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Chiral effects and transport theory in core-collapse supernovae

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(NCTS workshop : International Joint Workshop on the SM and Beyond, Oct. 14th, 2021)

Refs: [1] Naoki Yamamoto, Di-Lun Yang, APJ 895 (2020), 1, arXiv:2002.11348

[2] Naoki Yamamoto, Di-Lun Yang, arXiv:2103.13159.

[3] Kohei Kamada, Naoki Yamamoto, Di-Lun Yang, "Chiral Effects in Astrophysics and Cosmology" (review), arXiv:21XX.XXXX.

Core-collapse supernovae (CCSN)

- Core-collapse supernova explosions are one of most complicated astrophysical processes : neutrino reheating, different types of instabilities, magnetic fields, rotations, etc.

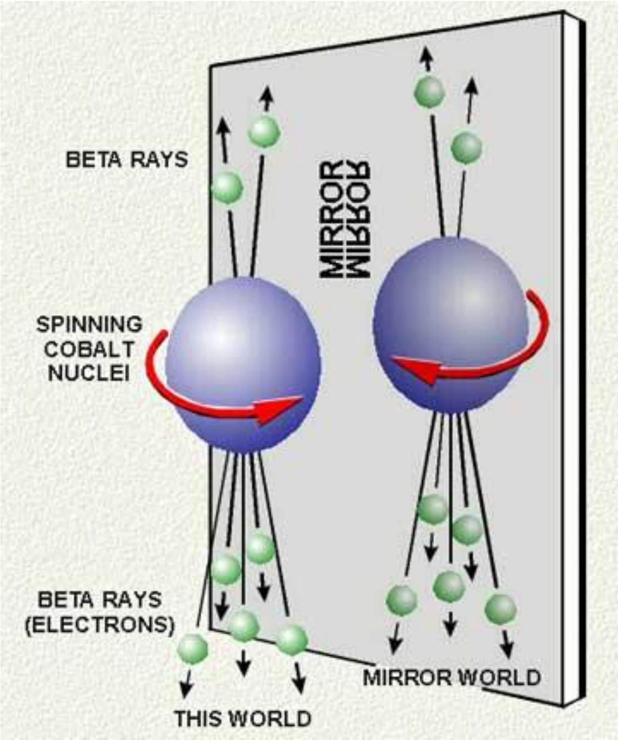
- 2-D simulations : successful explosions

- 3-D simulations : model dependent

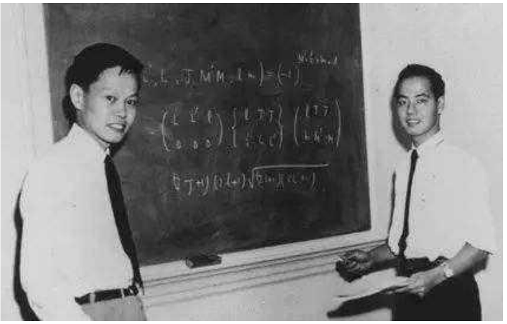
- Chiral effects on leptons : a new microscopic mechanism that could potentially affect the evolution of supernovae

credit : RIKEN

Parity violation & weak interaction



http://physics.nist.gov/GenInt/Parity/cover.html]



Lee & Yang

Wu, 1956

- Global parity violation in weak interaction
- Weak-interaction processes between leptons and nucleons are ubiquitous in CCSN.
- What will be the transport properties for (massless) femions under parity (chirality) violation?

Chiral transport for massless fermions

- Normal transport (massive) : $\boldsymbol{J}_V = \sigma_e \boldsymbol{E}$ (Ohmic current)
- Anomalous transport (massless) : $\boldsymbol{J}_V = \sigma_{BV} \boldsymbol{B}$ (chiral magnetic effect)

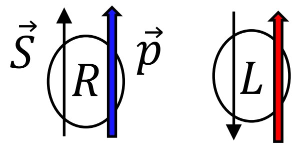
A. Vilenkin, PRD 22, 3080 (1980)

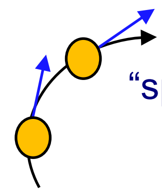
K. Fukushima, D. Kharzeev, H. Warringa, PRD78, 074033 (2008)

- Weyl (massless) fermions :

chirality=helicity

(no chirality mixing)





"spin locking"

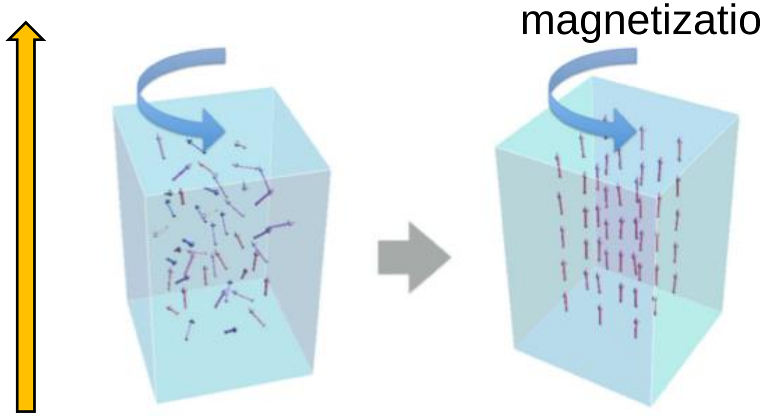
- An intuitive picture :

magnetic field: B

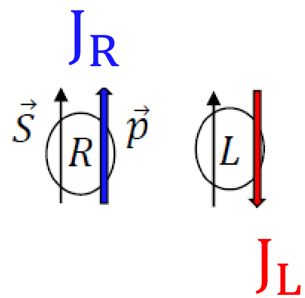
vorticity :

$$\boldsymbol{\omega} = \frac{1}{2}\boldsymbol{\nabla} \times \boldsymbol{v}$$

(Barnett effect)



M. Matsuo et al, fphy.2015.00054



$J_V = J_R + J_L \neq 0$

when $n_R - n_L \neq 0$

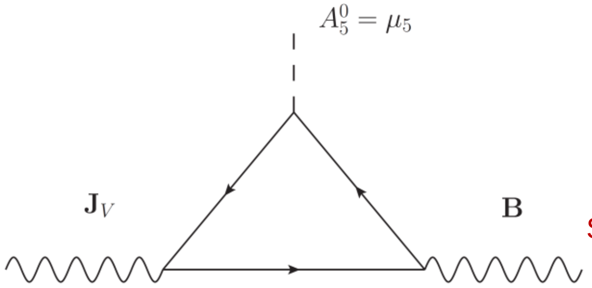
Chiral magnetic/vortical effect (CME/CVE)

- Chiral magnetic effect :

A. Vilenkin, PRD 22, 3080 (1980)
K. Fukushima, D. Kharzeev, H. Warringa, PRD78, 074033 (2008)

$$\mathbf{J}_V = \boxed{\frac{\mu_5}{2\pi^2}}\mathbf{B}$$

(parity odd, protected by chiral anomaly)



$$\partial_\mu J_5^\mu = \frac{\mathbf{E}\cdot\mathbf{B}}{2\pi^2}$$

S. Adler, J. Bell, R. Jackiw, 69
K. Fujikawa, 79

- Anomalous (quantum) transport of Weyl fermions :

$$J^\mu_{R/L} = \pm\frac{\mu_{R/L}}{4\pi^2}B^\mu \pm \left(\frac{T^2}{12} + \frac{\mu^2_{R/L}}{4\pi^2}\right)\omega^\mu$$

A. Vilenkin, PRD 20, 1807 (1979). PRD 22, 3080 (1980)
D. T. Son, P. Surowka, PRL 103, 191601 (2009)
K. Landsteiner, E. Megias, F. Pena-Benitez, PRL107,021601(2011)

$$\mathbf{J}_V = \mathbf{J}_R + \mathbf{J}_L$$
$$\mathbf{J}_5 = \mathbf{J}_R - \mathbf{J}_L$$

(spin current)

CME : $\mathbf{J}_V = \frac{1}{2\pi^2}\mu_5\mathbf{B}$

CVE : $\mathbf{J}_V = \frac{1}{\pi^2}\mu_5\mu_V\,\boldsymbol{\omega}$

CSE : $\mathbf{J}_5 = \frac{1}{2\pi^2}\mu_V\mathbf{B}$

(chiral separation effect)

aCVE : $\mathbf{J}_5 = \left(\frac{\mu_V^2+\mu_5^2}{2\pi^2} + \frac{T^2}{6}\right)\boldsymbol{\omega}$

(axial chiral vortical effect)

vector/axial chemical potentials : $\mu_{V/5} = (\mu_R \pm \mu_L)/2$

Chirality transfer & chiral plasma instability

- Chirality transfer : $\partial_\mu J_5^\mu = -\frac{E \cdot B}{2\pi^2}$ ⇒ $\frac{dN_5}{dt} = -\frac{d}{dt}\int d^3\boldsymbol{x}\,\frac{\boldsymbol{A}\cdot\boldsymbol{B}}{2\pi^2}$

  magnetic helicity

- Anomalous Maxwell's eq. : $\partial_t \boldsymbol{B} = -\boldsymbol{\nabla}\times\boldsymbol{E}\,, \quad \boldsymbol{\nabla}\times\boldsymbol{B} = \sigma\boldsymbol{E} + \frac{e^2\mu_5}{2\pi^2}\boldsymbol{B}\,.$

  CME

  ⇒ $\partial_t \boldsymbol{B} = \frac{1}{\sigma}\boldsymbol{\nabla}^2\boldsymbol{B} + \frac{e^2\mu_5}{2\pi^2\sigma}\boldsymbol{\nabla}\times\boldsymbol{B}$

  diffusion instability

- Chiral plasma instability (CPI) :

  M. Joyce and M. E. Shaposhnikov, PRL **79**, 1193 (1997).
  Y. Akamatsu and N. Yamamoto, PRL 111, 052002 (2013).

An unstable mode of magnetic fields :

$$\boldsymbol{B_k}(t) = \boldsymbol{B_k}(0)\exp\left[\frac{k}{\sigma}\left(\frac{e^2|\mu_5|}{2\pi^2} - k\right)t\right] \quad \text{(for small } k\text{)}$$

$$k_{\mathrm{CPI}} = \frac{e^2|\mu_5|}{4\pi^2}$$

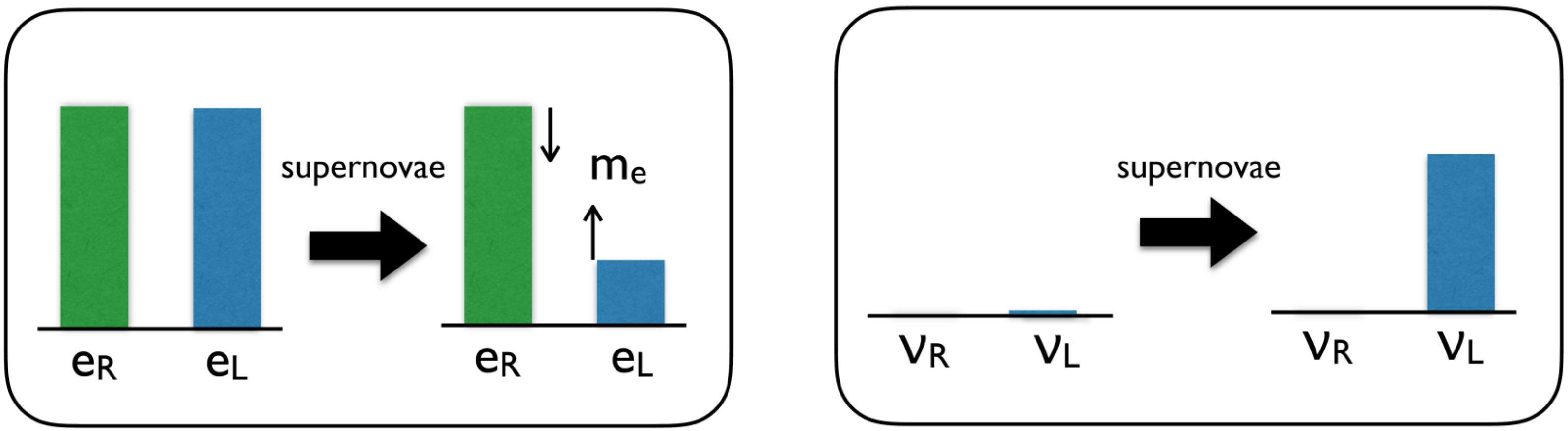
$$\Gamma_{\mathrm{CPI}} = \frac{1}{\sigma}\left(\frac{e^2\mu_5}{4\pi^2}\right)^2$$

Chiral imbalance in CCSN

- Chirality imbalance from the electron capture process:

an innate lefthander

$$p + e^{\mathrm{L}} \rightarrow n + \nu_e^{\mathrm{L}}$$



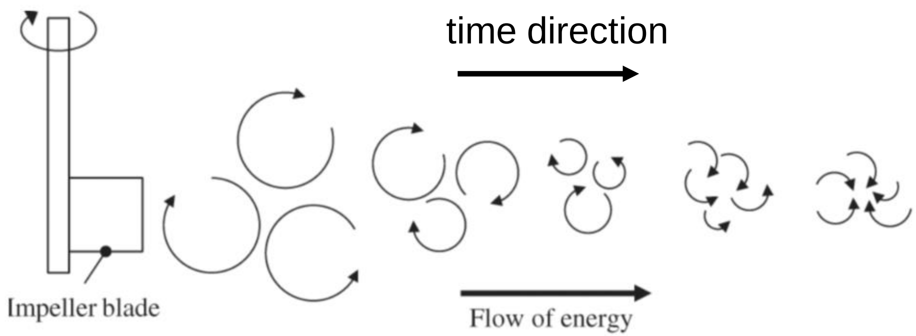
N. Yamamoto's talk, NTU, Taiwan, 18

- $\mathrm{B} \neq 0$ and $\mu_5 \neq 0$: what will be the consequence if there exists CPI in CCSN?

Direct & inverse energy cascades

- Intuitively, why is the 3D system more difficult to achieve explosion?
- Turbulence in 3D : direct energy cascade (difficult to explode)

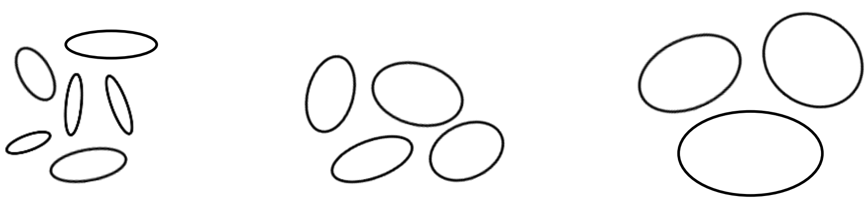
time direction



https://doi.org/10.1515/htmp-2016-0043

Kolmogorov 1941

- Turbulence in 2D : inverse energy cascade (easier to explode)



time direction

Kraichnan, Leith, Batchelor 1967-1969

Inverse cascade for chiral matter

- Electrons and nucleons could locally reach thermal equilibrium in CCSN
- wo chiral effects : magneto-hydrodynamics (MHD)
- wth chiral effects : chiral magneto-hydrodynamics (ChMHD)

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_{V\lambda}, \ \ \partial_\mu J^\mu_V = 0, \ \ \boxed{\partial_\mu J^\mu_5 = -\frac{E\cdot B}{2\pi^2}.}$$

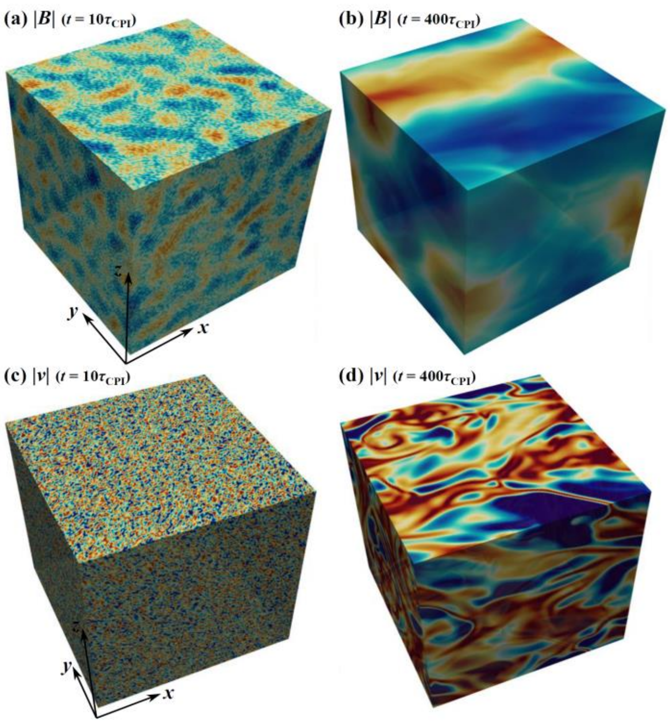
time →

- Inverse cascade is observed :

ChMHD simulations :

(with $\mu_5 \neq 0$)

Y. Masada, K. Kotake, T. Takiwaki, N. Yamamoto, PRD 98 (2018) 8, 083018



B field

Fluid velocity

- As expected from CPI : enhancement of B in the long-wavelength regime

Inclusion of neutrino radiation

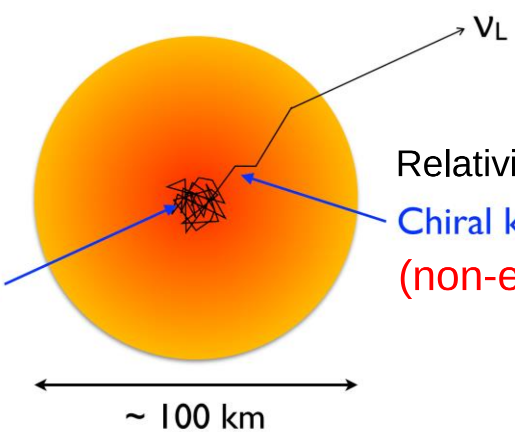
- Neutrino could reheat the matter and play an important role in the evolution of CCSN.
- Neutrinos are out of equilibrium away from the core : radiation hydrodynamics
- Radiation part is governed by the Boltzmann equation.

S. W. Bruenn, Astrophys. Jl. Suppl. 58 (1985) 771.

Magneto-hydro

Chiral hydro

(equilibrium)



Relativistic kinetic theory

Chiral kinetic theory

(non-equilibrium)

(CKT) : quantum kinetic theory incorporating chiral anomaly & spin-orbit int.

M. Stephanov & Y. Yin, PRL. 109, 162001 (2012)
D. T. Son & N. Yamamoto, PRL. 109, 181602 (2012)
J.-W. Chen, et al., PRL. 110, 262301 (2013)
Y. Hidaka, S. Pu, DY, PRD 95, 091901 (2017)

- Near the core : ChMHD (e, N, ν)
- Away from the core : ChMHD (e, N) + chiral kinetic theory (ν) ⟹ chiral radiation hydrodynamics

$$\nabla_\mu T^{\mu\nu}_{\text{rad}} + \nabla_\mu T^{\mu\nu}_{\text{mat}} = 0$$

(neutrinos) (electrons, nucleons : equilibrium)

Chiral radiation transport equation

- CKT for neutrinos : $\square_{\rm i} f_q^{(\nu)} = \frac{1}{E_{\rm i}} \left[(1 - f_q^{(\nu)}) \Gamma_q^< - f_q^{(\nu)} \Gamma_q^> \right]$

Boltzmann eq. in the inertial frame

collision term with quantum corrections

N. Yamamoto & DY, APJ 895 (2020), 1

- Neutrino absorption : $\bar{\Gamma}_q^{(\rm ab)\lessgtr} \approx \bar{\Gamma}_q^{(0)\lessgtr} + \hbar \bar{\Gamma}_q^{(\omega)\lessgtr}(q \cdot \omega) + \hbar \bar{\Gamma}_q^{(B)\lessgtr}(q \cdot B)$

$\nu_{\rm L}^{\rm e}(q) + {\rm n}(k) \rightleftharpoons {\rm e}_{\rm L}(q') + {\rm p}(k')$

isoenergetic approx.:

NR approx., $M_{\rm n} \approx M_{\rm p} \approx M$

small-energy transf.

vorticity & magnetic field corrections :

breaking spherical symmetry & axisymmetry

analytic expressions :

$$\bar{\Gamma}_q^{(0)>} \approx \frac{1}{\pi \hbar^4 c^4} (g_{\rm V}^2 + 3 g_{\rm A}^2) G_{\rm F}^2 (q \cdot u)^3 (1 - f_{0,q}^{(\rm e)}) \left(1 - \frac{3 q \cdot u}{M c^2} \right) \frac{n_{\rm n} - n_{\rm p}}{1 - {\rm e}^{\beta(\mu_{\rm p} - \mu_{\rm n})}},$$

$$\bar{\Gamma}_q^{(B)>} \approx \frac{1}{2\pi \hbar^4 c^4 M} (g_{\rm V}^2 + 3 g_{\rm A}^2) G_{\rm F}^2 (q \cdot u) (1 - f_{0,q}^{(\rm e)}) \left(1 - \frac{8 q \cdot u}{3 M c^2} \right) \frac{n_{\rm n} - n_{\rm p}}{1 - {\rm e}^{\beta(\mu_{\rm p} - \mu_{\rm n})}},$$

$$\bar{\Gamma}_q^{(\omega)>} \approx \frac{1}{2\pi \hbar^4 c^4} (g_{\rm V}^2 + 3 g_{\rm A}^2) G_{\rm F}^2 (q \cdot u)^2 (1 - f_{0,q}^{(\rm e)}) \left(\frac{2}{E_{\rm i}} + \beta f_{0,q}^{(\rm e)} \right) \frac{n_{\rm n} - n_{\rm p}}{1 - {\rm e}^{\beta(\mu_{\rm p} - \mu_{\rm n})}}$$

$\bar{\Gamma}_q^{(0)>}$: S. Reddy, M. Prakash, J. M. Lattimer, PRD58:013009,1998

Neutrinos near thermal equilibrium

- Neutrinos in local equilibrium : $\bar{f}_q^{(\nu)} = \frac{1}{e^h + 1}, \; h \approx \beta\left(q\cdot u - \mu_\nu + \hbar\frac{q\cdot\omega}{2q\cdot u}\right), \; \beta = 1/T.$

spin-vorticity coupling

J.-Y. Chen, et.al. PRL 113, 182302 (2014)

Y. Hidaka, S. Pu, DY, PRD 97, 016004 (2018)

- Neutrino near equilibrium : relaxation-time approximation

$$\frac{1}{E_\mathrm{i}}\left[(1-f_q^{(\nu)})\Gamma_q^< - f_q^{(\nu)}\Gamma_q^>\right] \approx -\frac{\delta f_q^{(\nu)}}{\tau}, \qquad \delta f_q^{(\nu)} \equiv f_q^{(\nu)} - \bar{f}_q^{(\nu)}$$

- Relaxation time : $\tau = \tau^{(0)} + \hbar\tau^{(1)}$

$$\tau^{(0)} = \frac{\kappa E_\mathrm{i}(1-f_{0,q}^{(\nu)})}{(q\cdot u)^3(1-f_{0,q}^{(\mathrm{e})})}, \quad \tau^{(1)} = -\tau^{(0)}\left[\left(\frac{2}{E_\mathrm{i}} + \beta(f_{0,q}^{(\mathrm{e})} - f_{0,q}^{(\nu)})\right)\frac{q\cdot\omega}{2q\cdot u} + \frac{q\cdot B}{2M(q\cdot u)^2}\right],$$

$$\kappa \equiv \frac{\pi}{G_\mathrm{F}^2(g_\mathrm{V}^2 + 3g_\mathrm{A}^2)\delta n}, \quad \delta n \equiv \frac{n_\mathrm{p} - n_\mathrm{n}}{1 - e^{\beta(\mu_\mathrm{n}-\mu_\mathrm{p})}}.$$

N. Yamamoto & DY, arXiv:2103.13159

- Near-equilibrium solution : $\delta f_q^{(\nu)} \approx -\tau\frac{q\cdot D}{E_\mathrm{i}}\bar{f}_q^{(\nu)}$

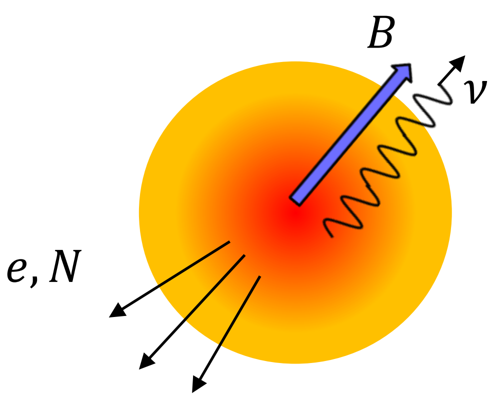
Neutrino flux driven by magnetic fields

- Non-equilibrium corrections from magnetic fields : N. Yamamoto & DY, arXiv:2103.13159

$$u_\rho \delta T^{\rho\mu}_{(\nu)B} \sim -\frac{\kappa I_1}{18MT}(\nabla\cdot u)B^\mu, \quad \delta J^{\mu}_{(\nu)B} \sim -\frac{\kappa I_2}{18MT}(\nabla\cdot u)B_\nu,$$

$$I_1 = \frac{T^2}{4\pi^2}\left[\frac{e^{\bar{\mu}_\nu} - e^{\bar{\mu}_e}}{1+e^{\bar{\mu}_\nu}} + (1+e^{\bar{\mu}_e-\bar{\mu}_\nu})\ln(1+e^{\bar{\mu}_\nu})\right], \quad I_2 = \frac{Te^{\bar{\mu}_\nu}(2+e^{\bar{\mu}_\nu}+e^{\bar{\mu}_e})}{4\pi^2(1+e^{\bar{\mu}_\nu})^2}.$$

❖ Neutrino momentum/number density currents driven by B fields!



(bulk viscous effect)

trigger the effective CME : $J^i_{(e)V} \sim \sigma_{\rm eff} B^i$

⇨ CPI & inverse cascade

The implications to pulsar kicks

- Pulsar kicks : fast-moving neutron stars (v~200–500 km/s)
- The source of momentum asymmetry?
- One hypothesis : neutrino momentum flux driven by magnetic fields

$$v_{\rm kick} \sim \frac{\delta T^{i0}_{(\nu)B}}{\rho_{\rm core}} \sim \left(\frac{B}{10^{13\text{-}14}\ {\rm G}}\right) \ {\rm km/s} \Rightarrow B \sim 10^{15-16}\ {\rm G}\ \text{for}\ v_{\rm kick} \sim 10^2\ {\rm km/s}$$

(in proto-neutron stars)

- Similar estimations with different theoretical setups :

e.g. A. Vilenkin, Astrophys. J. 451, 700 (1995).
D. Lai and Y.-Z. Qian, Astrophys. J. 505, 844 (1998), astro-ph/9802345.
P. Arras and D. Lai, Phys. Rev. D 60, 043001 (1999), astro-ph/9811371
M. Kaminski, C. F. Uhlemann, M. Bleicher, and J. Schaffner-Bielich, Phys. Lett. B 760, 170 (2016), 1410.3833

- A more practical estimation entails fully non-equilibrium contributions.

Conclusions & outlook

- ✓ The chiral effect for leptons may result in the inverse cascade and qualitatively affect the supernova evolution.
- ✓ The evolution of matter dictated by ChMHD follows the inverse cascade led by CPI, but we also need to include chiral transport of neutrinos.
- ✓ Chiral radiation transport equation for neutrinos is derived, which incorporates the quantum corrections from magnetic/vortical fields via collisions.
- ✓ Magnetic-field induced neutrino flux may trigger CPI and has the potential application to pulsar kicks.

- ❖ In the long run, we have to conduct full simulation with ChRHD : further collaborations with astrophysicists will be needed.
- ❖ Further simplifications, approximations, extensions, and test runs are needed.

Thank you!