Numerical Study on Controllability of Quantum Transition

蕭逸匀 Hsiao I-Yun

Institute of Physics, NCTU

Outline

- Introduction
- Theory
  - Optimal Control Theory
  - Controllability
- Transition Types
  - 2 level
  - 3 level
  - Ladder Type
  - Double Well
- Results
  - State to State
  - State to Superposition
  - Double Well
- Summary

Quantum Control

$\epsilon(t)$ external field

$$\psi(t_0) \longrightarrow \phi(T)$$

initial state

target state

t_0 : initial time

T : final time

Controllability: Is the control difficult or easy?

Optimal Control Theory : Formula

$$i\hbar\frac{\partial}{\partial t}|\psi(\vec{r},t)\rangle = \hat{H}|\psi(\vec{r},t)\rangle$$

$$\boxed{\hat{H} = \hat{H}_0 - \hat{\mu}\epsilon(t)}$$

$\hat{H}_0$: time independent Hamiltonian

$\hat{\mu}$: dipole operator

$\epsilon(t)$: electric field of a laser pulse

objective functional:

$$J[\psi,\epsilon] = \langle\psi(T)|\hat{O}|\psi(T)\rangle - \alpha\int_0^T \epsilon(t)^2 dt$$

$\hat{O}$: $|\phi\rangle\langle\phi|$ target state

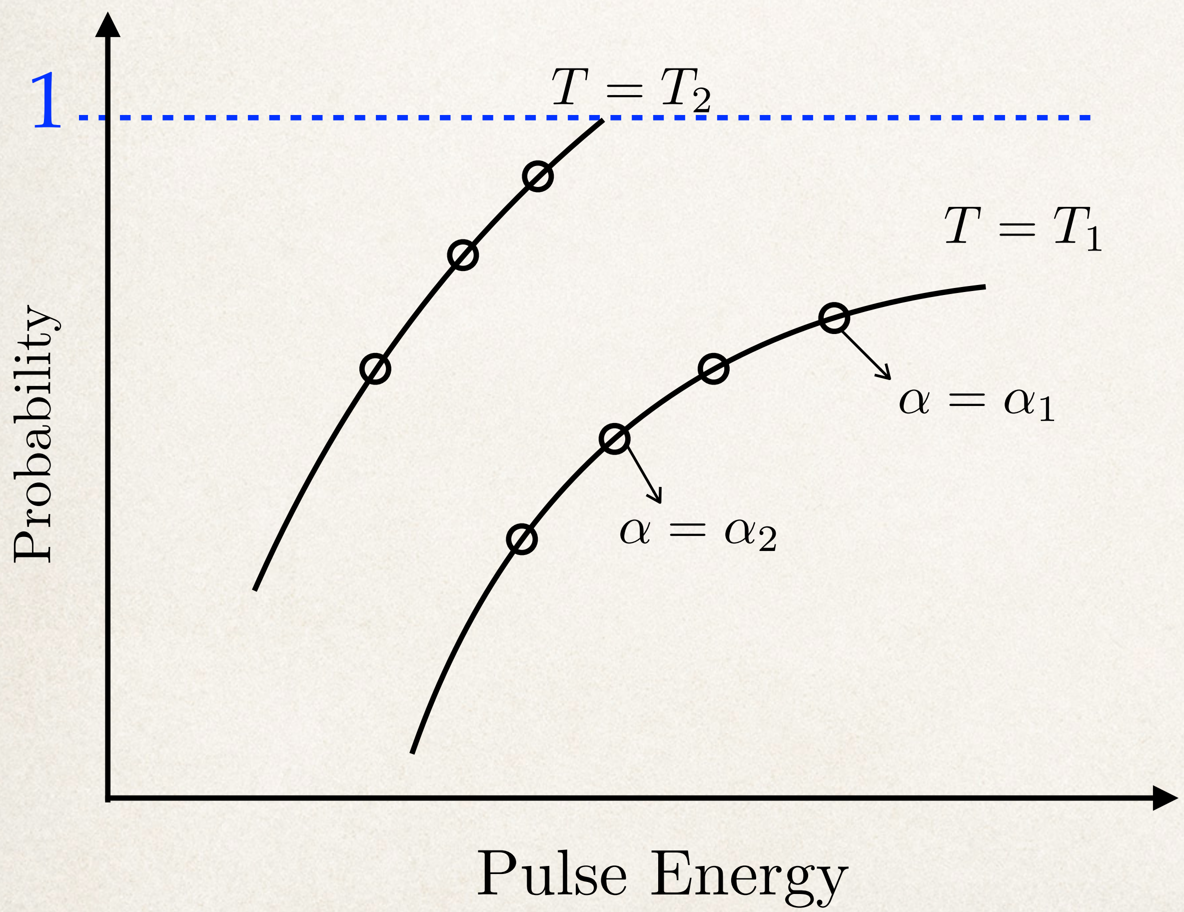
α : penalty factor (positive constant)

Controllability

$$J[\psi, \epsilon] = \langle \psi(T)|\hat{O}|\psi(T)\rangle - \alpha \int_0^T \epsilon(t)^2 dt$$

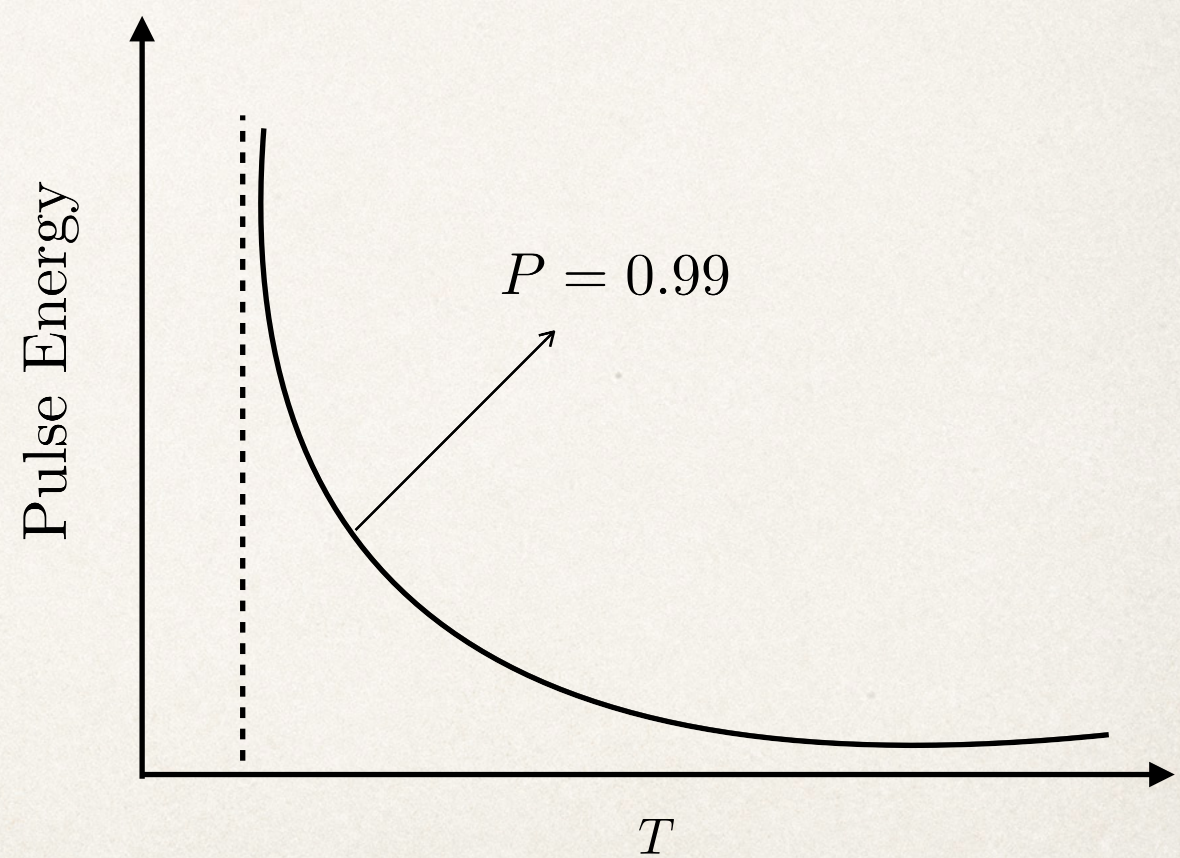
$$\text{Pulse Energy} = \int_0^T |\epsilon(t)|^2 dt$$

P-E

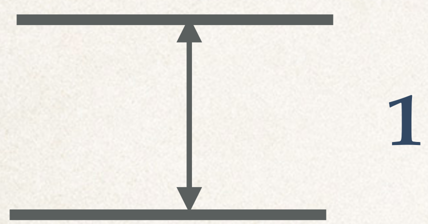


$T_2 > T_1$ $\alpha_2 > \alpha_1$

E-T

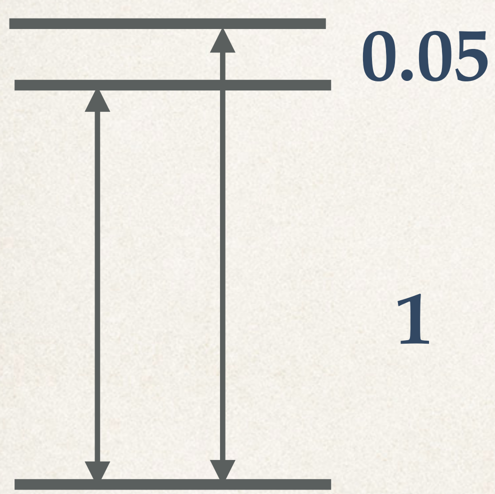


Level Diagram of Systems



Type 1

two level

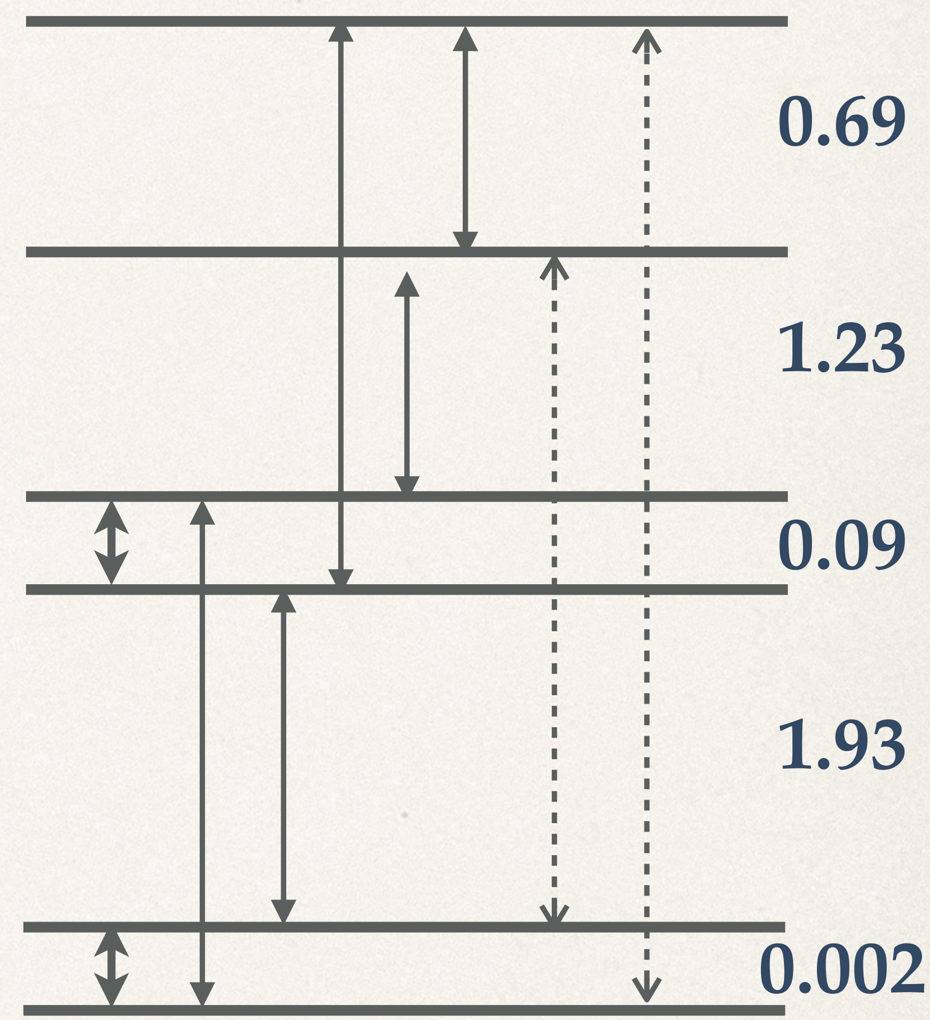


Type 2

three level

Type 3

ladder type



Type 4

double well

Transition Types

Case 1 : State to State

- 2 level
- Ladder Type
- 3 level

Case 2 : State to Superposition

- 3 level

Case 3 : Tunneling

- Double Well

Transition Types

Case 1 : State to State

2 level

Ladder Type

3 level

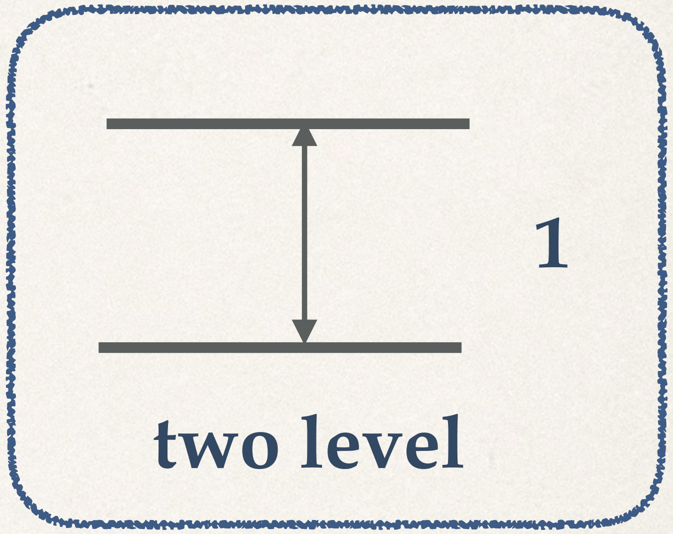
Case 2 : State to Superposition

3 level

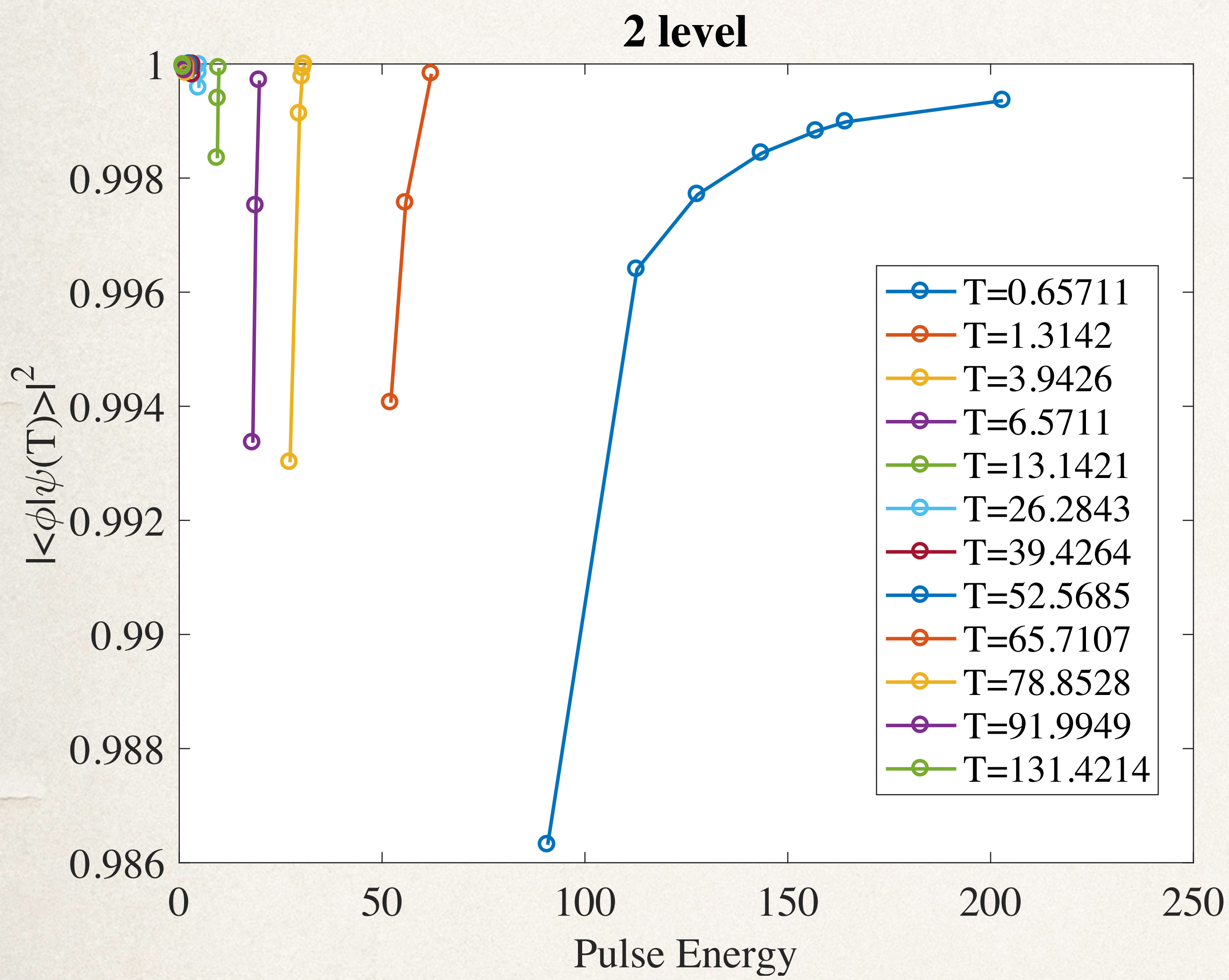
Case 3 : Tunneling

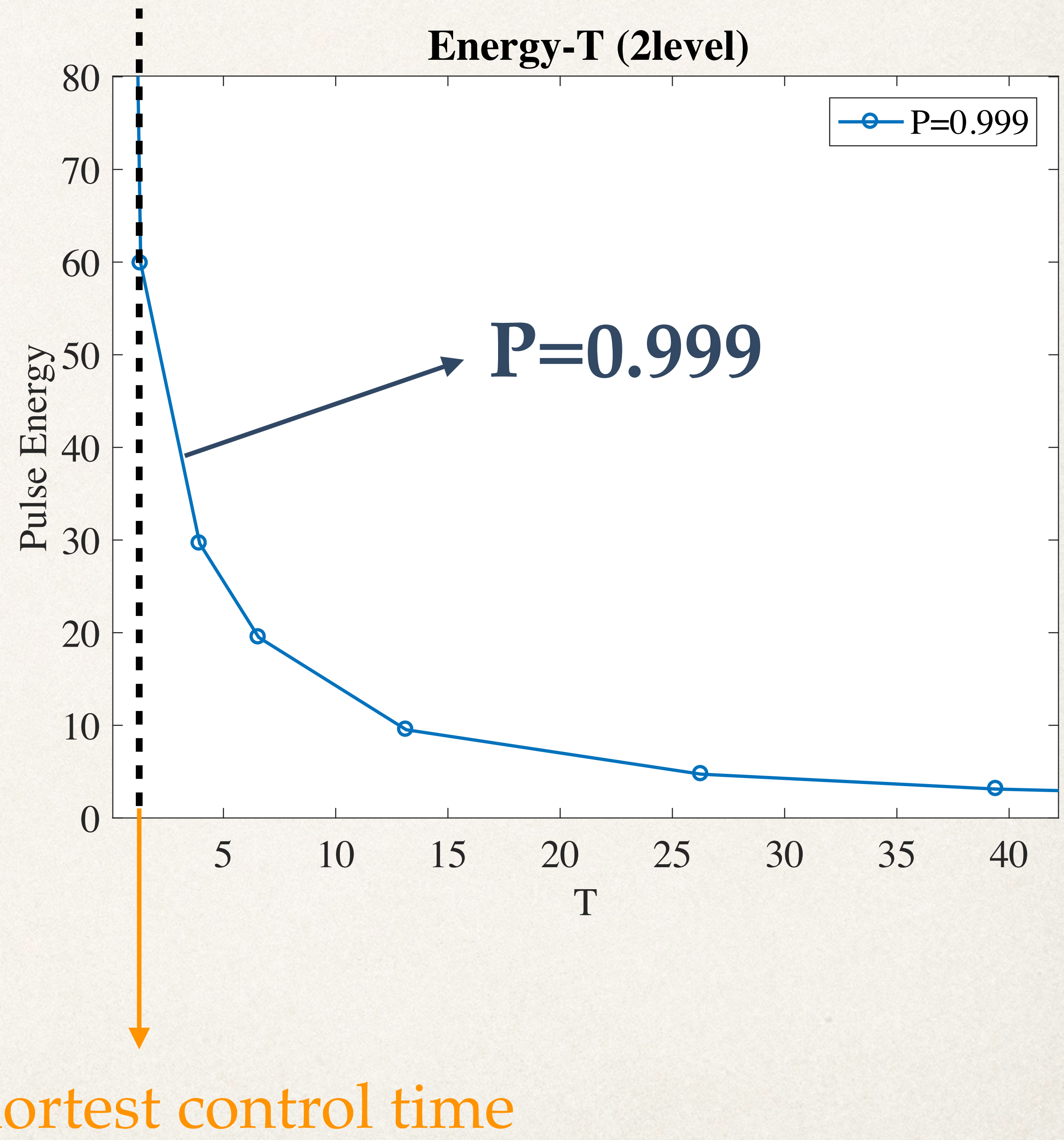
Double Well

Level Diagram

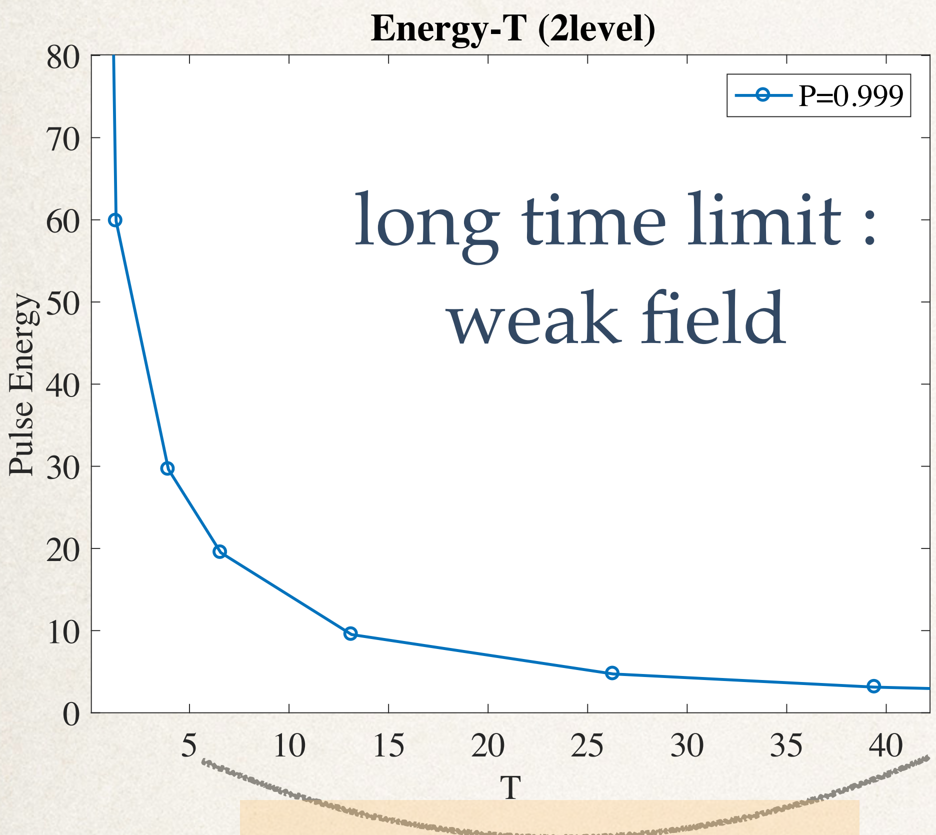


State to State : 2 level

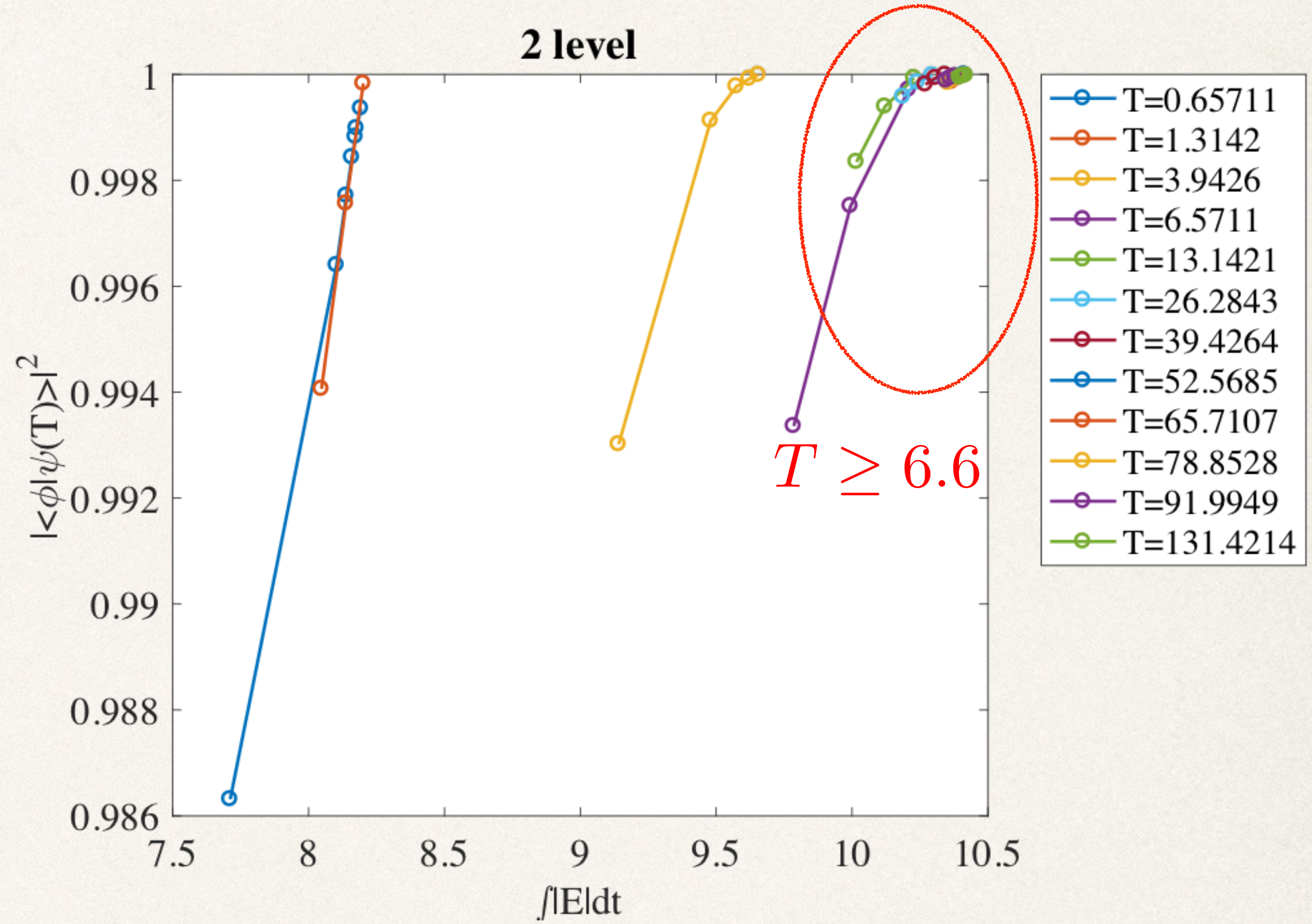




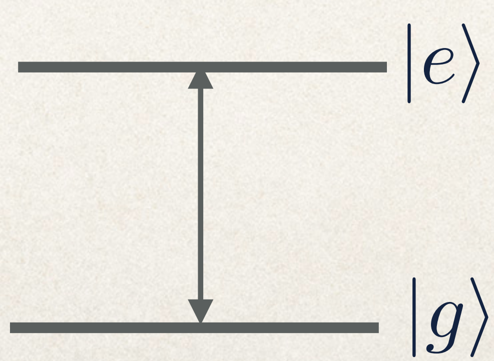
State to State : 2 level



resonance transition



long time ⇒ **resonance transition**



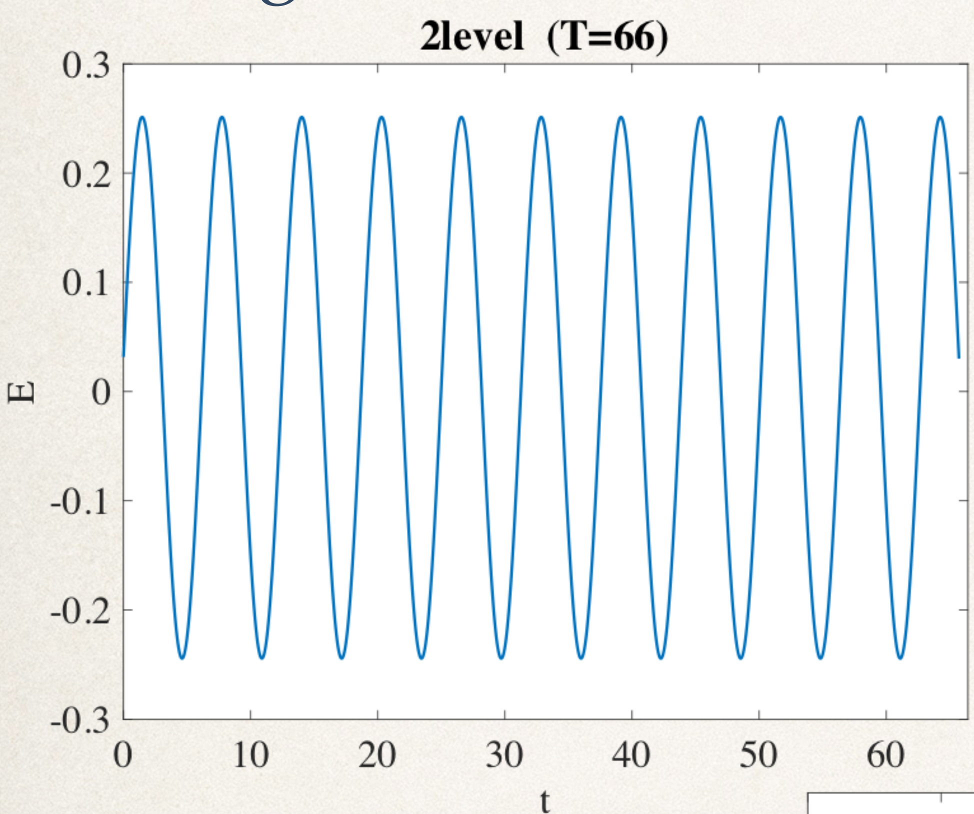
$$P_e = \sin^2\left(\frac{\Omega_{\mathrm{R}}}{2}\right)$$

(Rabi's formula)

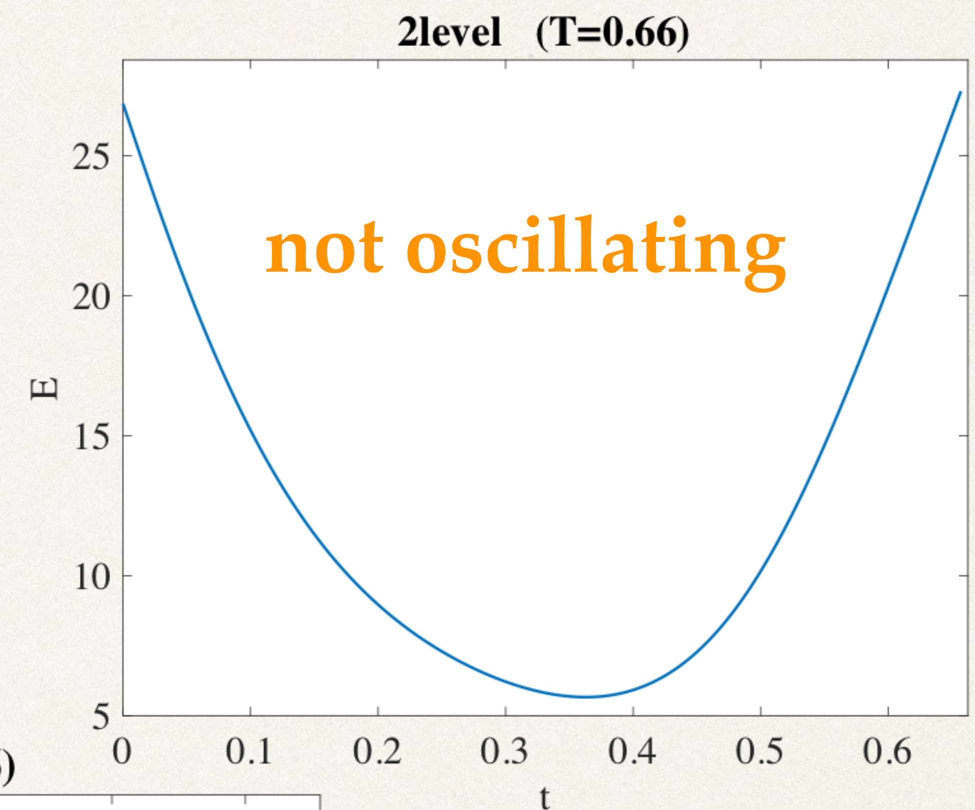
$$\Omega_R = \mu \int_0^T \epsilon(t)dt$$

State to State : 2 level

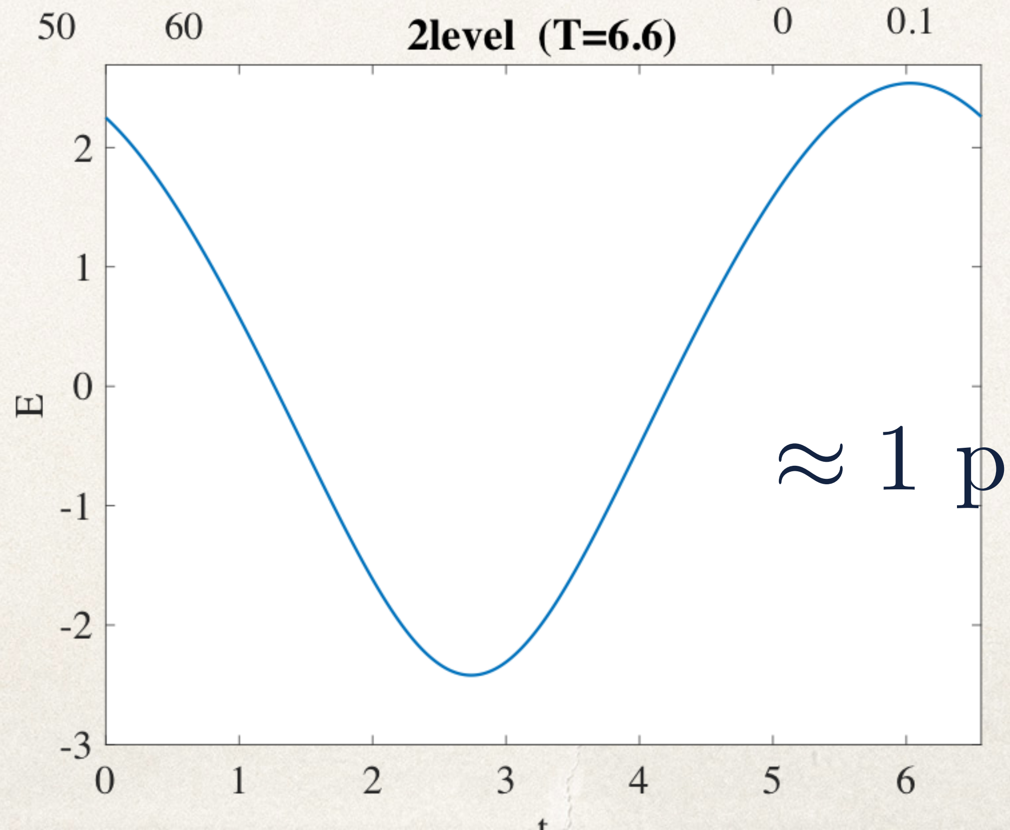
long time limit



short time limit



boundary



≈ 1 period

Transition Types

Case 1 : State to State

2 level

Ladder Type

3 level

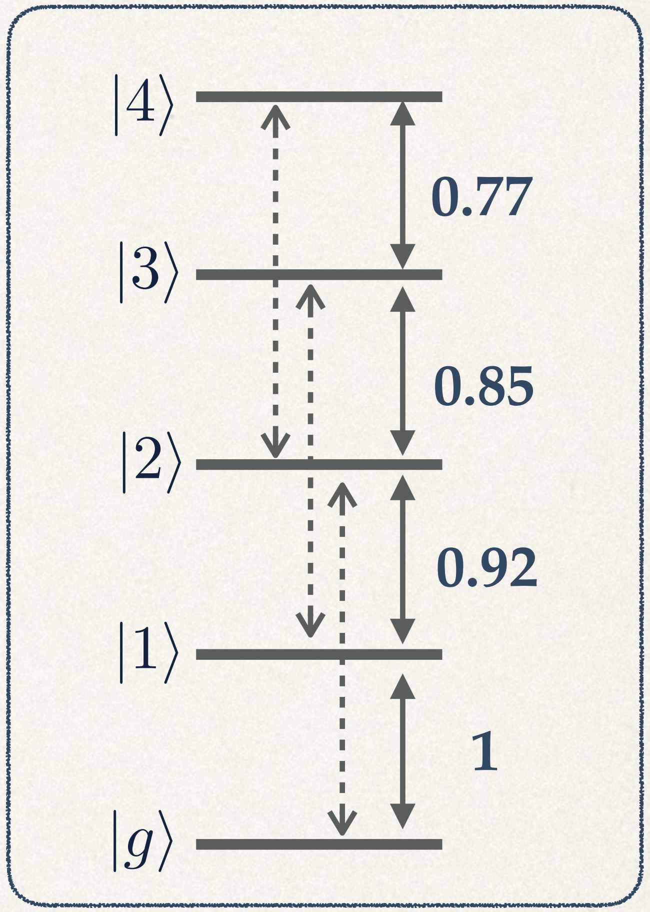
Case 2 : State to Superposition

3 level

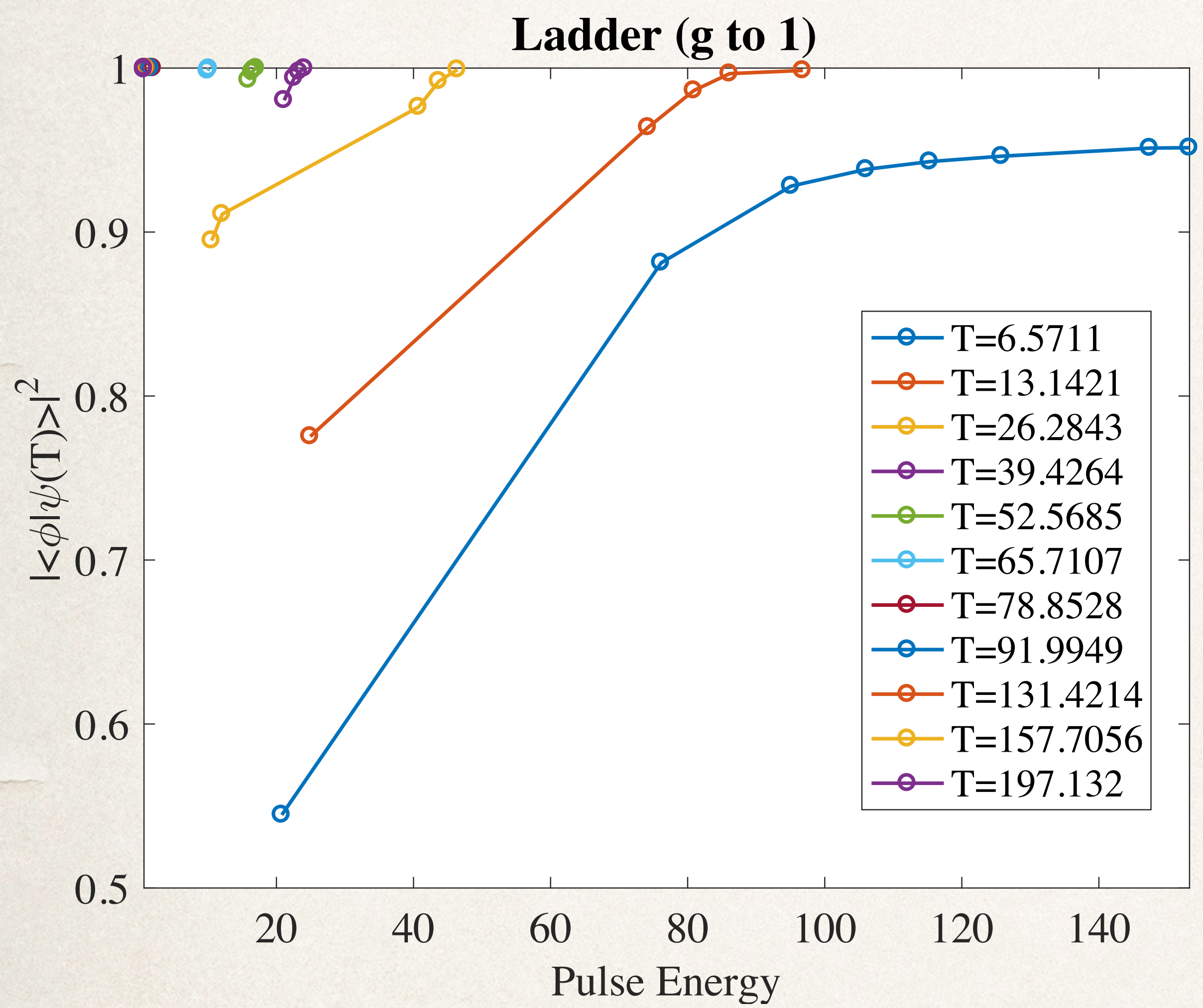
Case 3 : Tunneling

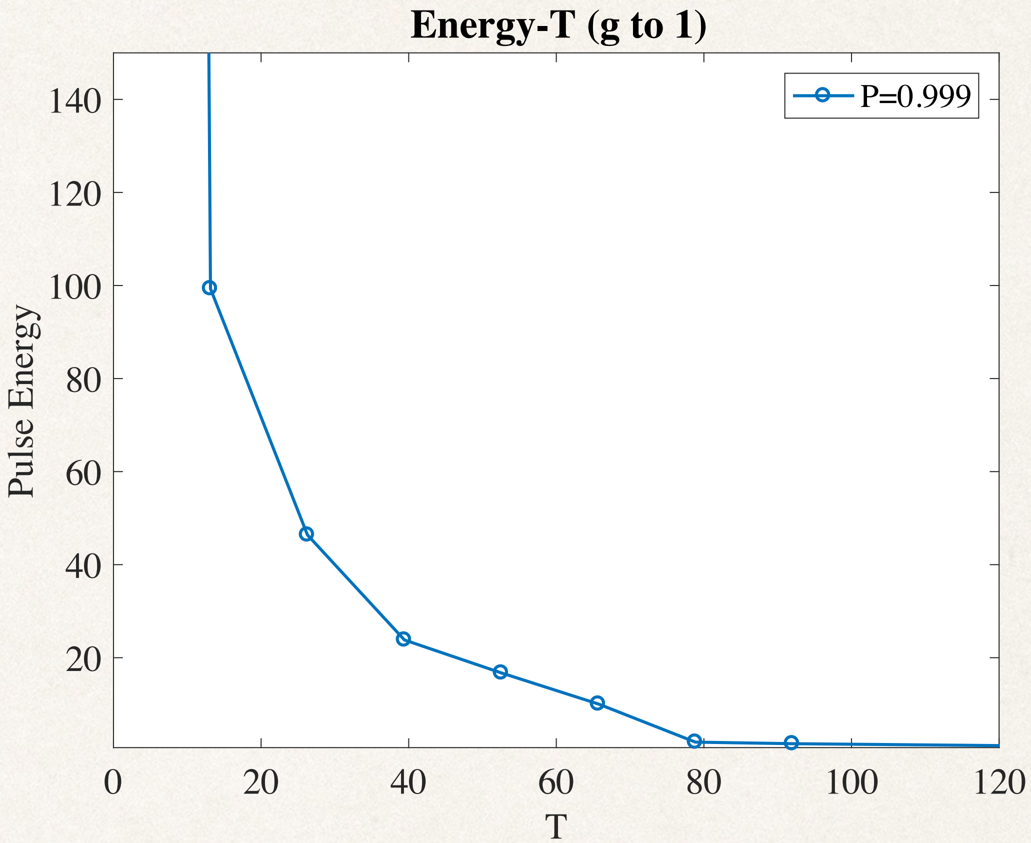
Double Well

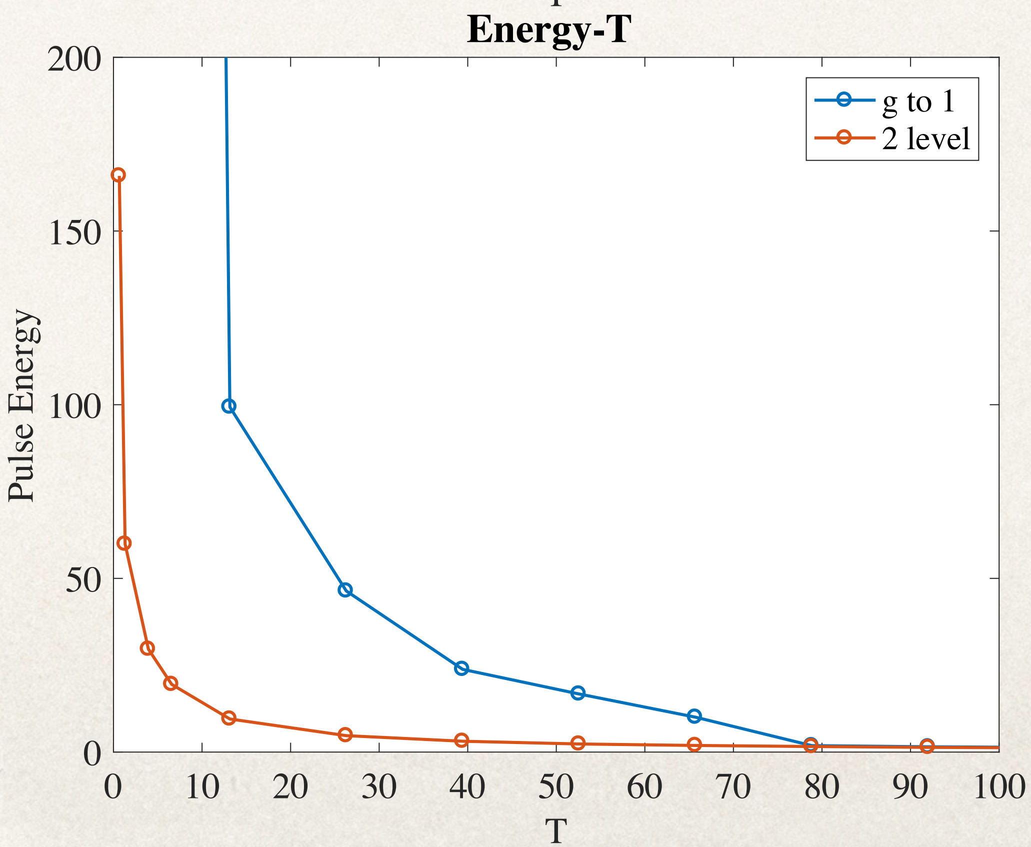
Level Diagram



State to State : Ladder Type $|g\rangle \rightarrow |1\rangle$

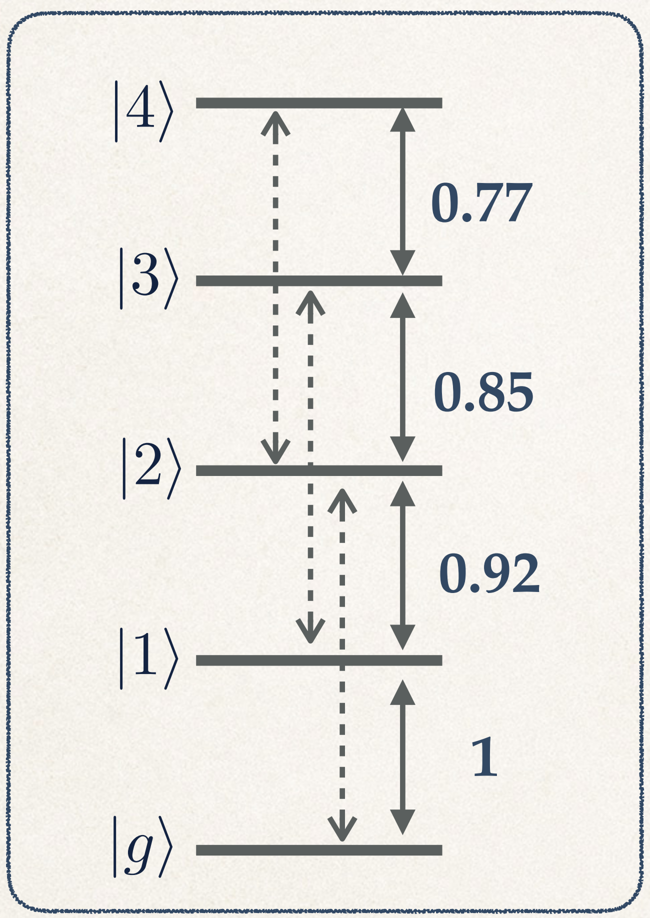






State to State : Ladder Type $|g\rangle \rightarrow |1\rangle$

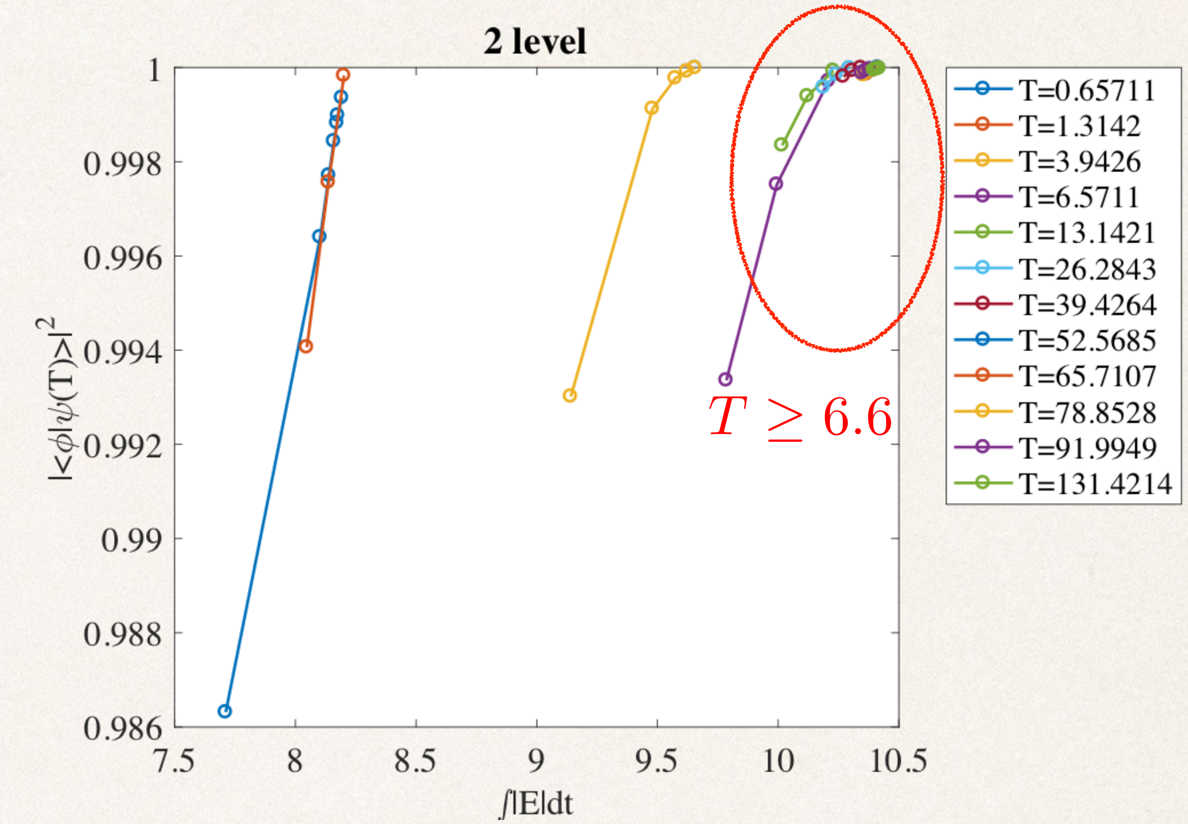
Level Diagram

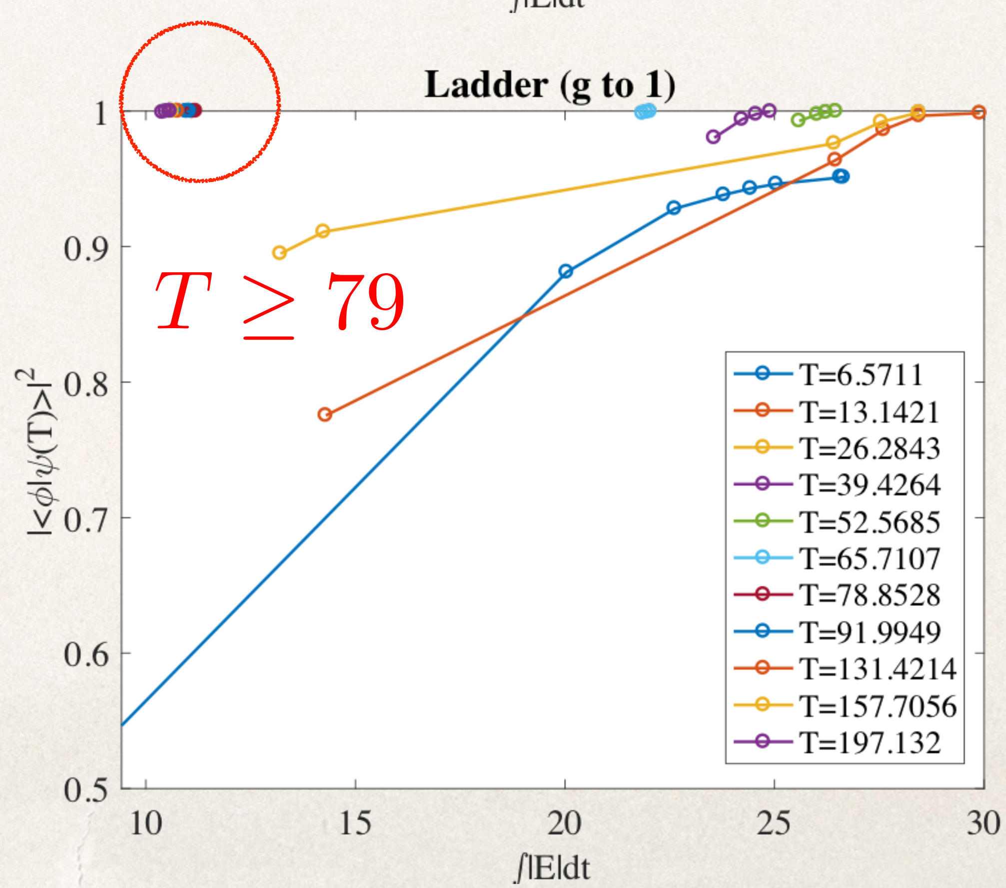


$$\Delta \equiv E_{g1} - E_{12}$$

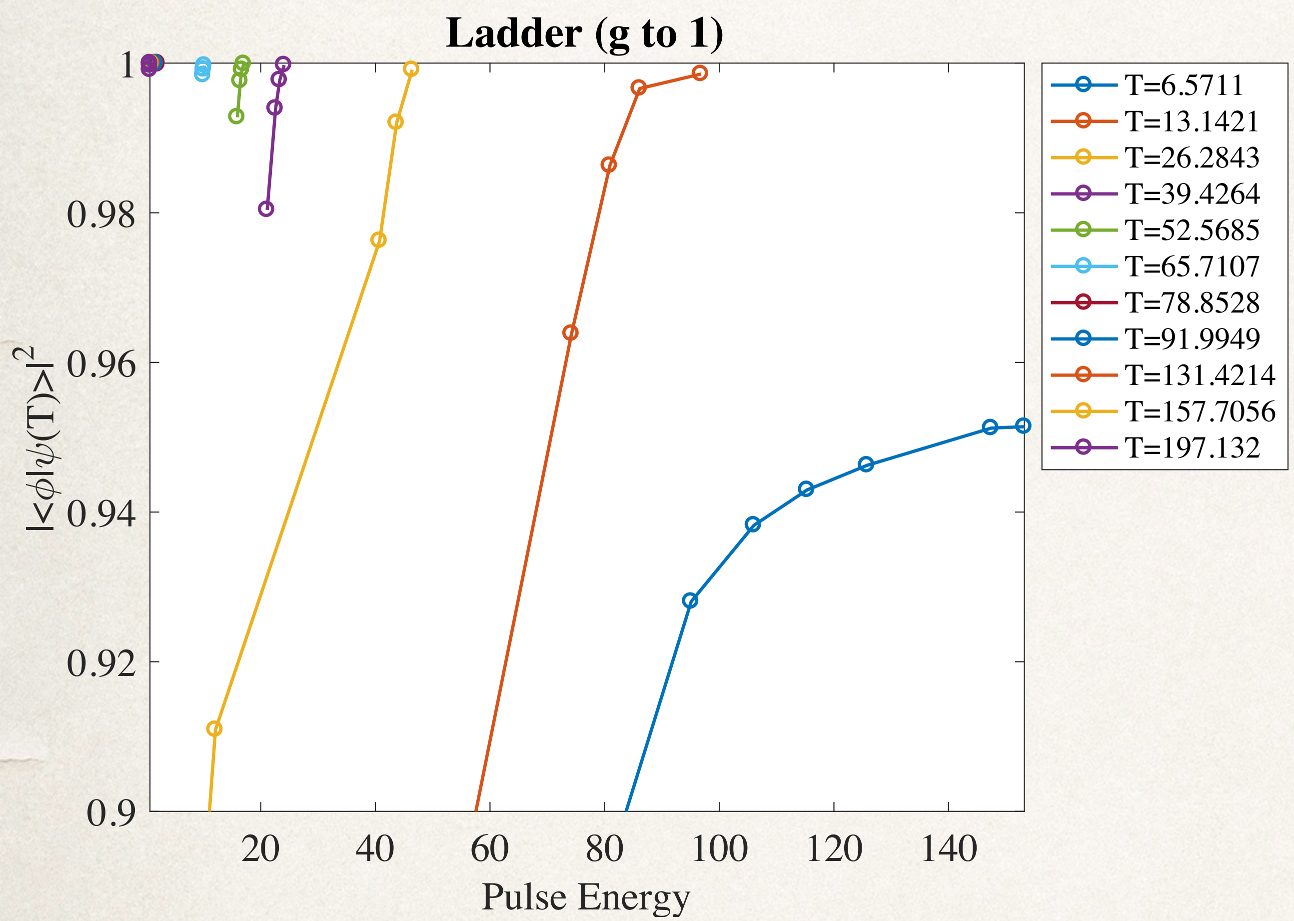
$$= 1 - 0.92 = 0.08$$

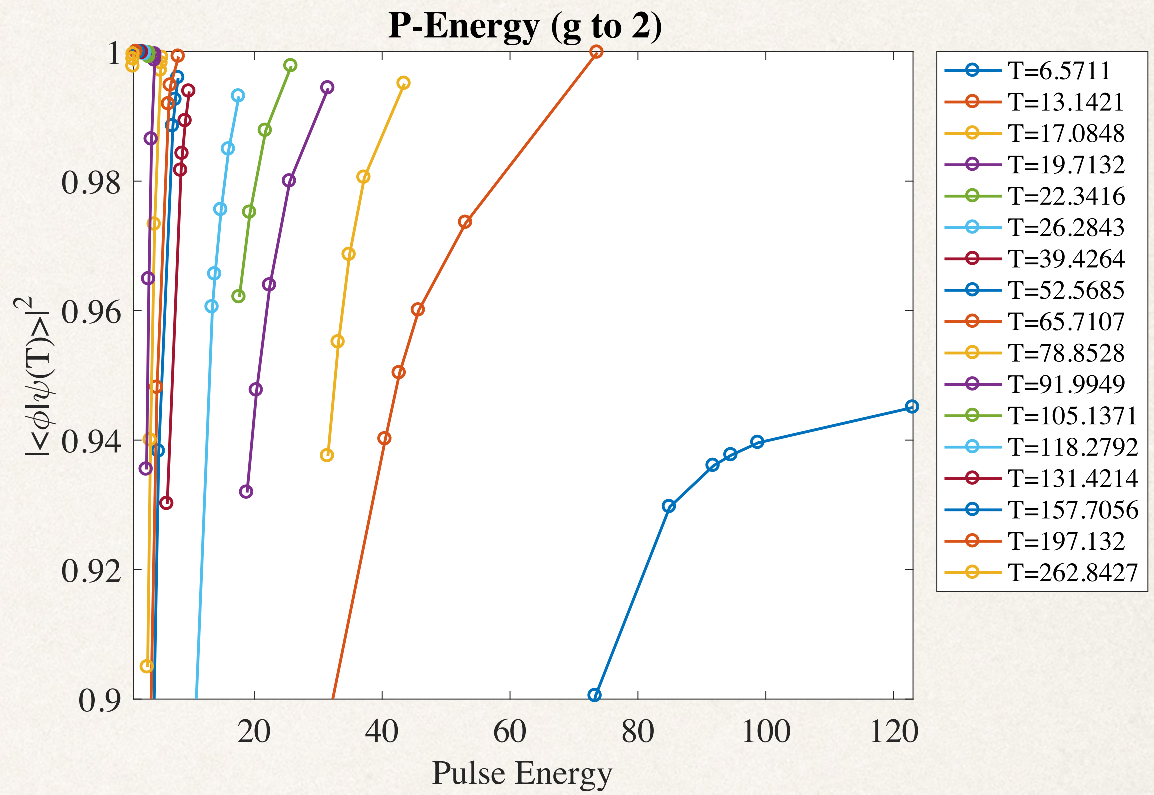
$$T > \frac{2\pi}{\Delta} \approx 78.5$$

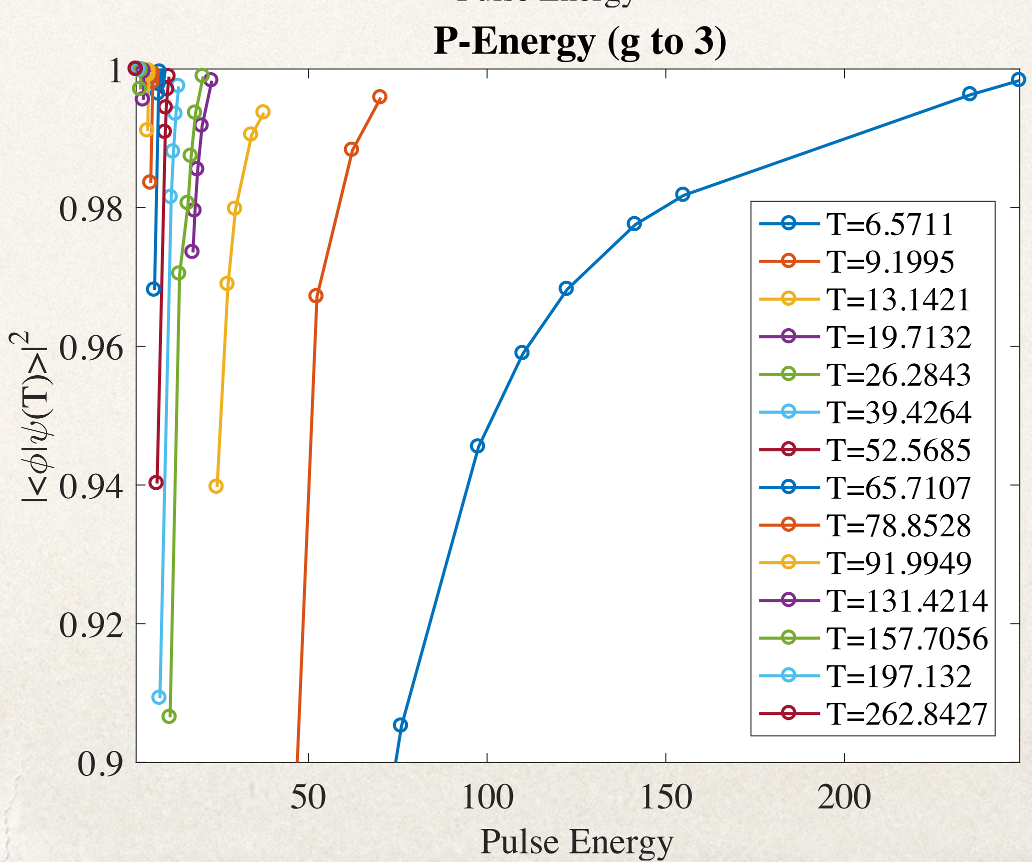




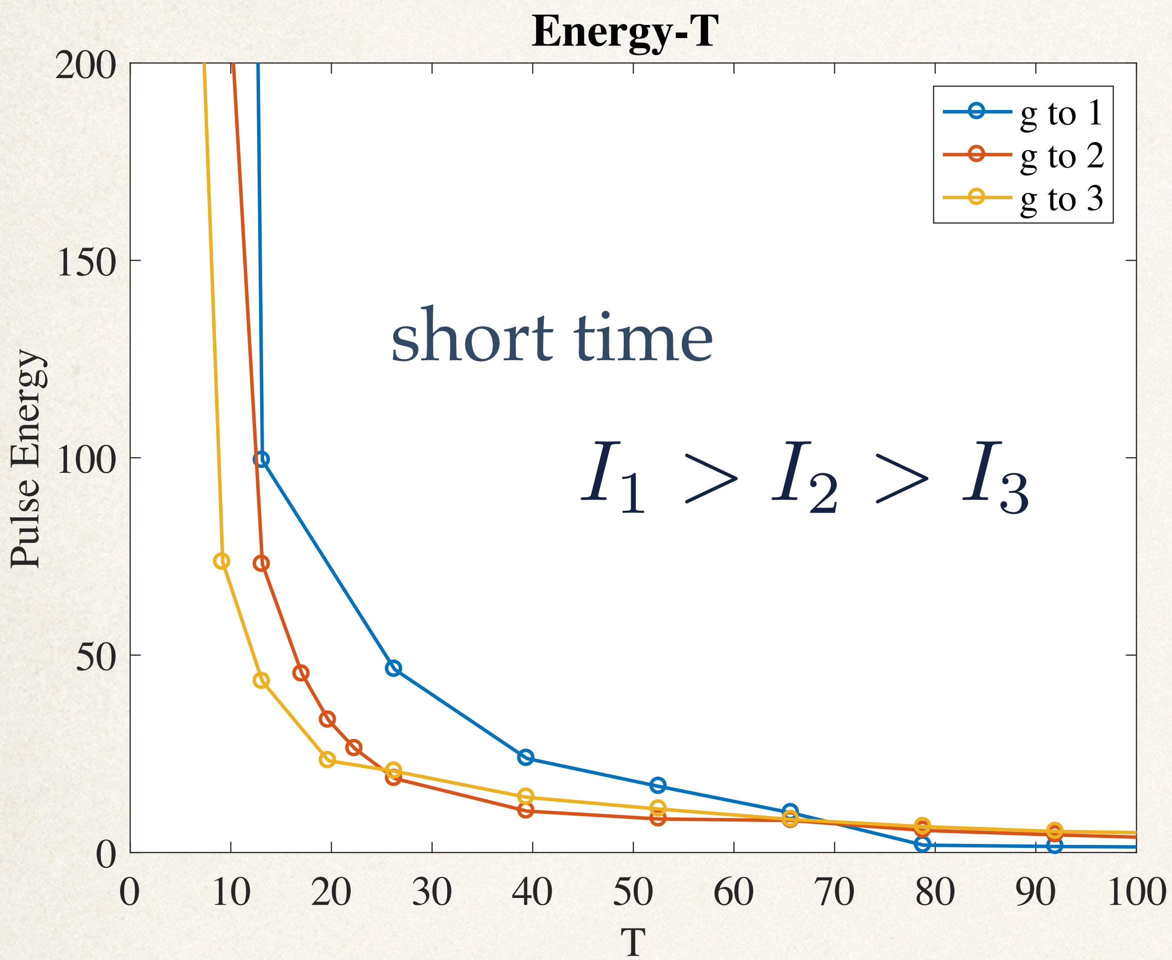
State to State : Ladder Type $|g\rangle \rightarrow |1\rangle, |2\rangle, |3\rangle$

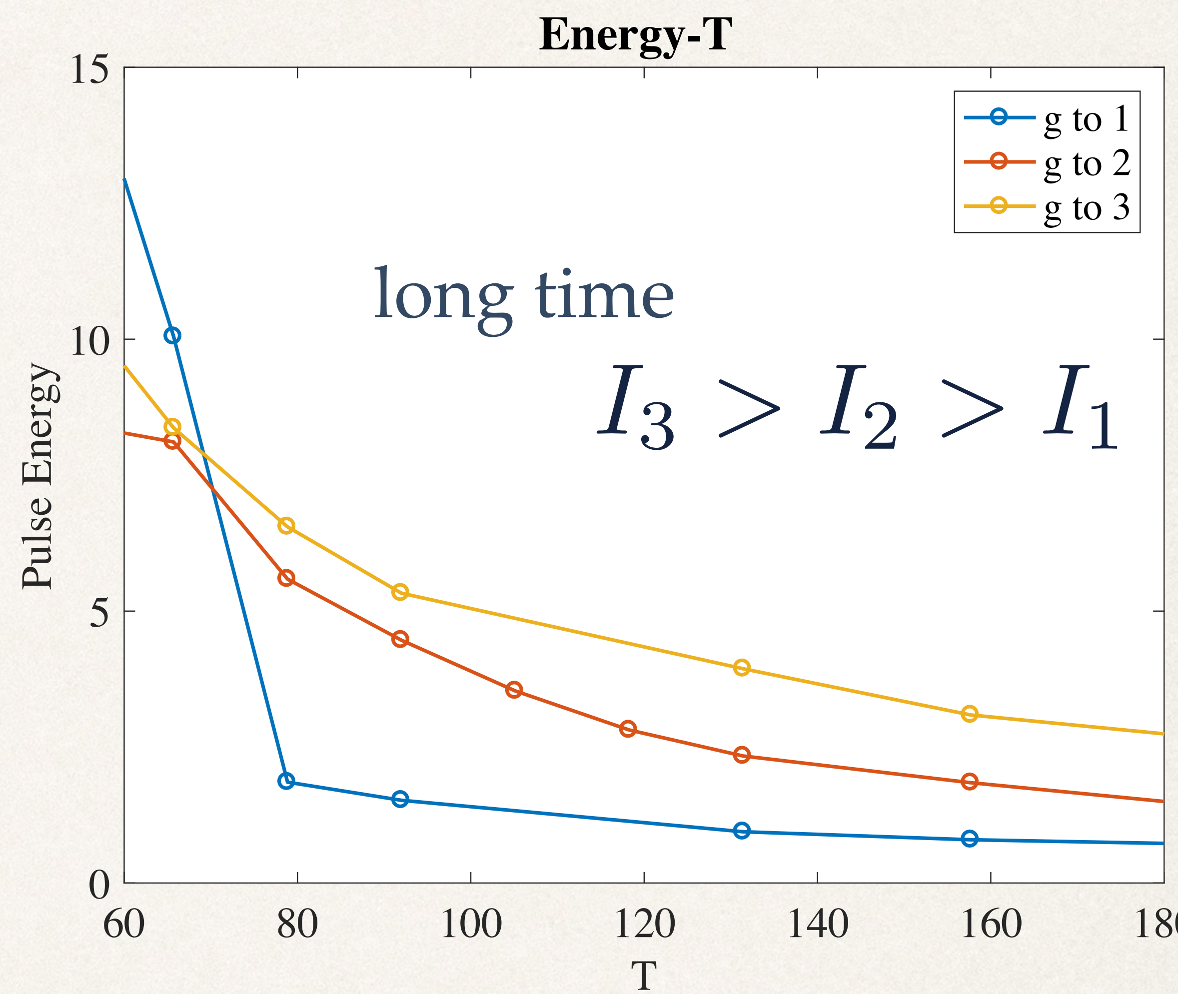






State to State : Ladder Type $|g\rangle \rightarrow |1\rangle, |2\rangle, |3\rangle$





pulse energy $I \equiv \int_0^T |\epsilon(t)|^2 dt$ $\qquad P_1 \propto I,\ P_2 \propto I^2,\ P_3 \propto I^3$

strong field: $I >> 1$

$\Rightarrow P_3 > P_2 > P_1$

same probability: $I_1 > I_2 > I_3$

weak field: $I << 1$

$\Rightarrow P_3 < P_2 < P_1$

same probability: $I_3 > I_2 > I_1$

Transition Types

Case 1 : State to State

2 level

Ladder Type

3 level

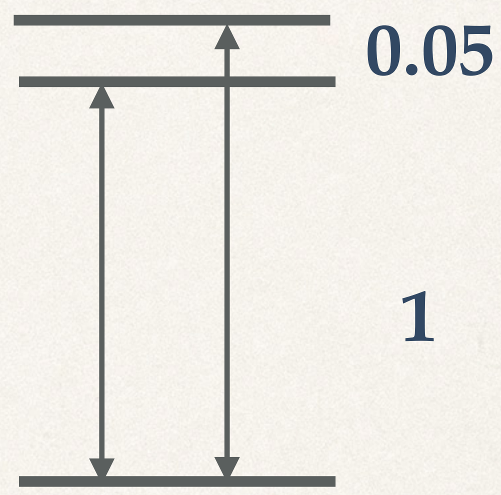
Case 2 : State to Superposition

3 level

Case 3 : Tunneling

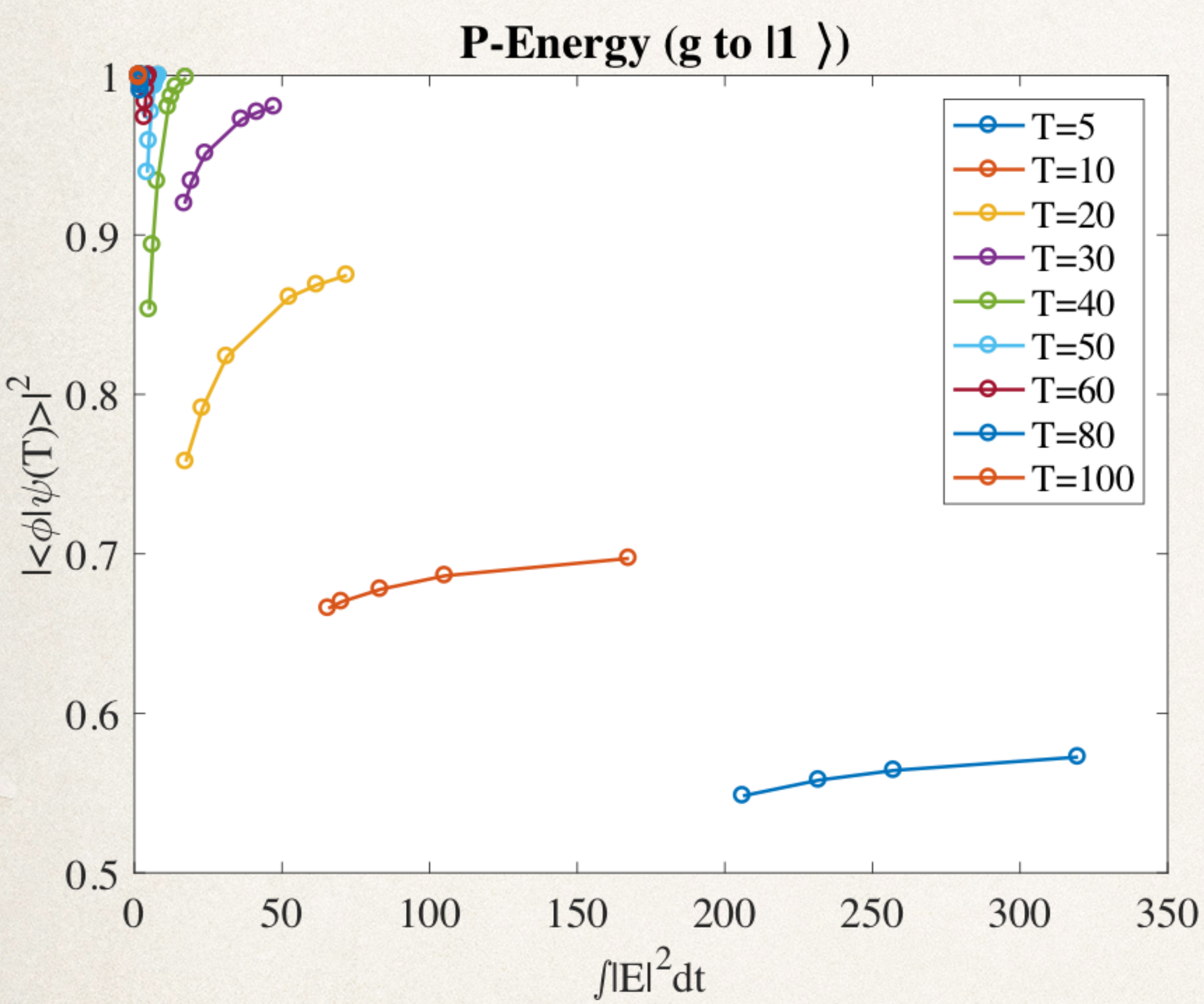
Double Well

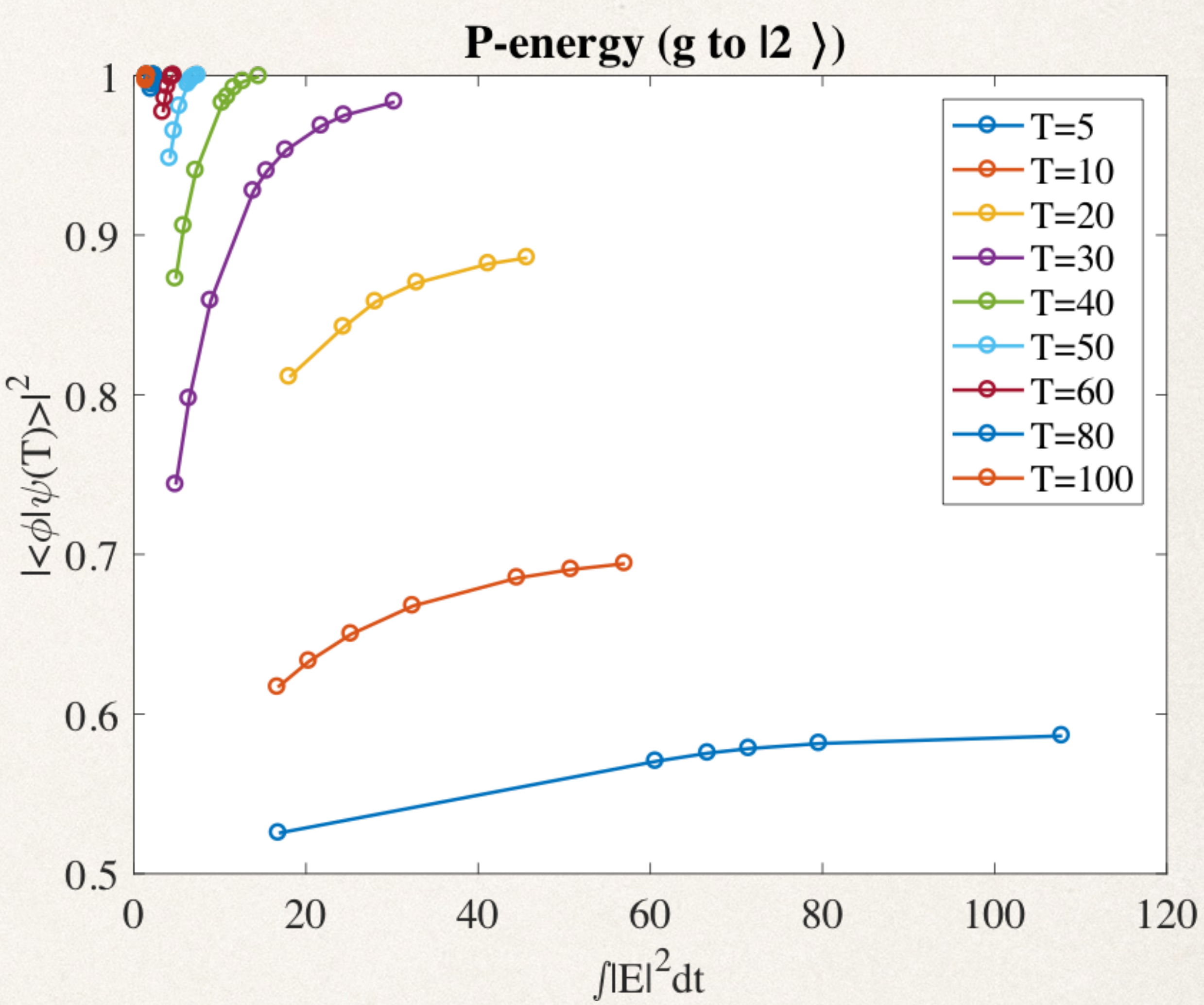
Level Diagram



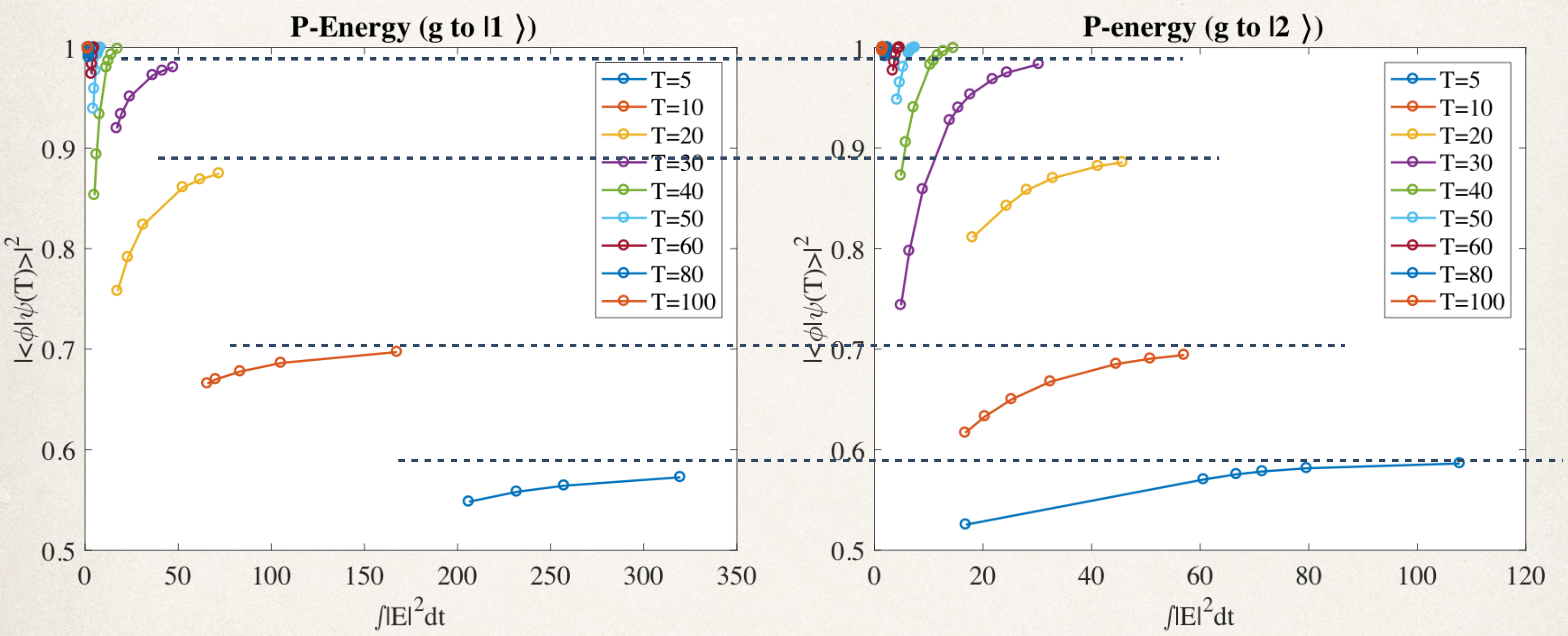
three level

State to State : 3 level $|g\rangle \rightarrow |1\rangle, |2\rangle$





State to State : 3 level $|g\rangle \to |1\rangle, |2\rangle$



Controllability does not depend on the final state !

Transition Types

Case 1 : State to State

2 level

Ladder Type

3 level

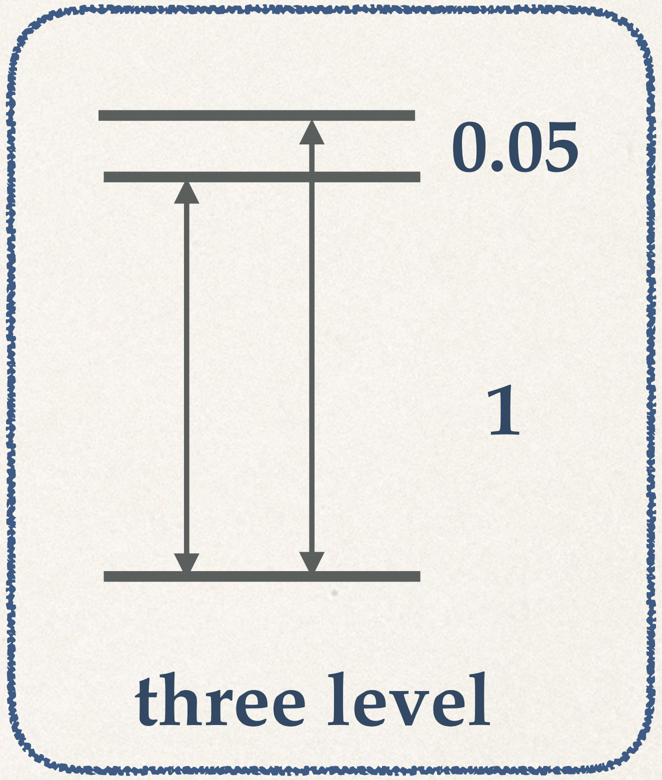
Case 2 : State to Superposition

3 level

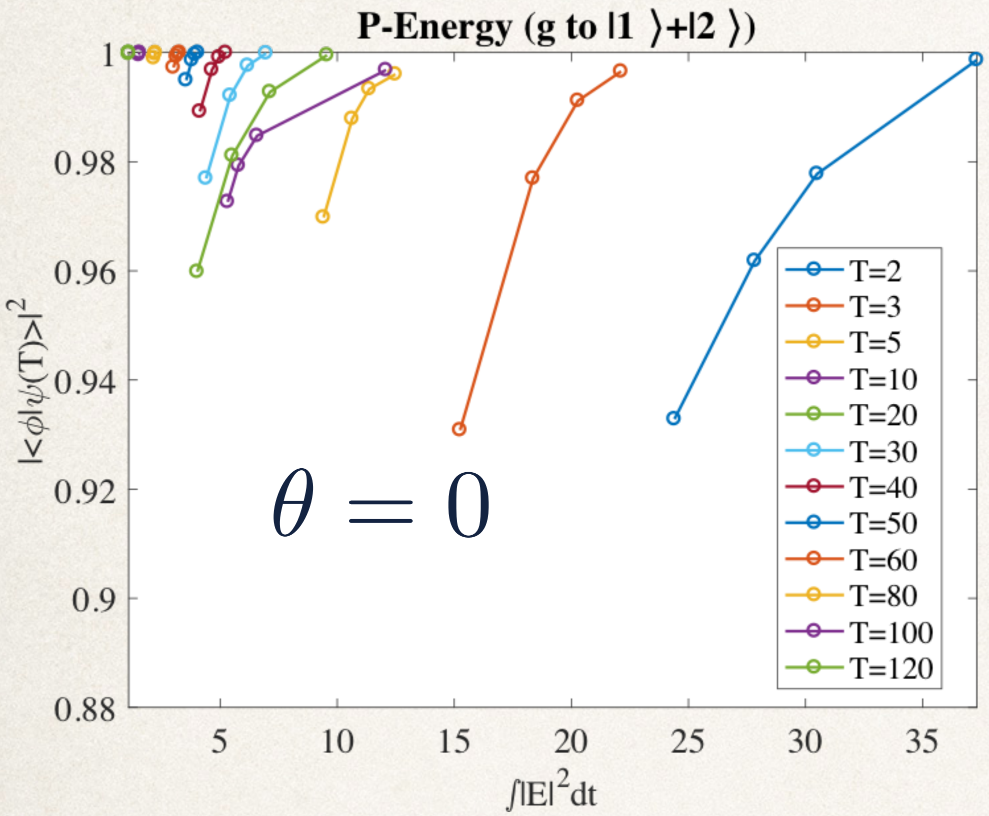
Case 3 : Tunneling

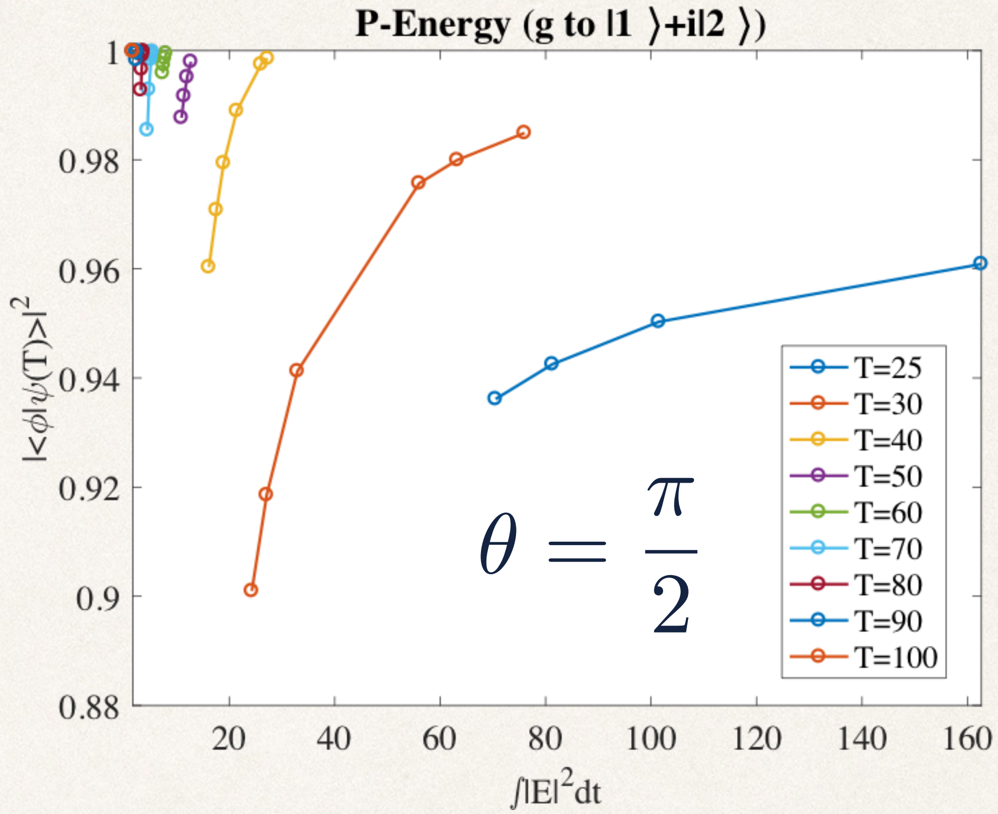
Double Well

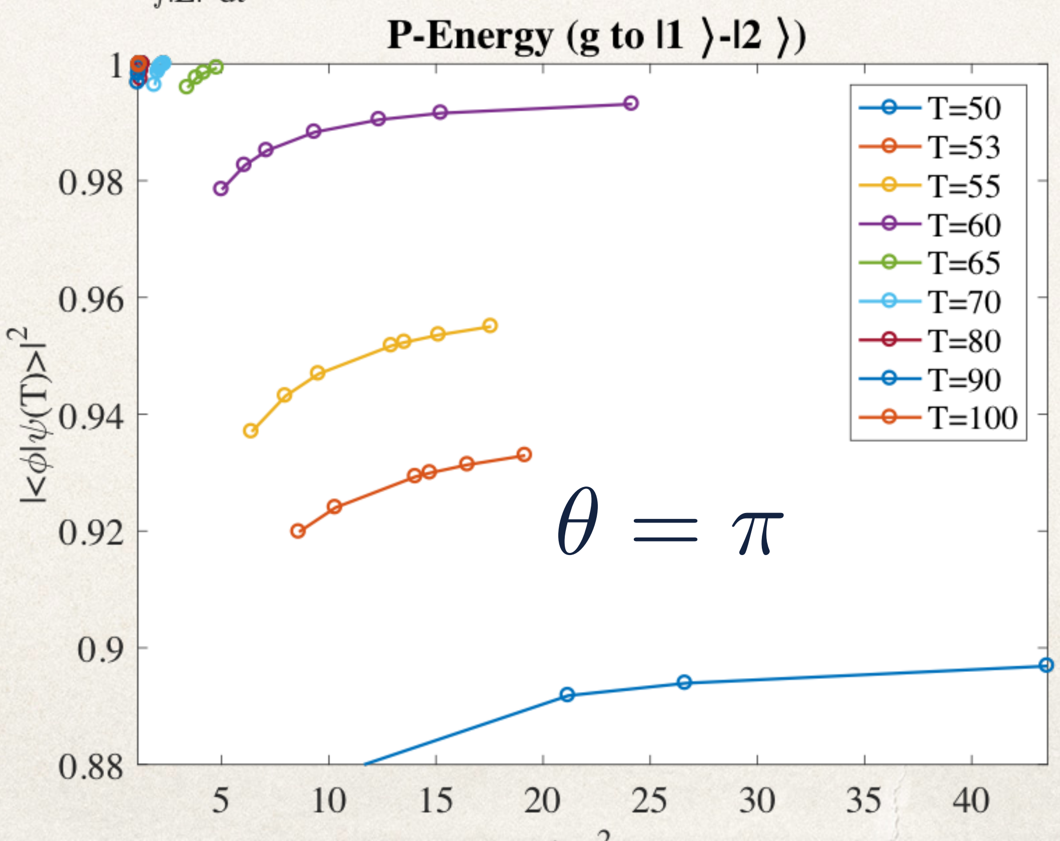
Level Diagram



State to Superposition : 3 level $|g\rangle \rightarrow |1\rangle + e^{i\theta}|2\rangle$

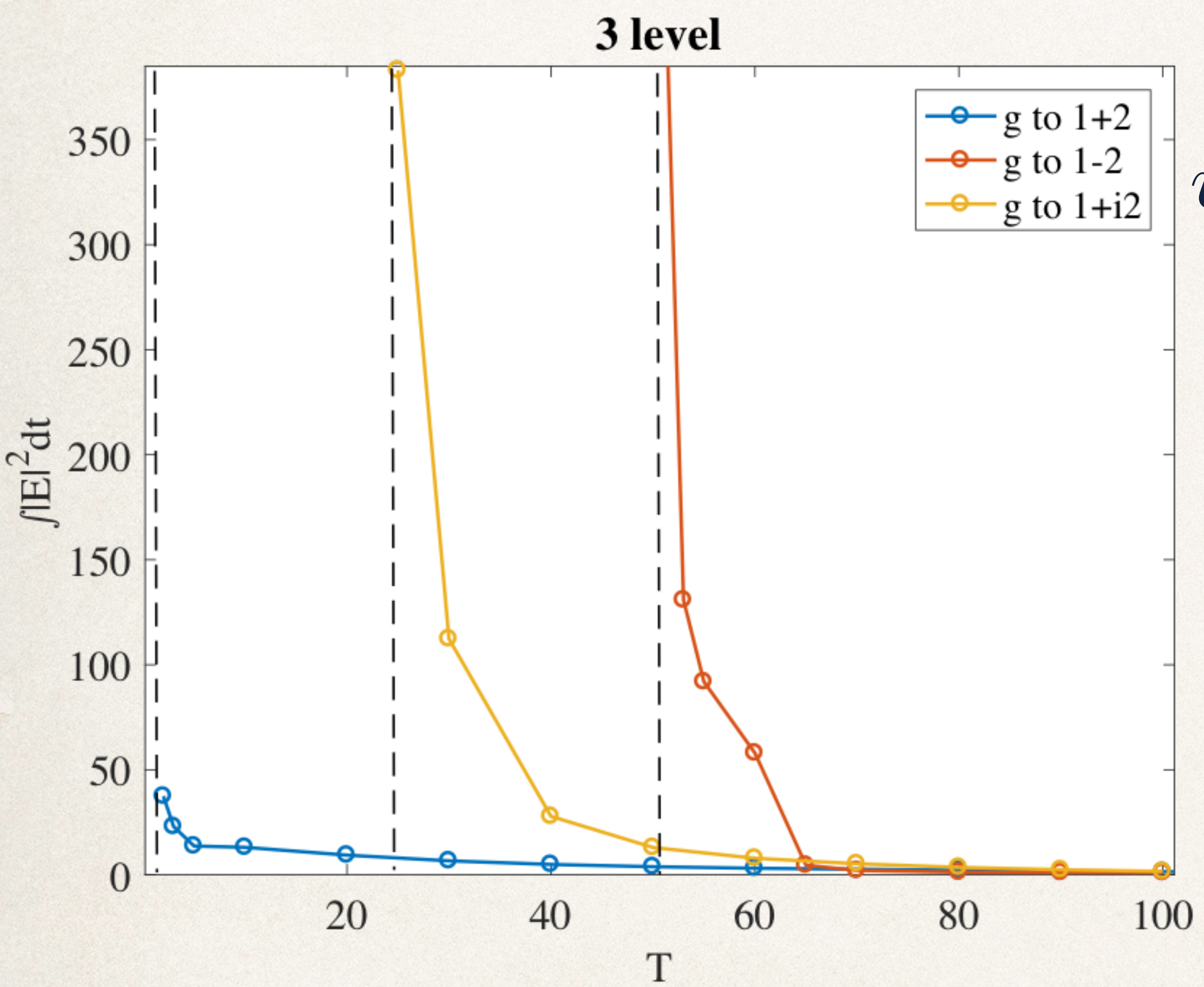






Strongly depends on the phase

State to Superposition : 3 level $\quad |g\rangle \rightarrow |1\rangle + e^{i\theta}|2\rangle$



1 25 50

$$\psi(t) = e^{-\frac{i}{\hbar}E_1 t}|1\rangle + e^{-\frac{i}{\hbar}E_2 t}|2\rangle$$

$$= e^{-\frac{i}{\hbar}E_1 t}(|1\rangle + e^{\frac{i}{\hbar}(E_1 - E_2)t}|2\rangle)$$

$$(E_1 - E_2)t = 0$$

$$(E_1 - E_2)t = \frac{\pi}{2}$$

$$(E_1 - E_2)t = \pi$$

Transition Types

Case 1 : State to State

2 level

Ladder Type

3 level

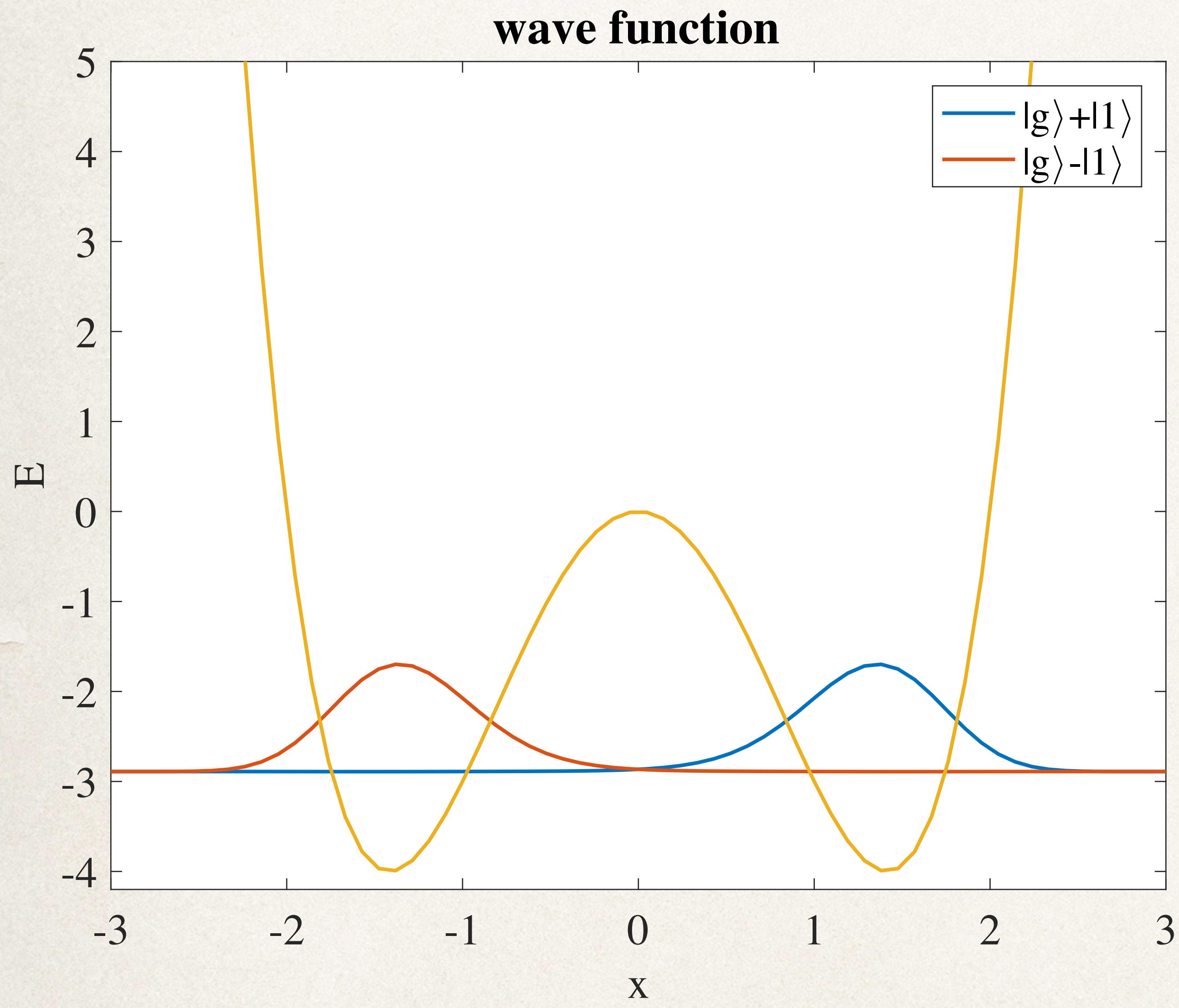
Case 2 : State to Superposition

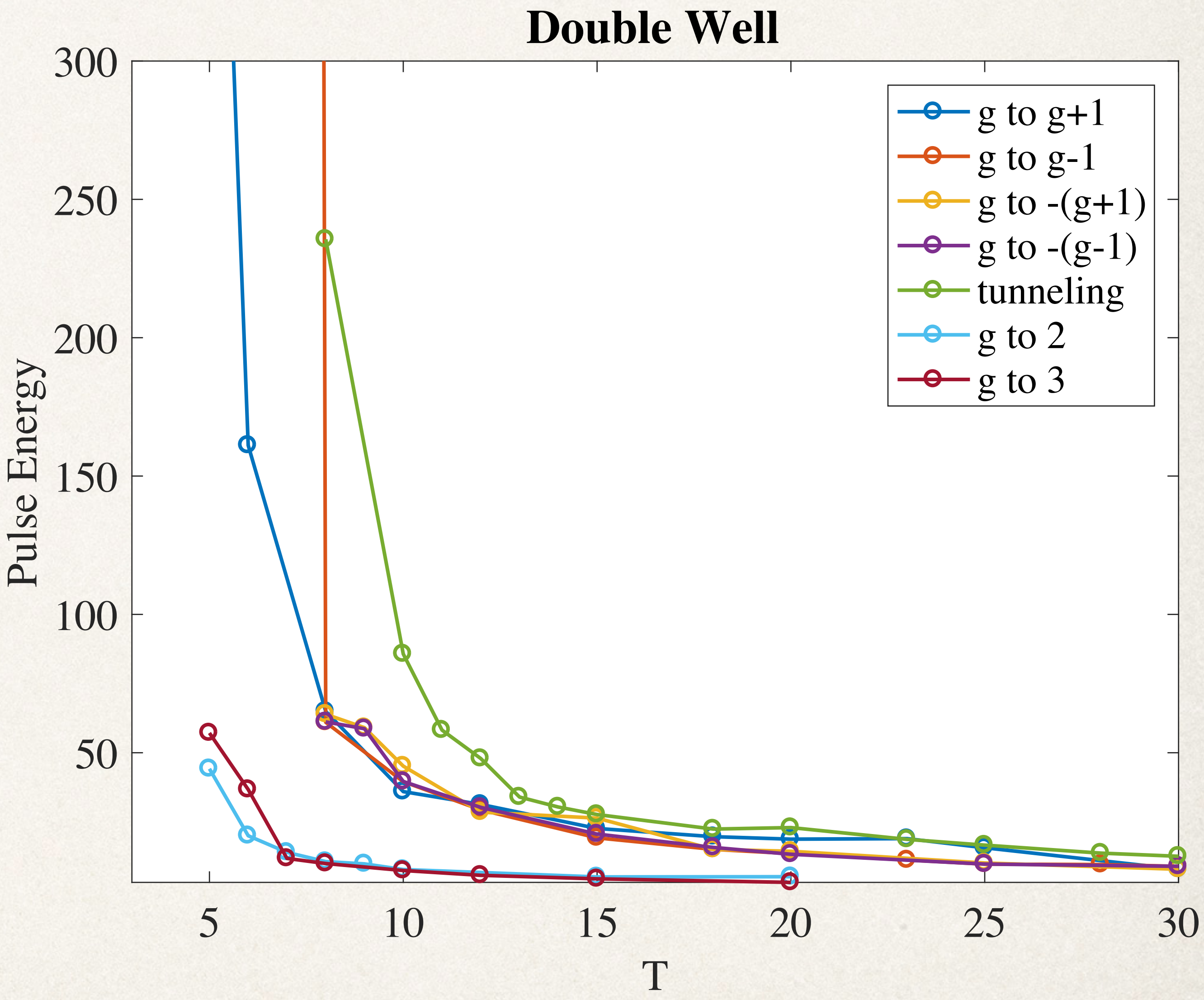
3 level

Case 3 : Tunneling

Double Well

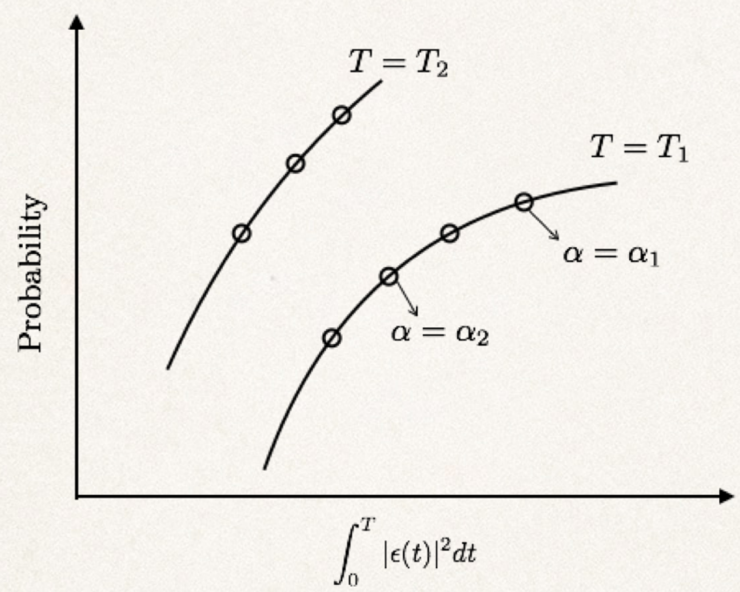
Double Well : Tunneling $|g\rangle + |1\rangle \rightarrow |g\rangle - |1\rangle$



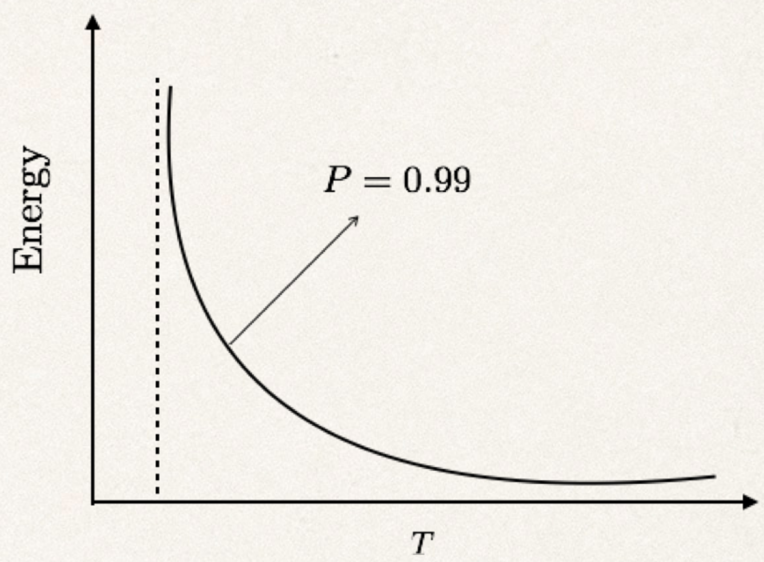


We propose 2 kinds of figure that can easily discuss controllability

P-E :



E-T :



(1). State to State

2 level	When $T \geq \dfrac{2\pi}{\Delta}$ the transition is resonance transition
Ladder Type	When $T \leq \dfrac{2\pi}{\Delta}$ we can see the multi level effect
3 level	The controllability is worse than 2 level system, but there is no target state dependence.

(2). State to Superposition

3 level	Controllability has strong phase dependence. $(E_1 - E_2)t = 0, \frac{\pi}{2}$, or π

(3). Tunneling

Double Well	Controlling tunneling is the most difficult The divergence T is similar to state to superposition

Thanks for your attentions