Entanglement Preserving Local Thermalization

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Framework

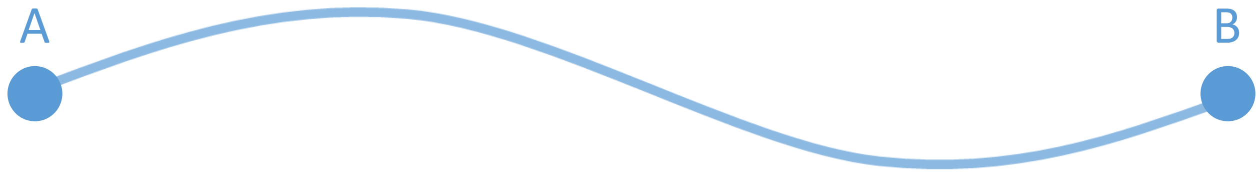
Framework

A

B

Framework

Framework



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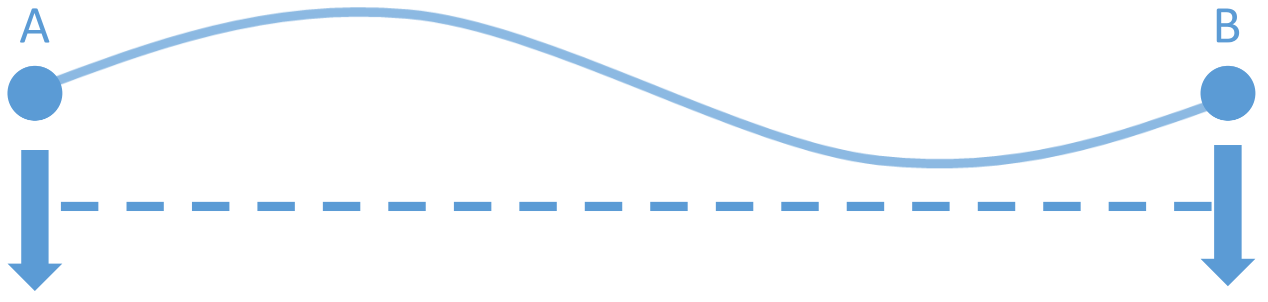

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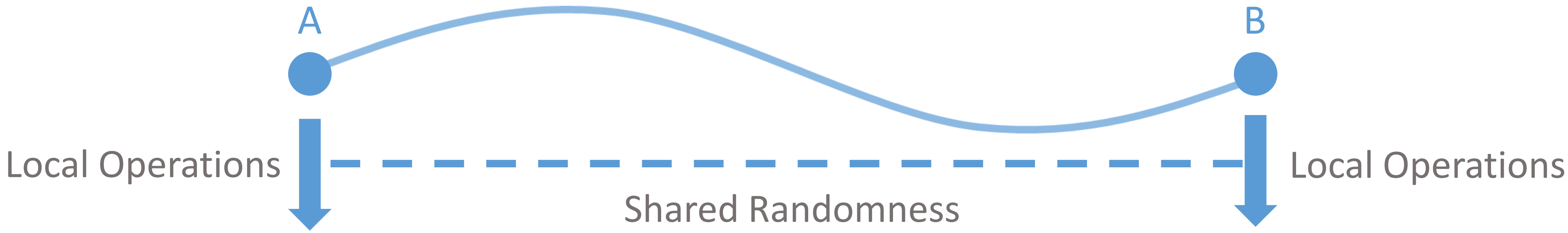

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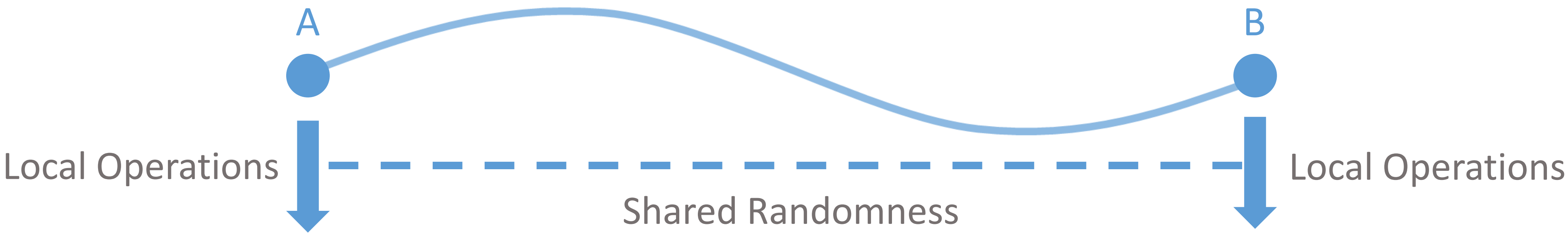
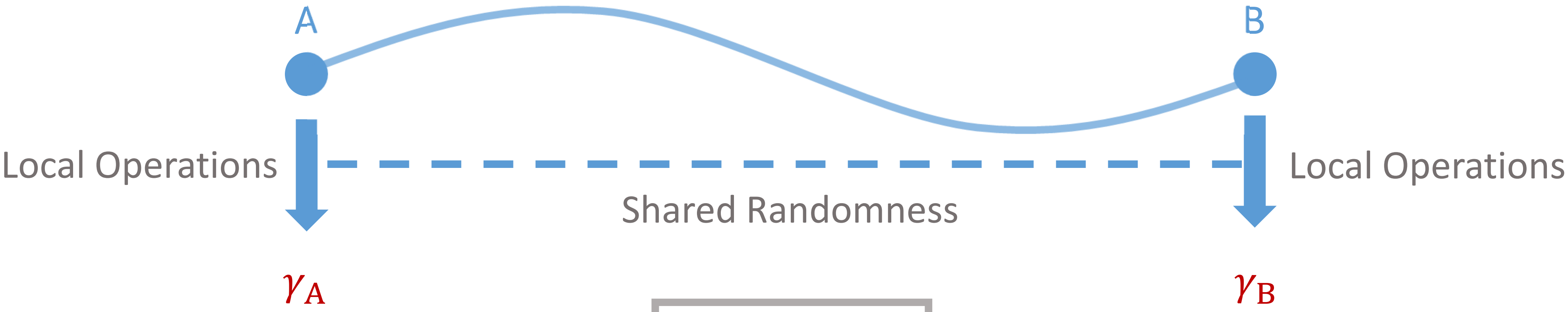


$$\int_{\lambda} \mathcal{E}_{\mathrm{A}}^{\lambda} \otimes \mathcal{E}_{B}^{\lambda} p_{\lambda} d\lambda$$

Framework



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Framework

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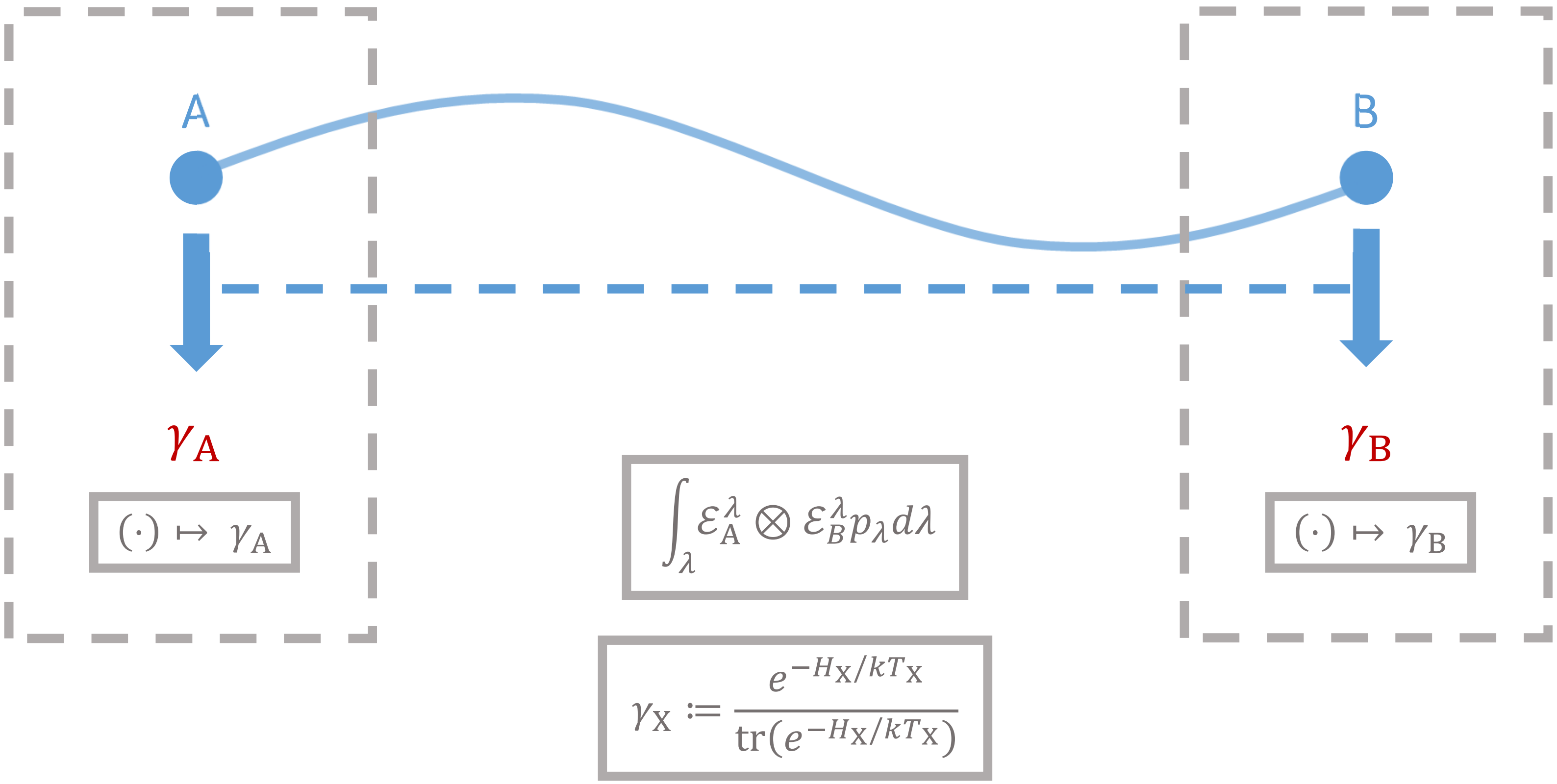
$$\gamma_{\mathrm{X}} := \frac{e^{-H_{\mathrm{X}}/kT_{\mathrm{X}}}}{\mathrm{tr}(e^{-H_{\mathrm{X}}/kT_{\mathrm{X}}})}$$

Framework


A
B
γ_A
$(\cdot) \mapsto \gamma_A$
γ_B
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$\int_\lambda \mathcal{E}_A^\lambda \otimes \mathcal{E}_B^\lambda p_\lambda d\lambda$
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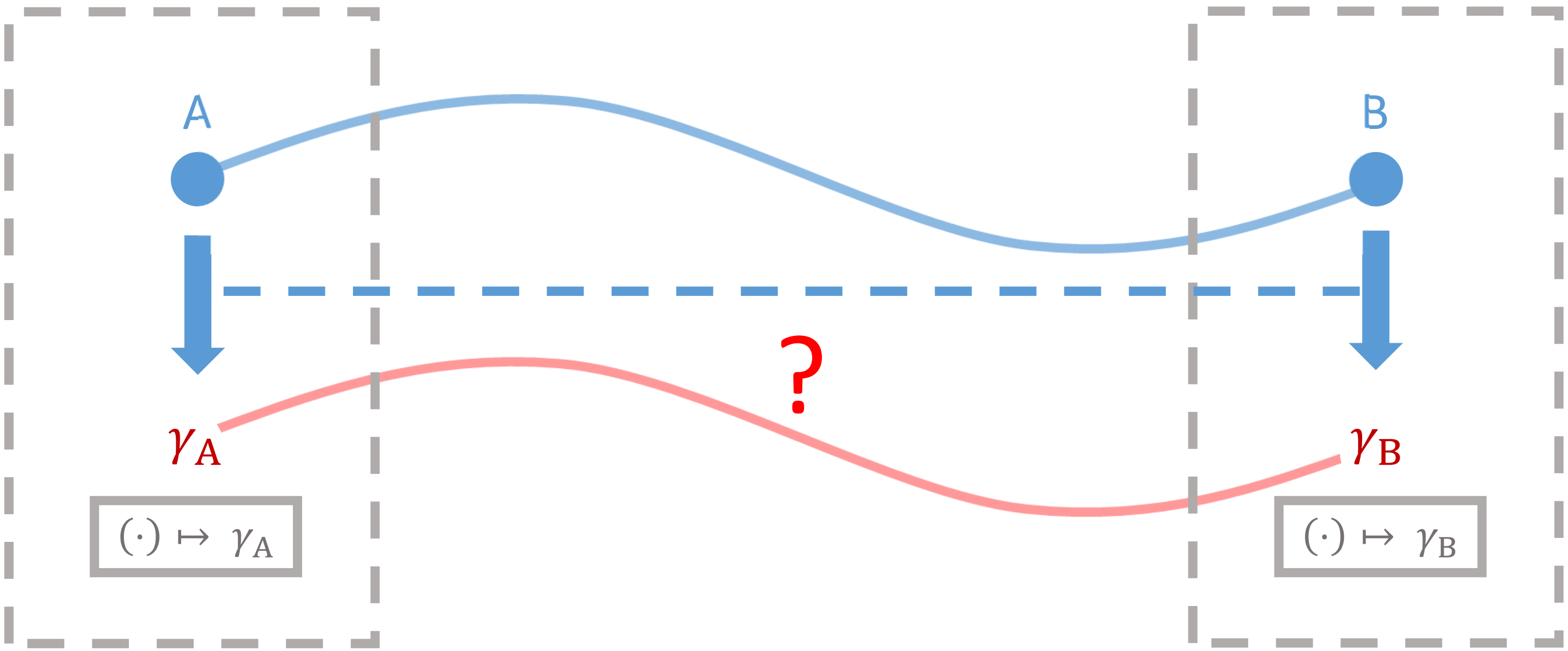

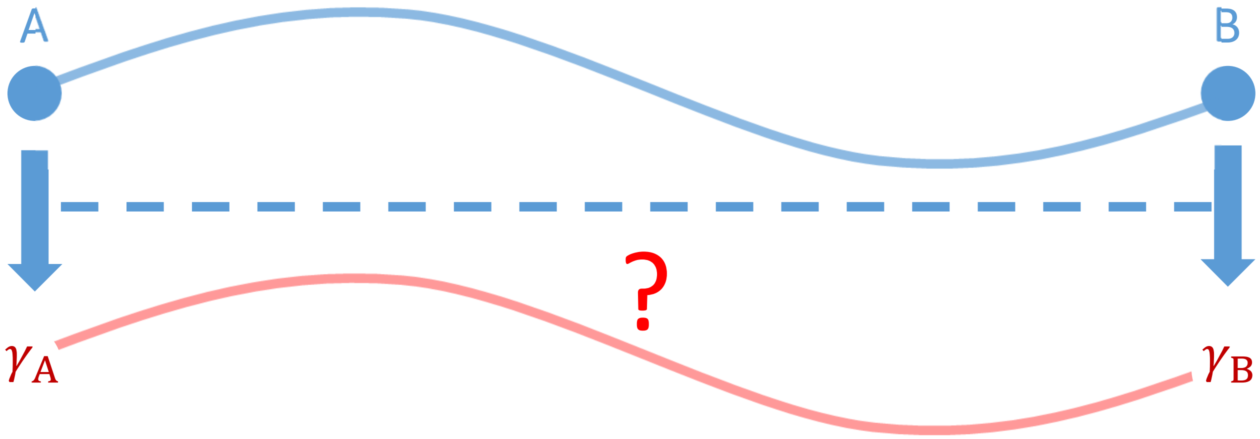

Local Thermalization

Local operations + shared randomness

Locally behaves as $(\cdot) \mapsto \gamma_A$ and $(\cdot) \mapsto \gamma_B$

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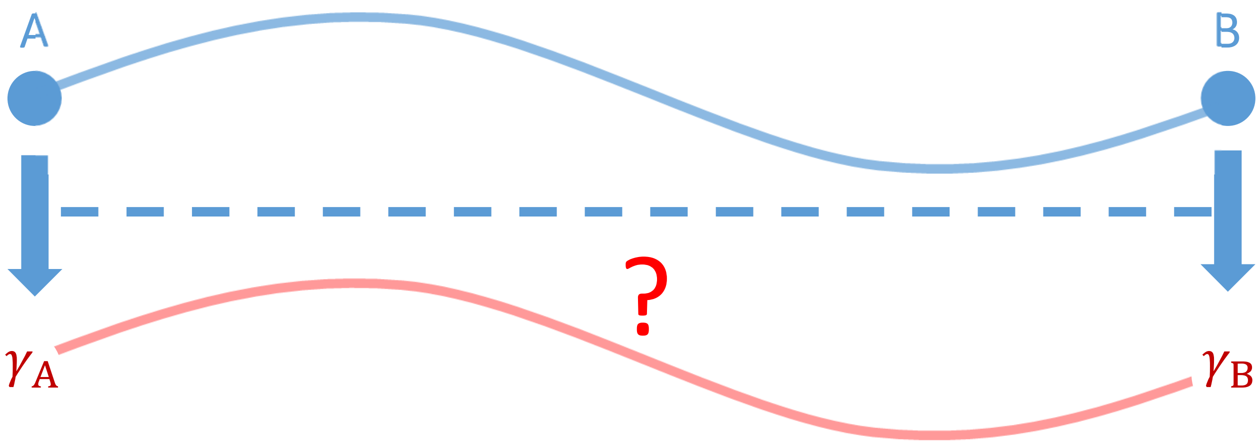

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Do entanglement preserving local thermalizations exist?

Result 1 | No Product Entanglement Preserving Local Thermalization

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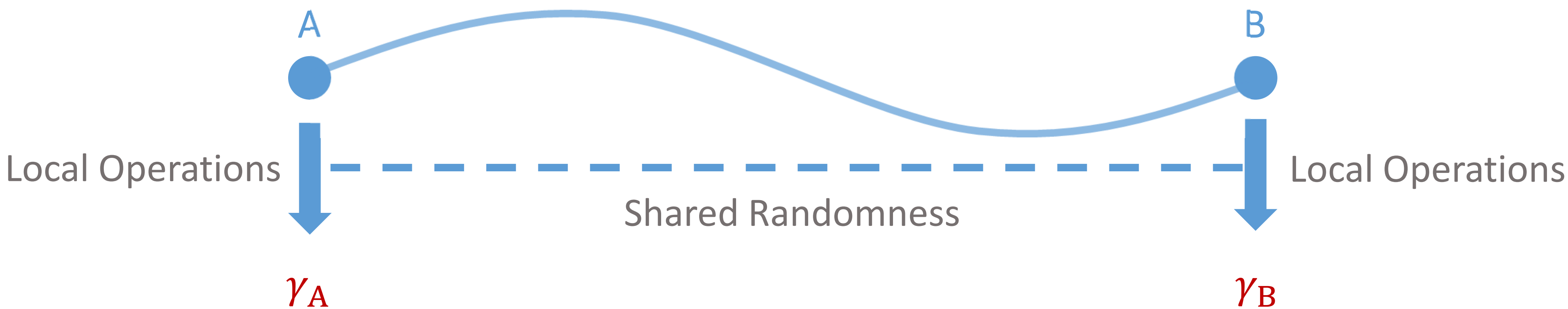

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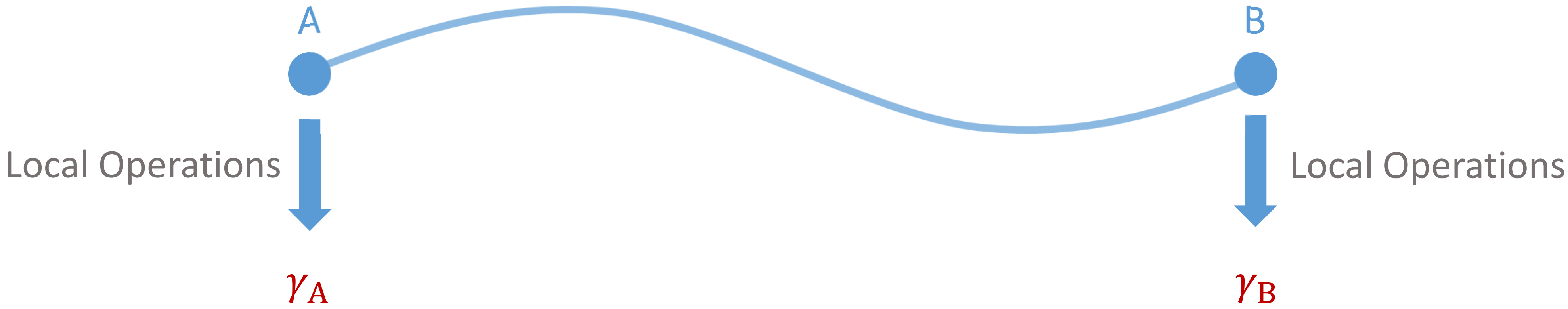

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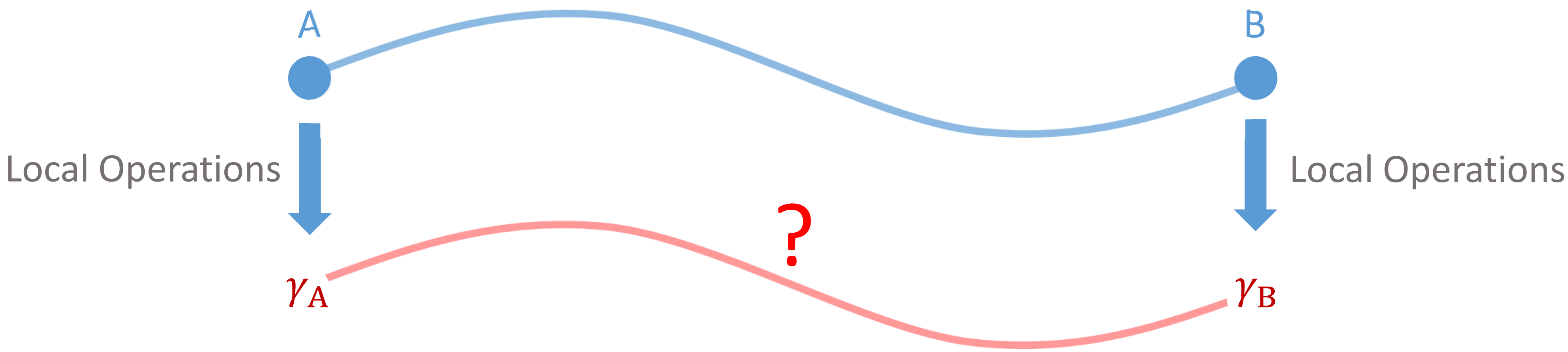

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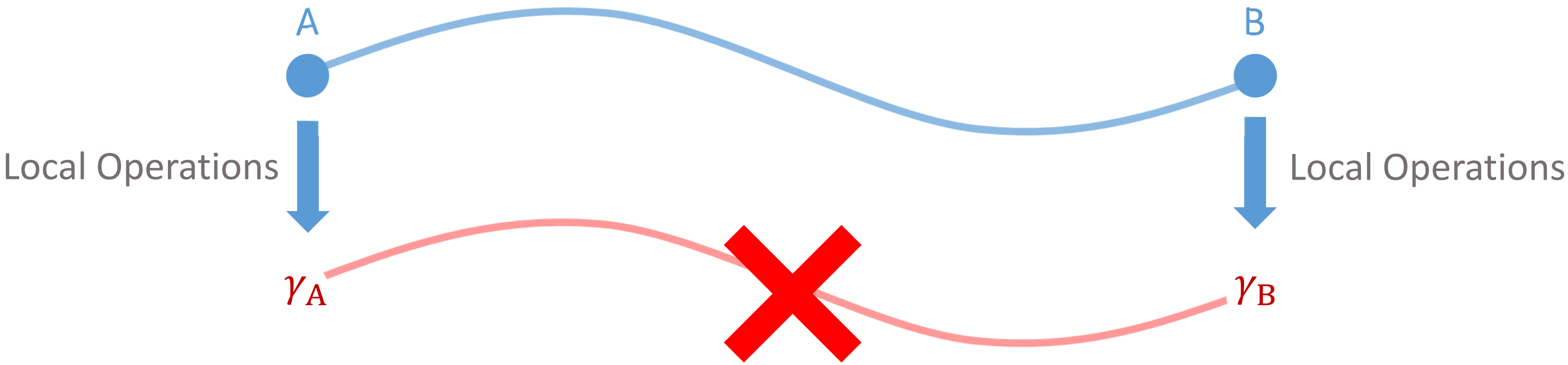

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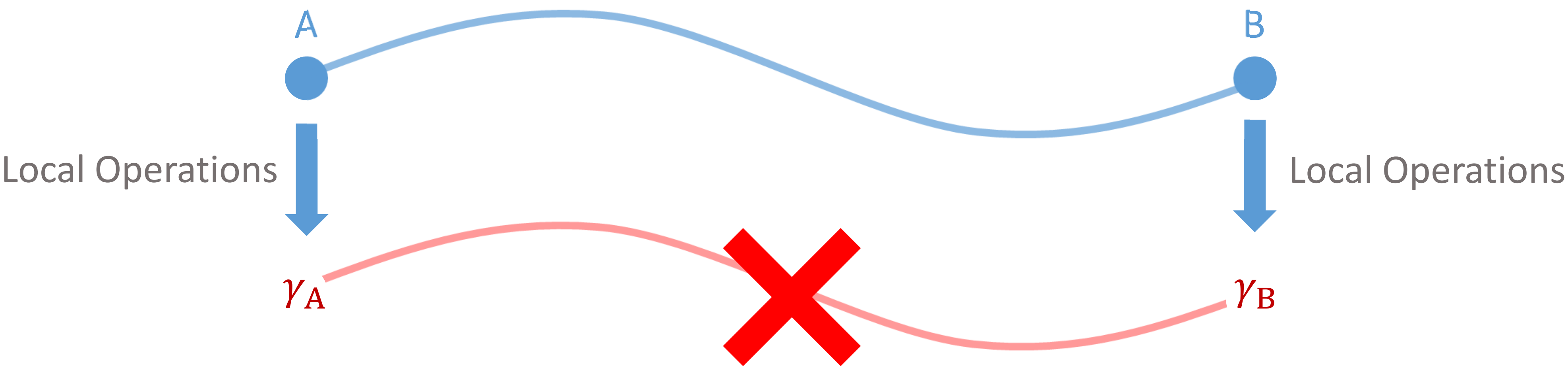

Any product local thermalization to (γ_A, γ_B) coincides with the channel $(\cdot) \mapsto \gamma_A \otimes \gamma_B$.

Result 2 | Existence of Entanglement Preserving Local Thermalization

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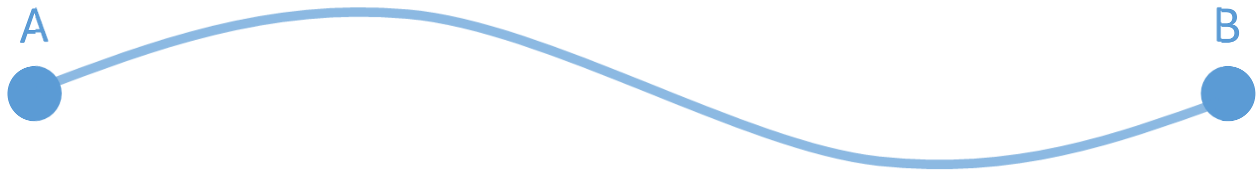

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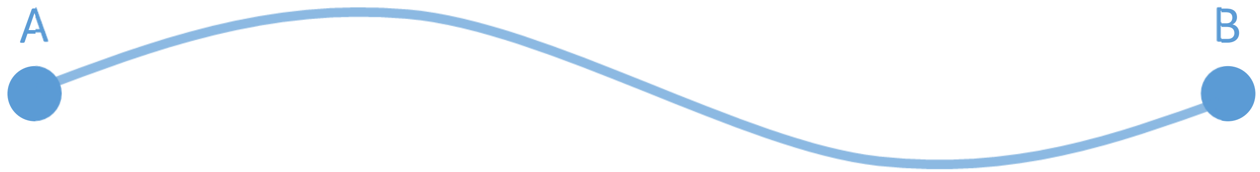

γ_{A} γ_{B}

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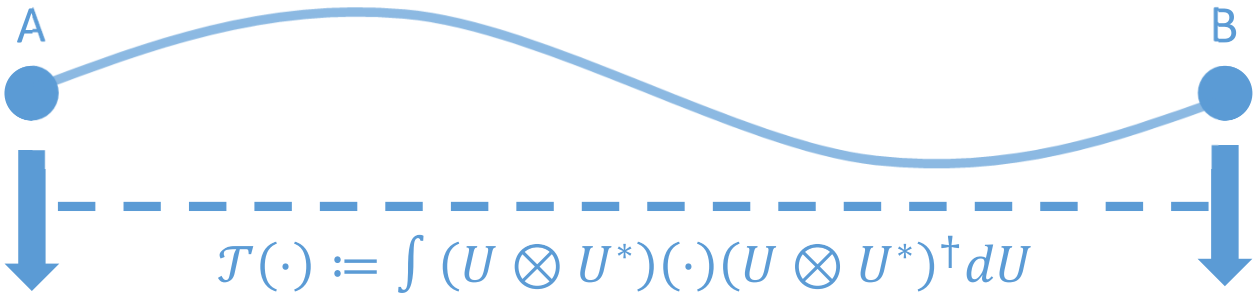

$$\mathcal{T}(\cdot) \coloneqq \int (U \otimes U^*)(\cdot)(U \otimes U^*)^\dagger dU$$

C. H. Bennett, et al., Phys. Rev. Lett. **76**, 722 (1996); M. Horodecki and P. Horodecki, Phys. Rev. A **59**, 4206 (1999).

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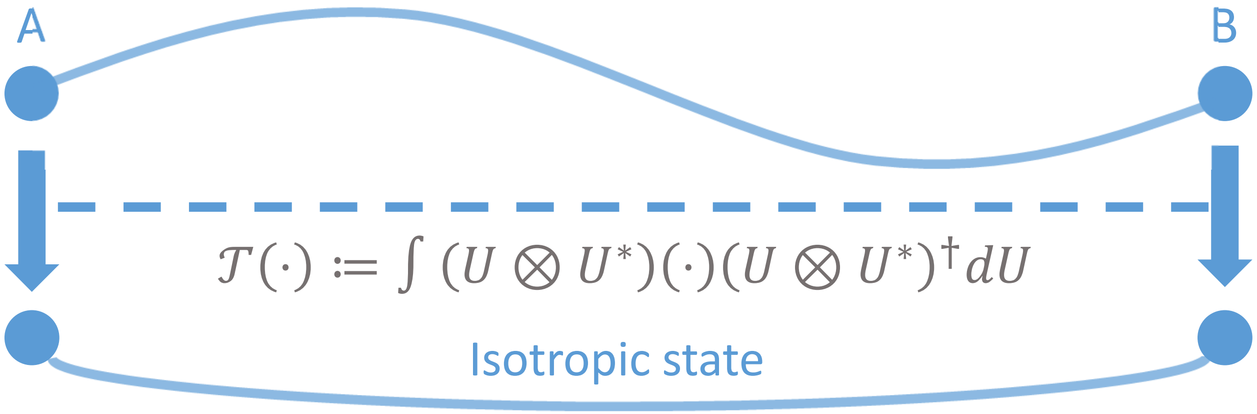

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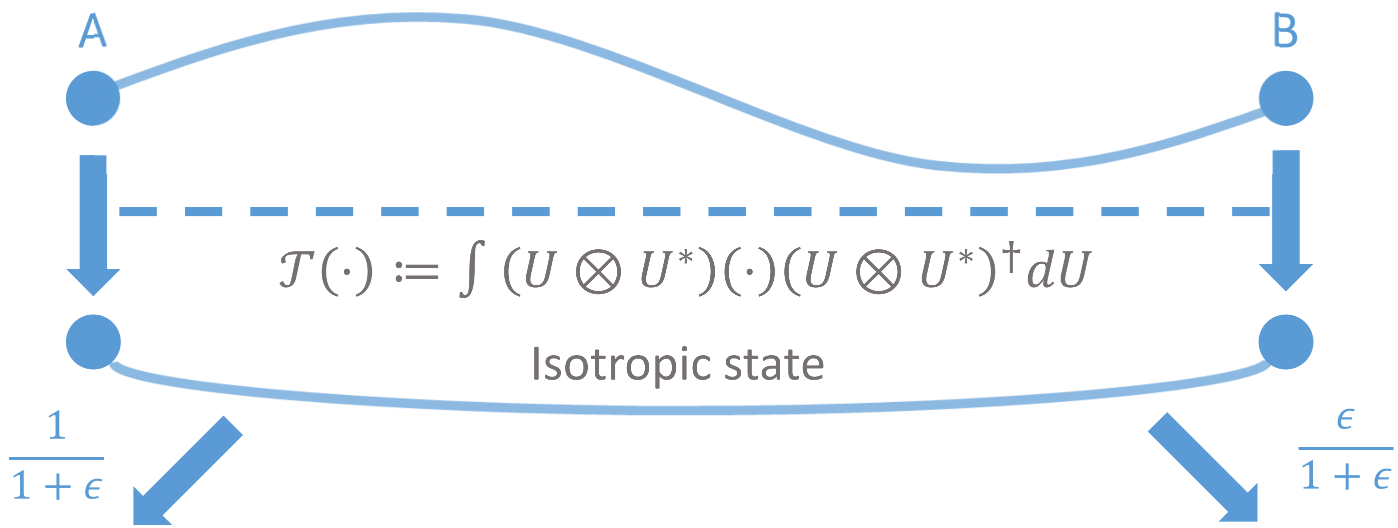

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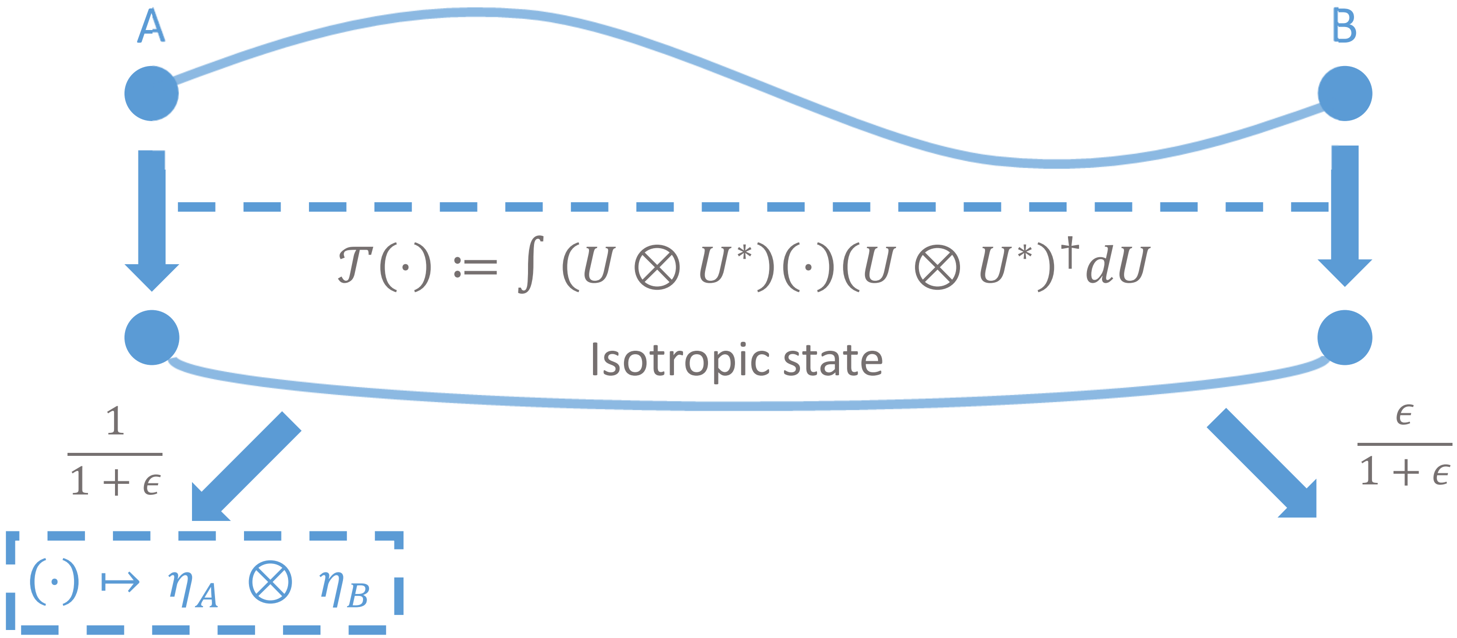

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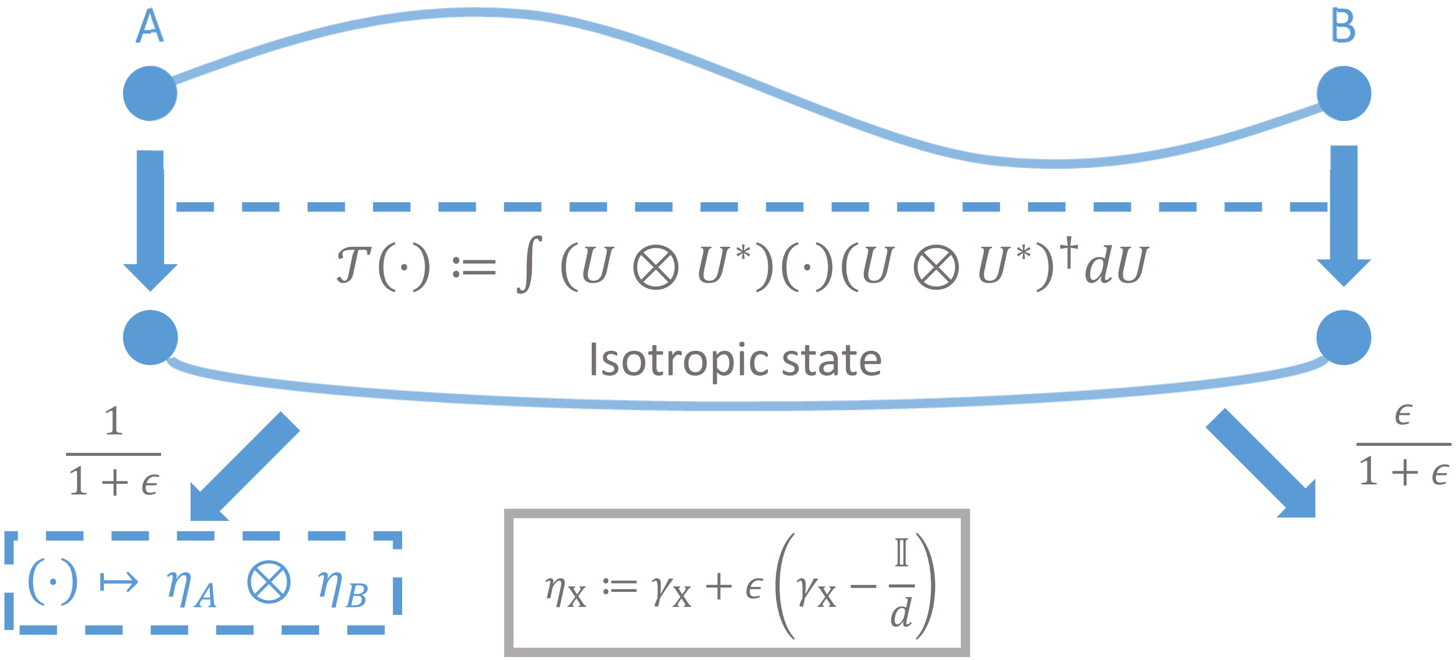

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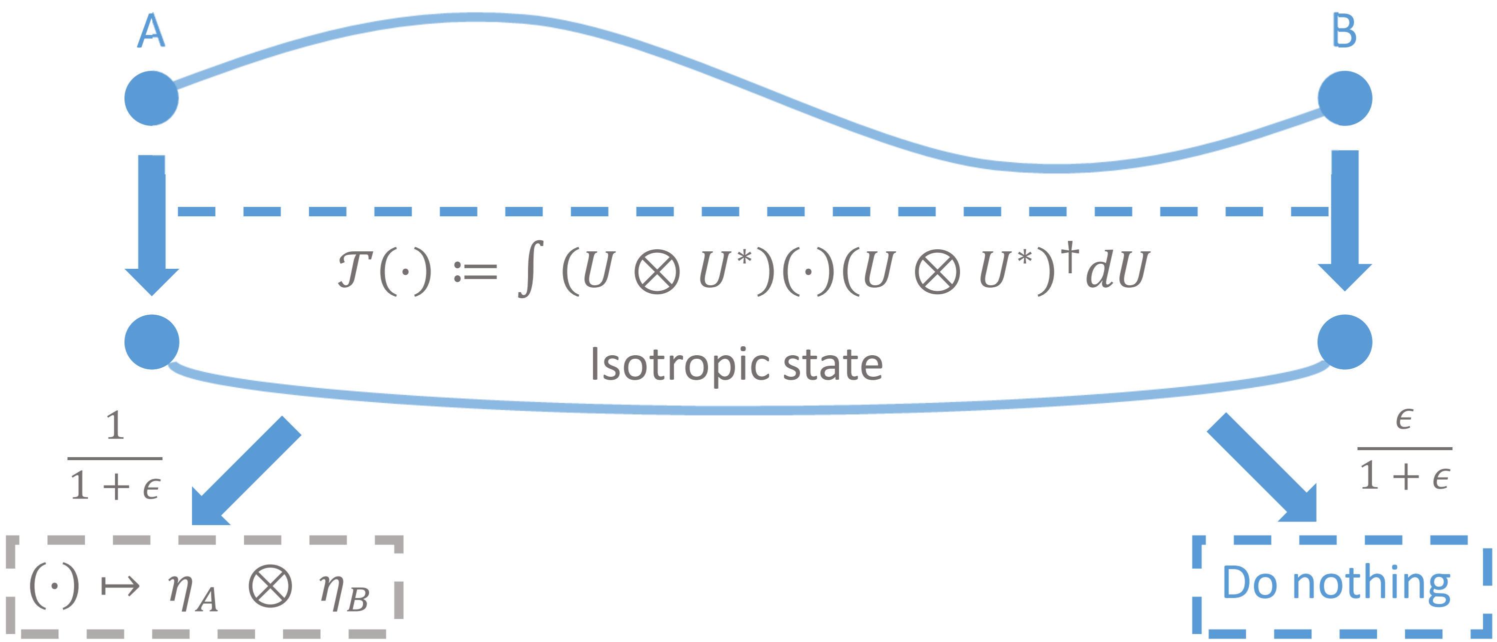

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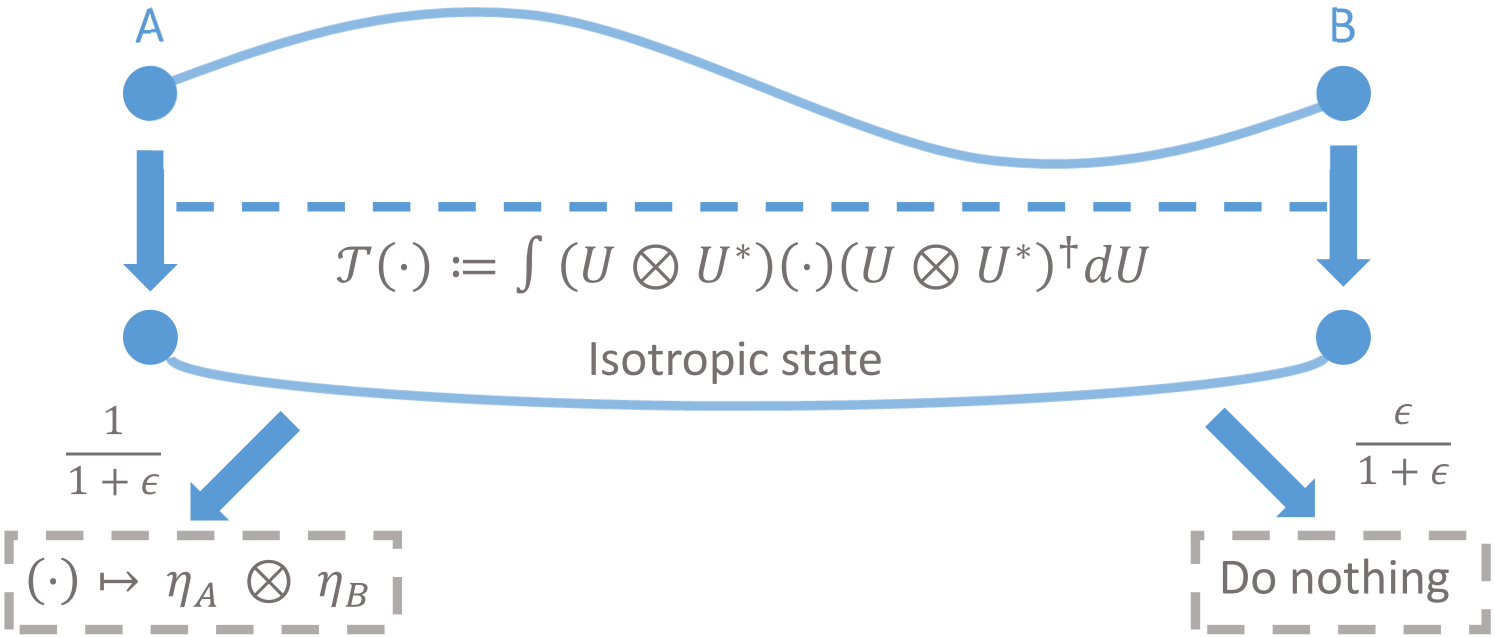

$$\mathcal{E}^{\epsilon}_{(\gamma_A,\gamma_B)}(\cdot) \coloneqq \frac{1}{1+\epsilon}\eta_A \otimes \eta_B + \frac{\epsilon}{1+\epsilon}\mathcal{T}(\cdot)$$

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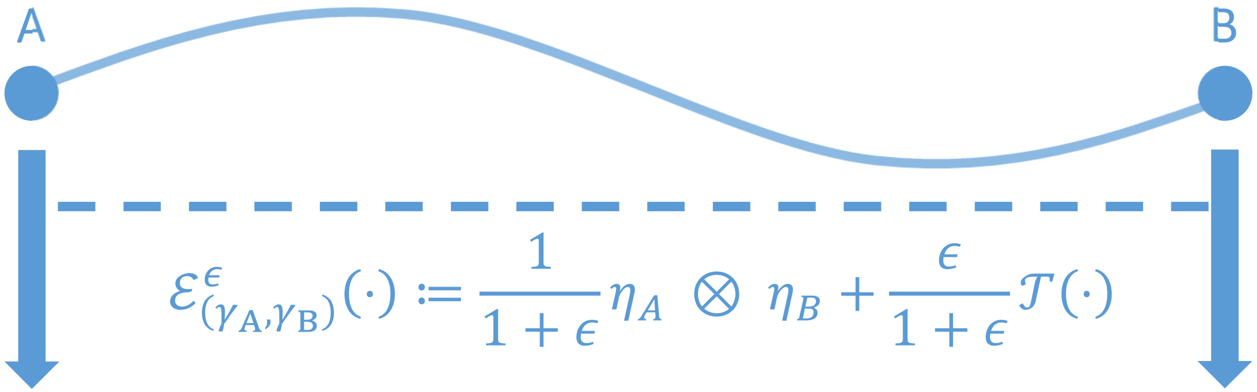

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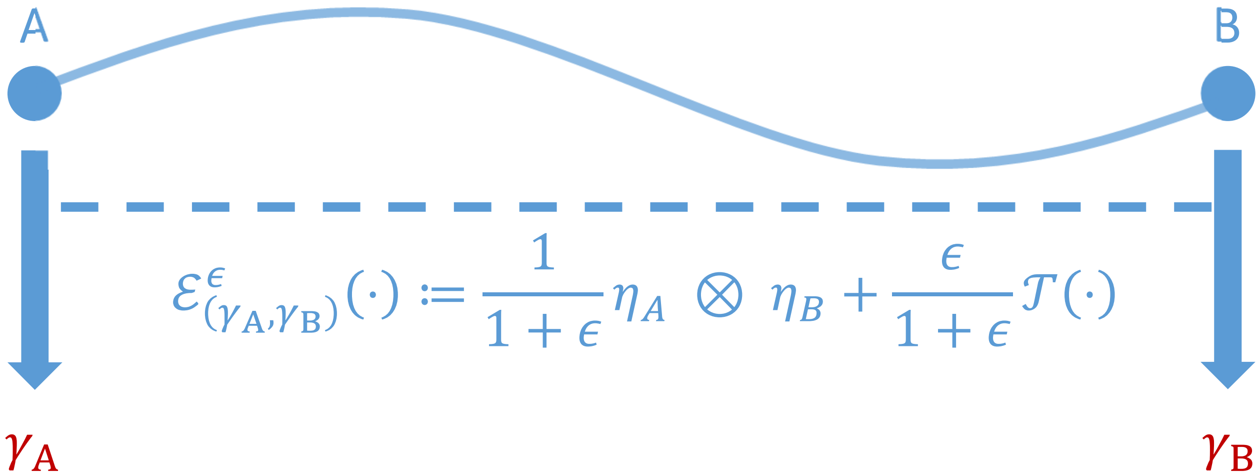

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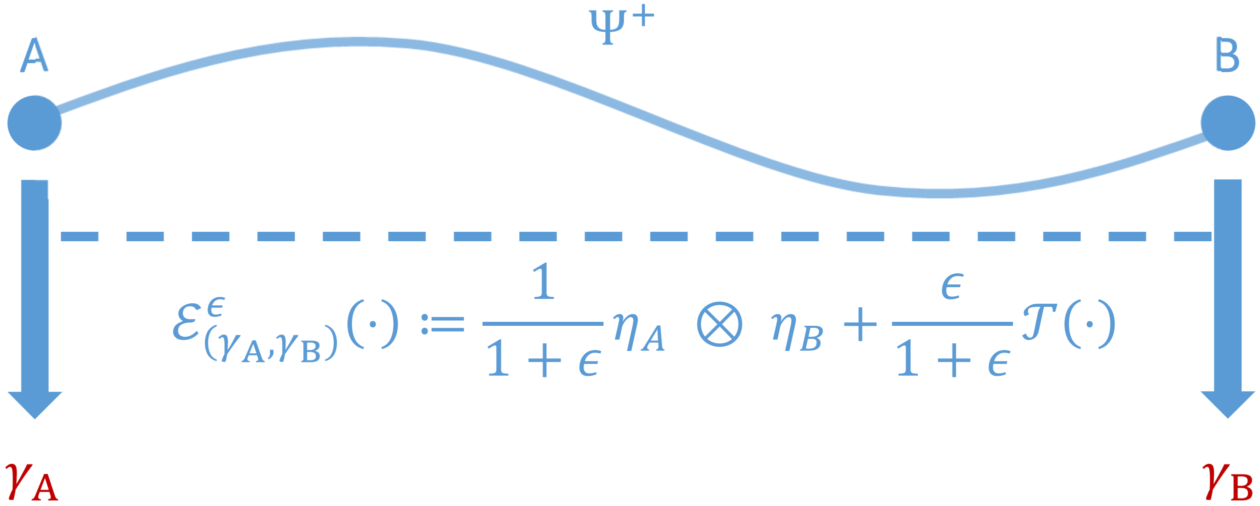

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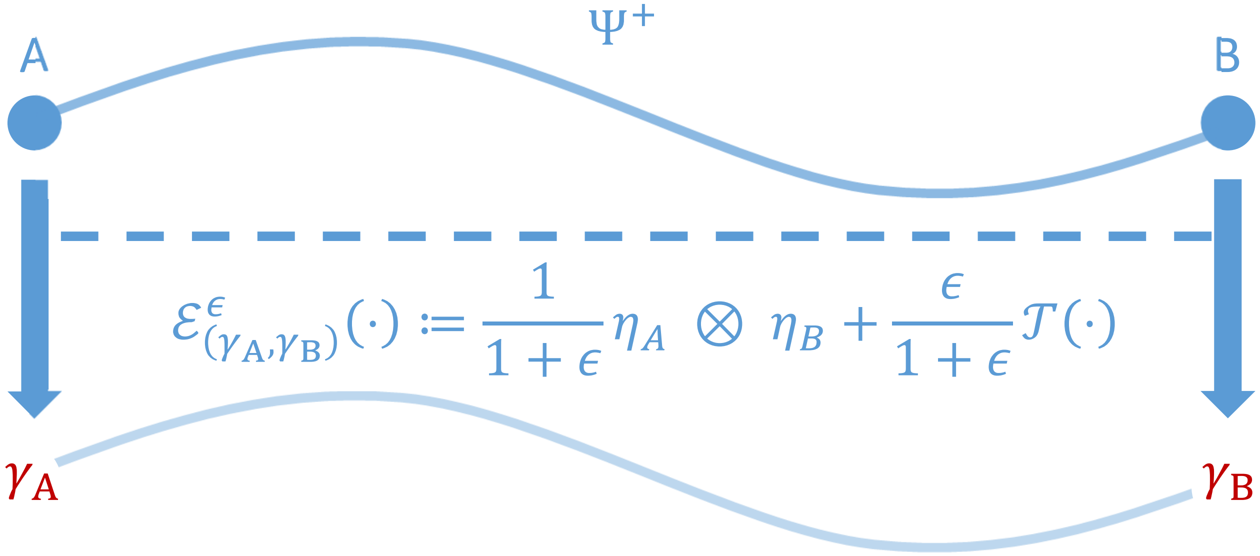

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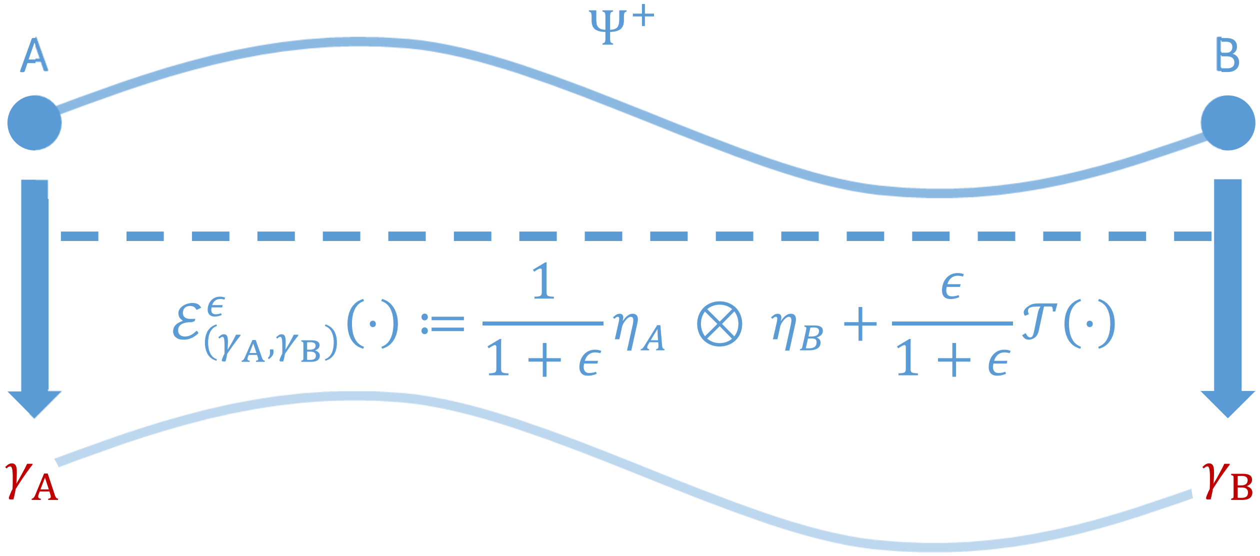

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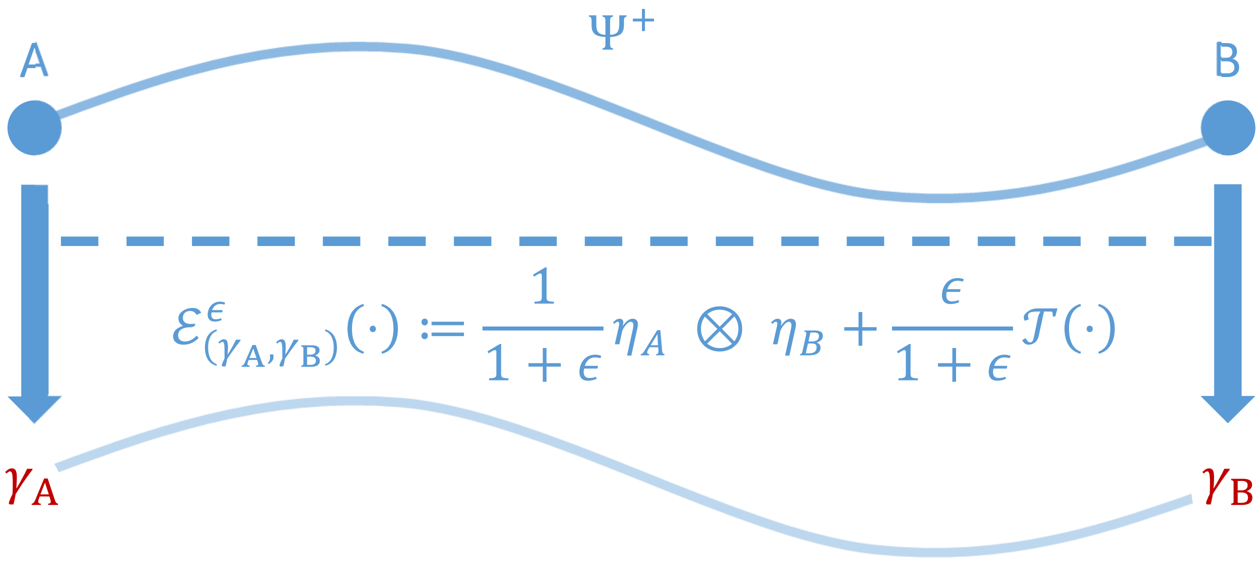

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Share randomness suffices for entanglement preserving local thermalization.

Result 3 | Ability of entanglement preservation

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Distance between channels & sets | $\mathcal{D}(\mathcal{E};\mathcal{S}) \coloneqq \inf_{\Lambda\in\mathcal{S}} \|\mathcal{E}-\Lambda\|_1$

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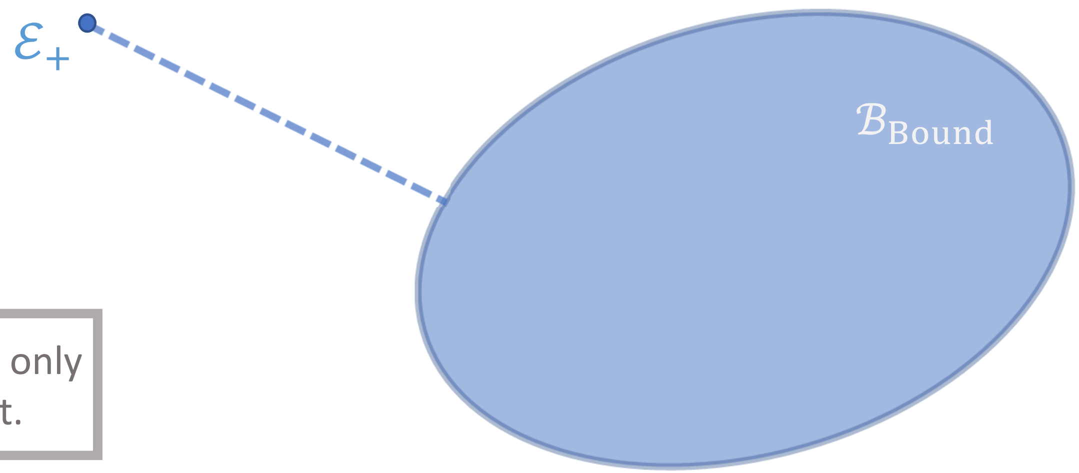

$\mathcal{B}_{\mathrm{Bound}}$ is the set of channels that can only at most preserve bound entanglement.

M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **80**, 5239 (1998).

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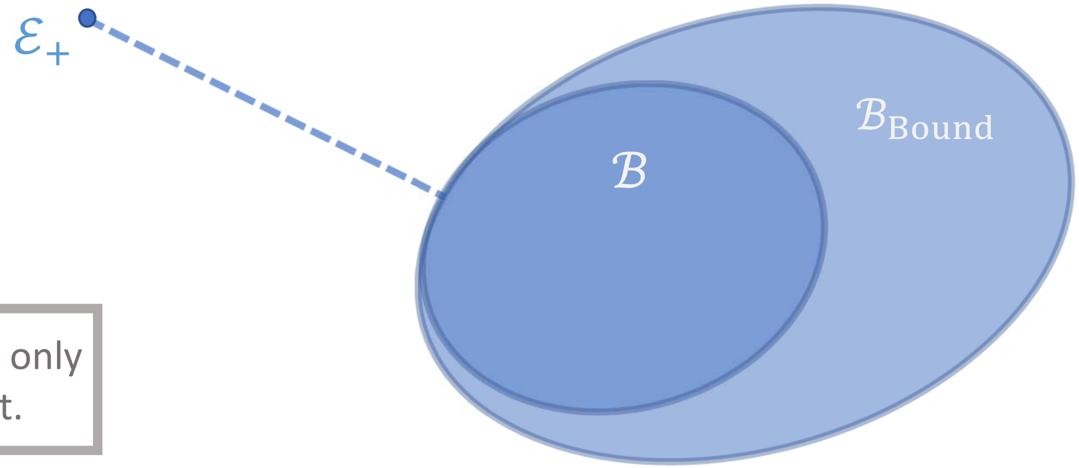

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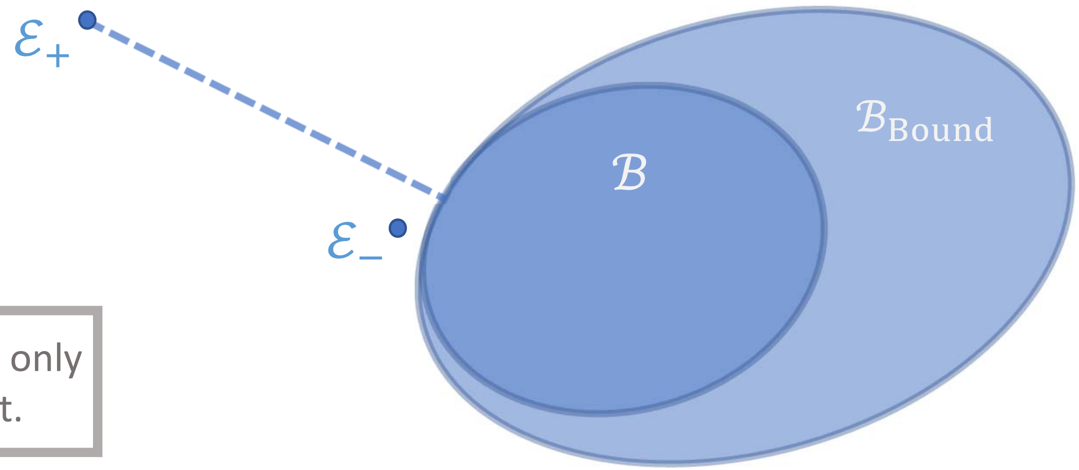

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For every $\delta > 0$, there exists a $\tau_\delta > 0$ such that for every $(\gamma_{\mathrm{A}},\gamma_{\mathrm{B}})$ with $\min_{\mathrm{X}} \tau_{\mathrm{X}} > \tau_\delta$, there exist two entanglement preserving local thermalizations $\mathcal{E}_+$ and $\mathcal{E}_-$ such that

$$\mathcal{D}(\mathcal{E}_+;\mathcal{B}_{\mathrm{Bound}}) \geq (3d-1)P_{\min} - 2$$

$$\mathcal{D}(\mathcal{E}_-;\mathcal{B}) < \delta \ \ \& \ \ \mathcal{E}_- \notin \mathcal{B}_{\mathrm{Bound}}$$

$\tau_{\mathrm{X}} \coloneqq kT_{\mathrm{X}}$/(Highest eigenenergy of the subsystem X), X=A,B.

$P_{\min}$ is the smallest eigenvalue amount γ_{A} and γ_{B}.

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For every $\delta > 0$, there exists a $\tau_\delta > 0$ such that for every (γ_A, γ_B) with $\min_X \tau_X > \tau_\delta$, there exist two entanglement preserving local thermalizations $\mathcal{E}_+$ and $\mathcal{E}_-$ such that

$$\mathcal{D}(\mathcal{E}_+;\mathcal{B}_{\mathrm{Bound}}) \geq (1-\delta)\left(1-\frac{1}{d}\right)$$

$$\mathcal{D}(\mathcal{E}_-;\mathcal{B}) < \delta \;\;\&\;\; \mathcal{E}_- \notin \mathcal{B}_{\mathrm{Bound}}$$

$\tau_X \coloneqq kT_X$/(Highest eigenenergy of the subsystem X), X=A,B.

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Result 4 | 2-qubits systems case & multipartite systems case

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Positive partial transpose criterion enables better results.

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Multipartite entanglement preserving local thermalizations exist for nonzero local temperatures.
Genuinely multipartite entanglement such as that of the Greenberger-Horne-Zeilinger state can be preserved in some cases.

D. M. Greenberger, M. A. Horne, and A. Zeilinger, *Going Beyond Bells Theorem in Bells Theorem, Quantum Theory, and Conceptions of the Universe* (Kluwer Academic, Dordrecht) (1989); D. M. Greenberger, M. A. Horne, and A. Zeilinger, arXiv:0712.0921.

Conclusion & Outlook

Conclusion

Share randomness suffices for entanglement preserving local thermalization.

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Outlook

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Share randomness suffices for entanglement preserving local thermalization.

Outlook

Realizing in particular physical systems, searching for a more specific dynamics, and understanding more properties.

Any possible application in quantum information & quantum computation.

Acknowledgements

This project is part of the ICFOstepstone - PhD Programme for Early-Stage Researchers in Photonics, funded by the Marie Skłodowska-Curie Co-funding of regional, national and international programmes (GA665884) of the European Commission, as well as by the 'Severo Ochoa 2016-2019' program at ICFO (SEV-2015-0522), funded by the Spanish Ministry of Economy, Industry, and Competitiveness (MINECO).

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Thank you for your attention and patience!