Cooperative spontaneous emissions: Superradiance, subradiance, and chiral quantum network

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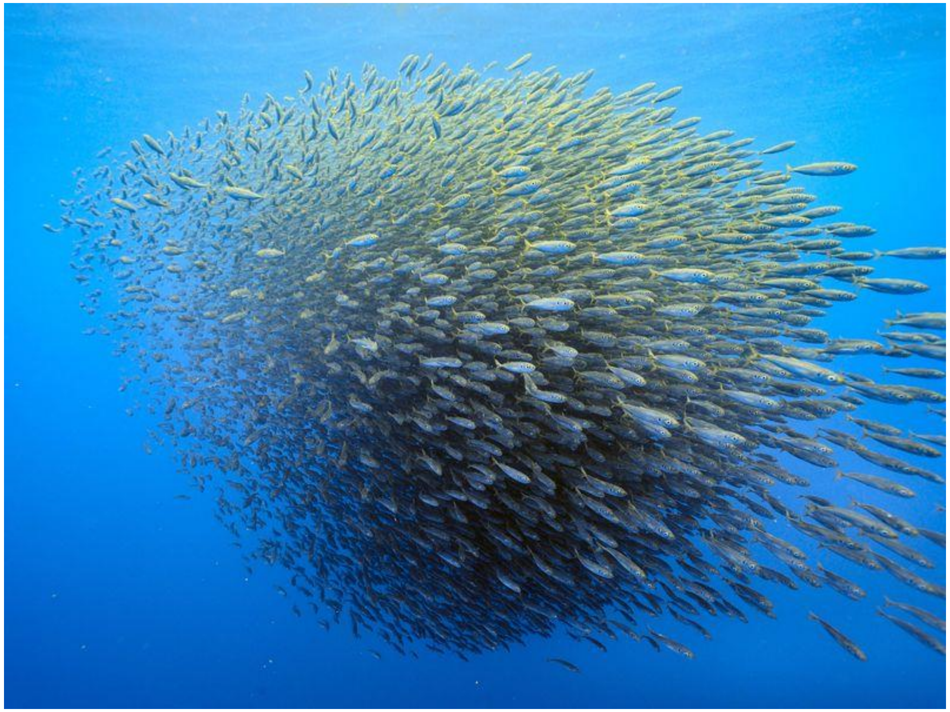
NCTS

Cooperativity or collectivity

Latin origin: cooperat- → worked together

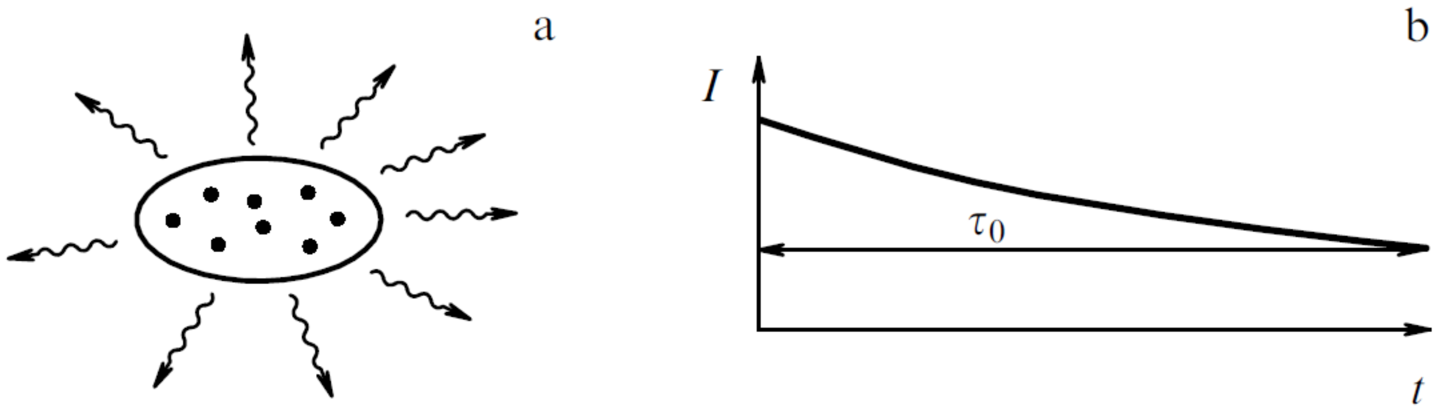
collect- → gathered together



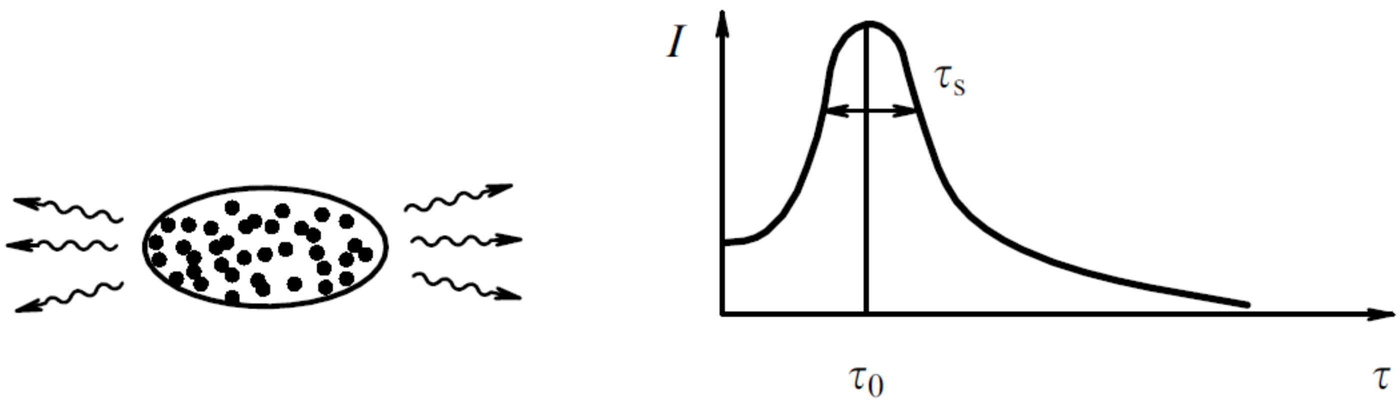
Outline

- **Introduction**
- Cooperative super- and subradiant states
- Far-field scattering
- Chiral quantum optics
- Conclusion

(1) Single atom effect:



(2) Multiple atoms effect:



L. I. Men'shikov, Uspekhi 42 (2) 107 (1999).

Dicke‘s superradiance: Two-level atoms.

Energy state: $|$---++---.......--+-$\rangle$, a spin1/2 system.

Since $\left[S_{x,y,z}, S^2 \right]$=0, choose common eigenstates of S_z and S^2.

$$\Rightarrow S_z |l,m\rangle = m|l,m\rangle \text{ and } S^2|l,m\rangle = l(l+1)|l,m\rangle$$

where $m = \dfrac{N_2 - N_1}{2}$ and $|m| \le l \le \dfrac{N}{2}$.

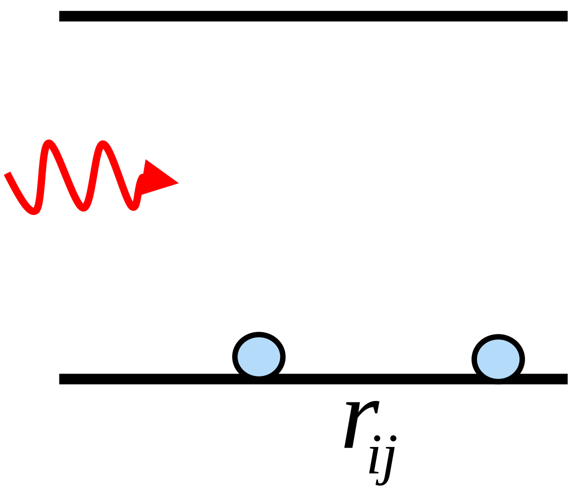
Rate of photon emission $\propto \langle l,m|S^+S^-|l,m\rangle = (l+m)(l-m+1)$.

Maximal when $l = N/2$ and $m = 0$.

R. H. Dicke, “Coherence in spontaneous radiation processes,” Phys. Rev **93**, 99-110 (1954);
L. Mandel and E. Wolf, Optical coherence and quantum optics, Chap. 16.

Hamiltonian: Lindblad form

(1) Rotating-wave:

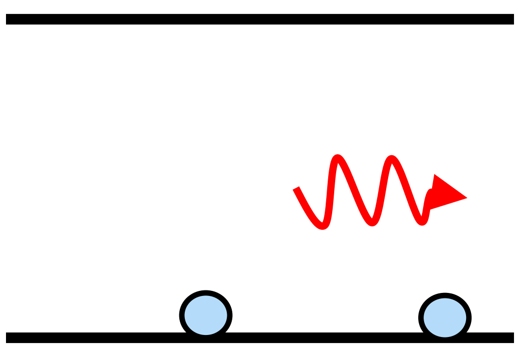


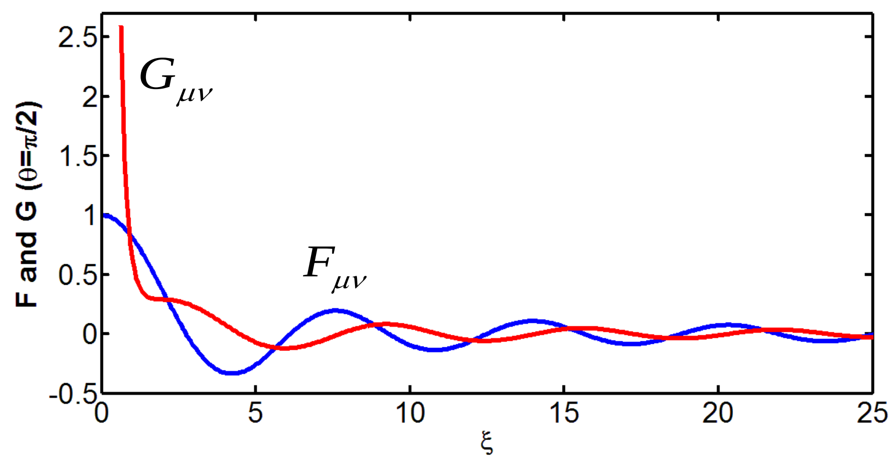
$$\frac{d\hat{Q}}{\Gamma dt} = i\sum_{\mu\neq\nu}^{N}\sum_{\nu=1}^{N} G_{\mu\nu}[\hat{\sigma}_{\mu}^{+}\hat{\sigma}_{\nu}, \hat{Q}]$$

$$+\sum_{\mu=1}^{N}\sum_{\nu=1}^{N} F_{\mu\nu}\left[\hat{\sigma}_{\mu}^{+}\hat{Q}\hat{\sigma}_{\nu} - \frac{1}{2}\left(\hat{\sigma}_{\mu}^{+}\hat{\sigma}_{\nu}\hat{Q} + \hat{Q}\hat{\sigma}_{\mu}^{+}\hat{\sigma}_{\nu}\right)\right].$$

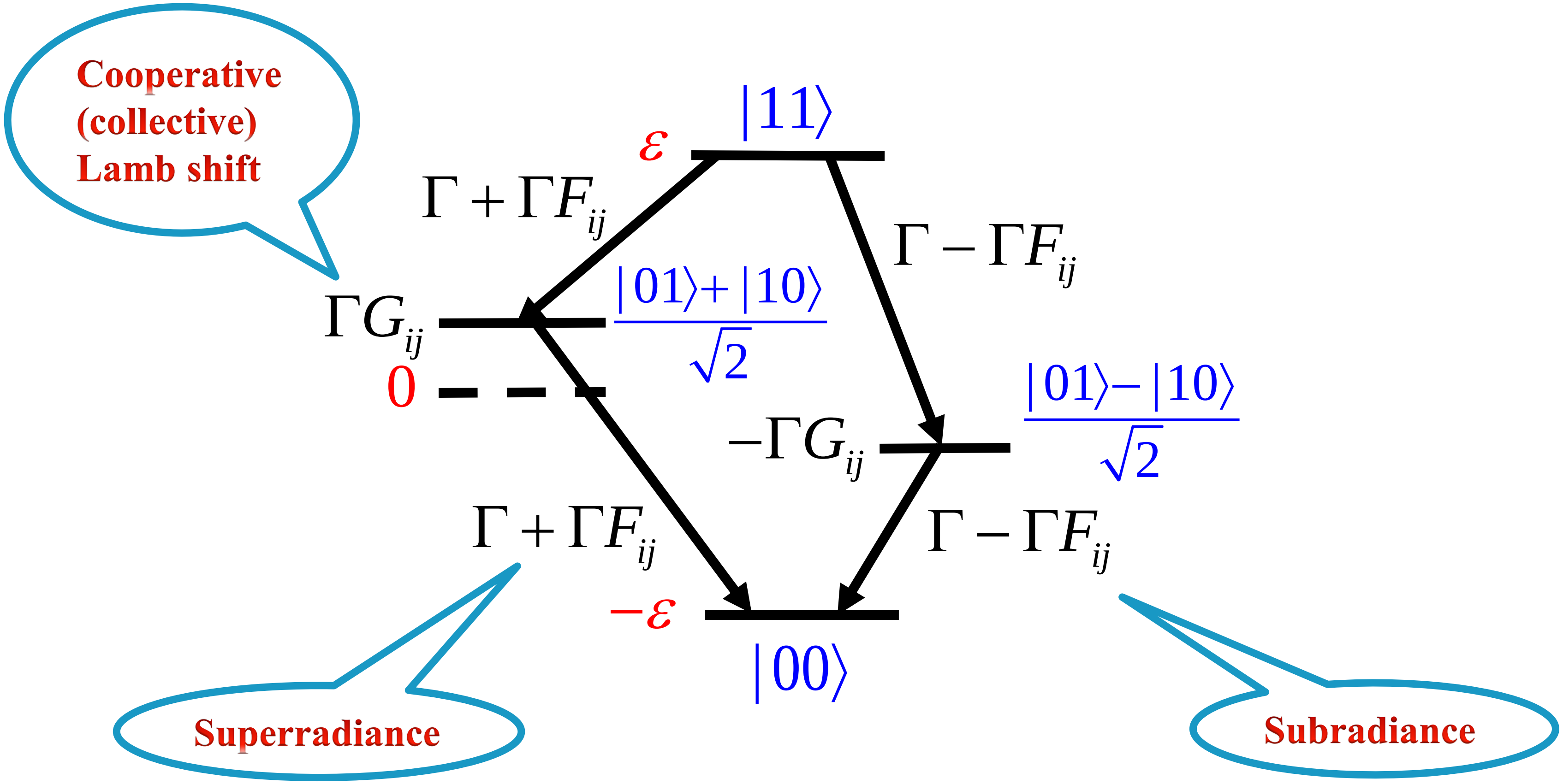
R. H. Lehmberg, Phys. Rev A **2**, 883-888 (1970).

(2) Counter-RWA:





Two-particle case



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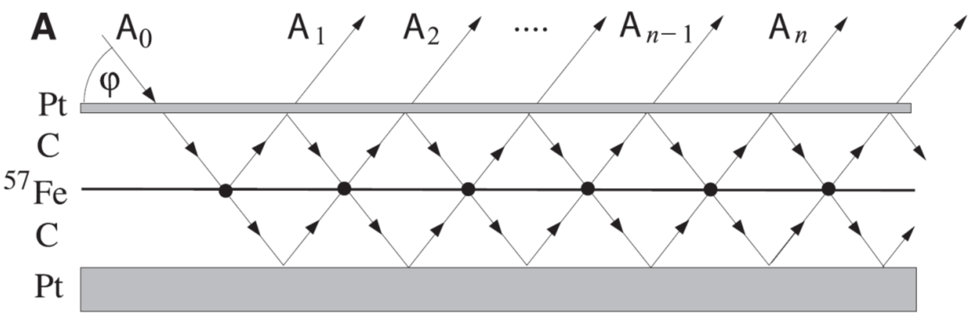
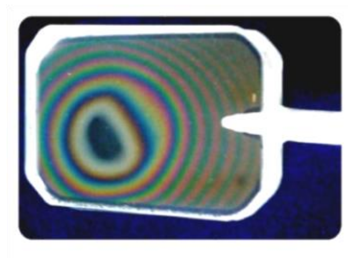
(1) Planar cavity:



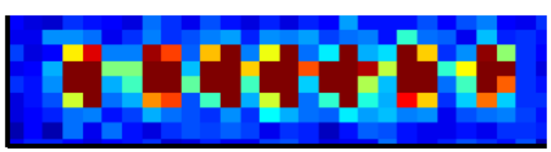
R. Röhlsberger, et. al., Science 328, 1248 (2010).

(2) Vapor cell:

J. Keaveney, et. al.,
PRL 108, 173601 (2012)

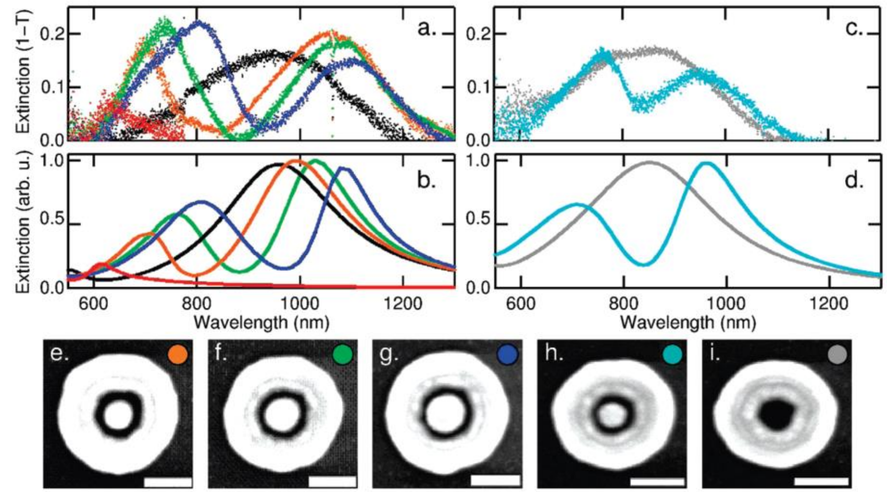


(3) Ion chains:



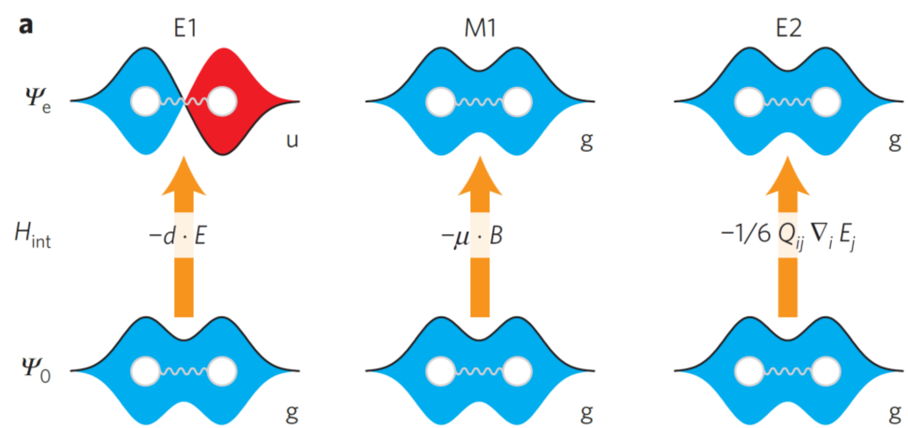
Z. Meir, et. al., PRL 113, 193002 (2014)

(4) Plasmonic modes:



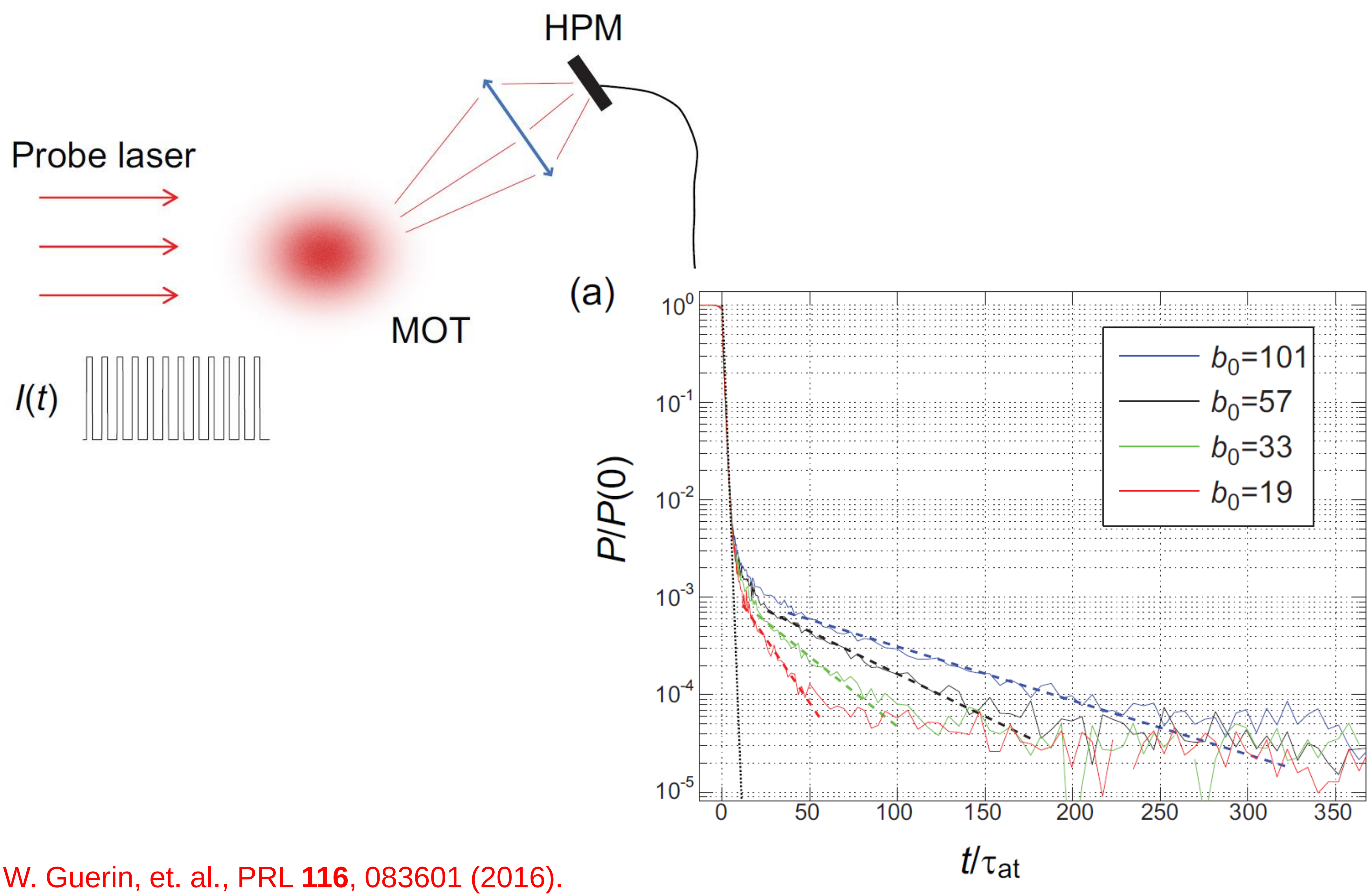
Y. Sonnefraud, et. al., ACS NANO 4, 1664 (2010)

(5) Molecules:



B. H. McGuyer, et. al., Nat. Phys. 11, 32 (2014).

Motivation: Large cloud subradiance



W. Guerin, et. al., PRL **116**, 083601 (2016).

Hilbert space candidates

Symmetric state: $\left|\phi_N\right\rangle = \frac{1}{\sqrt{N}}\sum_{\mu} e^{ik\cdot r_\mu}\left|e\right\rangle_\mu \left|g\right\rangle_{j\neq\mu}^{\otimes(N-1)}, \quad \left|\psi_\mu\right\rangle = \left|e\right\rangle_\mu \left|g\right\rangle_{j\neq\mu}^{\otimes(N-1)}.$

$$(1)\ \left|\phi_m\right\rangle = \sum_{\mu=1}^{N}\left(\frac{1+1/\sqrt{N}}{N-1} - \frac{\sqrt{N}+1}{N-1}\delta_{\mu N} - \delta_{\mu m}\right)e^{ik\cdot r_\mu}\left|\psi_\mu\right\rangle,$$

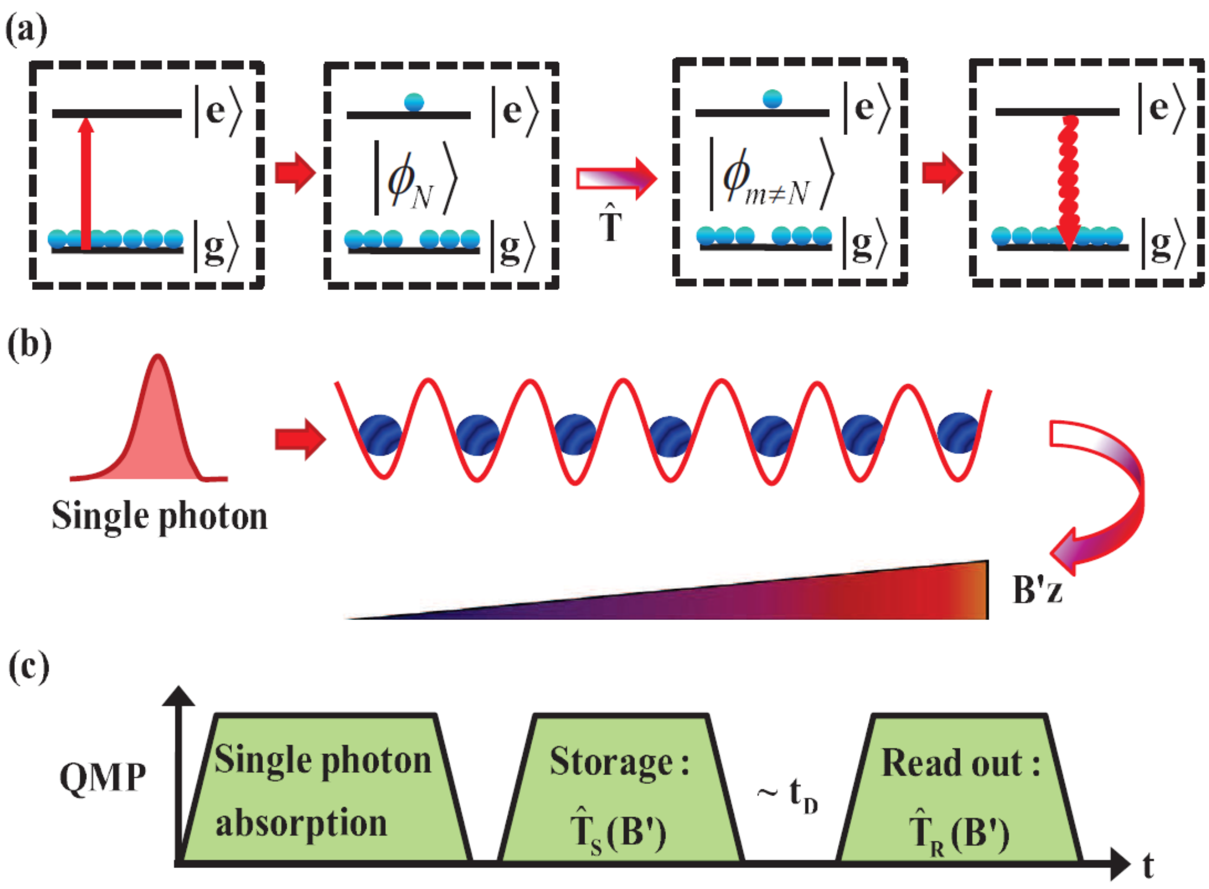
I. E. Mazets and G. Kurizki, JPB 40, F105 (2007).

$$(2)\ \left|\phi_m\right\rangle = \frac{\sum_{\mu=1}^{m} e^{ik\cdot r_\mu}\left|\psi_\mu\right\rangle - m e^{ik\cdot r_{m+1}}\left|\psi_{m+1}\right\rangle}{\sqrt{N(N-1)}},$$

Svidzinski, Chang, and Scully, PRL 100, 160504 (2008); Scully, PRL 115, 243602 (2015).

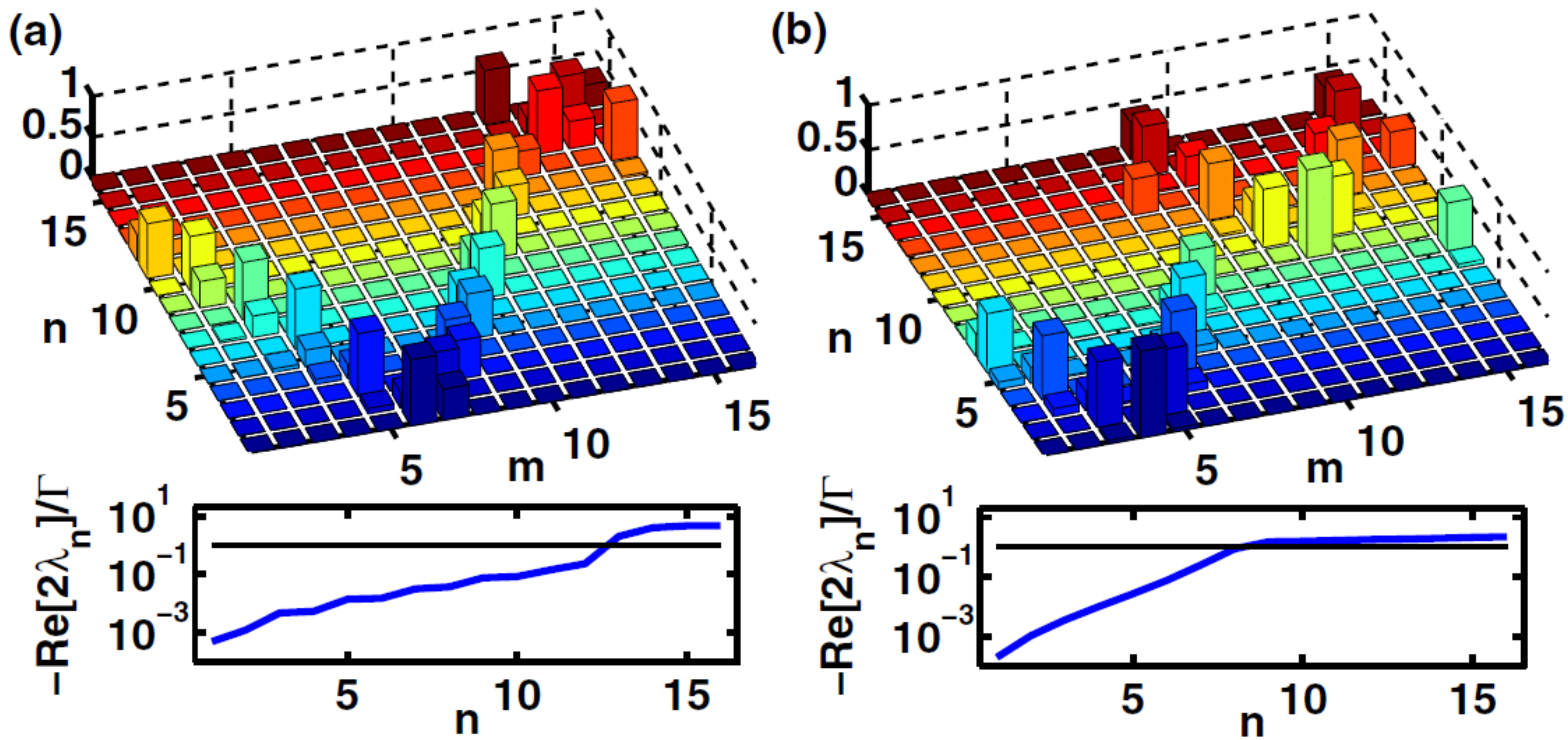
$$(3)\ \left|\phi_m\right\rangle = \sum_{\mu=1}^{N}\frac{e^{ik\cdot r_\mu}e^{i2m\pi(\mu-1)/N}}{\sqrt{N}}\left|\psi_\mu\right\rangle: \ \text{De Moivre states.}$$

Experimental proposal



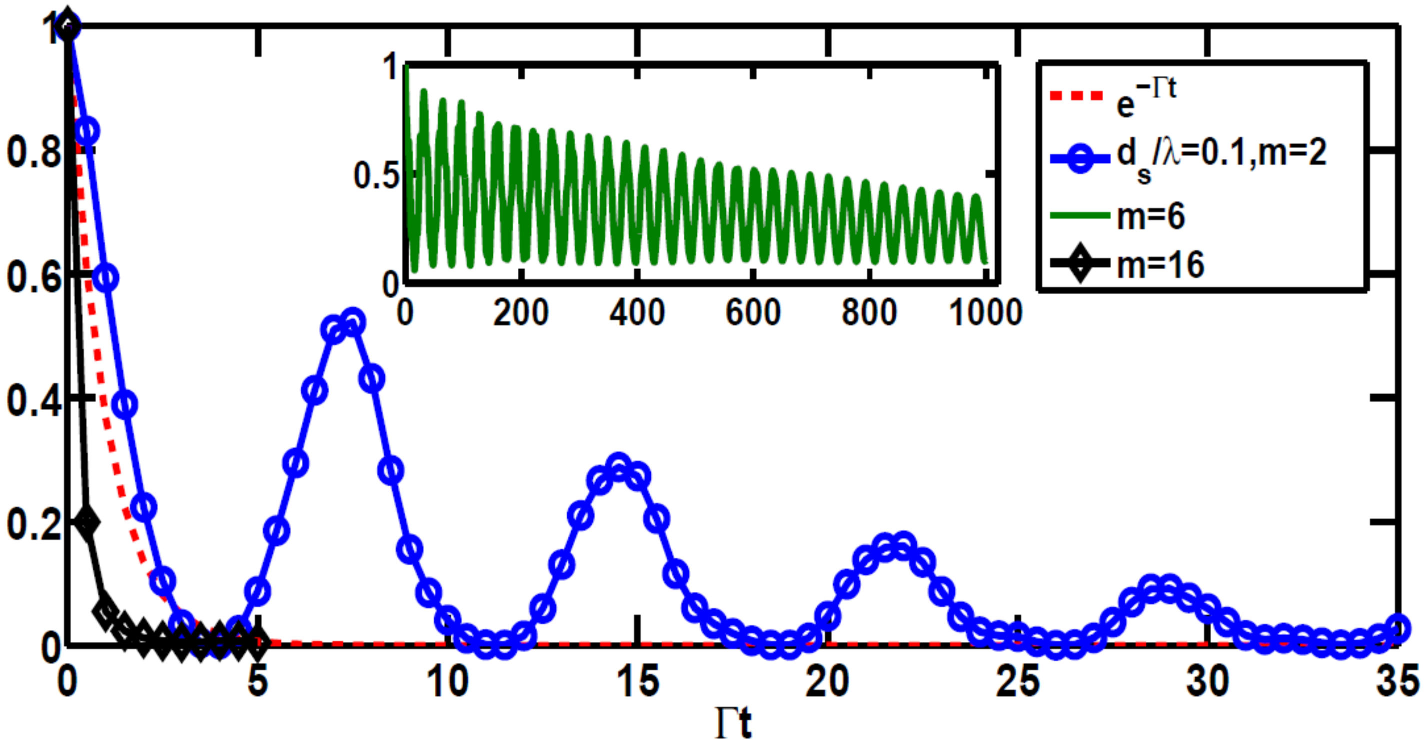
H. H. Jen, M.-S. Chang, and Y.-C. Chen, **PRA** 94, 013803 (2016)

Normalized weightings (projections) of DM states on the eigenstates for N = 16



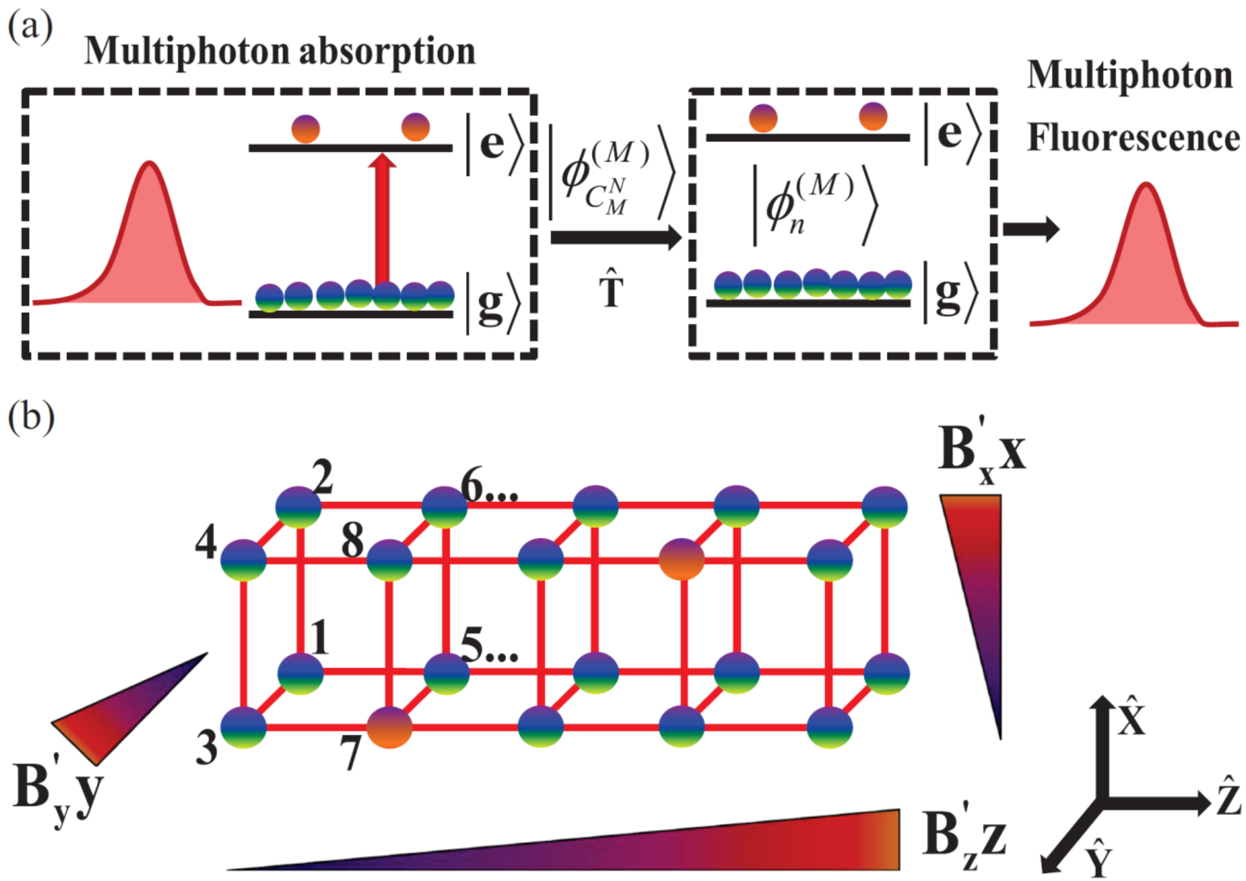
H. H. Jen, M.-S. Chang, and Y.-C. Chen, **PRA** 94, 013803 (2016)

Time evolutions



H. H. Jen, M.-S. Chang, and Y.-C. Chen, **PRA** 94, 013803 (2016)

Multiphoton states



$$|\phi_n^{(M)}\rangle = \prod_{j=1}^{M} \sum_{\mu_j=\mu_{j-1}+1}^{N-M+j} \frac{e^{i\mathbf{k}\cdot\mathbf{R}_M}}{\sqrt{C_M^N}} e^{i\frac{2n\pi}{C_M^N}[f(\vec{\mu})-1]} |\psi_N^{(M)}(\vec{\mu})\rangle$$

H. H. Jen, **PRA** 96, 023814 (2017)

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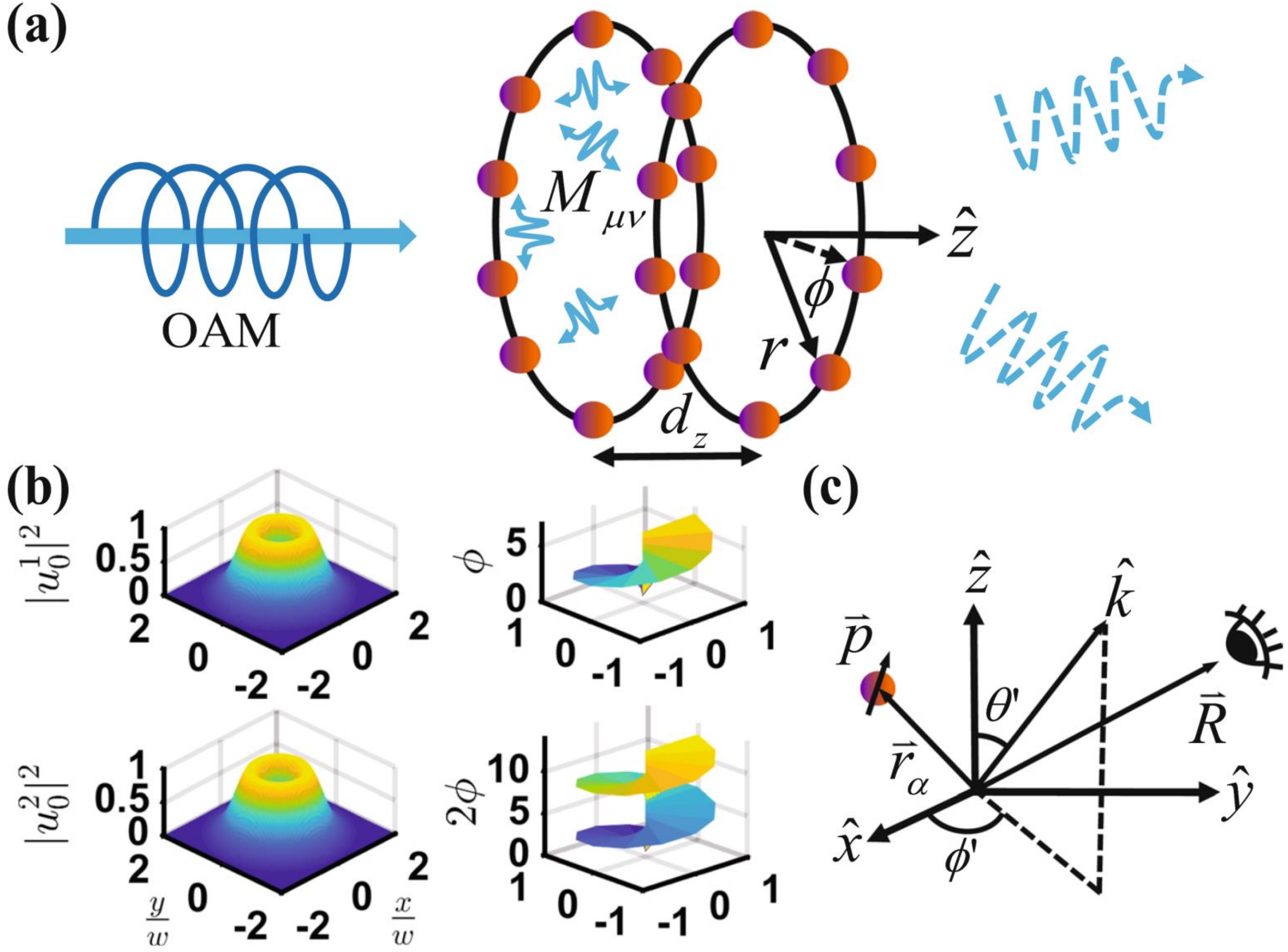
Helical-phase imprinted states

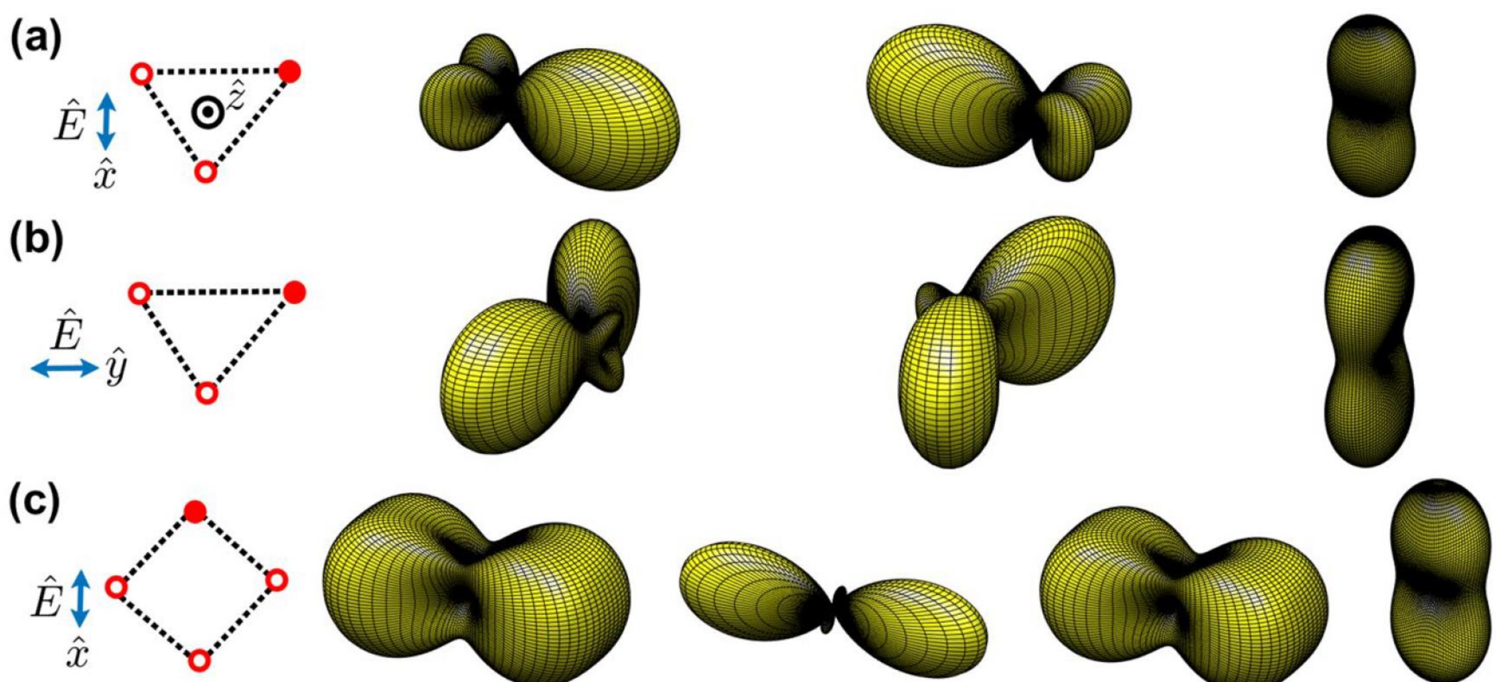


H. H. Jen, M.-S. Chang, and Y.-C. Chen, Scientific Reports 8, 9570 (2018).

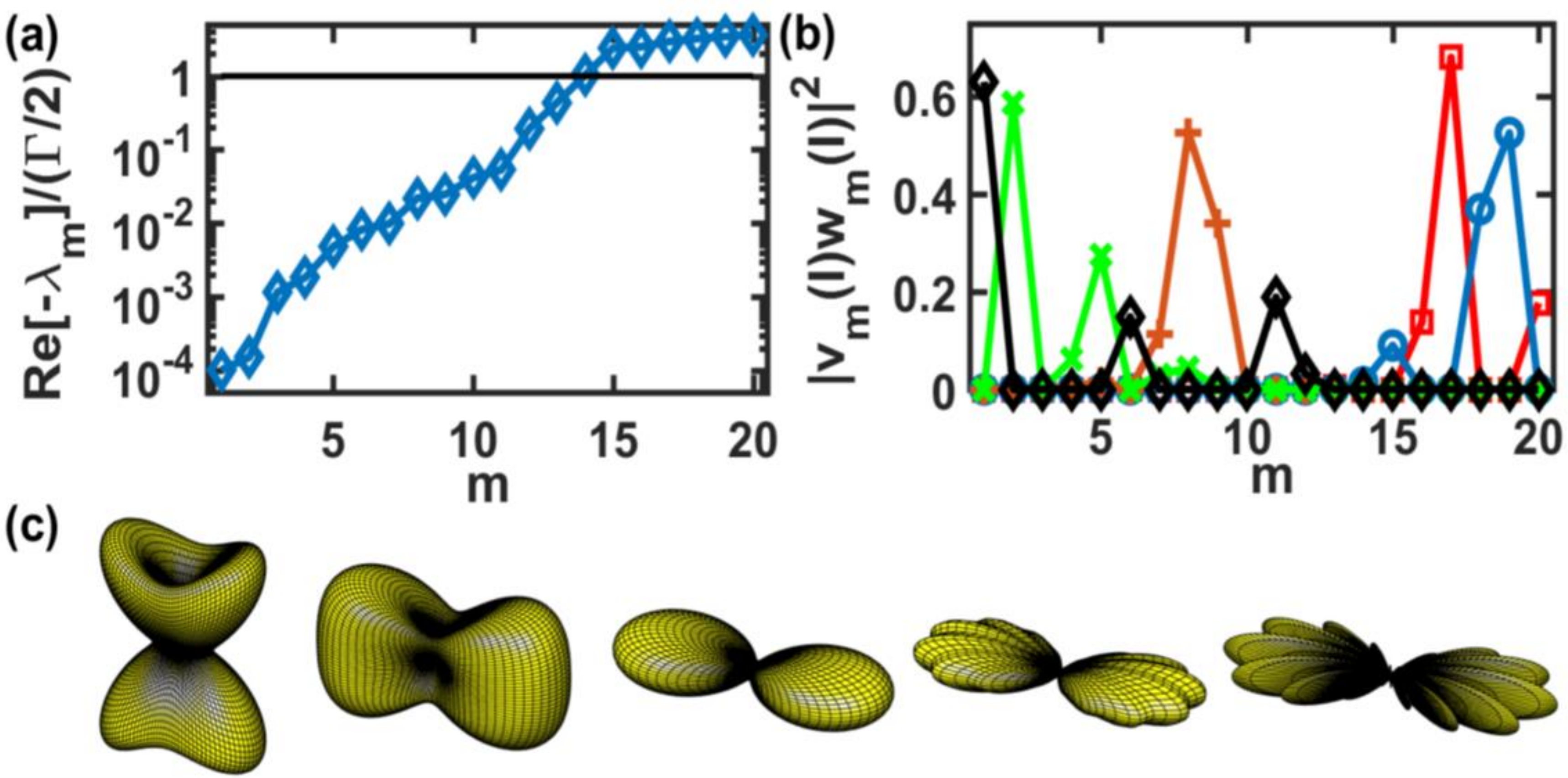
Far-field scattering for three atoms in a ring

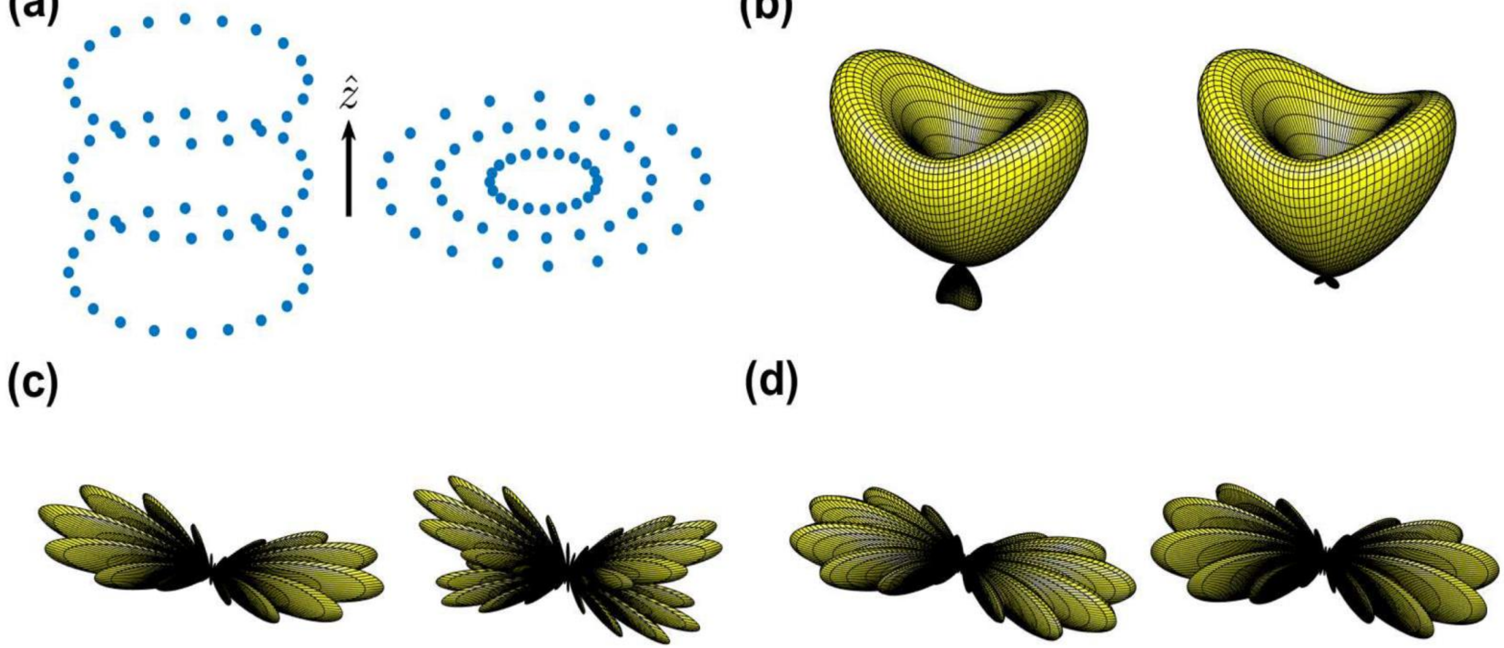
$$|\Phi_l\rangle_{\mathrm{HPI}} = \frac{1}{\sqrt{N}} \sum_{\mu_z=1}^{N_z} \sum_{\mu_\phi=1}^{N_\phi} e^{i\mathbf{k}_L\cdot\mathbf{r}_\mu} e^{i\frac{2l\pi}{N_\phi}(\mu_\phi-1)} |\psi_\mu\rangle$$

$$\frac{\left\langle \vec{E}^*(\mathbf{R},t)\vec{E}(\mathbf{R},t)\right\rangle}{I_0(t)} = \left[1-(\hat{\mathbf{R}}\cdot\hat{\mathbf{p}})^2\right] \sum_{\alpha,\beta=1}^{N} e^{i(\mathbf{k}_R-\mathbf{k}_L)\cdot\mathbf{r}_{\alpha\beta}} \frac{1}{N} e^{\frac{i2l\pi}{N_\phi}(\beta_\phi-\alpha_\phi)}$$

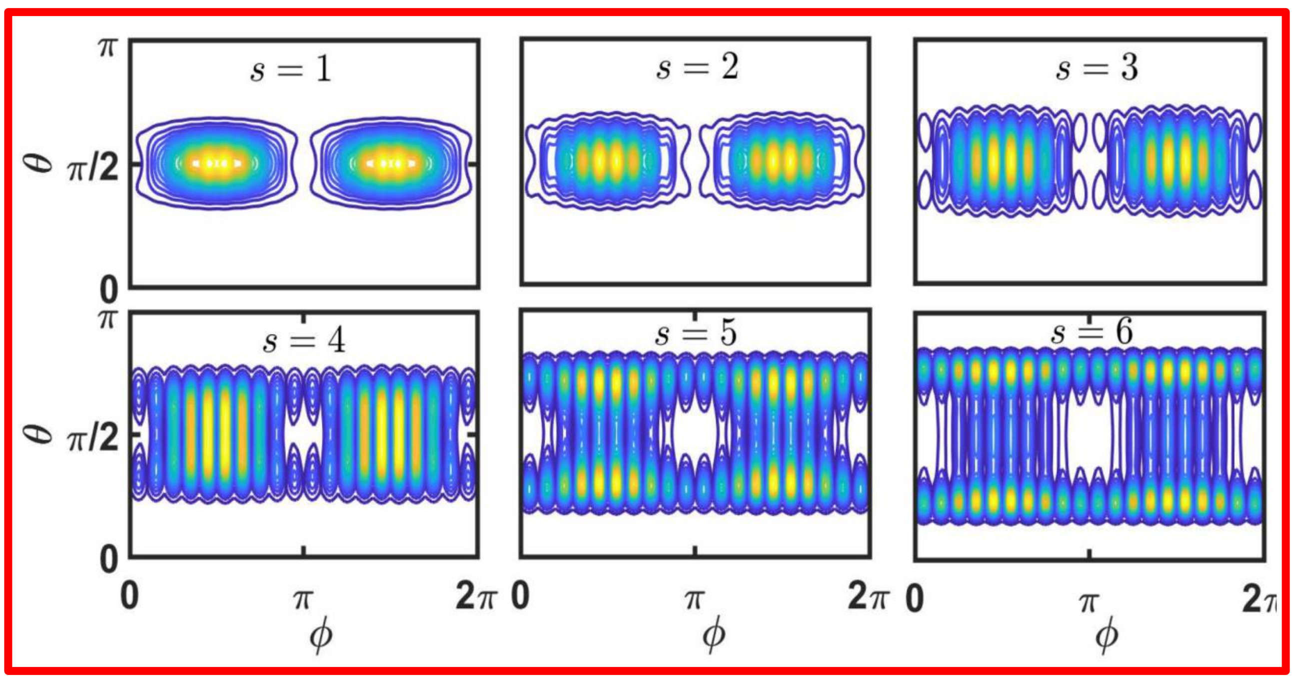


Far-field scattering for a single ring





(a)
(b)
$\hat{z}$
(c)
(d)



$s = 1$
$s = 2$
$s = 3$
$s = 4$
$s = 5$
$s = 6$
π
θ $\pi/2$
0
π
θ $\pi/2$
0
0 π 2π
0 π 2π
0 π 2π
ϕ
ϕ
ϕ

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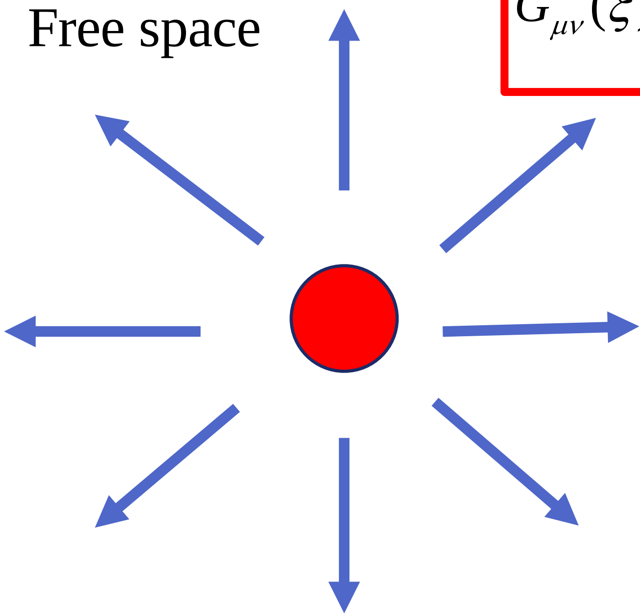
Infinite-range dipole-dipole interaction

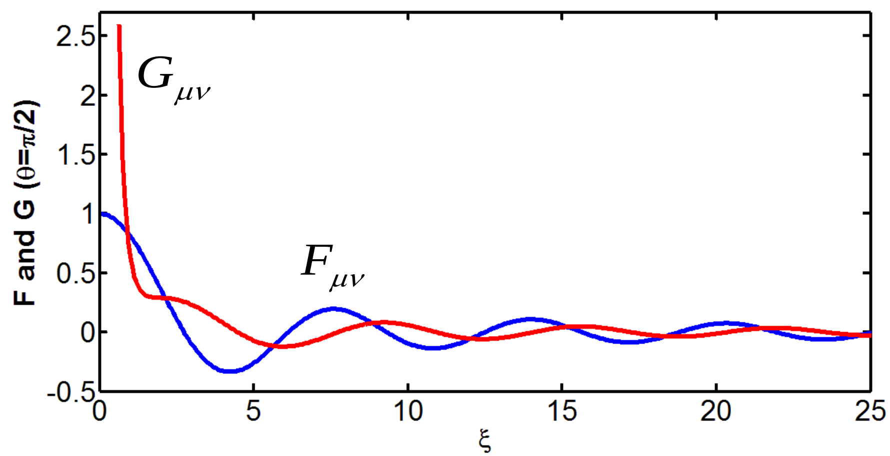
3D:

$$F_{\mu\nu}(\xi) = \frac{3}{2}\left\{\left[1-\left(\hat{d}\cdot\hat{r}_{\mu\nu}\right)^2\right]\frac{\sin\xi}{\xi} + \left[1-3\left(\hat{d}\cdot\hat{r}_{\mu\nu}\right)^2\right]\left(\frac{\cos\xi}{\xi^2} - \frac{\sin\xi}{\xi^3}\right)\right\},$$

$$G_{\mu\nu}(\xi) = \frac{3}{4}\left\{-\left[1-\left(\hat{d}\cdot\hat{r}_{\mu\nu}\right)^2\right]\frac{\cos\xi}{\xi} + \left[1-3\left(\hat{d}\cdot\hat{r}_{\mu\nu}\right)^2\right]\left(\frac{\sin\xi}{\xi^2} + \frac{\cos\xi}{\xi^3}\right)\right\}.$$

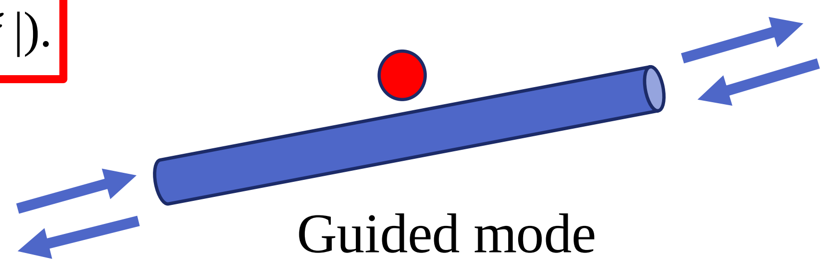
Free space





1D:

$$F_{\mu\nu}(\xi) = \cos(\xi),\ G_{\mu\nu}(\xi) = \sin(|\xi|).$$



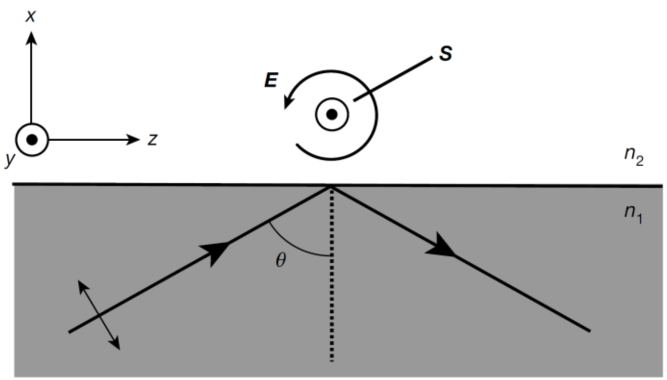
Guided mode

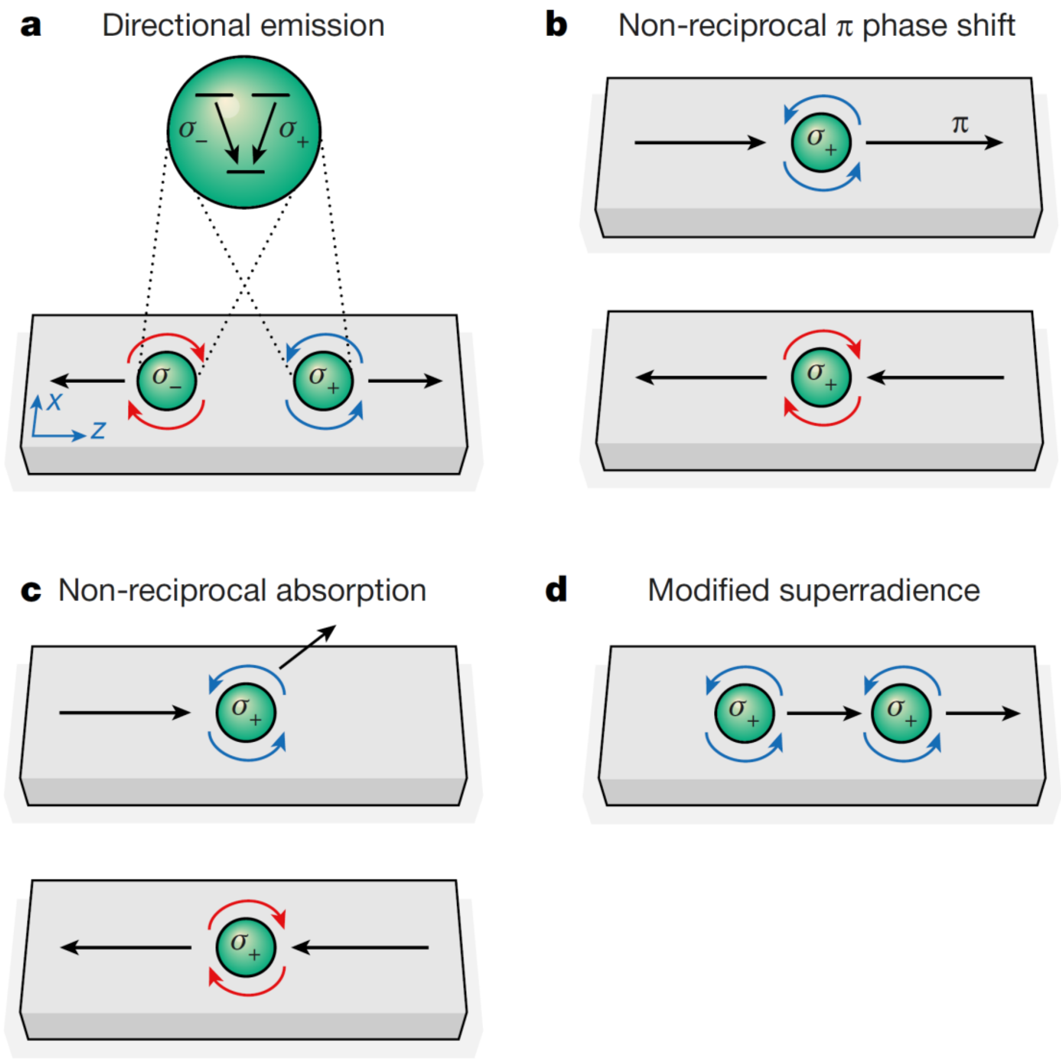
Basic chiral coupling process

Gauss law gives:

$$\mathcal{E}_{\pm,z} = \mp \frac{i}{k}\left(\frac{\partial \mathcal{E}_{\pm,x}}{\partial x} + \frac{\partial \mathcal{E}_{\pm,y}}{\partial y}\right)$$

Spin-momentum locking:





P. Lodahl, et. al., Nature 541, 473 (2017).

Chiral spontaneous emission

$$\dot{\rho} = -\frac{i}{\hbar}\left[H_{sys} + H_L + H_R, \rho\right] + \gamma_L D[c_L]\rho + \gamma_R D[c_R]\rho,$$

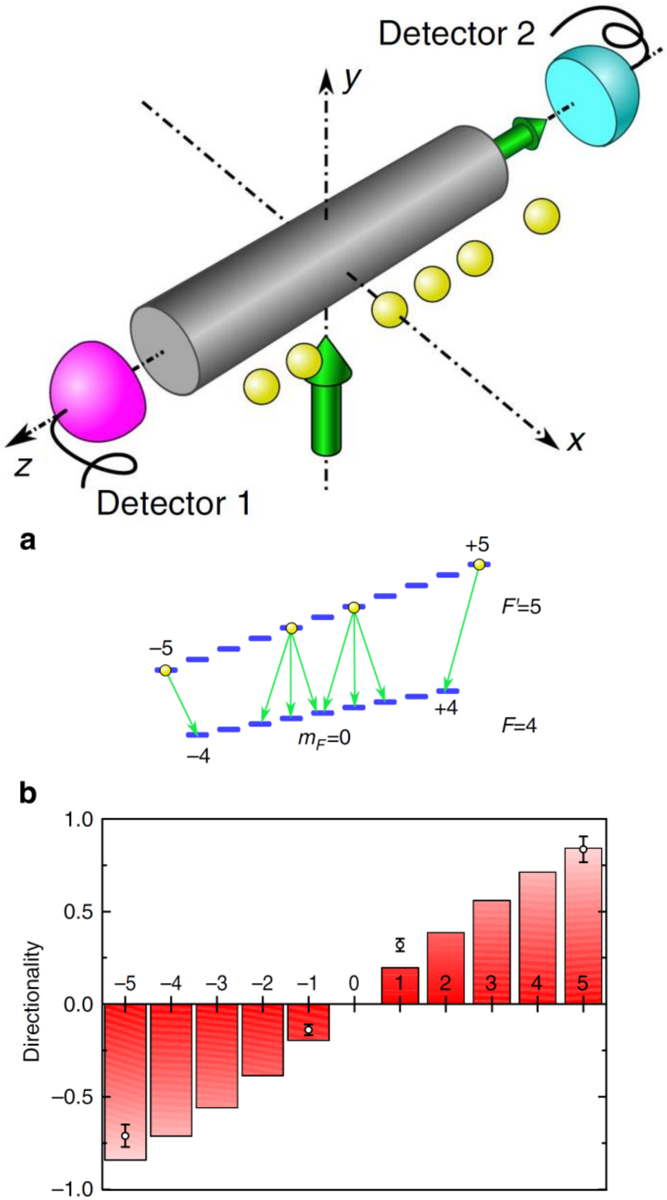
where $D[A]\rho = A\rho A^{+} - \{A^{+}A, \rho\}/2.$

$$H_L \equiv -\frac{i\hbar\gamma_L}{2}\sum_{j<l}(e^{ik|x_j - x_l|}\sigma_j^{\dagger}\sigma_l - \text{H.c.}),$$

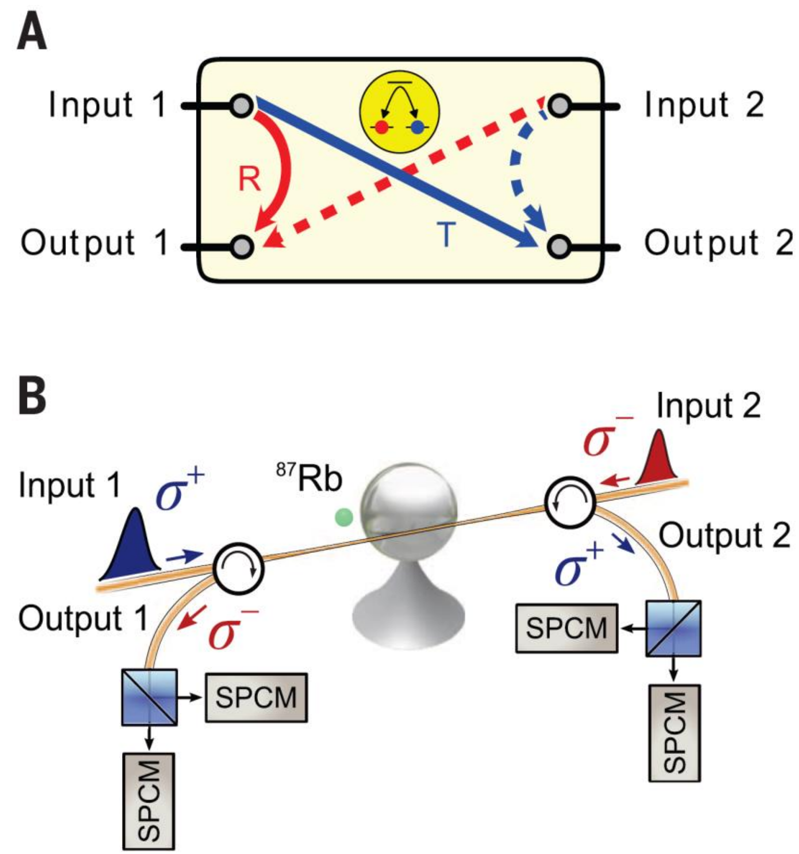
$$H_R \equiv -\frac{i\hbar\gamma_R}{2}\sum_{j>l}(e^{ik|x_j - x_l|}\sigma_j^{\dagger}\sigma_l - \text{H.c.})$$

H. Pichler, et. al., PRA 91, 042116 (2015)

Chiral coupling interfaces (atom-fiber/resonator)

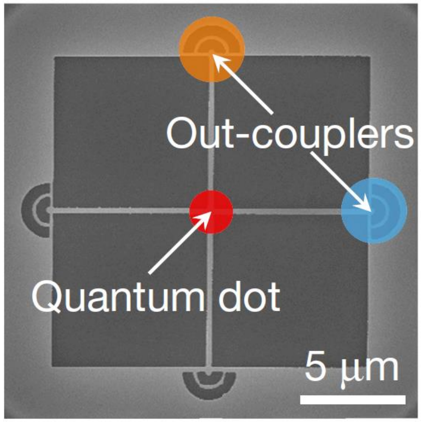


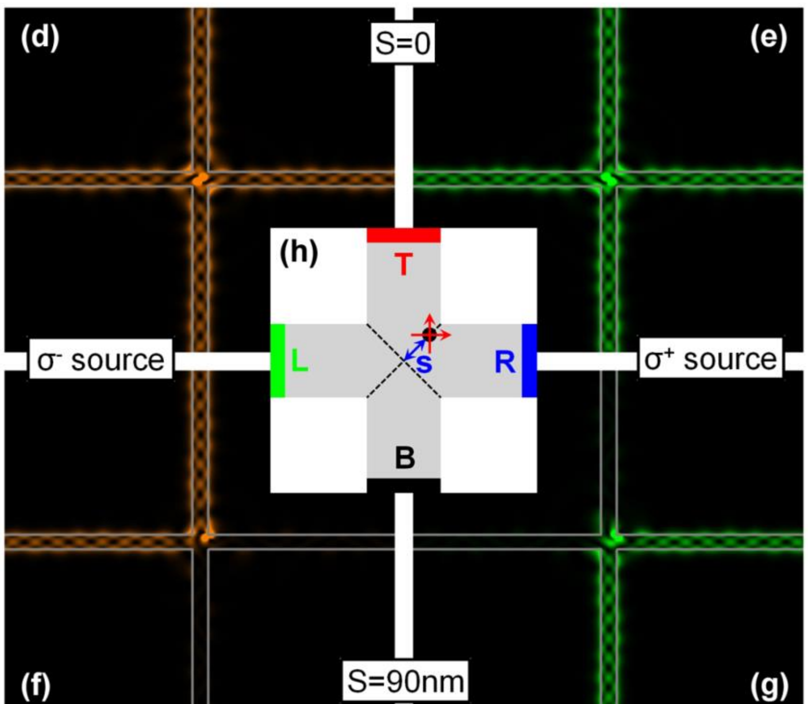
R. Mitsch, et. al., Nat. Comm. 5, 5713 (2014).



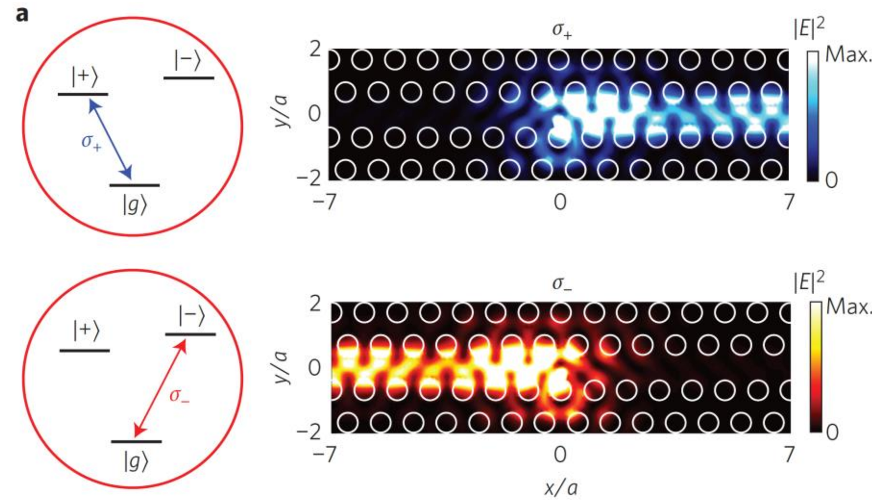
I. Shomroni, et. al., Science 345, 903 (2014).

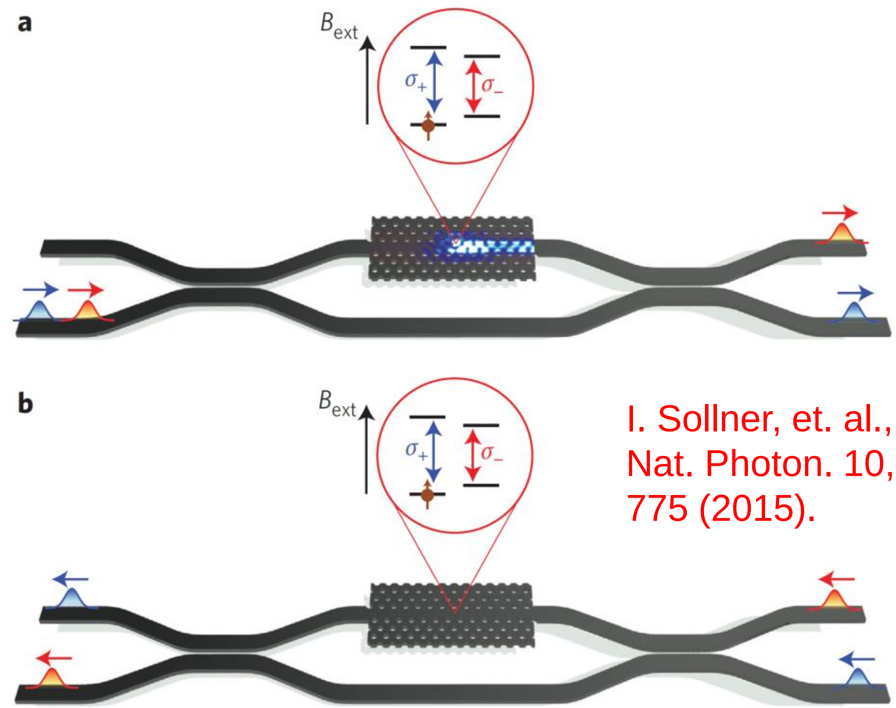
Chiral coupling interfaces (QD-waveguide)





I. J. Luxmoore, et. al., PRL 110, 037402 (2013).





I. Sollner, et. al., Nat. Photon. 10, 775 (2015).

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Conclusions

(1) Cooperative single and multi-photon states can be prepared and manipulated in an array of two-level atoms, and **systematically studied** by varying the spatially increased phases we imprint on the atoms.

(2) The HPI subradiant states show **side scattering** patterns, which have the **narrowing effects** in azimuthal and polar angles when more concentric rings are added in the radial direction.

(3) HPI states can be served as **quantum storage of light with OAM**, with infinite capacity of entanglement.

(4) Chirality of light-matter interaction can be manipulated to realize spin-photon interfaces and simulate novel quantum many-body states.

Thank you for your attentions.