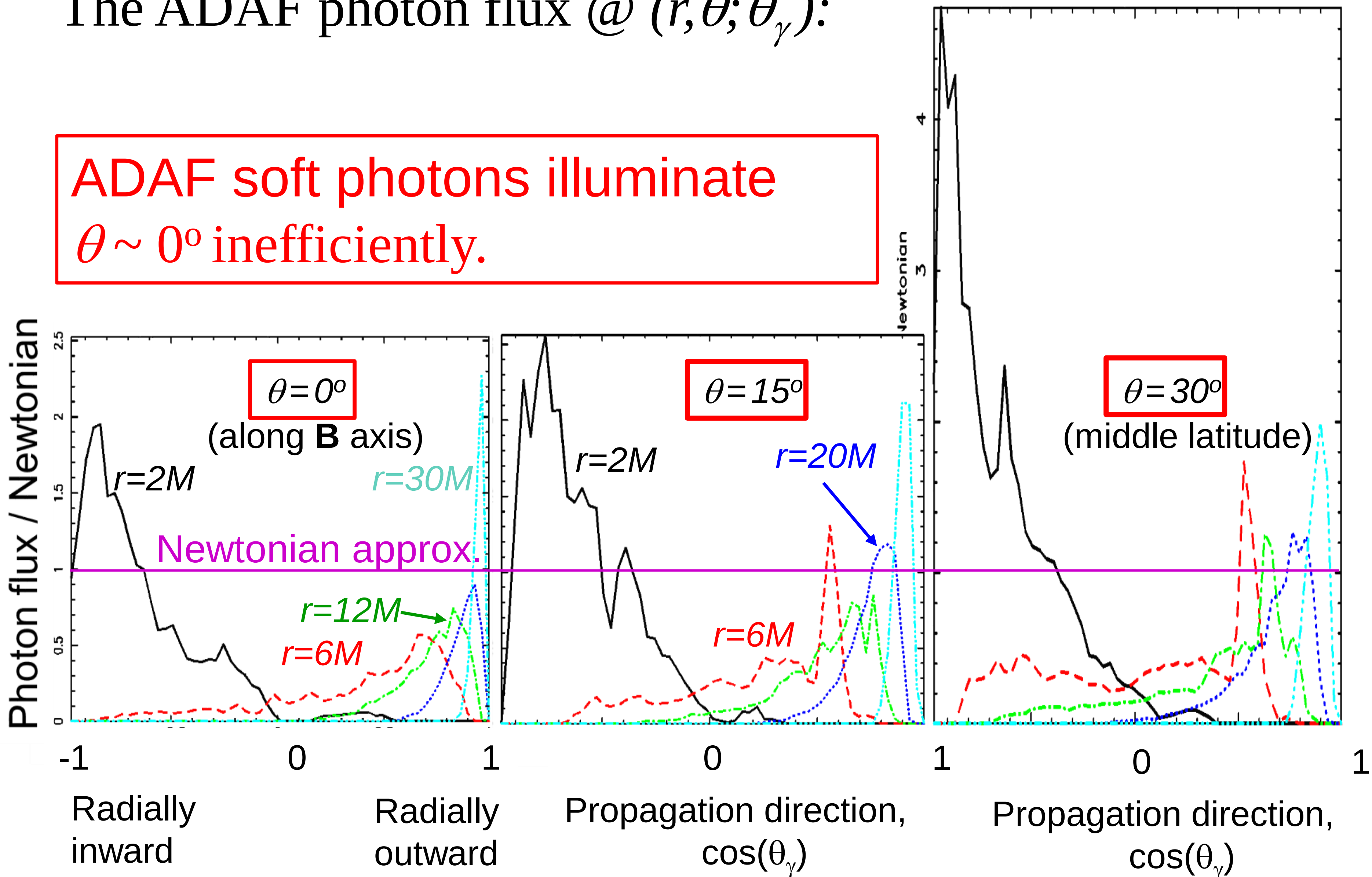
Finally, integrate it over the solid angle in each photon propagation direction to compute the soft photon flux at each point in each direction.

§ 2 ADAF soft photon field

The ADAF photon flux @ $(r,\theta;\theta_\gamma)$:

KH+ ('17, ApJ **845**, 77)

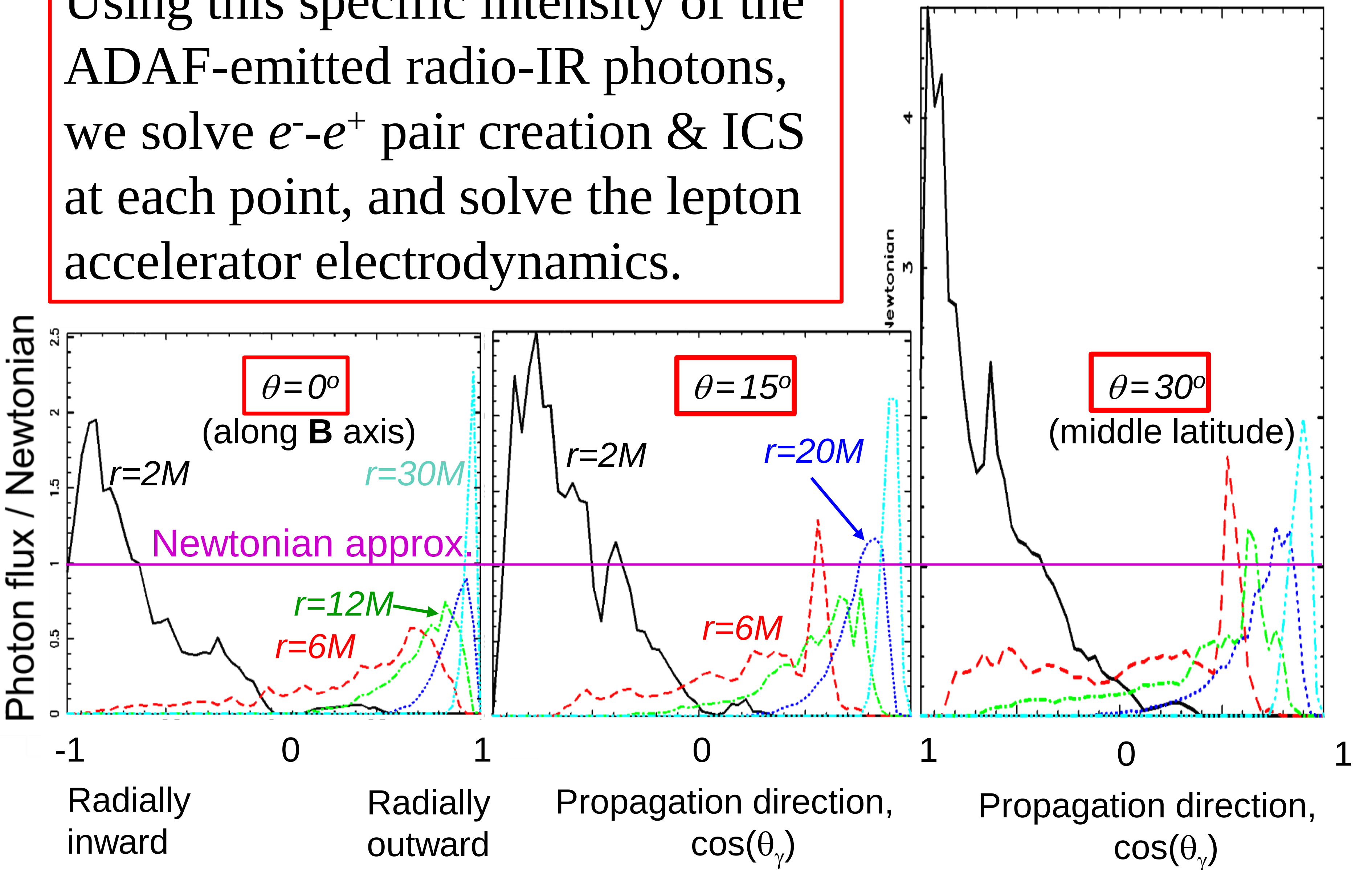
ADAF soft photons illuminate $\theta \sim 0^{\mathrm{o}}$ inefficiently.



§ 2 ADAF soft photon field

KH+ ('17, ApJ **845**, 77)

Using this specific intensity of the ADAF-emitted radio-IR photons, we solve e^--e^+ pair creation & ICS at each point, and solve the lepton accelerator electrodynamics.



§ 2 ADAF soft photon field

Created $e^{\pm}$'s are accelerated by $E_{\parallel}$, emitting TeV photons via ICS process and GeV photons via curvature process.

ADAF-emitted soft photons are up-scattered by ultra-relativistic e^{-}'s.

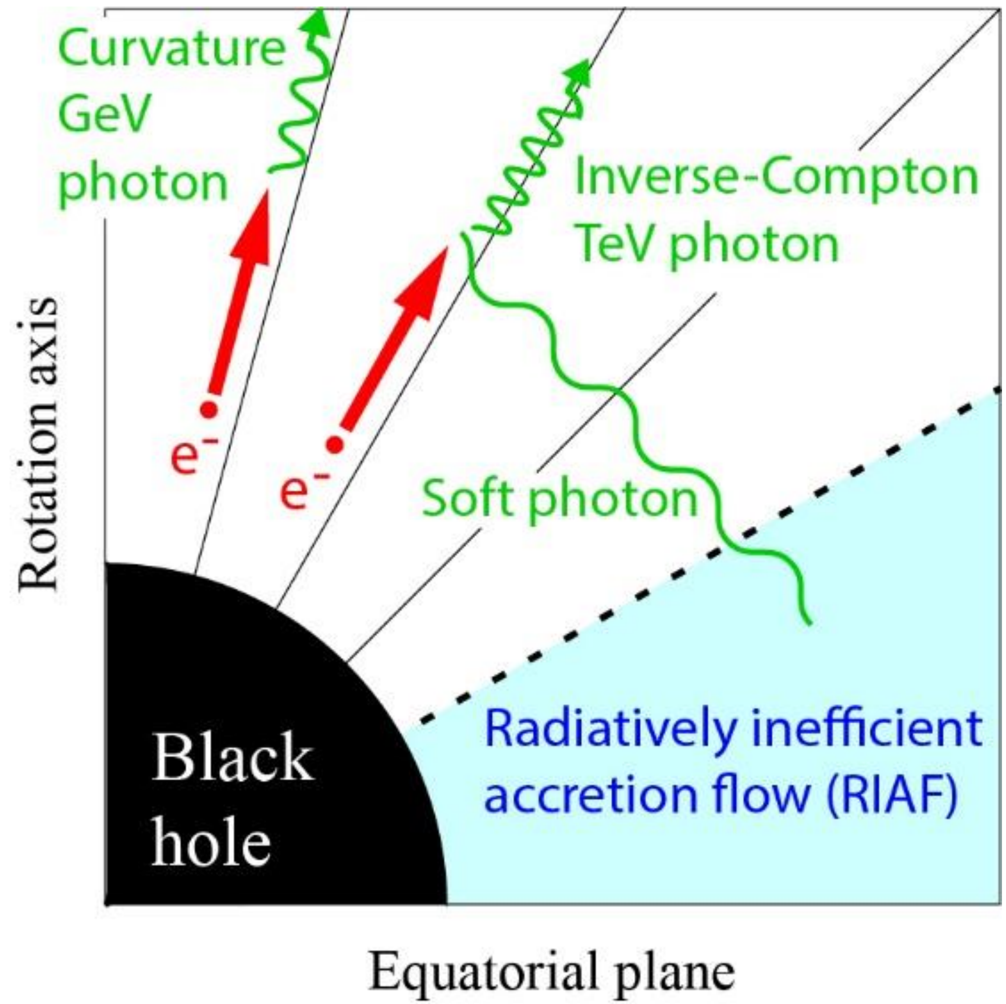


Fig. Side view of a BH magnetosphere.

END OF CHAPTER 4